

CLASS 3 - EXERCICES

EXERCICE 1 Let A be a $n \times n$ non-zero matrix whose rank equals k .

1/ If $k = n$ and $b \in \mathbb{R}^n$, explain why there is only one vector x such that

$$Ax = b$$

2/ Suppose $k < n$, show that there are vectors $b \in \mathbb{R}^n$ for which the equation $Ax = b$ has no solutions.

EXERCICE 2 Show that $\forall a, b \in \mathbb{R}^n$

$$(a+b) \cdot (a+b) = \|a\|_2^2 + 2a \cdot b + \|b\|_2^2$$

EXERCICE 3 Let v_1, \dots, v_m be a list of orthogonal non zero vectors, that is for all $i, j \in \{1, \dots, m\}$ $v_i \cdot v_j = 0$. Prove that they are linearly independent.