An entropy-consistent colocated finite volume method for solving two-dimensional Lagrangian hydrodynamics

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Keywords: Finite Volume scheme; Entropy conservative scheme; Gas dynamics; Colocated Lagrangian Hydrodynamics.

ABSTRACT

The Colocated Lagrangian Hydrodynamics (CLH) scheme is a *cell-centered moving mesh Finite Volume method* that has been developed for solving the multidimensional *Lagrangian gas dynamics* equations [1, 2]. In this approach, the cell interface numerical fluxes are evaluated at the nodes of the computational grid by means of a nodal solver. The resulting numerical method satisfies by construction the geometrical conservation law, *i.e.*, the zone deformation and the volume flux are computed in a compatible manner. In addition, the conservation of momentum and total energy is ensured. Finally, a local entropy inequality is fulfilled at the semi discrete level. This guarantees the dissipation of kinetic energy into internal energy through shock waves consistently with the Second Law of thermodynamics.

On the other hand, the CLH method, like all the *Godunov-type approach*, is unable to conserve entropy for smooth flows. This flaw might be partially cured developing high-order extension. However, it implies severe inaccuracies in the simulation of *isentropic flows*. In this talk, we describe a hybrid Finite Volume scheme, which has the capability of both conserving entropy for isentropic flows and dissipating entropy for shock waves. This method also conserves momentum and total energy. *Entropy-consistent numerical fluxes* are constructed extending the methodology introduced in [3] to multi-dimensional Lagrangian gas dynamics. The efficiency of this novel approach shall be assessed against various challenging problems.

References

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This work was partially supported by l'Agence Nationale de la Recherche, France (grant number ANR-14-ASTR-0016-01).