Modélisation et estimation du comportement électromécanique du coeur avec OpenFEM

D. Chapelle
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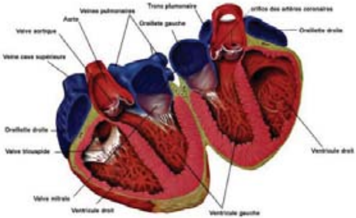
Journée Gamni sur FreeFEM++/OpenFEM, 23 septembre 2005

Motivation

- Modeling
  - Complex : (multiphysics, multiscale. . .)
  - Experiments difficult or impossible (living organs, patient-specific. . .)
- Clinical measurements:
  - Abundant and complex (images, ECG. . .)
  - But also sparse and noisy
  - "Hidden variables" (stresses, pressures. . .)
- ▶ "Data/model coupling" desired

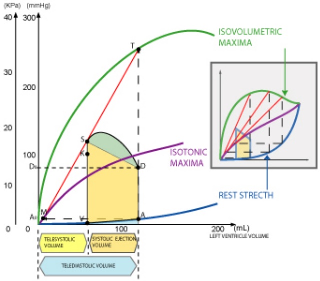
1. Overview of the cardiac function
    - Physiological description
2. Heart model
3. Data assimilation

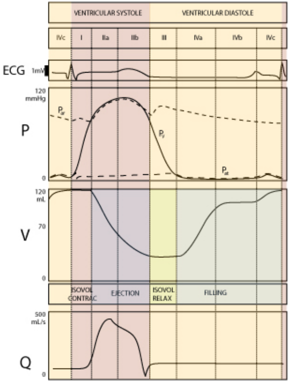
A biological pump



- Two chambers and two circulations.

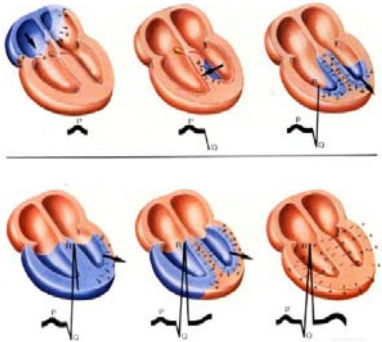
A biological pump





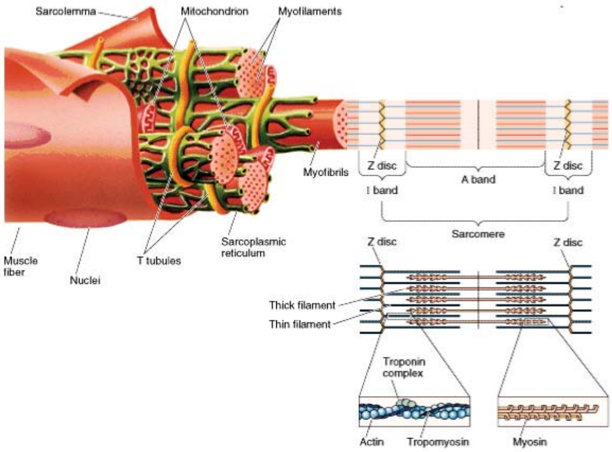
- Two chambers and two circulations.
- Thermodynamical system with PV cycle.

A biological pump



- Two chambers and two circulations.
- Thermodynamical system with PV cycle.
- Electrically-activated contraction.

Multiscale fibers



1. Overview of the cardiac function
2. Heart model
   - Constitutive law
   - Closure of the system
   - Input data
   - Discretized linear system
   - Results
3. Data assimilation

Heart model

1. Fundamental law of dynamics (virtual work): 3D

$$\int_{\Omega_0} \rho \underline{\ddot{y}}.\delta \underline{v} d\Omega_0 + \int_{\Omega_0} \underline{\underline{\Sigma}} : \delta \underline{\underline{e}} d\Omega_0 + \int_{\Gamma} P_V \underline{\nu}.\underline{\underline{F}}^{-1}.\delta \underline{v} J d\Gamma = 0 \quad \forall \delta \underline{v} \in V$$

