Derivative-Free Optimization
Multiobjective Optimization, 2^{nd} part

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Anne Auger
INRIA Saclay – Ile-de-France

Dimo Brockhoff
INRIA Saclay – Ile-de-France

Overview

Last time:

- fundamentals of multiobjective optimization
- algorithm design principles and concepts

Today: selected advanced concepts

- performance assessment
- preference articulation
- visualization aspects

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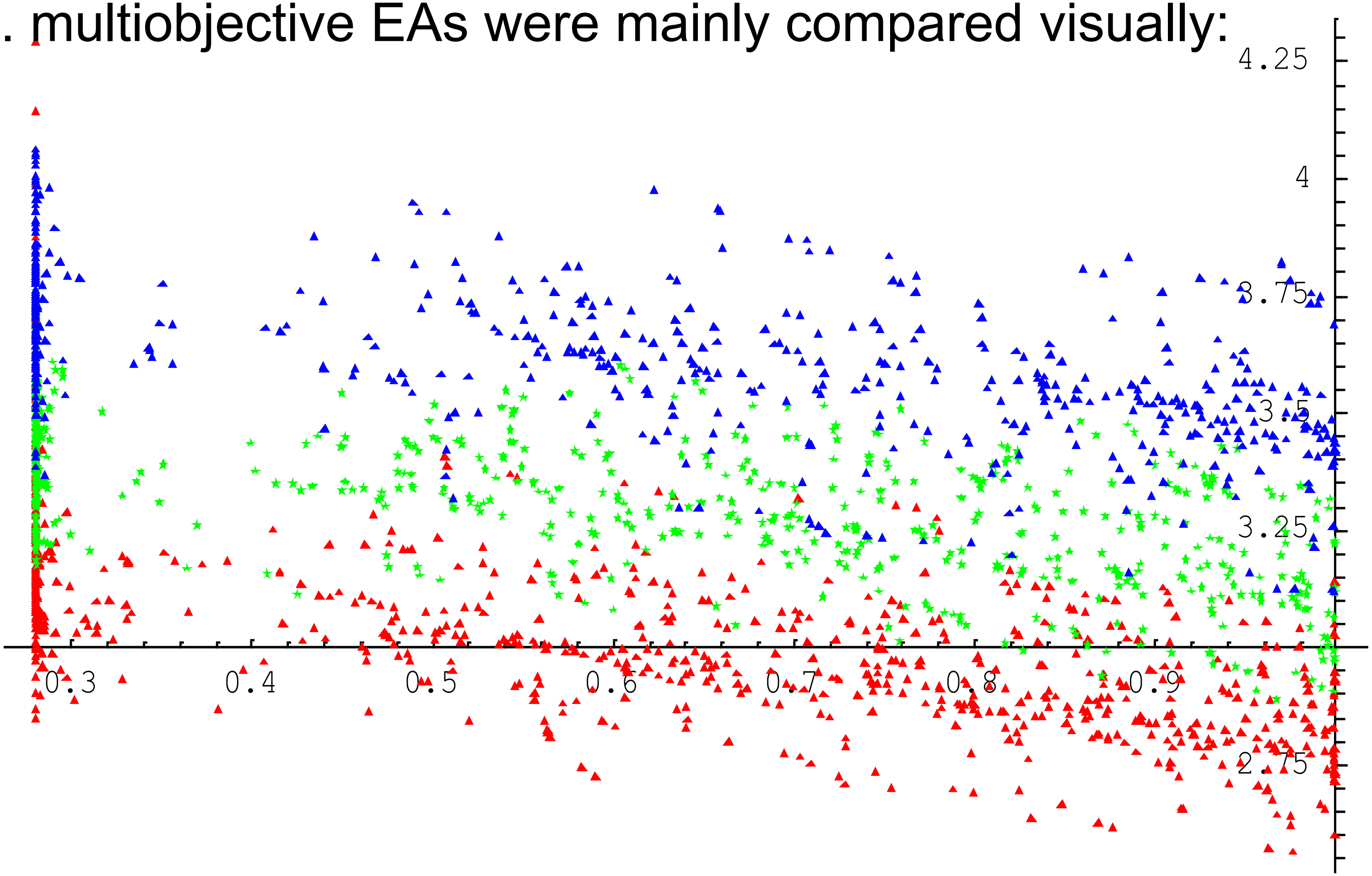
Once Upon a Time...

... multiobjective EAs were mainly compared visually:



Two Main Approaches for Empirical Studies

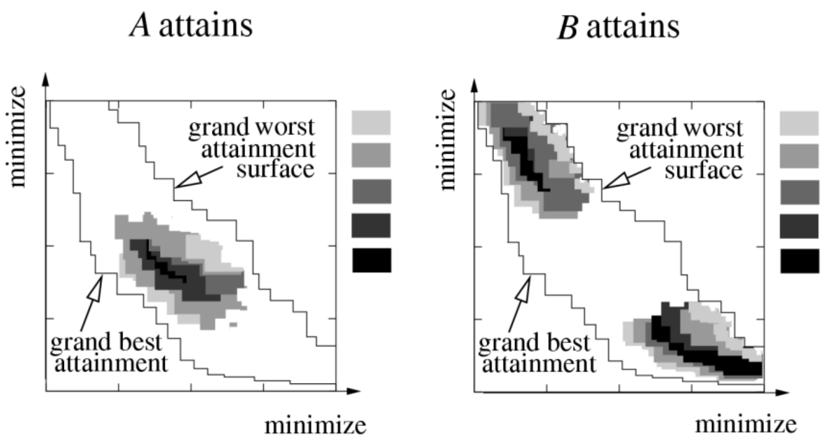
Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

reduces each approximation set to a single quality value

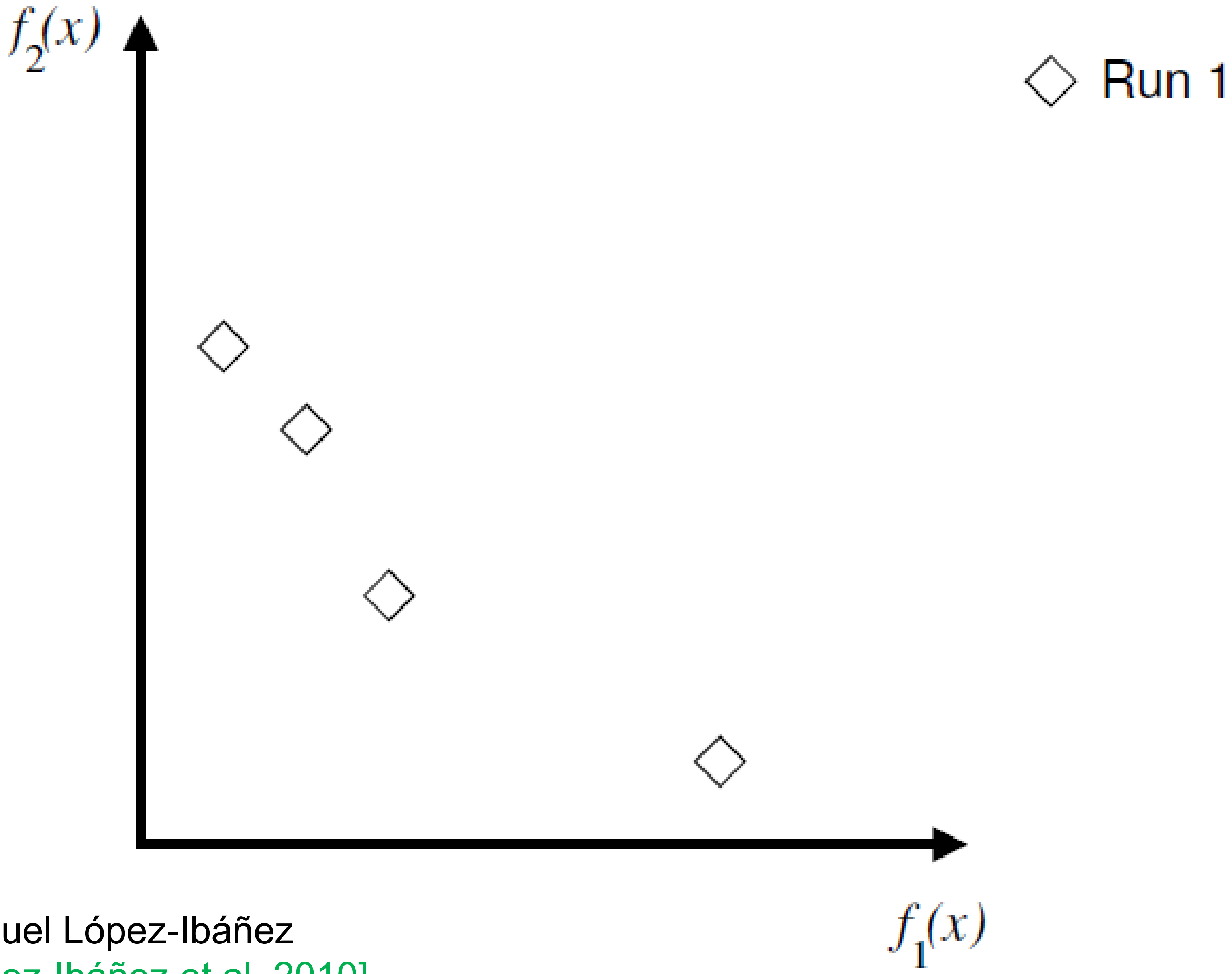
applies statistical tests to the quality values



Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions: Idea



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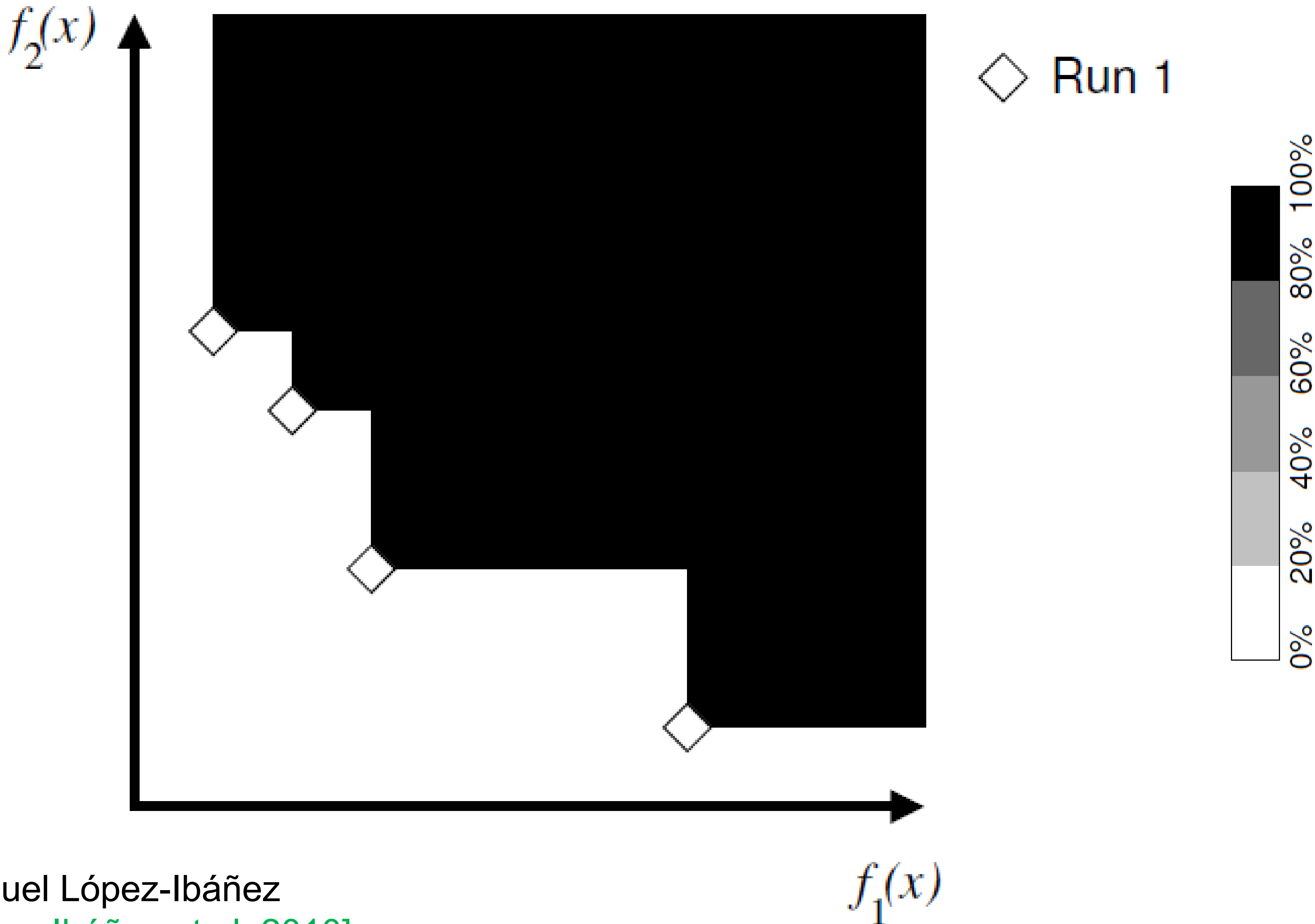
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



