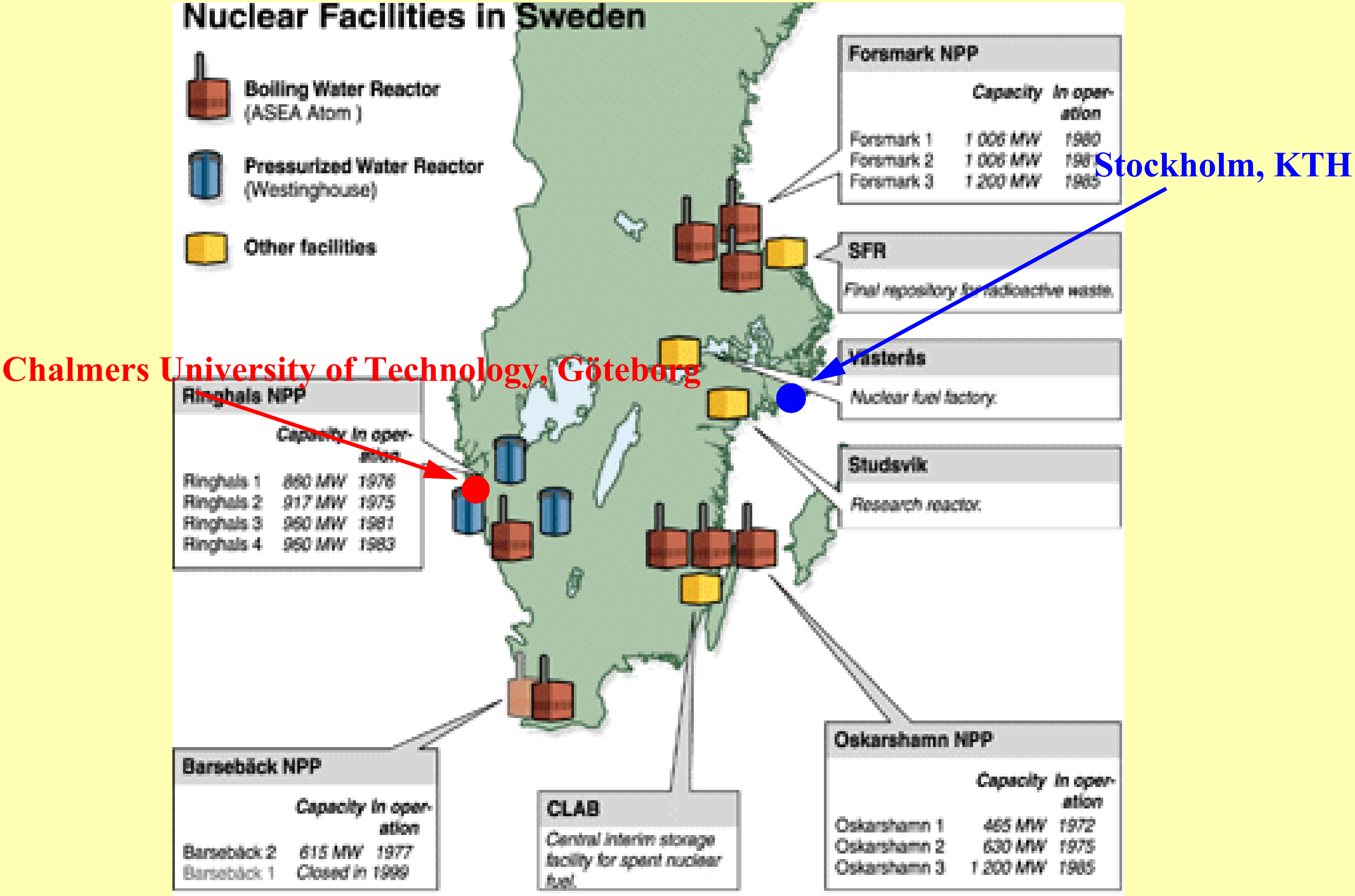
The invariant embedding method of particle transport and its extension to particle fluctuations

