LINEAR ALGEBRA I
EXERCICES

EXERCICE 1 – LINEAR COMBINATION

For $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $w = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

describe all points cv with.

a) c being an integer, i.e., $c \in \mathbb{Z} = \{\dots, -3, -2, \dots, 1, 2, \dots\}$

b) c nonnegative numbers, $c \geq 0$

describe $cv + dw$ where $d \in \mathbb{R}$ and c in like in a) OR b).

EXERCICE 2:

Is $z = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ in the span of

$x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. If so, find

$\underset{\mathbb{R}}{\underset{\cap}{\alpha}}$ and $\underset{\mathbb{R}}{\underset{\cap}{\beta}}$ such that $z = \alpha x + \beta y$

EXERCICE 3 :

1) Prove that $u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $u_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, $u_3 = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ are linearly independent.

2) Is $\{u_1, u_2, u_3\}$ a basis of $\mathbb{R}^3$?

EXERCICE 4 :

Consider the following transformations in $\mathbb{R}^2$

$$L_1 : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} y \\ x \end{pmatrix} \qquad L_2 = \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x^2 \\ y \end{pmatrix}$$

1/ Is L_1 a linear transformation ? [If YES make a formal proof)]

2/ Same question for L_2

3/ Interpret geometrically L_1