MAP562 Optimal design of structures

by Grégoire Allaire, Thomas Wick

Ecole Polytechnique
Academic year 2016-2017

Homework 5, Feb 01, 2017

Exercise 1

In this exercise we optimize the thickness h of a two-dimensional elastic body in Ω. As in the other exercises, we have two boundary parts Γ_D and Γ_N. For an homogeneous elastic body the governing equations satisfy linearized elasticity: Find $u \in V$ such that

$$-\nabla \cdot (\sigma(u)) = f \quad \text{in } \Omega, \tag{1}$$

$$\sigma(u) \cdot n = g_N \quad \text{on } \Gamma_{N1}, \tag{2}$$

$$\sigma(u) \cdot n = 0 \quad \text{on } \Gamma_N, \tag{3}$$

$$u_y = 0 \quad \text{on } \Gamma_{top}, \tag{4}$$

$$u = u_D \quad \text{on } \Gamma_D, \tag{5}$$

where the stress is given by the classical law:

$$\sigma(u) = h(2\mu e(u) + \lambda tr(e(u))I),$$

with the linearized strain tensor $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$, I is the identity matrix, and μ and λ are the Lamé coefficients. Furthermore, $f = 0$ and $u_D = 0$.

As objective functional we take

$$J(h) = \frac{1}{2}\int_\Omega k(x)|\sigma - \sigma_0|^2 \, dx,$$

where $k(x) = 1$ in the small brown domain (see the Figure) and $k(x) = 0$ otherwise. Furthermore, $\sigma_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. In particular, the absolute value of a tensor is defined via the Frobenius norm:

$$|\sigma| := \sqrt{\sigma : \sigma} = \sqrt{\sum_i \sum_j \sigma_{ij}^2}.$$

The goal is

$$\inf_{h \in U_{ad}} J(h) \text{ s.t. } (1) - (5).$$

Recall that U_{ad} features a volume constraint and bound constraints for the thickness.

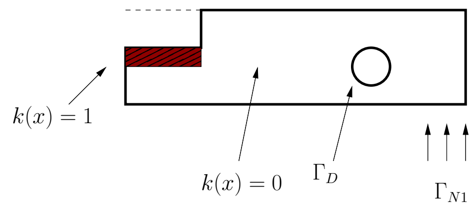

Figure 1: Geometry, boundaries, and forces.

Exercise 2

In this second exercise we consider the counter-example of a non-existing optimal height h (recapitulate the lecture notes chapter5.pdf pages 7-10 for all details). Implement this example and verify by yourself that not only the theory yields non-existence but also the numerical solution will not converge.

Remark:
Please upload your solutions as seperate files on

`http://www.cmap.polytechnique.fr/~MAP562/`