Imre Pázsit

Chalmers University of Technology
SE-412 96 Göteborg, Sweden

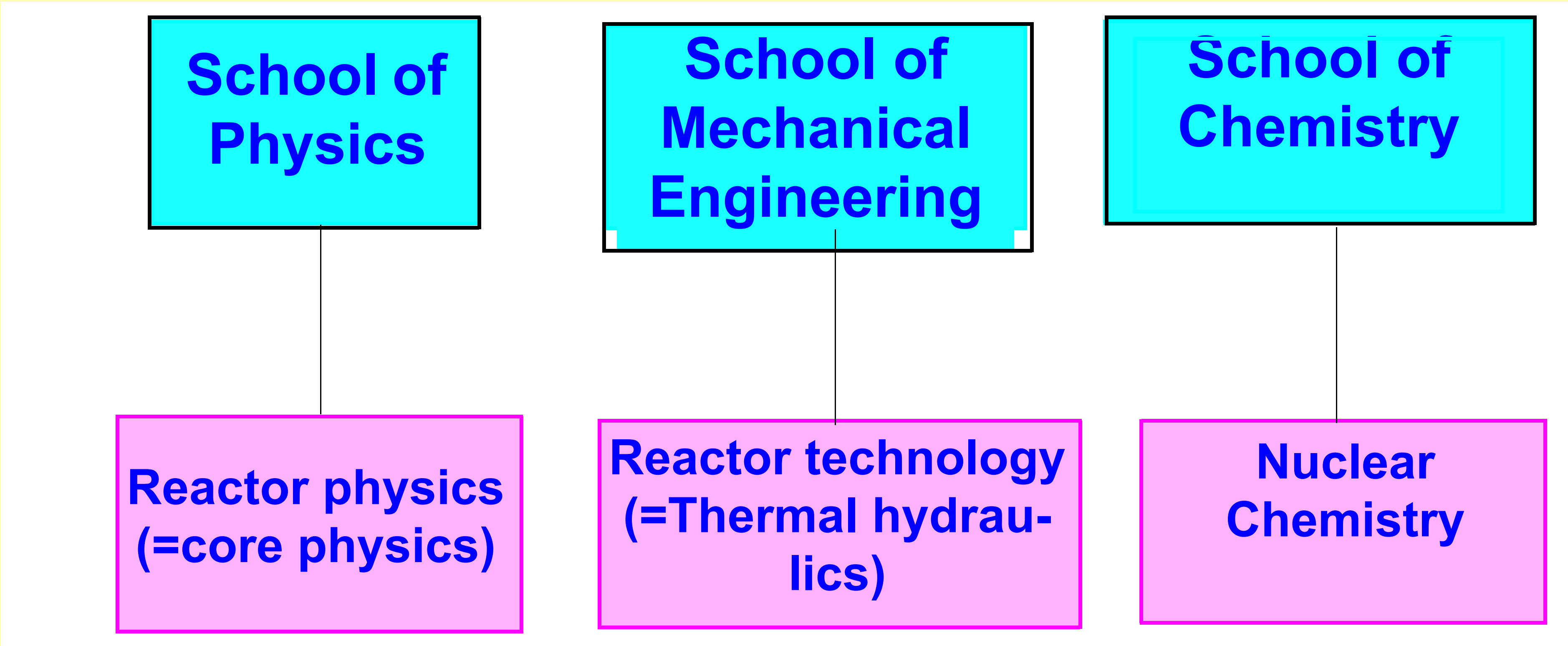
CEA Paris, 29 September 2008

Geographical



Nuclear engineering subjects in Chalmers

The traditional Swedish system:



The above separate units were collected into a Dept of Nucl. Engineering under the Institute of Applied Physics.

Department of Nuclear Engineering

Its predecessor (Dept. of Reactor Physics) was founded in 1961

Staff:

