Entropy stable and positivity preserving Godunov-type schemes for multidimensional hyperbolic systems on unstructured grids

Agnes Chan[2*], Gérard Gallice[1], Raphael Loubère[3] and Pierre-Henri Maire[1]

[1] CEA Cesta, Le Barp, France, pierre-henri.maire@cea.fr, gerard.gallice@gmail.com
[2] Université de Bordeaux, Bordeaux INP, and CEA Cesta, Le Barp, agnes.chan@u-bordeaux.fr
[3] Université de Bordeaux, Bordeaux INP, CNRS, Bordeaux, raphael.loubere@math.u-bordeaux.fr

In this presentation, a class of cell-centered Finite Volume scheme based upon the Lagrange-to-Euler mapping will be introduced. This mapping establishes a fundamental relation bridging the Lagrangian and Eulerian descriptions. The construction of the scheme originates from the idea of extending the cell-centered Lagrangian scheme [4] for multidimensional gas dynamics on the Eulerian framework at the same time benefiting from the approach introduced by [5]. The numerical fluxes of the scheme are evaluated by means of an approximate Riemann solver located at the grid nodes, which provides the nodal velocity required to move the grid in a compatible manner. The conservation condition and entropy stability of the scheme result respectively from a node-based vectorial equation and a scalar inequation. The Lagrange-to-Euler mapping introduced by [3] and revisited in [1] then allows to build positive and entropic Eulerian Riemann solvers from their Lagrangian counterparts, provided that an explicit time step condition is fulfilled [2].
An associated Finite Volume simulation code has been built in multi-dimensions for unstructured meshes. Parallelization has been accomplished using the MPI library embedded in PETSc. A large set of 2D/3D numerical experiments show that the proposed solver is less sensitive to spurious instabilities such as the infamous carbuncle, compared to the classical one. The numerical assessment of this new method by means of representative test cases is very promising in terms of robustness.

REFERENCES

[1] A. Chan, G. Gallice, R. Loubère and P.-H Maire, Positivity preserving and entropy consistent approximate Riemann solvers dedicated to the high-order MOOD-based Finite Volume discretization of Lagrangian and Eulerian gas dynamics. *Computers & Fluids*, 2021, **229**.

[2] G. Gallice, A. Chan, R. Loubère and P.-H Maire, Entropy stable and positivity preserving Godunov-type schemes for multidimensional hyperbolic systems on unstructured grid. *J. Comp. Phys*, 2022.

[3] G. Gallice, Positive and Entropy Stable Godunov-Type Schemes for Gas Dynamics and MHD Equations in Lagrangian or Eulerian Coordinates. *Numer. Math.*, 2003, **94**.

[4] R. Loubere, P.-H. Maire and B. Rebourcet, Staggered and Colocated Finite Volume Schemes for Lagrangian Hydrodynamics, Handbook of Numerical Methods for Hyperbolic Problems: Basic and Fundamental Issues, 2016, **17**.

[5] Z.J. Shen, W. Yan and G.W. Yuan, A robust and contact resolving Riemann solver on unstructured mesh, Part I, Euler method. *J.Comput. Phys.*, 2014, **268**.