ECOLE POLYTECHNIQUE
Master M2 "Mathematical modelling"
PDE constrained optimization (G. Allaire)

Exercise 2

Let Ω be a smooth bounded open set in $\mathbb{R}^d$, for $1 \leq d \leq 3$. For given $f \in L^2(\Omega)$, consider the following energy for any $v \in H_0^1(\Omega)$

$$J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 dx + \int_\Omega v^2(1-v^2)^2 dx - \int_\Omega f v dx.$$

1. Prove that J admits at least one minimizer in H.
2. Compute the derivative of J and write the optimality condition.
3. Deduce the boundary value problem satisfied by any minimizer.
4. Compute the second order derivative of J and explains how to implement a Newton algorithm to compute minimizers of J.