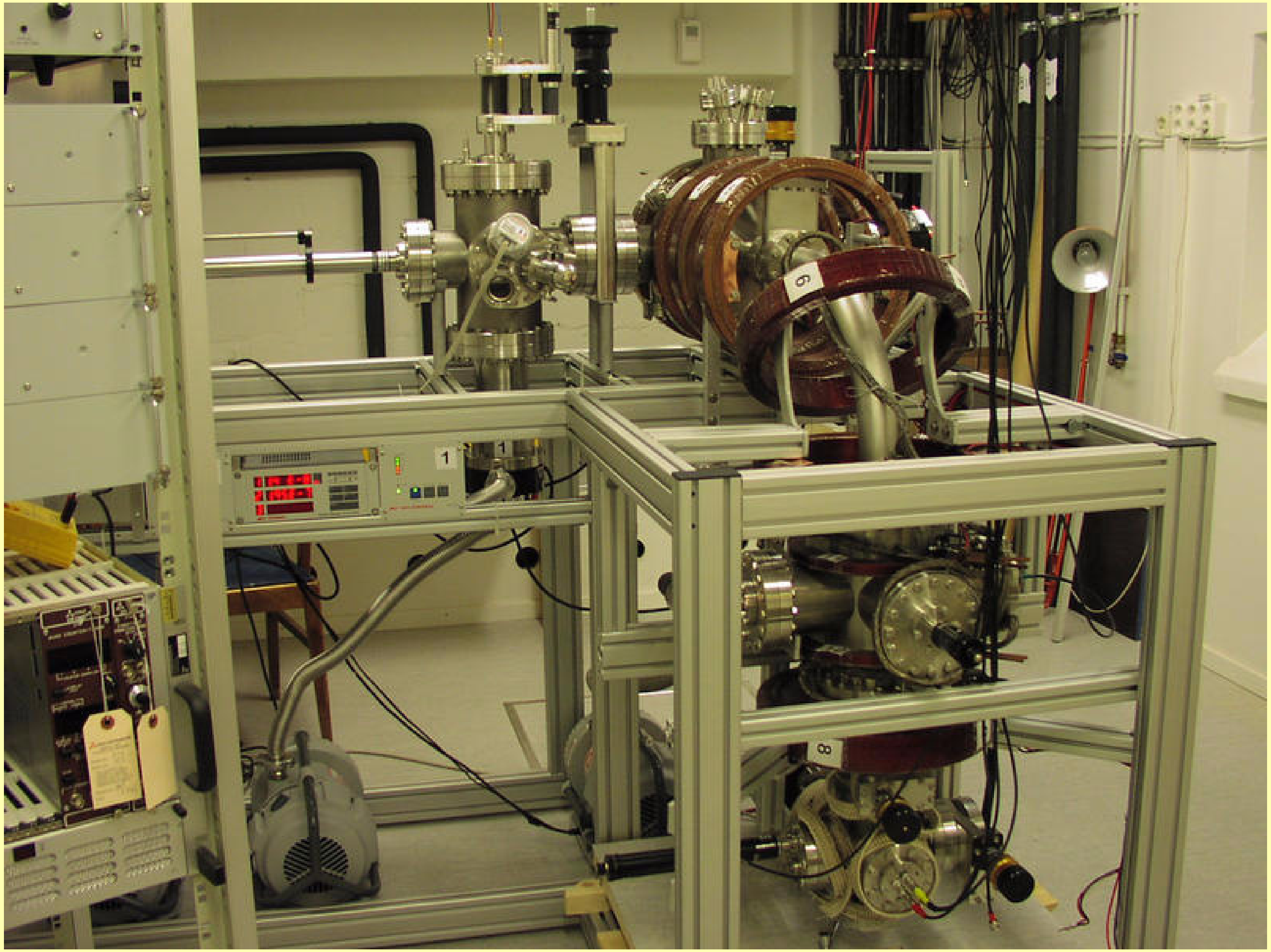
2 professors + 1 prof. emeritus
2 associate professors
2 research associate
1 postdoc
5 PhD students (+2 under recruiting)
2 technicians
1 secretary
7MSc thesis students

Facilities:

Portable pulsed neutron generator, 14 MeV neutrons
Pulsed beam for slow neutrons

The neutron generators (~~the stationary~~ and the portable)

The pulsed beam for slow positrons



Courses

Until last year:

5 elective courses in the undergraduate education

- reactor physics
- neutron physics
- radiation protection
- reactor technology (thermal hydraulics
- transport theory and stochastic processes

PhD program in Reactor Physics

Last year: Minor course (1 semester, 20 credit points)

2009: Full master's program

1. Research activities at the Department

R&D work within the fields of core phsyics, ICFM, reactor noise diagnostics, and reactor thermal hydraulics CFD, two-phase flow etc.

There is both academic and pragmatic research.

Most of the work is performed in co-operation with, and supported by, the safety authority and the power plants in Sweden, and aims at direct applications at the plants.

The purpose is to solve certain practical problems by developing new methods, and to apply them in practice. However, the challenge of solving new problems, doing good quality research, are important to us.

PÁZSIT
PÁL

ELSEVIER

The transport and of neutrons in a multiplying system is an area of branching processes with an aesthetically pleasing and clear formalism. The theory has very concrete and useful applications for diagnostics of nuclear systems. Yet, this beautiful theory has never been compiled in a self-contained and complete monograph. The lack of such a treatise has become apparent recently when new fields of applications were opened with the appearance of new reactor concepts. This monograph is intended to fill this vacancy.

This book was written with two objectives in mind, and correspondingly it consists of two parts. The first part presents an account of the mathematical tools used in describing branching processes, which are then used to derive a large number of properties of the neutron distribution in multiplying systems with or without an external source. The emphasis is, however, not so much on giving lengthy derivations or a complete collection of formulae, rather to expose the reader to the methodology of setting up and solving master equations of particle transport through the completeness of the treatment.

In the second part the theory is applied to the description of the neutron fluctuations in nuclear reactor cores as well as in small samples of fissile material. The question of how to extract information about the system in question is discussed. In particular the measurement of the reactivity of subcritical cores, driven with various Poisson and non-Poisson (pulsed) sources, and the identification of fissile material samples, is illustrated. This part of the book gives pragmatic information for those planning and executing and evaluating experiments on such systems.

This book will be of interest for graduate students as well as researchers in the fields of nuclear engineering, reactor physics, safeguards, as well as physicists and biologists working with branching processes.