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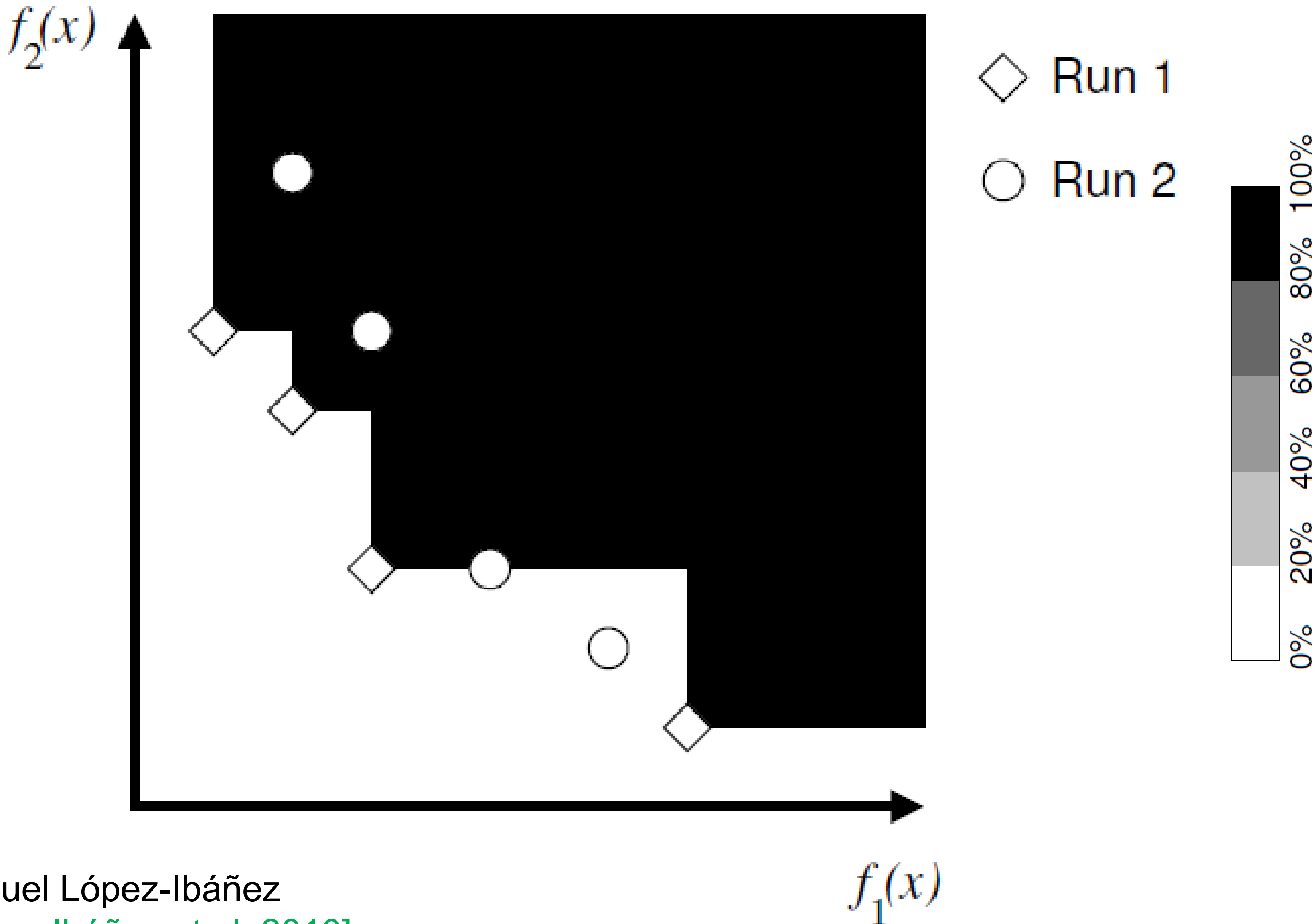
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



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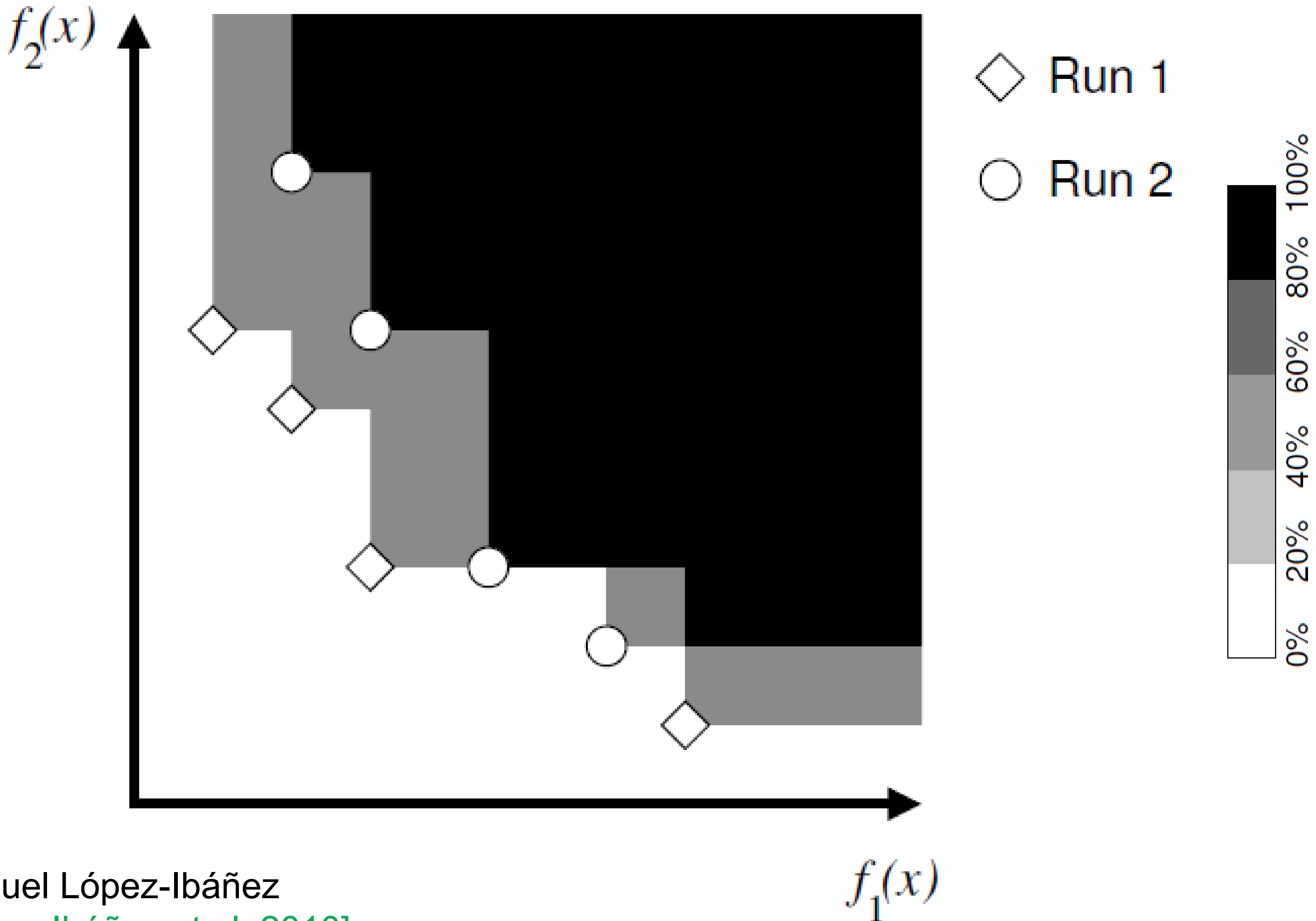
Empirical Attainment Functions: Idea



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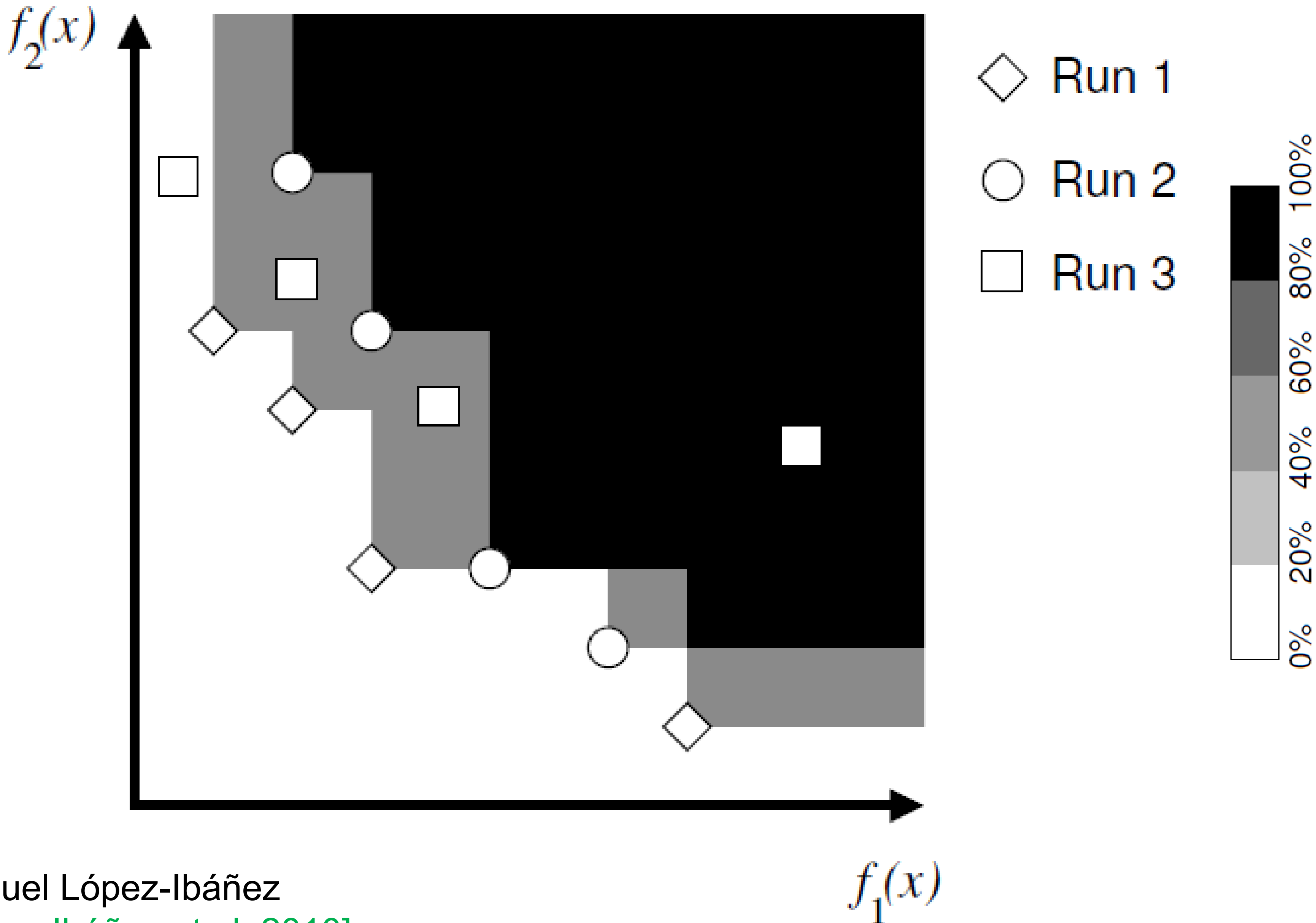
Empirical Attainment Functions: Idea



