Proposition for a Master's thesis

Linear Convergence of some Derivative-Free Randomized Optimization Algorithms

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Context and objective of the Master's thesis:

Important problems of the 21st century in domains like medicine or energy critically rely on the resolution of challenging numerical optimization problems. They commonly stem from complex simulations or use noisy experimental data such that derivatives are not available or not useful. Recently, powerful methods have emerged to solve those problems. The algorithms are adaptive, randomized (or stochastic) and derivative-free. One subclass of those algorithms are so-called evolution strategies (ES). Among them the CMA-ES is recognized as the state-of-the-art method with successful applications in many domains [1].

Yet, the development of such methods has been mainly driven by heuristics and practice rather than a general theoretical framework. Undoubtedly, this has been an asset as the scope of possibilities for design was not restricted by current mathematical frameworks for proving convergence.

To date, however, convergence proofs of even simple ES variants are still lacking. One approach that has been explored to prove the linear convergence rely on investigating the stability of Markov chains underlying the algorithm [2]. Yet, this approach restricts the class of functions where convergence can be proven to scaling-invariant functions.

In this Master's thesis, we want to explore an alternative approach relying on so-called drift analysis from which we can deduce bounds on the hitting time to reach an ϵ-ball around the optimum [3, 4]. This approach has been mostly used so far to analyze algorithms optimizing functions defined on a finite search space.

We will first investigate the so-called hit-and-run algorithm that samples randomly a search direction and assumes optimal step-size along this direction. Lower-bounds on the hitting time to reach an ϵ-ball around the optimum have been investigated in [5]. We are here interested to derive lower *and* upper bounds on strongly convex functions first and see how the strongly convex assumption can be relaxed. In a second part, we will investigate the $(1 + 1)$-ES with one-fifth success rule [6].

The questions investigated in this Master's thesis are fully open and in case of success will lead to a scientific publication. Additionally, this Master's thesis can be followed by a PhD thesis.

Practical aspects:

The internship will take place in the applied math laboratory of Ecole Polytechnique (CMAP) within the Inria RandOpt team. The intern will receive a "gratification" (ca. 650 euros per month) and will benefit of all the facilities of the Ecole Polytechnique campus.

References:

[1] N. Hansen and A. Ostermeier. Completely Derandomized Self-Adaptation in Evolution Strategies. *Evolutionary Computation*, 9(2):159–195, 2001.

[2] Anne Auger and Nikolaus Hansen. Linear Convergence of Comparison-based Step-size Adaptive Randomized Search via Stability of Markov Chains. *SIAM Journal on Optimization*, 26(3):1589–1624, 2016.

[3] Per Kristian Lehre, Carsten Witt. General Drift Analysis with Tail Bounds. https://arxiv.org/abs/1307.2559

[4] Johannes Lengler. Drift Analysis. https://arxiv.org/abs/1712.00964

[5] Jens Jägersküpper Lower bounds for hit-and-run direct search. Proceedings of the 4th Int'l Symposium on Stochastic Algorithms: Foundations and Applications (SAGA 2007).

[6] A. Auger and N. Hansen. Linear convergence on positively homogeneous functions of a comparison based step-size adaptive randomized search: the (1+1) ES with generalized one-fifth success rule, 2013. ArXiv eprint.