ELSEVIER

books.elsevier.com

ISBN 0-08-045064-4

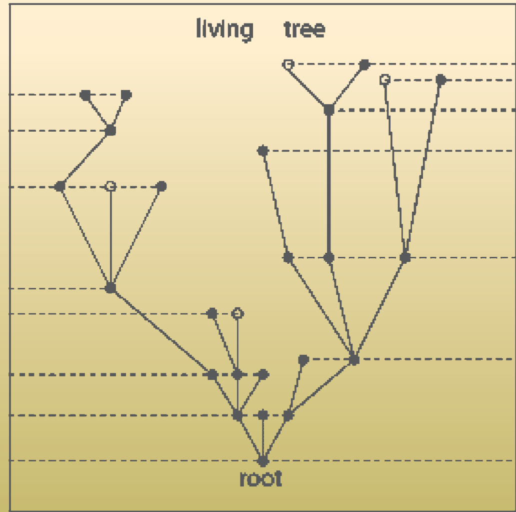
9 780080 450643

Neutron Fluctuations
A TREATISE ON THE PHYSICS OF BRANCHING PROCESSES

ELSEVIER

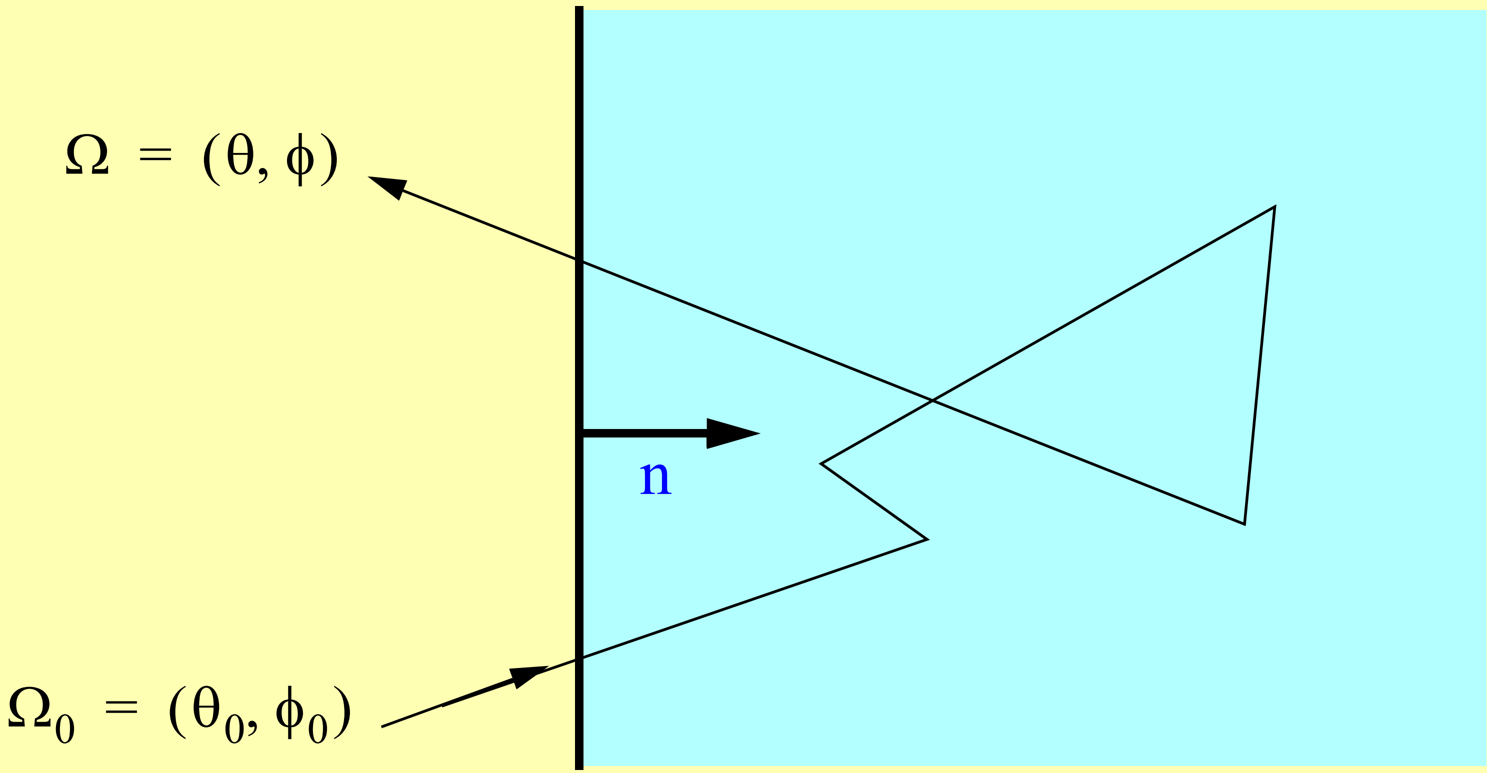
Neutron Fluctuations

A TREATISE ON THE PHYSICS OF BRANCHING PROCESSES



IMRE PÁZSIT AND LÉNÁRD PÁL

Goal of invariant embedding: to calculate the flux of backscattered particles, bombarding a surface of a homogeneous, semi-infinite medium.