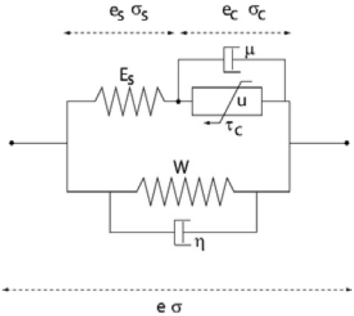
2. Constitutive law of the fiber and the matrix: 1D and 3D

$$\underline{\underline{\Sigma}} = -pJ\underline{\underline{C}}^{-1} + \frac{\partial W^e}{\partial \underline{\underline{e}}} + \frac{\partial W^\eta}{\partial \underline{\underline{\dot{e}}}} + \sigma_{1D}(e_{1D}, e_c)\underline{n} \otimes \underline{n}$$

   - Hyperelastic components : $-pJ\underline{\underline{C}}^{-1} + \frac{\partial W^e}{\partial \underline{\underline{e}}}$
   - Viscous pseudo potential : $\frac{\partial W^\eta}{\partial \underline{\underline{\dot{e}}}}$
   - Contraction model contained in $\sigma_{1D}(e_{1D}, e_c)$: physiological grounds

3. Closure of the system with valves and circulation modeling : 0D

Hill-Maxwell rheological model



- Small strains $e_{1D} = e_s + e_c$
  $\sigma_{1D} = \sigma_c = \sigma_s$
- Large strains $1 + 2e_{1D} = (1 + 2e_s)(1 + 2e_c)$
  $\sigma_{1D} = \frac{\sigma_c}{1+2e_s} = \frac{\sigma_s}{1+2e_c}$

Multiscale contraction modeling (1)

- Huxley Model (1957)

$$\frac{Dn}{Dt} = \frac{\partial n}{\partial t} + \dot{\varepsilon}_c \frac{\partial n}{\partial \xi} = (1 - n)f - ng.$$

  - ξ: position of the bridge
  - $n(\xi, t)$: ratio of attached bridges
  - $f(\xi, t)$: attachment rate
  - $g(\xi, t)$: detachment rate
  - $\varepsilon_c(t)$: strain along sarcomere

- Distribution moment approach (Zahalak, 1981)

$$\begin{cases} k_c(t) & = k_0 \int_{-\infty}^{+\infty} n(\xi, t)\; d\xi \\ \tau_c(t) & = \sigma_0 \int_{-\infty}^{+\infty} \xi\; n(\xi, t)\; d\xi \end{cases}$$

Multiscale contraction modeling (2)

- Closure (Bestel, Clément and Sorine, 2001)
  - $u(t)$ action potential
  - $f(\xi, t) = |u(t)|$ for $\xi \in [0, 1]$, $= 0$ otherwise
  - $g(\xi, t) = |u(t)| + \alpha|\dot{\varepsilon}_c(t)| - f(\xi, t)$

$$\begin{cases} \dot{k}_c &= -(\alpha|\dot{\varepsilon}_c| + |u|)k_c + k_0|u|_+ \\ \dot{\tau}_c &= k_c\dot{\varepsilon}_c - (\alpha|\dot{\varepsilon}_c| + |u|)\tau_c + \sigma_0|u|_+ \\ \sigma_c &= \tau_c + \mu\dot{\varepsilon}_c + k_c\xi_0 \end{cases}$$

"Active constitutive law" (with visco-elasto-plastic features)

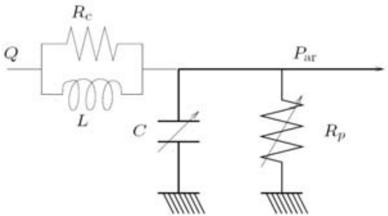
- Cyclic behaviour
  - Contraction $(u > 0, \dot{\varepsilon}_c < 0)$: $k_c \rightarrow k_0$, $\tau_c \rightarrow\approx \sigma_0$
  - Relaxation $(u \leq 0, \dot{\varepsilon}_c > 0)$: $k_c \rightarrow 0$, $\tau_c \rightarrow 0$