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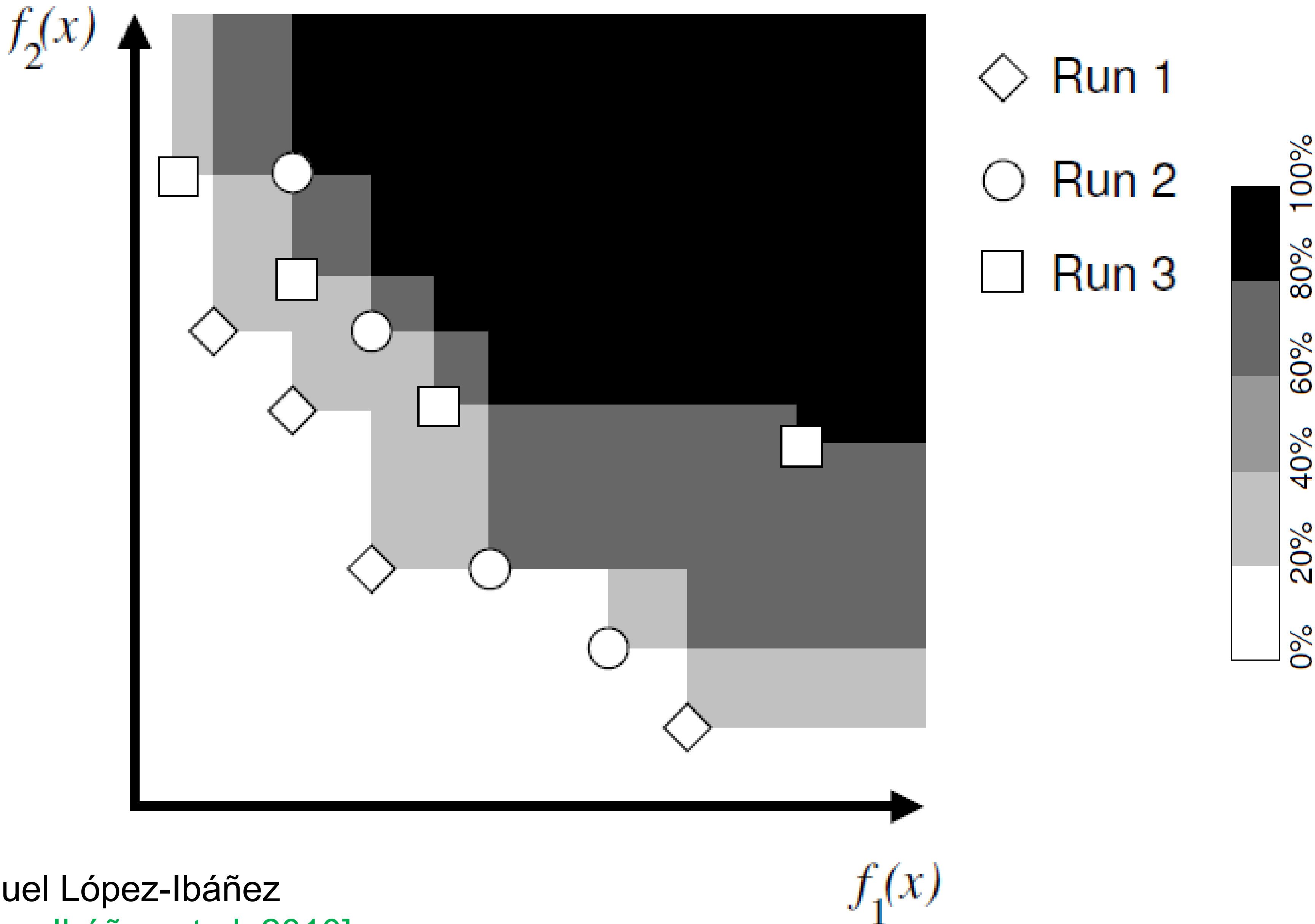
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Idea



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[López-Ibáñez et al. 2010]

Definition Empirical Attainment Function

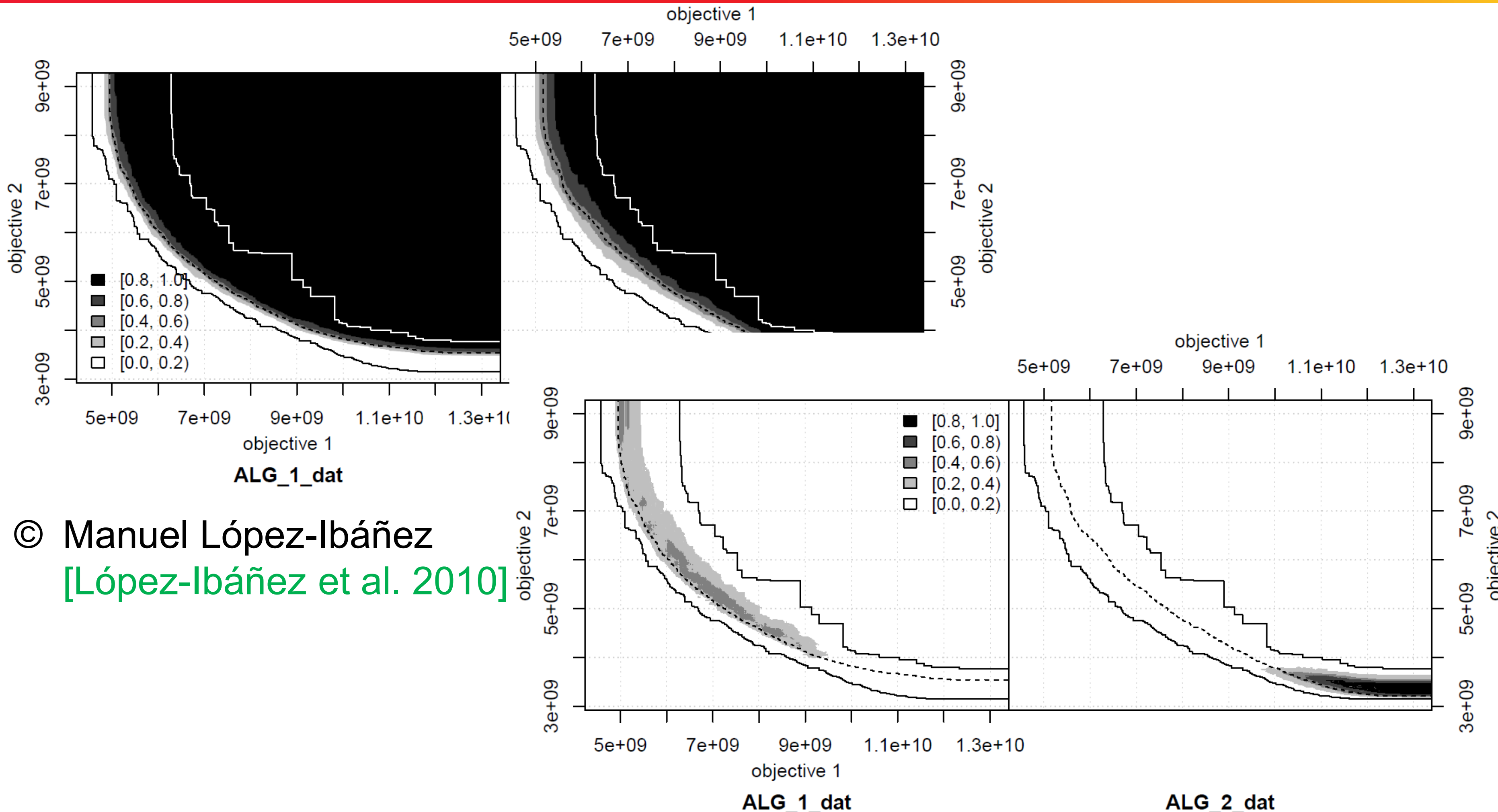
The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector z at time T:

$$\alpha_T(z) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time T.

Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time T in the single-objective case ("fixed budget scenario").

Attainment Plots in Practice



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[López-Ibáñez et al. 2010]

latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

Most Used Approach: Quality Indicators

A quality indicator

- maps a solution set to a real number
- can be used with standard performance assessment
  - report median, variance, ...
  - boxplots
  - statistical tests
- should optimally refine the dominance relation on sets

Recommendation:

- use hypervolume (refinement, i.e. it does not contradict the dominance relation)
- or epsilon indicator or R2 indicator (are weak refinements)

Also important:

- interpretation of the results (by knowing theoretical properties of the used indicator)

Quality Indicator Approach

Idea:

transfer multiobjective problem into a set problem

define an objective function ("quality indicator") on sets

use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation $A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \leq_{par} y$ should be reflected!

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements…

[Zitzler et al. 2010]

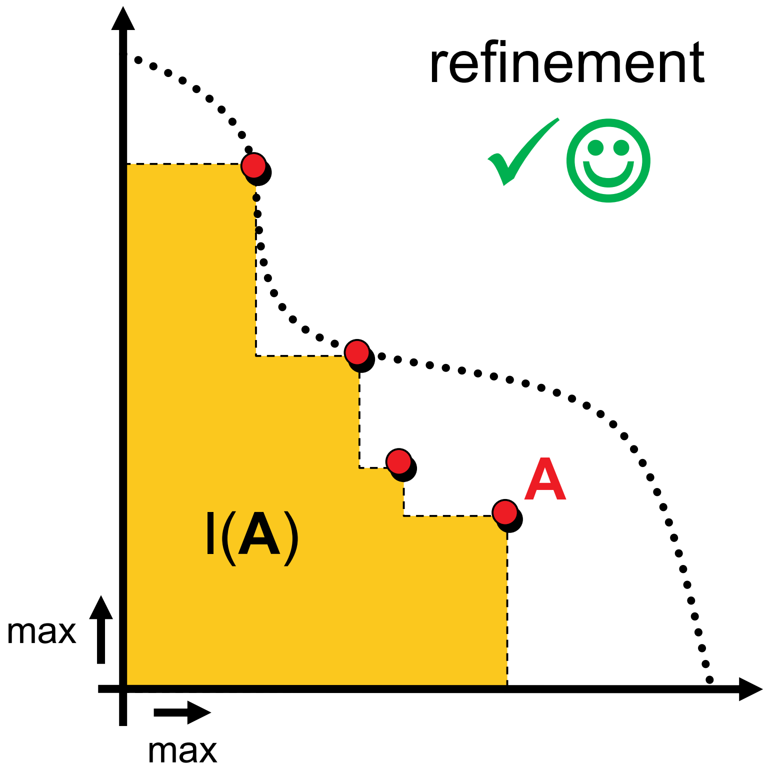
Example: Refinements Using Indicators

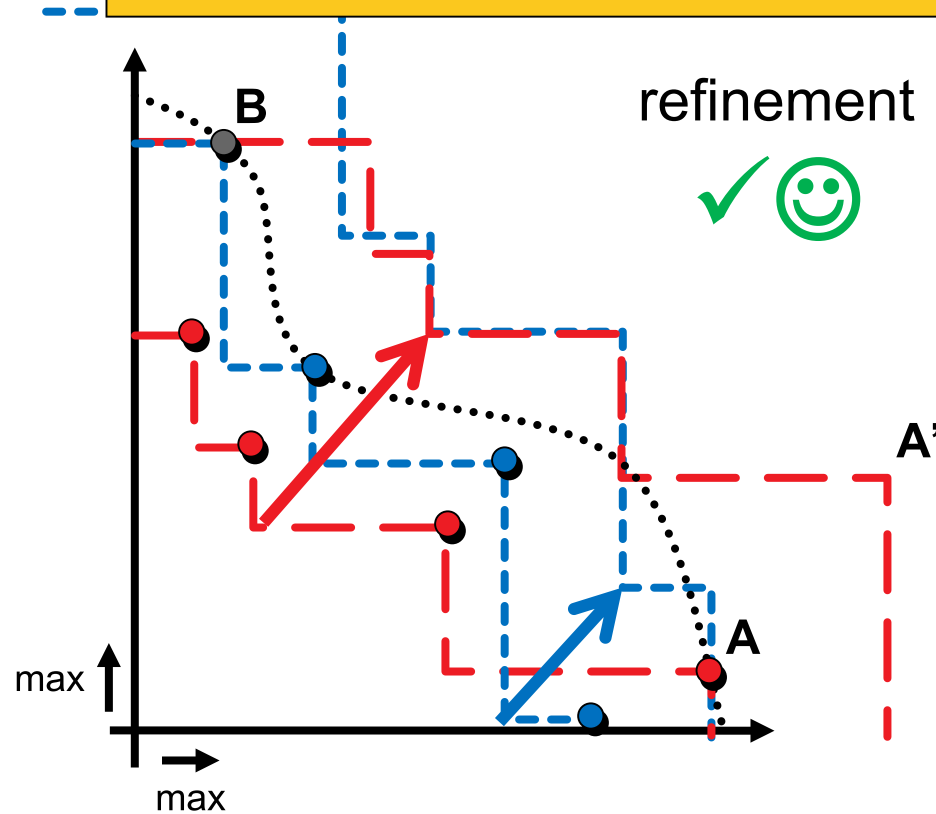
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B





