MAP562 Optimal design of structures

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Exercise 1

In this exercise we investigate some properties of a composite material of rang 2 in 2-d. The configuration is displayed in Figure 1. The equations are the same as in the practical exercise, see `composite.edp` online.

This exercise has the following goals:

1. Implement the geometry in Figure 1 into the given program `composite.edp`. This program will then yield the values A^*_{ij} of the homogenized tensor A^*.
2. Secondly take the formula from Lemma 7.14 (see the lecture notes) using the specific values for m_1 and m_2. With that formula compute as well the values of A^* (possibly see Lemma 7.11) in order to see how we specifically obtain A^*_{ij}.
3. Compare the two previous results.

We outline the specific tasks in the following:

1. For task 2 from above, we need to find the proportions of the lamination, which yield a lamination of rang 2 with two orthogonal directions. Here the two phases are assumed to be isotropic. Specifically, show that

$$m_1 = \frac{1-\theta_1}{1-\theta}, \quad m_2 = \frac{\theta_1(1-\theta_2)}{1-\theta_1\theta_2}.$$

   What are the values of θ_1, θ_2 that yield an isotropic composite material for a given volume fraction θ ?
2. Implement task 1 in FreeFem.

Exercise 2

We consider the following heat optimization problem. Let $\Omega = (0,1)^2$ and let T the temperature. Find T such that

$$\begin{aligned} -div(A^*\nabla T) &= 0 \quad \text{in } \Omega, \\ A^*\nabla T \cdot n &= 1 \quad \text{on } \Gamma_N, \\ A^*\nabla T \cdot n &= 0 \quad \text{on } \Gamma, \\ T &= 0 \quad \text{on } \Gamma_D. \end{aligned}$$

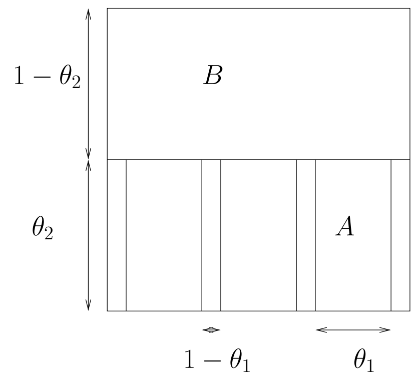

Figure 1: Exercise 2: Geometry of the rang-2 material.

In order to optimize the heater, we want to minimize the temperature on the top boundary:

$$\min_{\theta, A^*}\Big\{J(\theta, A^*) = \int_{\Gamma_N} T\, ds.\Big\}$$

Moreover, we need to penalize the problem such that θ only takes the values 0 and 1. Rather than working with the true θ_{opt} we introduce

$$\theta_{pen} = \frac{1-\cos(\pi\theta_{opt})}{2}.$$

This will yields $\theta_{pen} < \theta_{opt}$ for $0 < \theta_{opt} < 0.5$ and $\theta_{opt} < \theta_{pen}$ for $0.5 < \theta_{opt} < 1$.

1. (possibly part of the TP) We notice that this problem is again self-adjoint and thus for the optimization, we can work with an optimality criteria method. Implement this strategy in FreeFem.
2. We change the cost functional to

$$\min_{\theta, A^*}\Big\{J(\theta, A^*) = \int_{\Gamma_N} T^2\, ds.\Big\}$$

   In this case, the problem is not any more self-adjoint. This means we must again adopt the general approach the gradient algorithm requires the solution of the adjoint state. Implement this solution algorithm in FreeFem.

Remark:

Please upload your solutions as seperate files on

`http://www.cmap.polytechnique.fr/~MAP562/`