Probability of backscattering: $\phi(E_0, \Omega_0, E, \Omega) dEd^2\Omega$ - proportional to the traditional particle flux.

ϕ is non-vanishing only for $E < E_0$ and $0 \le \theta_0 \le \pi/2 \le \theta \le \pi$, i.e.

$$E < E_0 \tag{1}$$

and

$$0 \le \theta_0 \le \pi/2 \le \theta \le \pi\ . \tag{2}$$

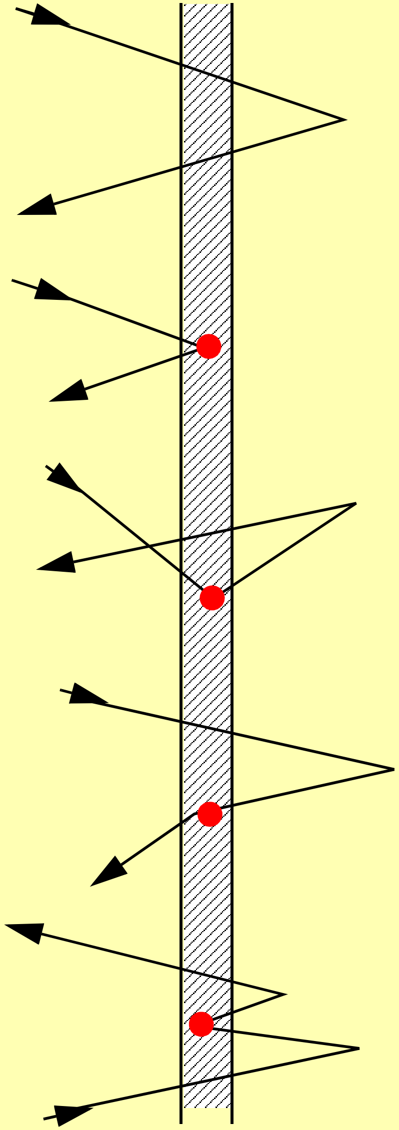
The particle interactions are described by the differential scattering cross section (includes all possible reaction types)

$$\Sigma(E, \Omega, E', \Omega') \tag{3}$$

and the total cross section

$$\Sigma_t(E) \equiv \int dE' \int d^2\Omega' \, \Sigma(E, \Omega, E', \Omega') \tag{4}$$

Derivation of the invariant embedding equation for ϕ is based on the principle that adding an infinitesimally thin (dx) layer of the same material will not change the backscatterig probability. Then ϕ can be formulated, in first order of *dx,* as a sum of the probabilities of no scattering or one scattering in the added layer. One scattering can happen in 4 different ways:



1. No collision

2. Direct backscattering from surface layer

3. Scattering inbound

4. Scattering outbound

5. Backscattering from layer to material. It is seen that this term will be non-linear in Y.

Probability of collision in the layer

Probability of collision within an infinitesimal path ds is

$$\Sigma_t ds = \frac{\Sigma_t dx}{\cos\theta_0} \quad \text{or} \quad \frac{\Sigma_t dx}{|\cos\theta|} \tag{5}$$

where θ_0 and θ are the inbound and outbound angles, respectively.

The probability density of scattering into given directions and energy is given as

$$\frac{\Sigma(E_0, \Omega_0, E, \Omega)}{|\cos\theta_0|} \equiv G(E_0, \Omega_0, E, \Omega) \tag{6}$$

With these, the following equation can be formulated.

$$\phi = Prob(1) + Prob(2) + Prob(3) + Prob(4) + Prob(5) \tag{7}$$

Here $Prob$ stands for probability density, to get rid of the factor $dEd^2\Omega$.

1. Probability density of backscattering without scattering in the layer:

$$\left[1 - dx\left(\frac{\Sigma_t(E_0)}{\mu_0} + \frac{\Sigma_t(E)}{|\mu|}\right)\right]\phi(E_0, \Omega_0, E, \Omega) \tag{8}$$

2. Probability density of direct backscattering from the layer:

$$G(E_0, \Omega_0, E, \Omega) \tag{9}$$

3. Prob. density of backscattering with an inward scattering in the layer:

$$\int_E^{E_0} dE' \int d\Omega'^2 G(E_0, \Omega_0, E', \Omega')\phi(E', \Omega', E, \Omega) \equiv G \times \phi \tag{10}$$

where the operator $\times$ is defined by the equation.

4. Prob. density of backscattering with an outbound scattering from layer:

$$\int_E^{E_0} dE' \int d\Omega'^2 \phi(E_0, \Omega_0, E', \Omega')G(E', \Omega', E, \Omega) \equiv \phi \times G \tag{11}$$

5. Prob. density of backscattering into the material:

$$\int dE' \int d\Omega'^2 \int dE'' \int d\Omega''^2 \phi(E_0, \Omega_0, E', \Omega') G(E', \Omega', E'', \Omega'') \phi(E'', \Omega'', E, \Omega) \tag{12}$$

$$\equiv \phi \times G \times \phi$$

One obtains after rearranging

$$\left(\frac{\Sigma_t(E_0)}{\mu_0} + \frac{\Sigma_t(E)}{|\mu|}\right)\phi(E_0, \Omega_0, E, \Omega) = \tag{13}$$

$$= G + G \times \phi + \phi \times G + \phi \times G \times \phi$$

The integrations in the last three terms have different domains for obvious reasons.

