MAP562 Optimal design of structures

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Ecole Polytechnique

Academic year 2016-2017

Homework 9, Mar 08, 2017

Choose one of the four following exercises:

Exercise 1

In this first exercise we consider a conduction problem. The cell problem in $Y = (0,1)^2$ is given by:

$$-\, div_y(A(y)(e_i + \nabla_y w_i(y))) = 0 \quad \text{in } Y, \quad y \to w_i(y)\; Y - period. \tag{1}$$

Here $A(y) = 1$ for x in $(0; 0.4) \cup (0.6; 1)$ and $A(y) = 10$ for x in $(0.4; 0.6)$; see also Figure 1.

1. Compute the homogenized tensor A^*.
2. We introduce the rotation matrix $R(\alpha)$ defined as

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

Then the problem in the macroscopic cantilever domain Ω (take a cantilever configuration from one of our previous exercises) is given by

$$\begin{aligned} -div_x(RA^*R^T\nabla_x u(x)) &= 0 \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \Gamma_{fixed}, \\ \nabla u \cdot n &= 0 \quad \text{on } \Gamma_N. \end{aligned}$$

The objective functional is to minimize the compliance with respect to the angle $\alpha := \alpha(x)$ for $x \in \Omega$. Implement this problem into FreeFem.

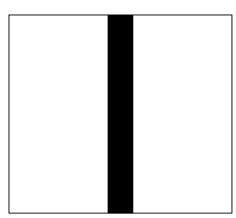

Figure 1: Homework 9, Ex. 1: domain and proportions of the stiff material (black zone) and the smoother material.

Exercise 2

Consider the bridge problem (pont).

1. Using SIMP, find another (better?!) penalization strategy, e.g., $\sqrt{\theta}A$. and implement this strategy into FreeFem.
2. Implement a three phases problem with the proportions θ_1 and θ_2 and the material coefficients (Young's moduli) $A = 1, B = 2$ and $C = 20$ while keeping Poisson's ratio $\nu = 0.3$ fixed. Their relation is given by:

$$\theta_1 A + (1 - \theta_1)\theta_2 B + (1 - \theta_1)(1 - \theta_2)C,$$

   with $0 \leq \theta_i \leq 1$. The different materials are distributed as 60% for A; 30% for B; and 10% for C.
   Hint: For the algorithm we need to implement updates for θ_1 and θ_2. Furthermore, two Lagrange multipliers are necessary. Here, compute first the first Lagrange multiplier, and with the obtained value, fix then the second one.

Exercise 3

Consider the Stokes problem from fluid mechanics. For a basic implementation we refer to the FreeFem documentation. The equations read: Find a vector-valued velocity v and a scalar-valued pressure p such that

$$\begin{aligned}
-\mu\Delta v - \nabla p &= 0 \quad \text{in } \Omega, \\
\nabla \cdot v &= 0 \quad \text{in } \Omega, \\
v &= g \quad \text{on } \Gamma_{in} \cup \Gamma_{out}, \\
v &= 0 \quad \text{on } \Gamma_{remaining}.
\end{aligned}$$

The domain is sketched in Figure 2 The objective functional is defined as:

$$\min \int_\Omega \mu |\nabla v|^2 \, dx.$$

Furthermore, we need a constraint that the volume of the fluid is $\leq 30\%$. In order to realize this optimization problem again with SIMP, we extend the Stokes equations by one term resulting in the so-called Brinkmann problem:

$$\begin{aligned}
-\mu\Delta v - \nabla p + \kappa u &= 0 \quad \text{in } \Omega, \\
\nabla \cdot v &= 0 \quad \text{in } \Omega, \\
v &= g \quad \text{on } \Gamma_{in} \cup \Gamma_{out}, \\
v &= 0 \quad \text{on } \Gamma_{remaining}.
\end{aligned}$$

where κ is variable and e.g., $\kappa = 0$ in the optimal flow way and $\kappa = 10^9$ in the remaining zone, yielding a solid domain.
Hint: In this example κ takes the role of θ.

1. Implement this situation in FreeFem.

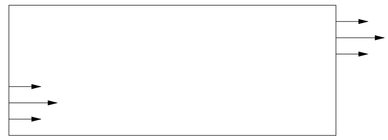

Figure 2: Homework 9, Ex. 3: domain and inflow/outflow conditions.

Exercise 4

In this final exercise we consider an eigenvalue problem. The geometry is displayed in Figure 3. For the basics we refer to Session 2 (FreeFem script and also the theoretical exercise). The problem is given by:

$$\begin{aligned} -div(A^*\sigma(u)) &= \lambda\rho u \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \Gamma_{fix}, \\ \sigma \cdot n &= 0 \quad \text{on } \Gamma_{fix}, \end{aligned}$$

where σ is the tensor of linearized elasticity and $l \in \mathbb{R}$ and ρ is the density. The density in the non-optimizable region is $\rho = 100$ and in the remaining, large, domain $\rho = 1$. We use the SIMP method and re-define

$$A \to \theta A, \quad \rho \to \theta^p \rho,$$

where $p = 2$ for example. For θ it holds:

$$10^{-3} \le \theta \le 1.$$

The first eigenvalue (see again Session 2) can be computed as:

$$\lambda_1 = \min \frac{\int_\Omega Ae(u) : e(u)}{\int_\Omega \rho |u|^2}$$

where $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$.

1. For the cost functional

$$\min(-\lambda_1)$$

implement this task in FreeFem.

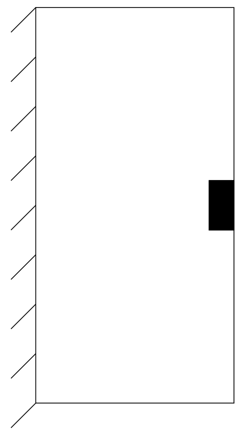

Figure 3: Homework 9, Ex. 4: domain and boundary conditions. The black zone is non-optimizable.

Remark:

Please upload your solutions as seperate files on

`http://www.cmap.polytechnique.fr/~MAP562/`