unary hypervolume indicator

binary epsilon indicator

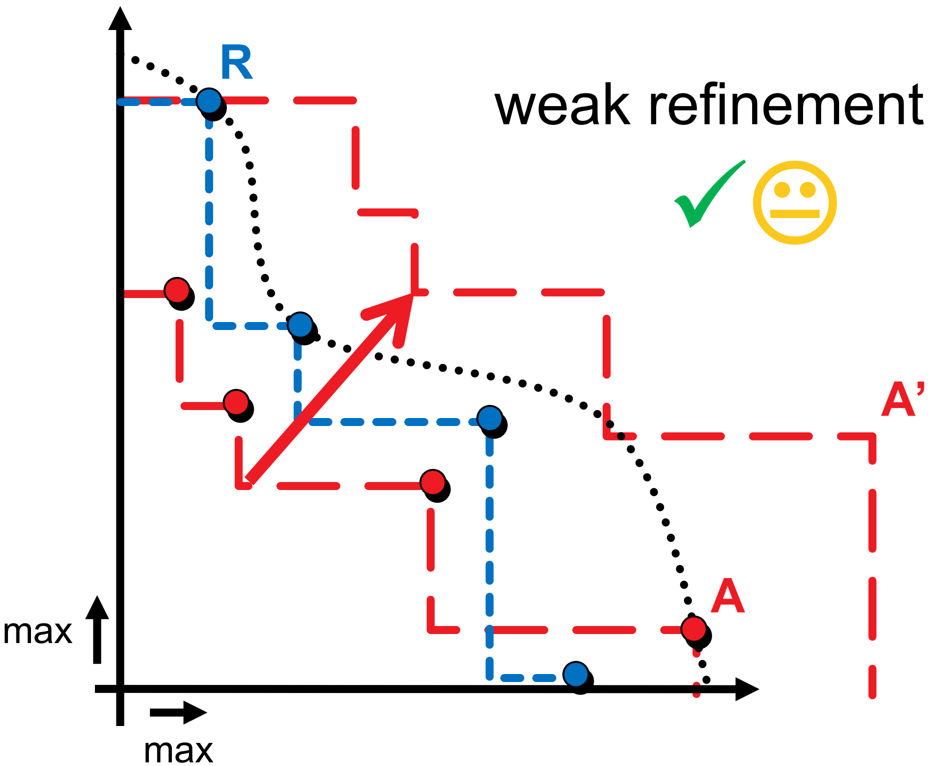
Example: Weak Refinement / No Refinement

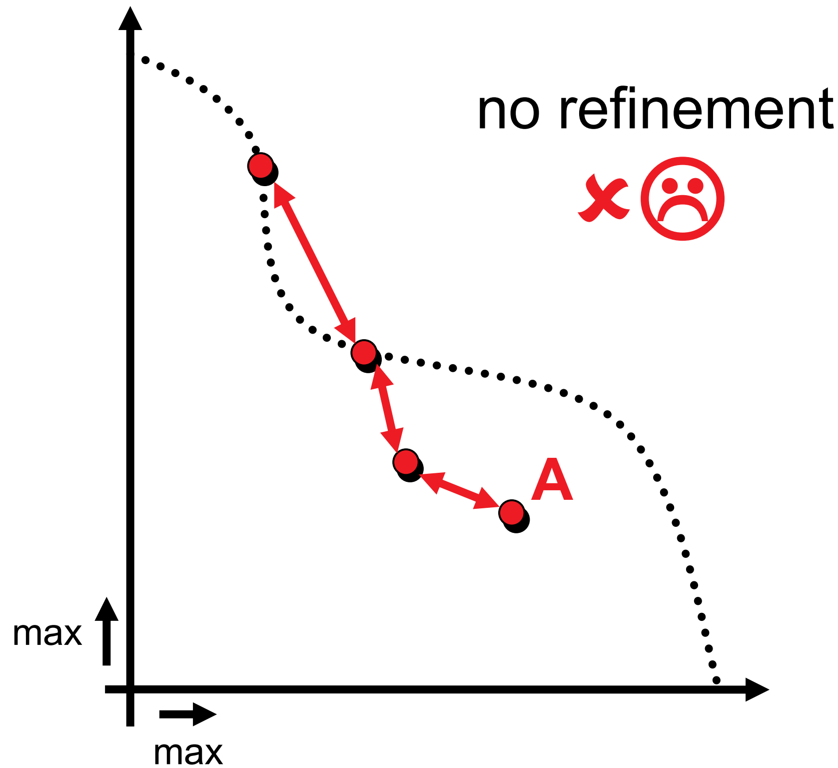
$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A,R) \leq I(B,R)$$

$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A) \leq I(B)$$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



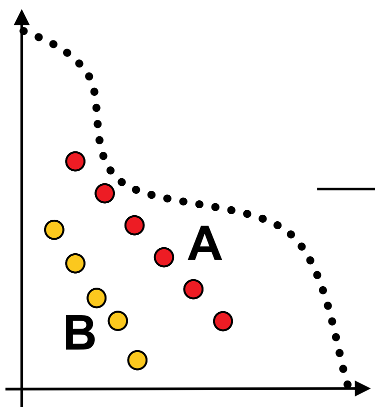


unary epsilon indicator

unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



→

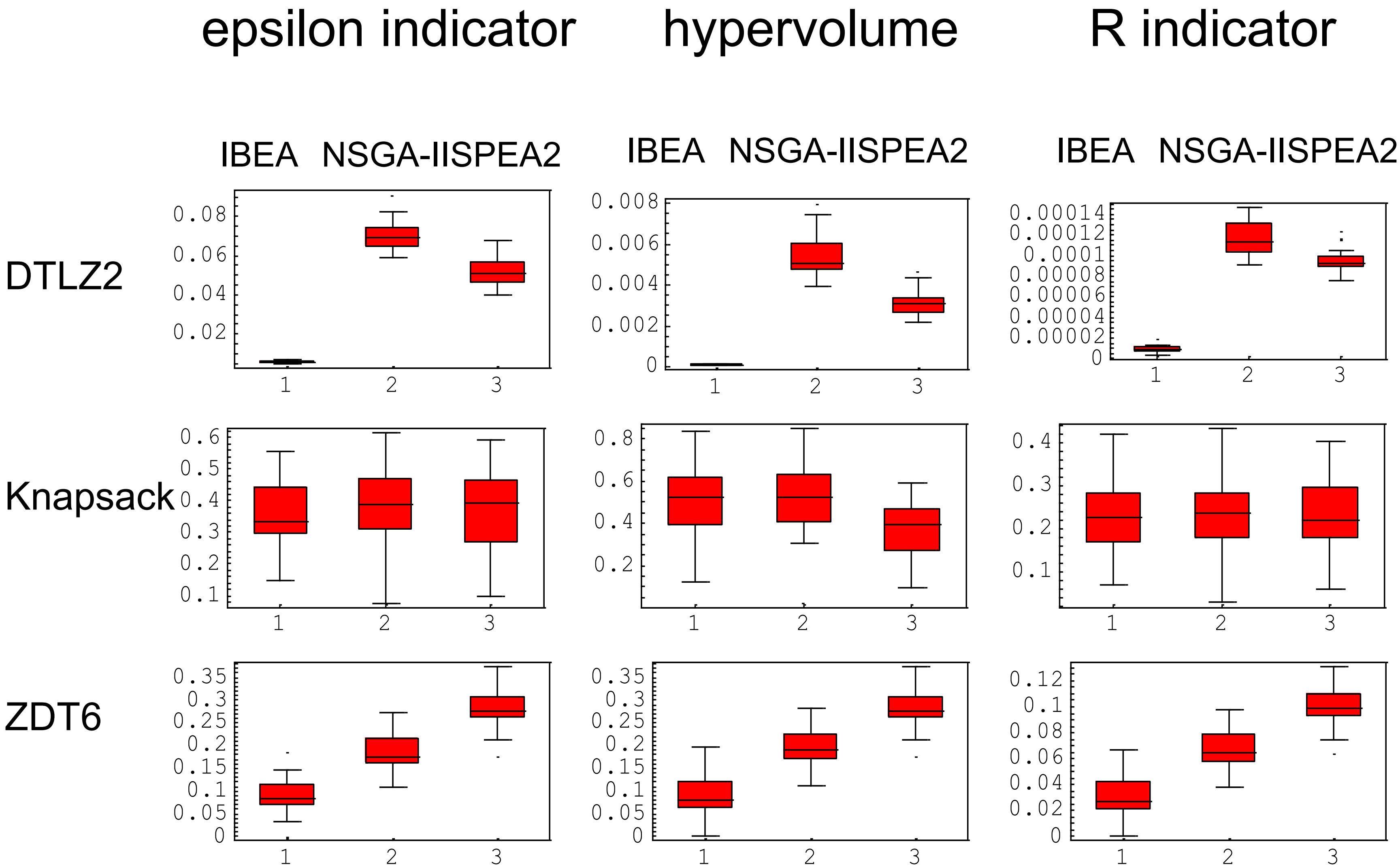
	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow[\textbf{reduction}]{\text{quality measure}} \mathbb{R}^n \xrightarrow[\textbf{interpretation}]{\text{Boolean function}} \text{statement}$$

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than	IBEA	NSGA2	SPEA2
IBEA		~0	~0
NSGA2	1		~0
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than	IBEA	NSGA2	SPEA2
IBEA		~0	~0
NSGA2	1		1
SPEA2	1	~0	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

Automated Benchmarking

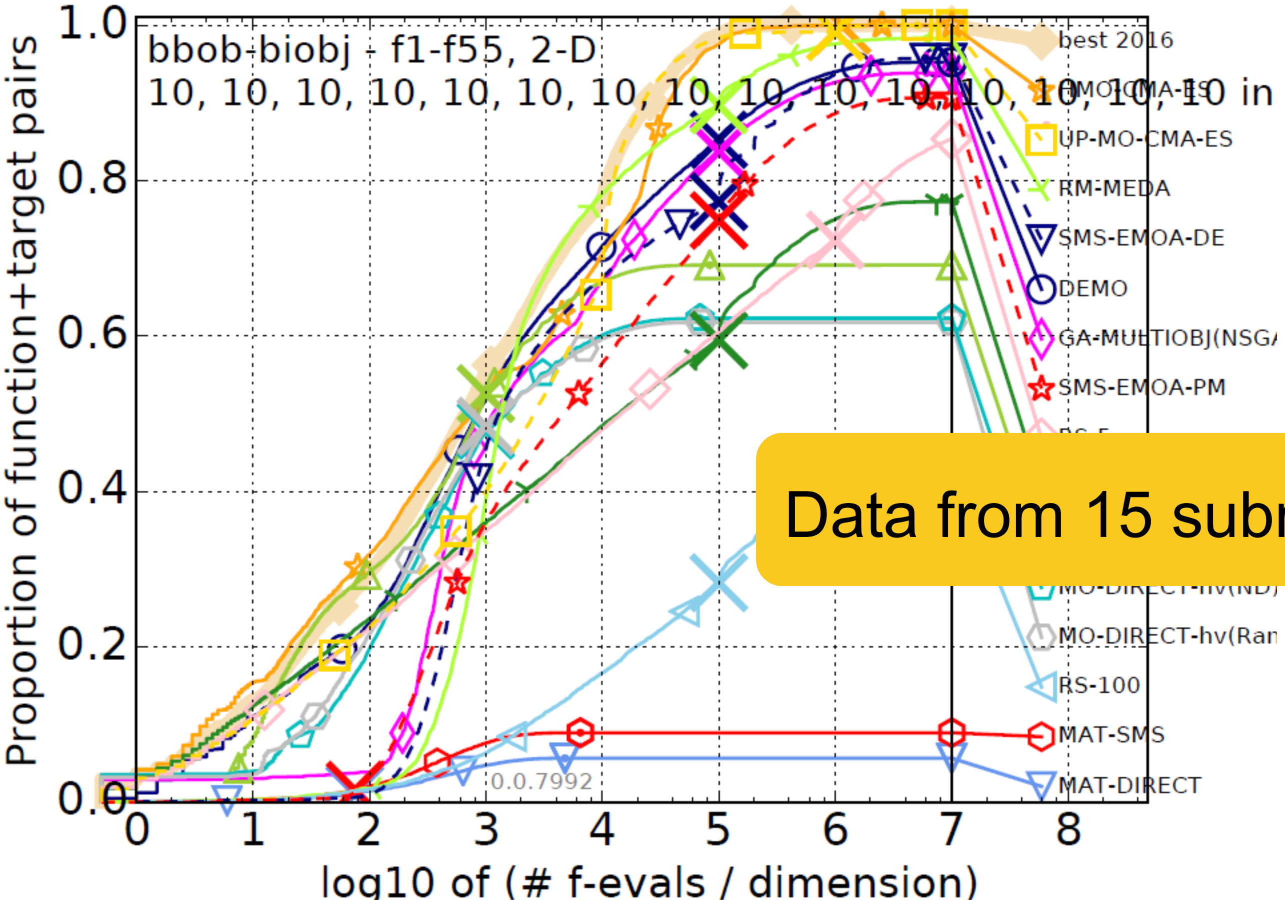
- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform

  `https://github.com/numbbo/coco`

- In 2016: first release of a bi-objective test suite and corresponding BBOB-2016 workshop @ GECCO

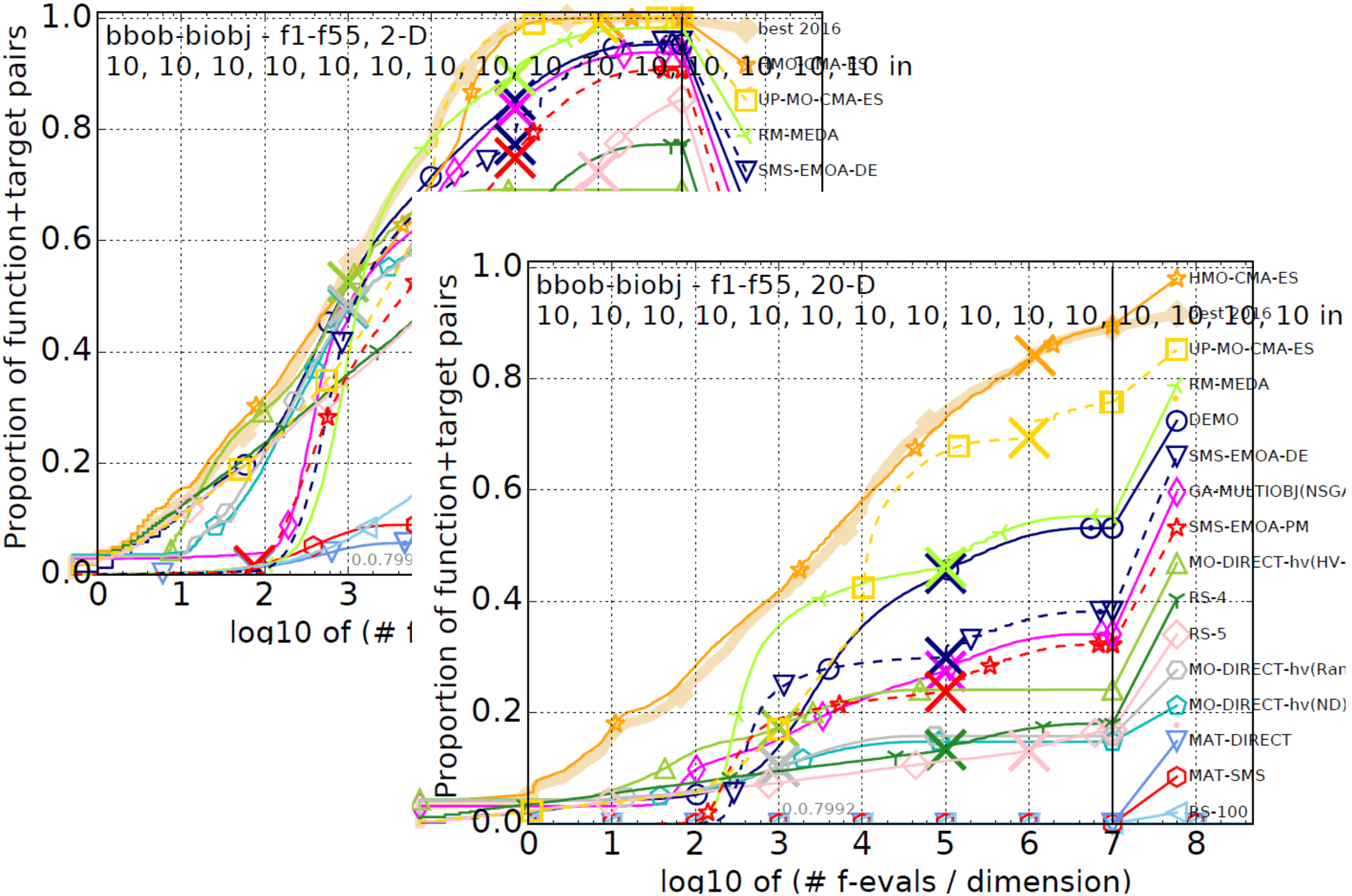
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports average runtimes (aRT) to reach target precision
  - hence: not really a difference to single-objective optimization anymore

Exemplary BBOB-2016 Results



Data from 15 submitted algorithms

Exemplary BBOB-2016 Results



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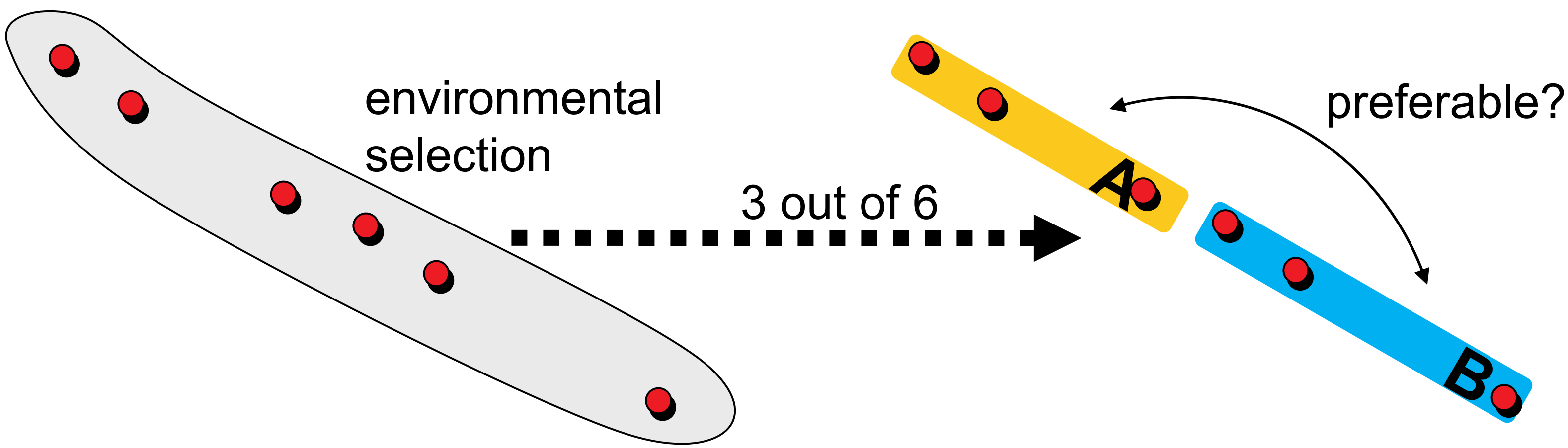
What we thought: EMO is preference-less

> **Search before decision making:** Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

What we learnt: EMO just uses weaker preference information



Incorporation of Preferences During Search

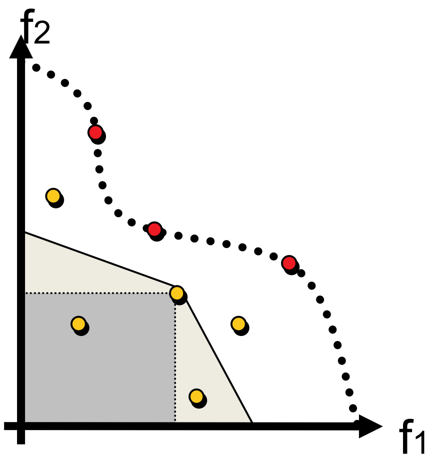
Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]

❶ Refine/modify dominance relation, e.g.:

- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of dominance cones
  [Branke and Deb 2004]

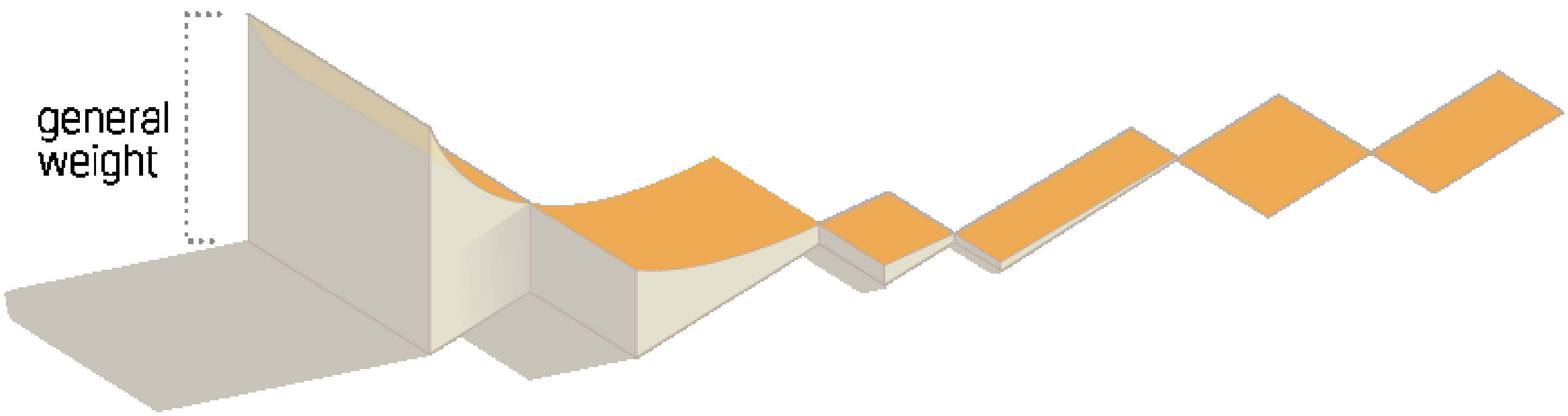


❷ Use quality indicators, e.g.:

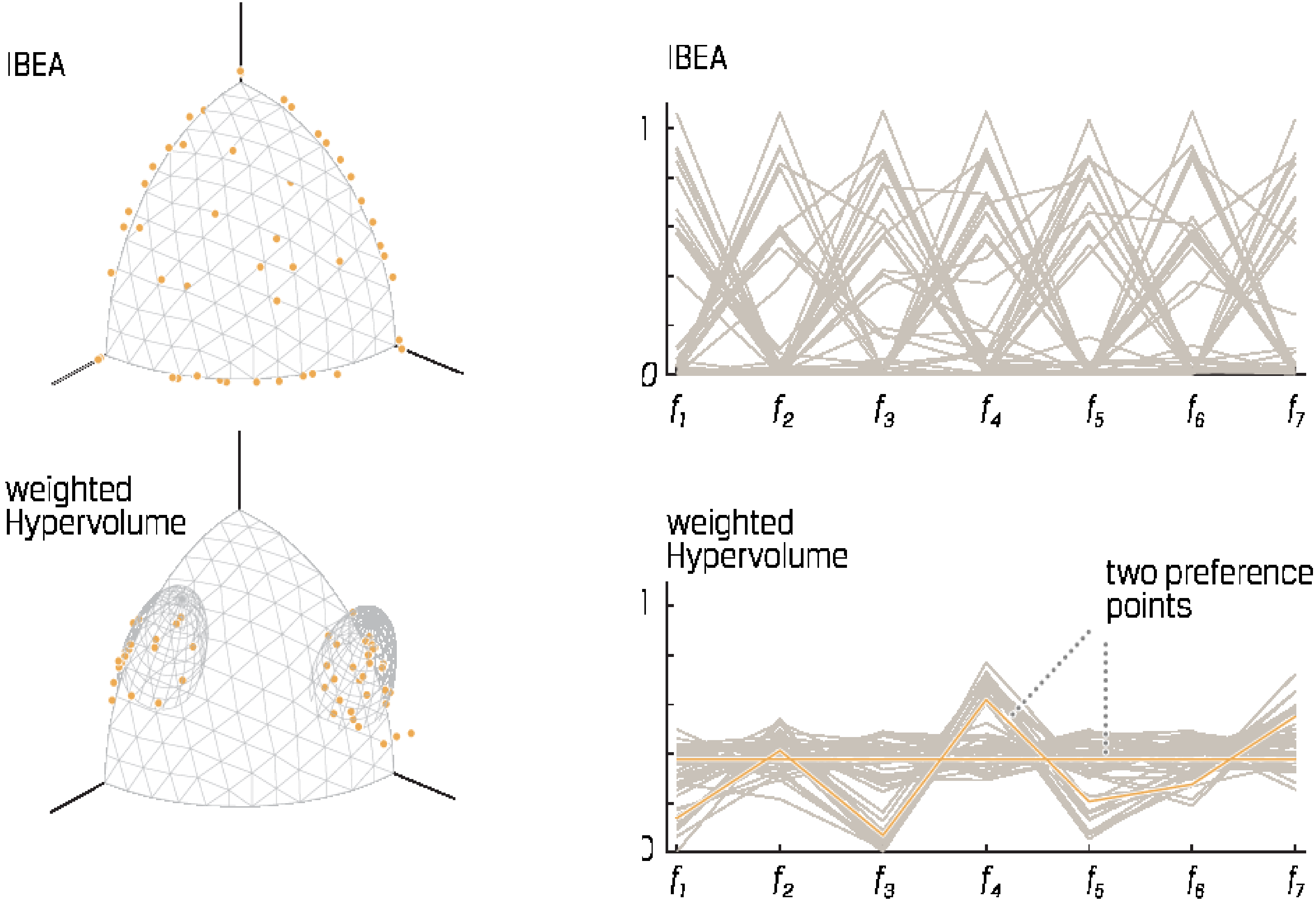
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator
  [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

Example: Weighted Hypervolume Indicator

[Brockhoff et al. 2013]



Weighted Hypervolume in Practice

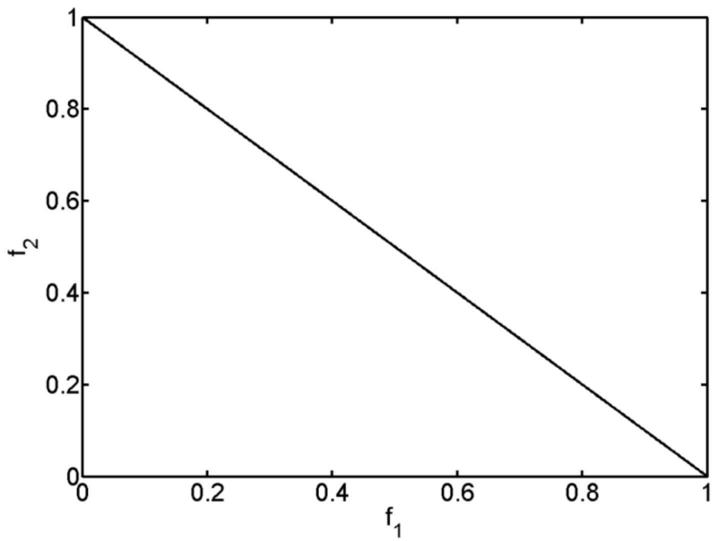


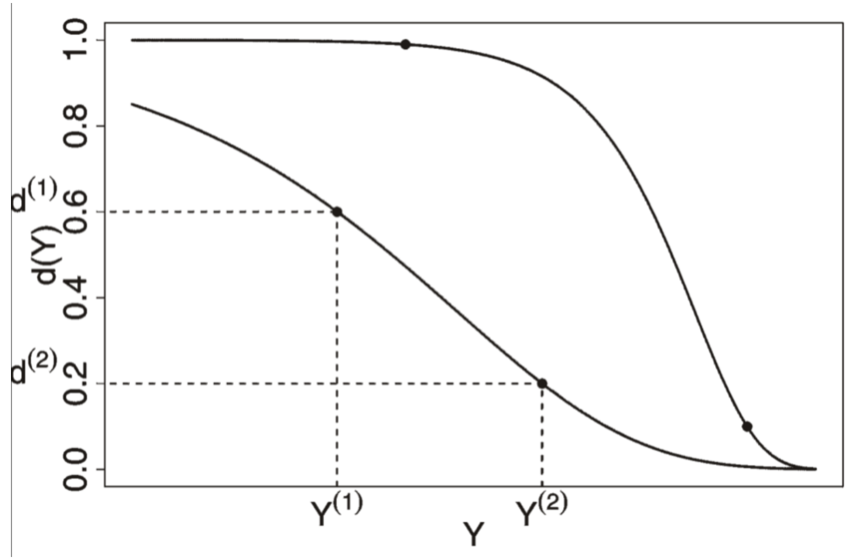
[Auger et al. 2009b]

Example: Desirability Function (DF)-SMS-EMOA

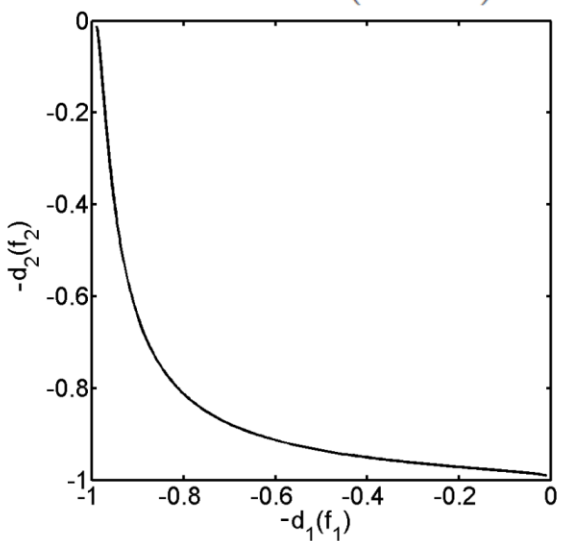
[Wagner and Trautmann 2010]

Shape of the untransformed Pareto front

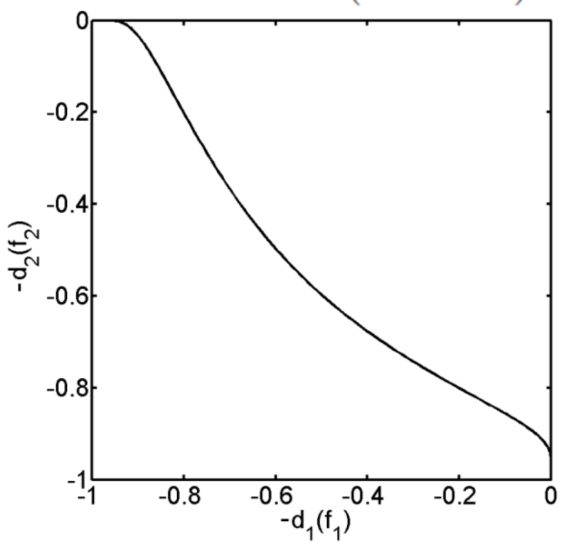




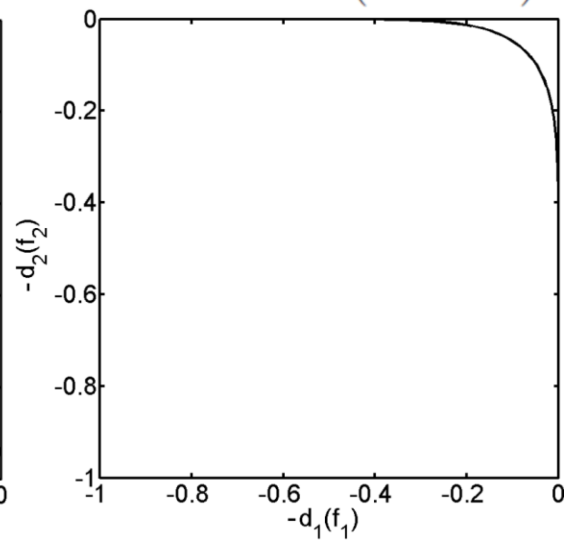
Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 1 & 0.01 \end{pmatrix}$



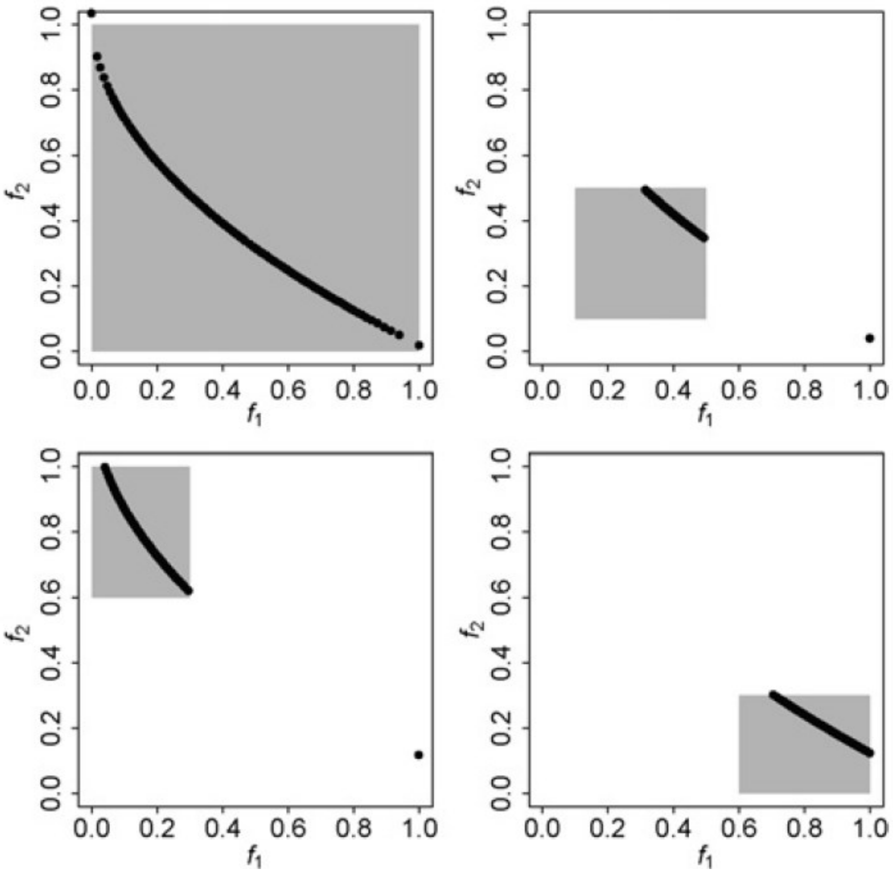
Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.75 & 0.01 \end{pmatrix}$

