A robust high-order discontinuous Galerkin method with large time steps for the compressible Euler equations

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Abstract: We present a high-order Lagrange-projection like method for the approximation of the Euler equations with general equations of state [1, 2]. The method is based on a decomposition between acoustic and transport operators associated to an implicit-explicit time integration, thus relaxing the constraint of acoustic waves on the time step as proposed in [3, 4]. A discontinuous Galerkin method is used for the space approximation.

We derive conditions on the time step and on a local numerical dissipation parameter to keep positivity of the mean value of the discrete density and internal energy in each element of the mesh and to satisfy a discrete inequality for the physical entropy at any approximation order in space. These results are then used to design limiting procedures in order to restore these properties at nodal values within elements.

Numerical experiments support the conclusions of the analysis and highlight stability and robustness of the present method when applied to either discontinuous flows or vacuum. Moreover, large time steps are allowed while keeping accuracy on smooth solutions even for low Mach number flows.

References

[1] F. Renac, A robust high-order Lagrange-projection like scheme with large time steps for the isentropic Euler equations, Numer. Math., 2016, DOI 10.1007/s00211-016-0807-0.

[2] F. Renac, A robust high-order discontinuous Galerkin method with large time steps for the compressible Euler equations, to appear in Comm. Math. Sci.

[3] F. Coquel, Q. Long-Nguyen, M. Postel and Q. H. Tran, Entropy-satisfying relaxation method with large time-steps for Euler IBVPs, Math. Comput., 79 (2010), pp. 1493–1533.

[4] B. Després, Entropy inequality for high order discontinuous Galerkin approximation of Euler equations, in VII conference on hyperbolic problems. ETHZ-Zurich, 1998.

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