Blood circulation

- External circulation needs to be modeled if we want to describe the 4 phases of the heart beat
- We use a "Windkessel model":

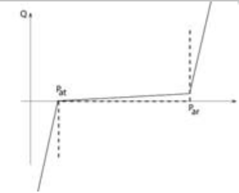
$$-\dot{V} = Q = C\dot{P}_{ar} + \frac{P_{ar} - P_{sv}}{R_p}$$

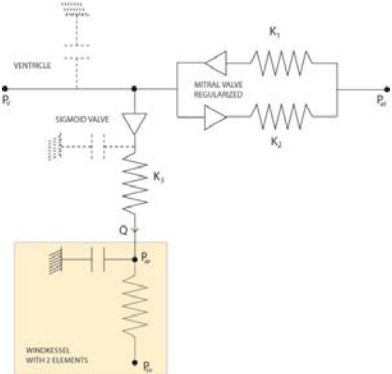
- Can be more complex (PDE could be considered ...)



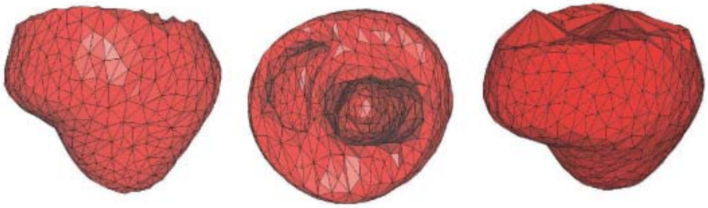
Valve modeling

$$\begin{cases} Q \geq 0 & \text{when } P_V = P_{ar} \quad \text{(ejection)} \\ Q = 0 & \text{when } P_{at} < P_V < P_{ar} \quad \text{(isovol.)} \\ Q \leq 0 & \text{when } P_V = P_{at} \quad \text{(filling)} \end{cases}$$



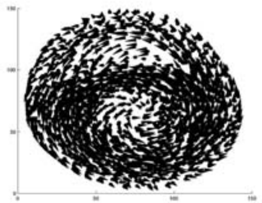


Input Data



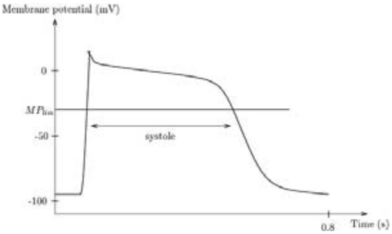
1. Anatomical data
   - Geometry (note: YAMS and GHS3D used)

Input Data



1. Anatomical data
    - Geometry (note: YAMS and GHS3D used)
    - Fibers

Input Data



1. Anatomical data
   - Geometry (note: YAMS and GHS3D used)
   - Fibers
2. Action potential input
   - Wave propagation of an excitation profile : $u(\underline{x}, t) = u^0(\underline{x} - \underline{V} t)$

Input Data

courtesy Yves Coudière

1. Anatomical data
   - Geometry (note: YAMS and GHS3D used)
   - Fibers
2. Action potential input
   - Wave propagation of an excitation profile : $u(\underline{x}, t) = u^0(\underline{x} - \underline{V}t)$
   - From electrical simulations

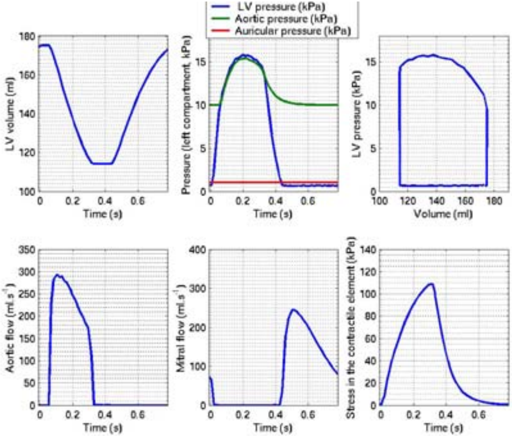
Discretized linear system

Note: time integration (mid-point) and Newton loops written in Matlab. Linear system to be solved at each step:

$$\begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{\Pi}_1 \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{0} \\ \mathbf{\Pi}_2 & \mathbf{0} & \mathbf{\Pi}_3 \end{pmatrix} \begin{pmatrix} \mathbf{\Delta y} \\ \mathbf{\Delta e_c} \\ \mathbf{\Delta P} \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ \mathbf{g} \\ \mathbf{h} \end{pmatrix},$$