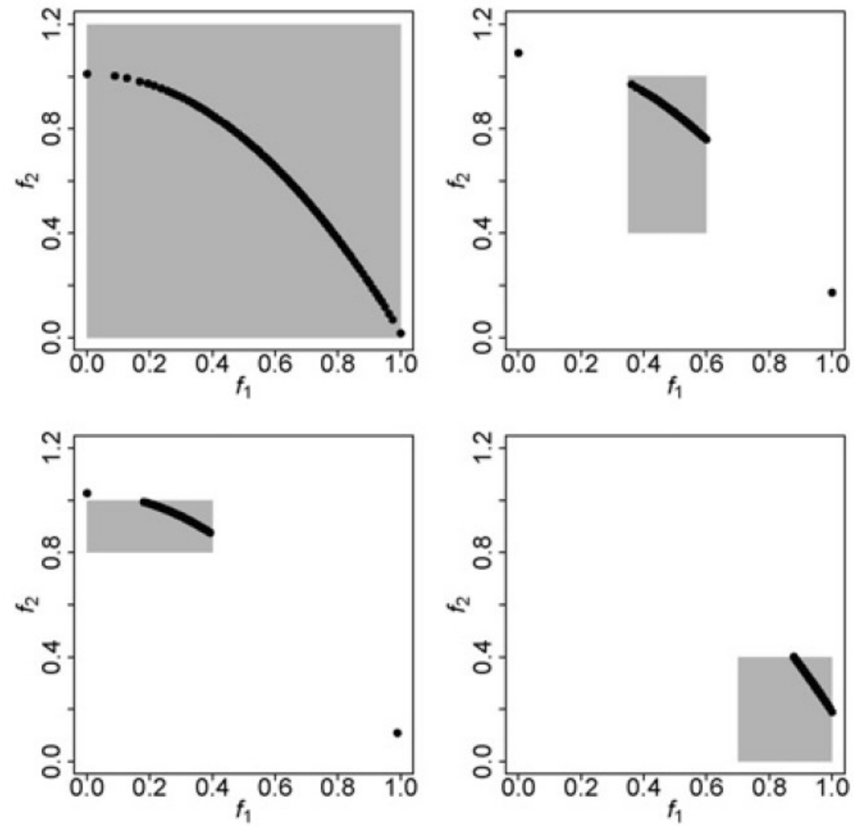


Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.55 & 0.01 \end{pmatrix}$



DF-SMS-EMOA in Practice



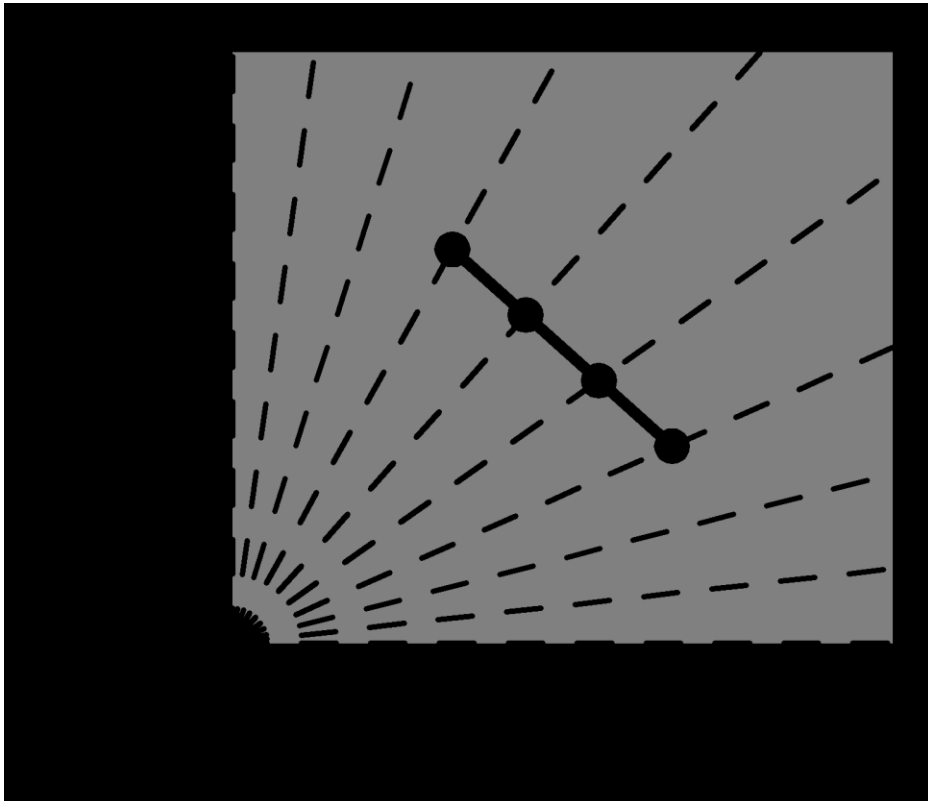


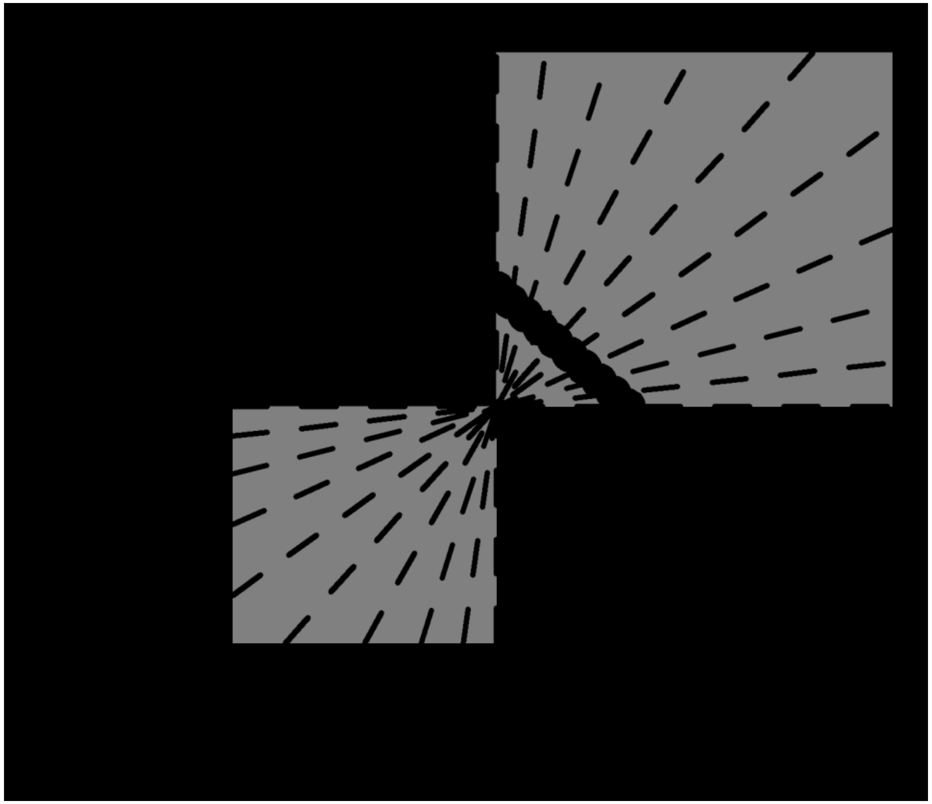
Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Position of ideal point



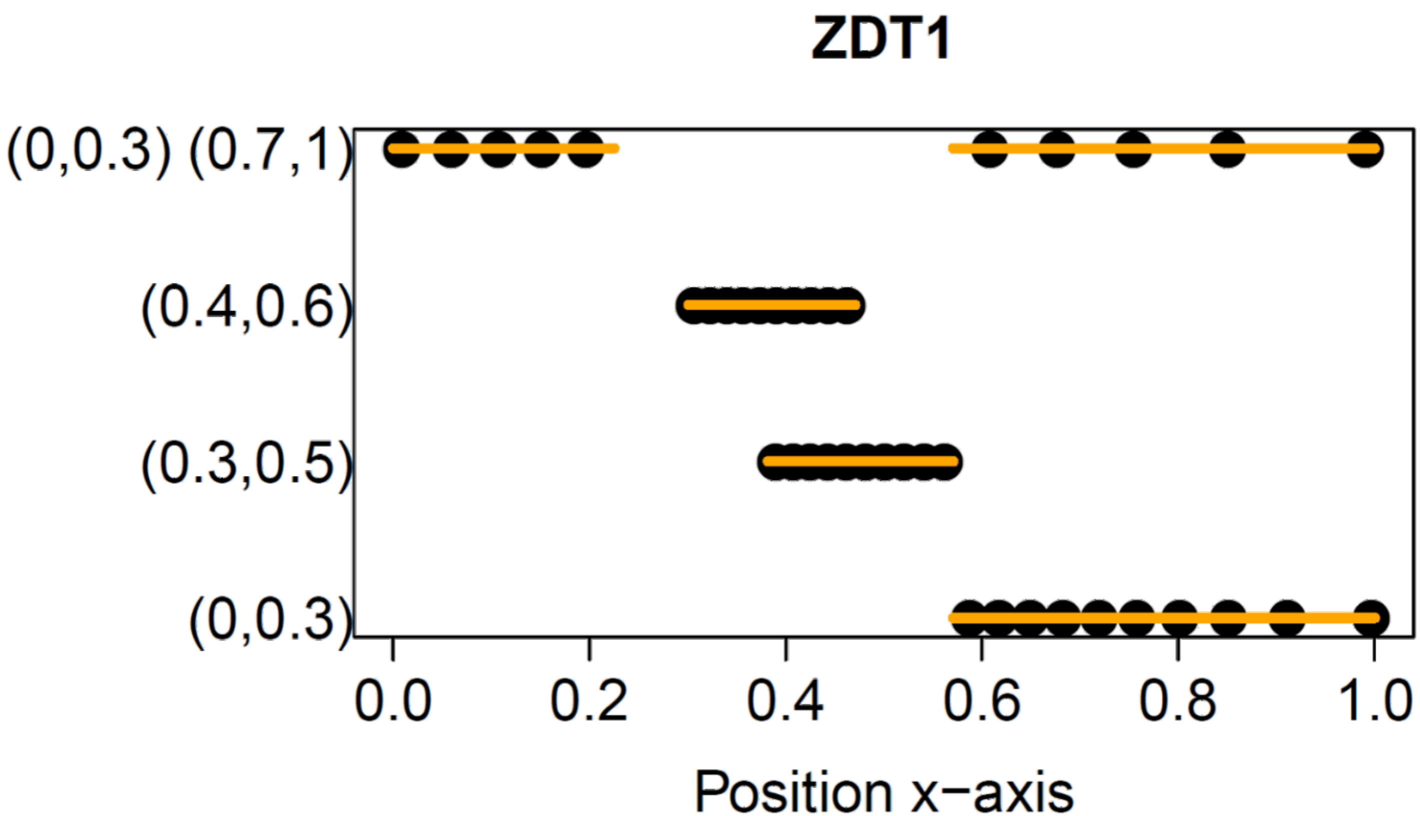


Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Restriction of the weight space



Interactive Approaches

Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

Examples

- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

Issues/Open Questions

- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

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Visualization is Difficult for Many Objectives

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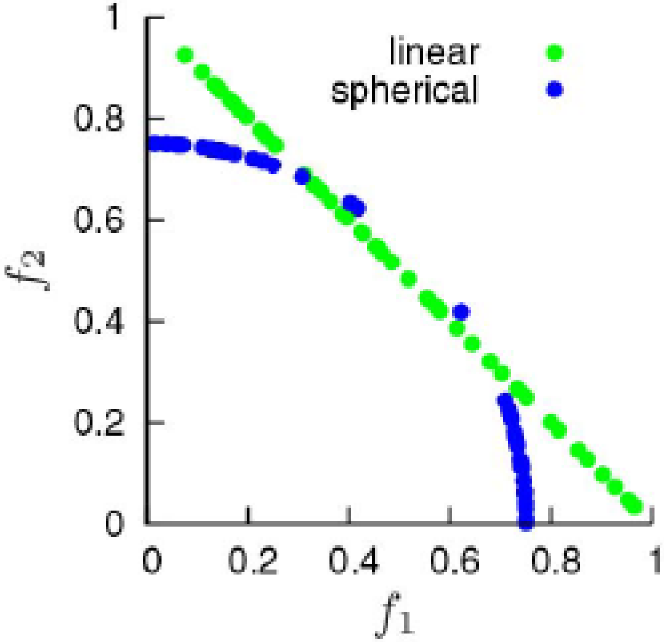
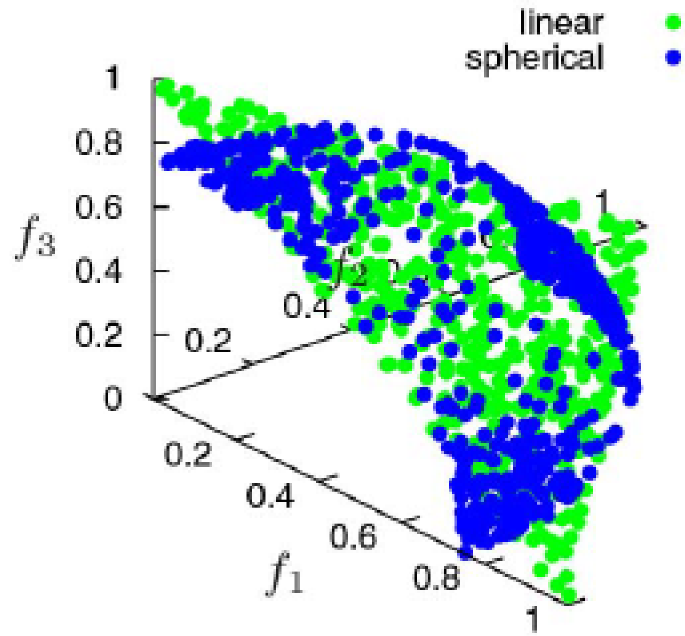