Eqn (13) has the advantage that it does not contain the depth or path length variable, only angular and energy variables.

On the other hand, it is non-linear. This presents a difficulty even in case of a numerical solution.

However, a solution scheme can be elaborated if the equation is expanded such than iterative equations for the contributions ϕ_n from n-times collided particles are derived.

For $n \geq 2$ one obtains easily the equation system for $\phi_n(E_0, \Omega_0, E, \Omega)$ as

$$\left(\frac{\Sigma_t(E_0)}{\mu_0} + \frac{\Sigma_t(E)}{|\mu|}\right)\phi_n(E_0, \Omega_0, E, \Omega) = \tag{14}$$
$$= G \times \phi_{n-1} + \phi_{n-1} \times G + \sum_m \phi_m \times G \times \phi_{n-1-m}$$

For $n = 1$ one has the direct solution

$$\phi_1(E_0, \Omega_0, E, \Omega) = \left(\frac{\Sigma_t(E_0)}{\mu_0} + \frac{\Sigma_t(E)}{|\mu|}\right)^{-1} G(E_0, \Omega_0, E, \Omega) \tag{15}$$

Eqn (14) represents a recurrence relationship, where the higher order terms can be obtained from the lower ones with integration. The equation is linear in the highest order, and the r.h.s. only contains lower order terms.

Quasielastic backscattering

In electron and positron transport it is customary to decouple the elastic collisions from the inelastic ones by assuming that the inelastic collisions to not change the trajectory.

Nearly elastically backscattered positrons (small energy loss):

- the distribution W_n of positrons having n elastic collisions can be calculated using only the elastic cross section with constant energy;
- all other quantities can be derived from the W_n: the path length distribution, and the probability of m inelastic collisions. From the latter the energy loss just below the initial energy can also be calculated.

The equations for the W_n were solved in slab geometry:

$$W_n = W_n(\eta_0, \eta) \tag{16}$$

where $\eta_0 = \cos\theta_0$ and $\eta = \cos\theta$. Realistic elastic cross sections

$\sigma_{el}(\Omega\Omega')$ were used from the literature. The relationship between the W_n and the other quantities can be derived, from further embedding equations, as follows.

Path length distribution: probability of backscattering after having travelled a path length R in the medium:

$$Q(\eta_0, \eta, R) = \frac{1}{\lambda_{el}} e^{-R/\lambda_{el}} \sum_{n=0}^{\infty} \frac{(R/\lambda_{el})^{n-1}}{(n-1)!} W_n(\eta_0, \eta) \tag{17}$$

where $\lambda_{el} = \left(N\int\sigma_{el}(\Omega\Omega')d\Omega'\right)^{-1}$.

Distribution of particles backscattered after m inelastic collisions:

$$V_m(\eta_0, \eta) = \sum_{n=0}^{\infty} \frac{(n+m-1)!}{m!(n-1)!} \frac{\lambda_{el}^m \lambda_{in}^n}{(\lambda_{el} + \lambda_{in})^{m+n}} W_n(\eta_0, \eta) \tag{18}$$