- $\mathbf{K}_{22}$ is diagonal $\longrightarrow$ static condensation of $\mathbf{\Delta e_c}$.
- We use a sparse solver (UMFPACK, PARDISO...) to invert the resulting non-symmetric system.

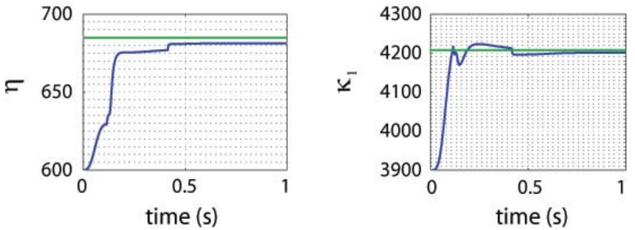
Results


LV pressure (kPa)
Aortic pressure (kPa)
Auricular pressure (kPa)
LV volume (ml)
Time (s)
Pressure (left compartment, kPa)
LV pressure (kPa)
Volume (ml)
Aortic flow (ml.s^{-1})
Mitral flow (ml.s^{-1})
Stress in the contractile element (kPa)

Principles

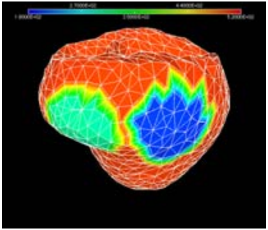
1. Dynamical system $\begin{cases} \dot{X} = F(X, U, t) \\ X(t_0) = X_0 \end{cases}$ with measurem^{ts} $Y = H(X)$.
2. Principle: Use
$\begin{cases} \dot{\hat{X}} = F(\hat{X}, \hat{U}, t) \\ \hat{X}(t_0) = \hat{X}_0 \end{cases}$ and adjust $\hat{U}$, $\hat{X}_0$,... by compar. Y and $H(\hat{X})$.
3. Two major families of approaches explored:
    - Variational: minimize
    $J(\hat{U}) = \frac{1}{2}\int_0^T \|Y(t) - H(\hat{X}(\hat{U}, t))\|_\Omega^2 dt + penalty.$
    - Sequential: (observer) consider instead
    $\begin{cases} \dot{\hat{X}} = F(\hat{X}, \hat{U}, t) + K_X(Y - H(\hat{X})) \\ \dot{\hat{U}} = K_U(Y - H(\hat{X})) \end{cases}$
    K: filter, e.g. Kalman-Bucy filter and various extensions, etc.
    - These two methods are equivalent for linear system
4. Of course, for both approaches observability issues are crucial.

Results



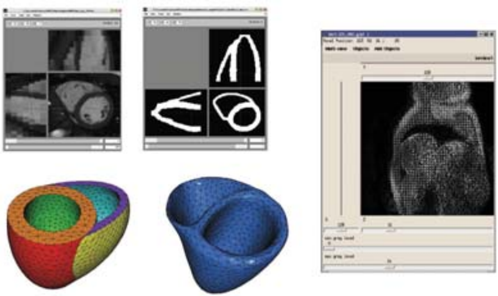
- Results in simplified configuration
  - 0 D model

Results



- Results in simplified configuration
  - 0 D model
  - 3 D with one parameter constant by regions
    Variational assimilation with Matlab Gauss-Newton

Results



- Results in simplified configuration
  - 0 D model
  - 3 D with one parameter constant by regions
- Real framework with King's Hospital

Summary of available advanced OpenFEM features

Already available in CVS version:

- Generic hyperelastic materials
- Non-linear pressure
- Effective assembling procedure (`fe_mknl`)
- Interface with state-of-the-art sparse solvers (`ofact`)