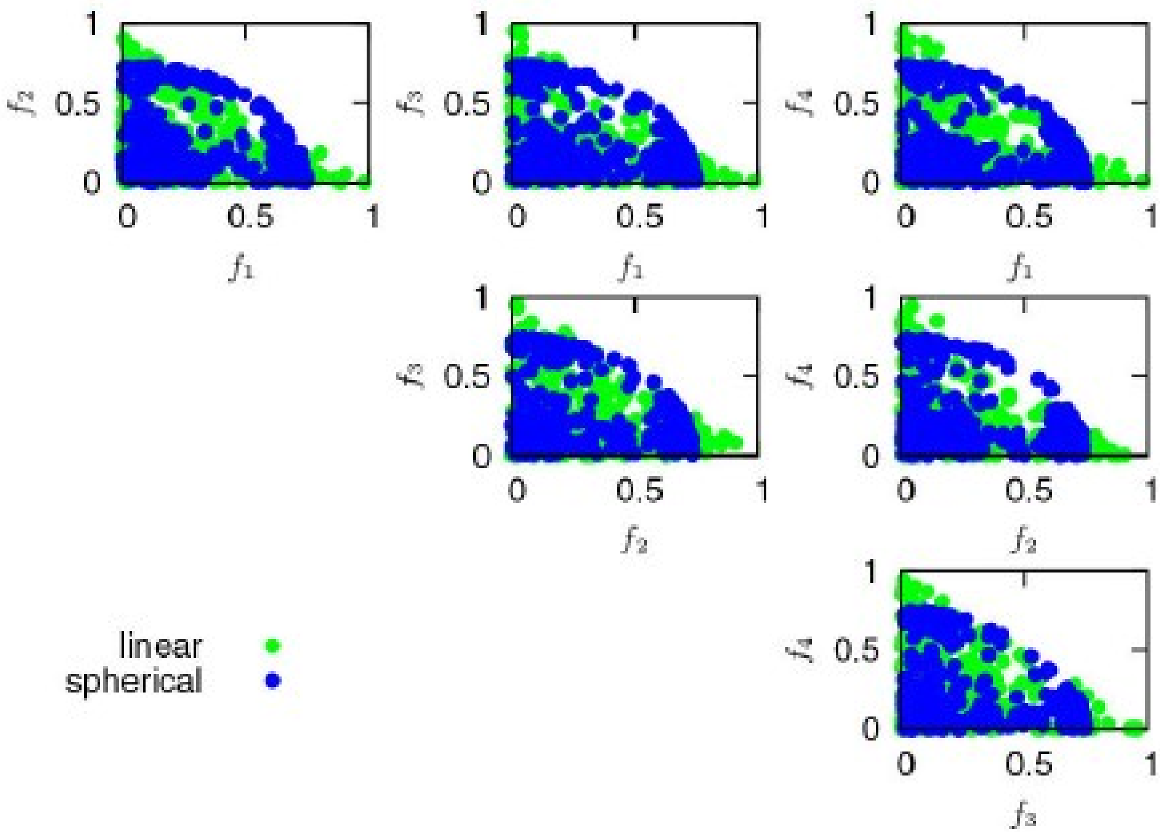
2 objective functions



3 objective functions

>3 objective functions?

Scatter Plots for all Objective Combinations

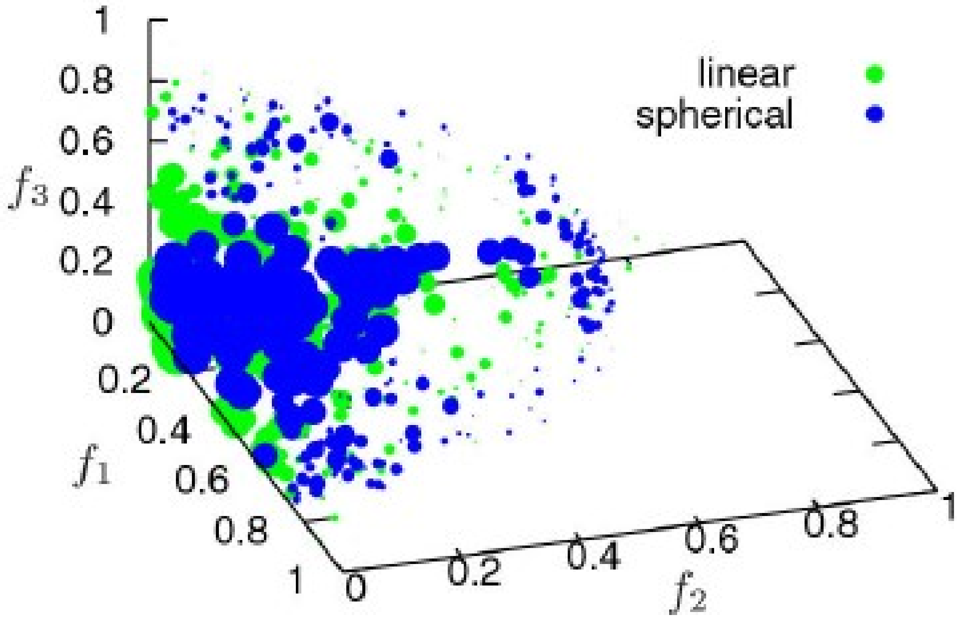


These and the following plots are taken from

Tea Tušar and Bogdan Filipic: "Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method". *IEEE Transactions in Evolutionary Computation*, 19(2):225-245, 2015.

Bubble chart:

size of bubble = forth objective



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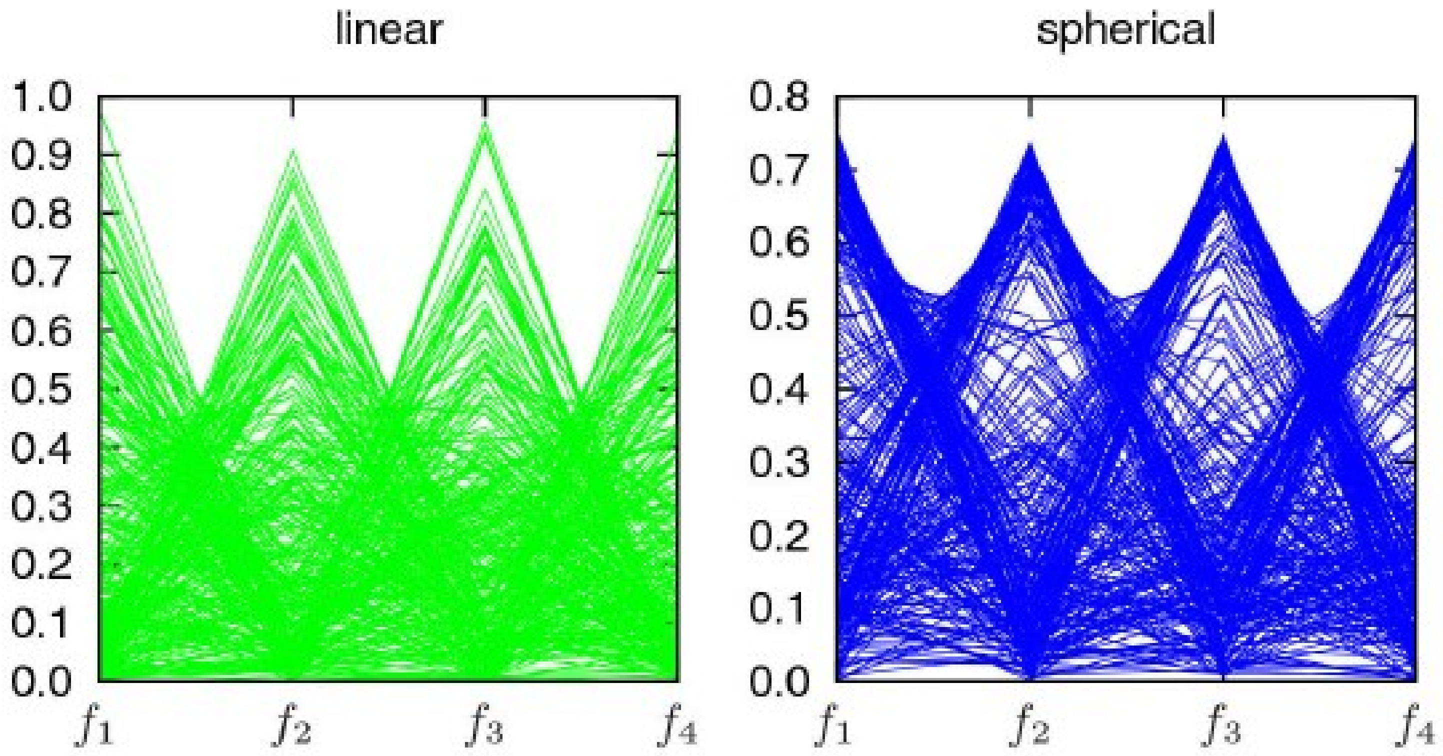
Parallel Coordinates



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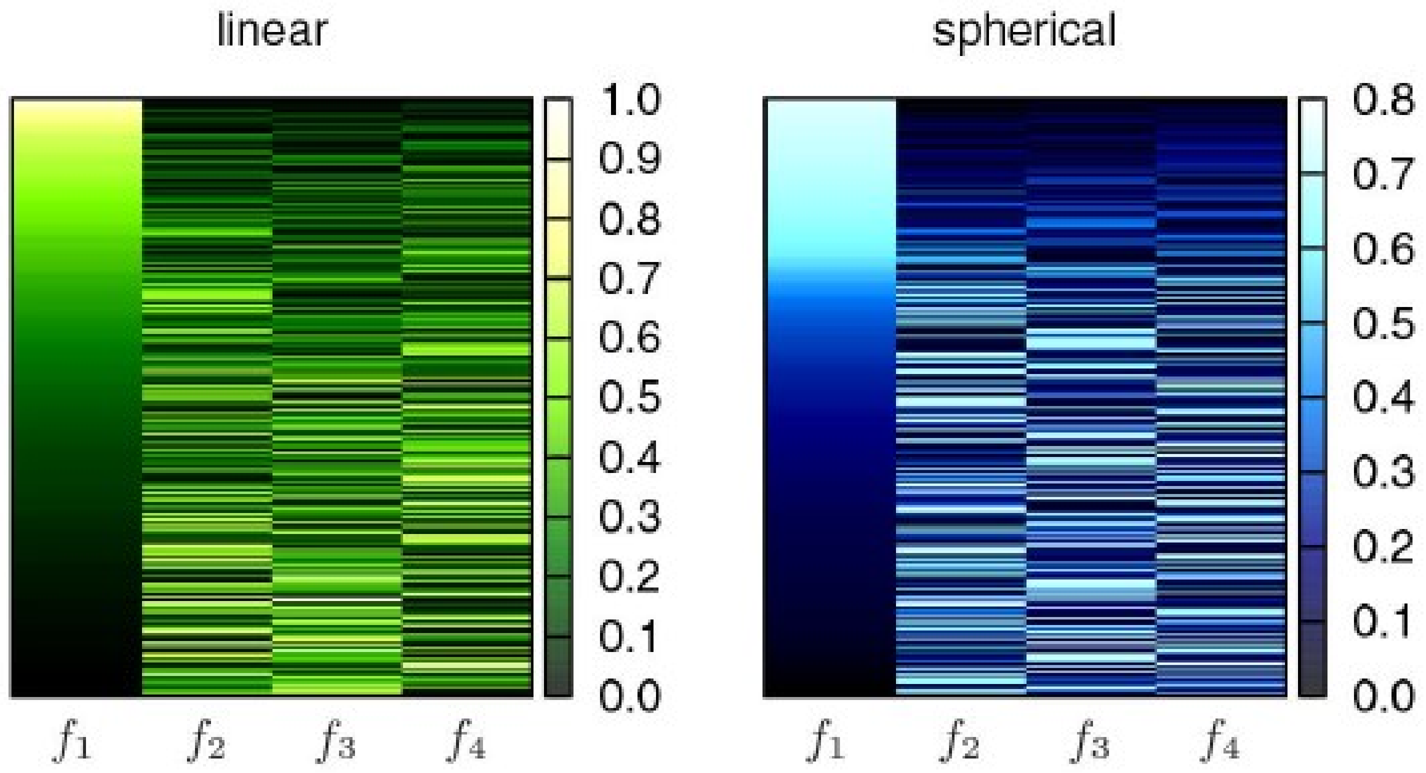
and many more...



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The EMO Community

Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

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