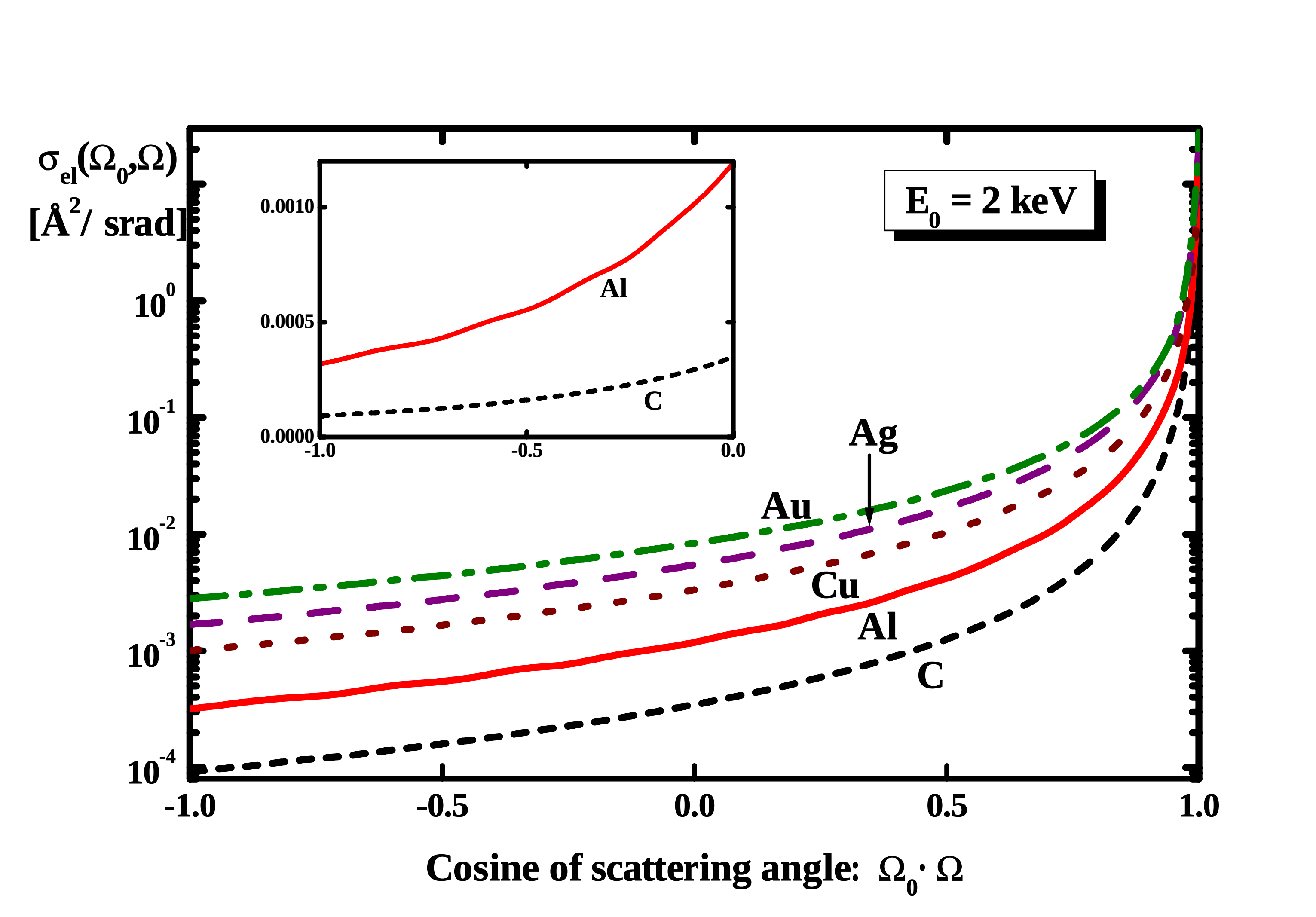
Results

Calculations were made for several incident energies, elements, and incident and exiting angles. Both azimuthally resolved, and azimuthally integrated distributions can be calculated.

Glazov L.G. and Pázsit I. Applications of invariant embedding: positron backscattering from surfaces. *Nucl. Instr. Meth.* **B 215**, 509-524 (2004)

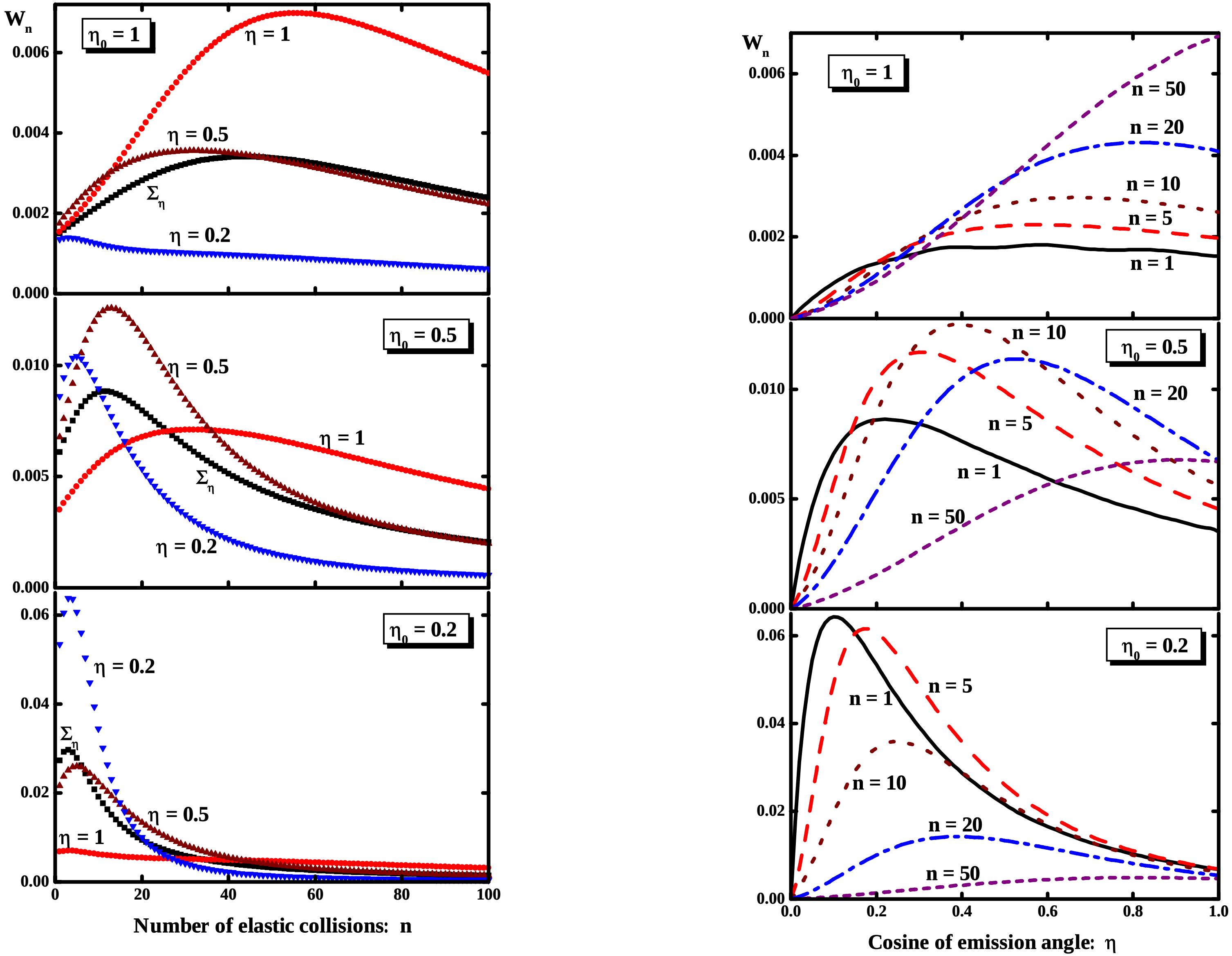
Glazov L.G. and Pázsit I. Kinetic theory of particle reflection/emission from surfaces: Invariant-embedding and cognate descriptions. *Nucl. Instr. Meth.* **B 256**, Issue 2, pp. 638 - 658 (2007)

Angular dependence of some representative cross sections

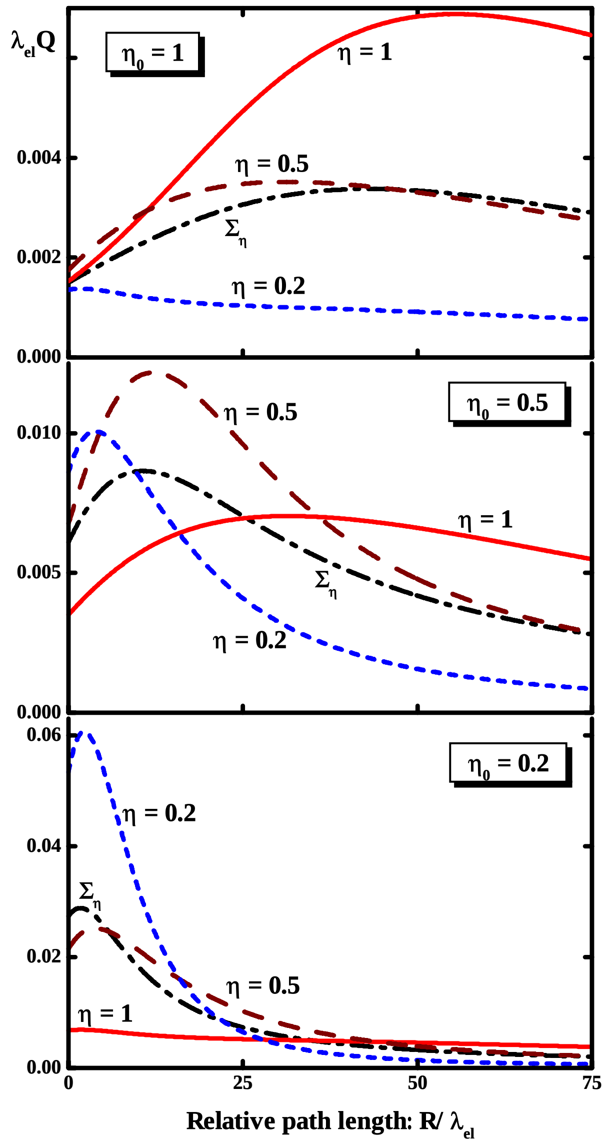


Strongly forward peaked scattering

W_n as functions of collision number and the emission angle



Path length distribution for various angles



Conclusions

The backscattering problem could be solved very effectively with the invariant embedding method.

For the case with the given simplifications (quasielastic backscattering; equivalent with ones-speed time dependent theory in a half space with highly anisotropic scattering), the method is exact. It is yields solutions resolved in several parameters in one sweep. It is therefore several orders of magnitude faster than Monte-Carlo methods, which are the dominating method in electron and positron backscattering calculations.