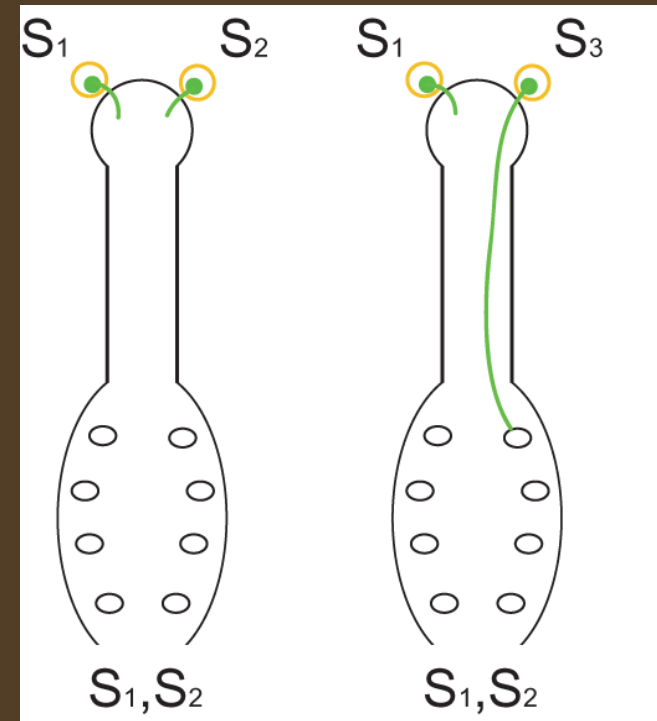
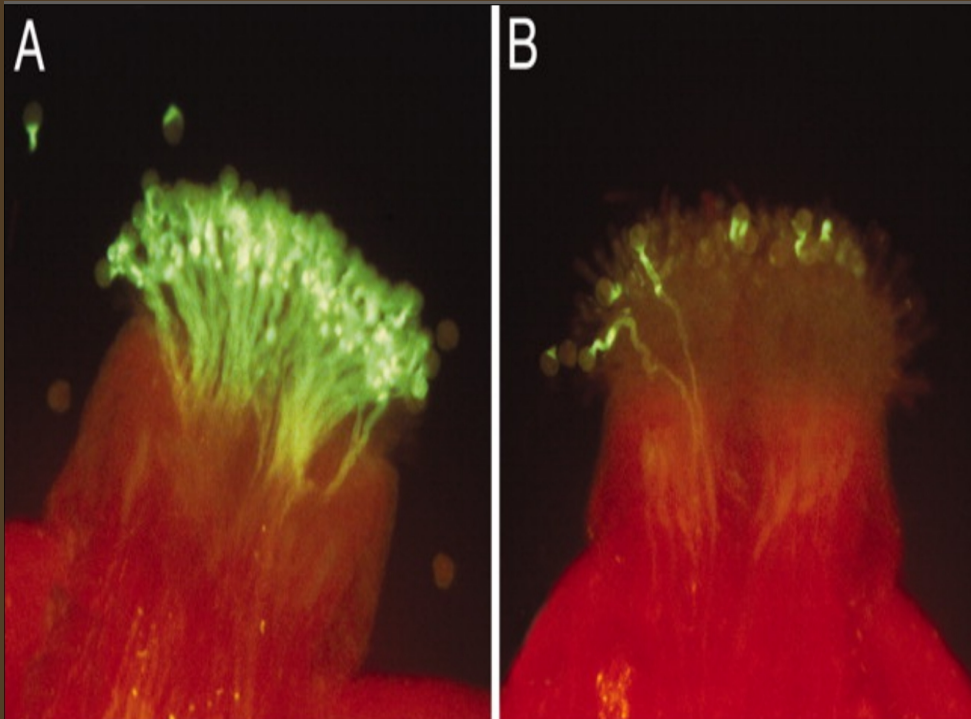


# Origins and dynamics of new Self-Incompatibility alleles in plants

*An example of diversification with coevolution*

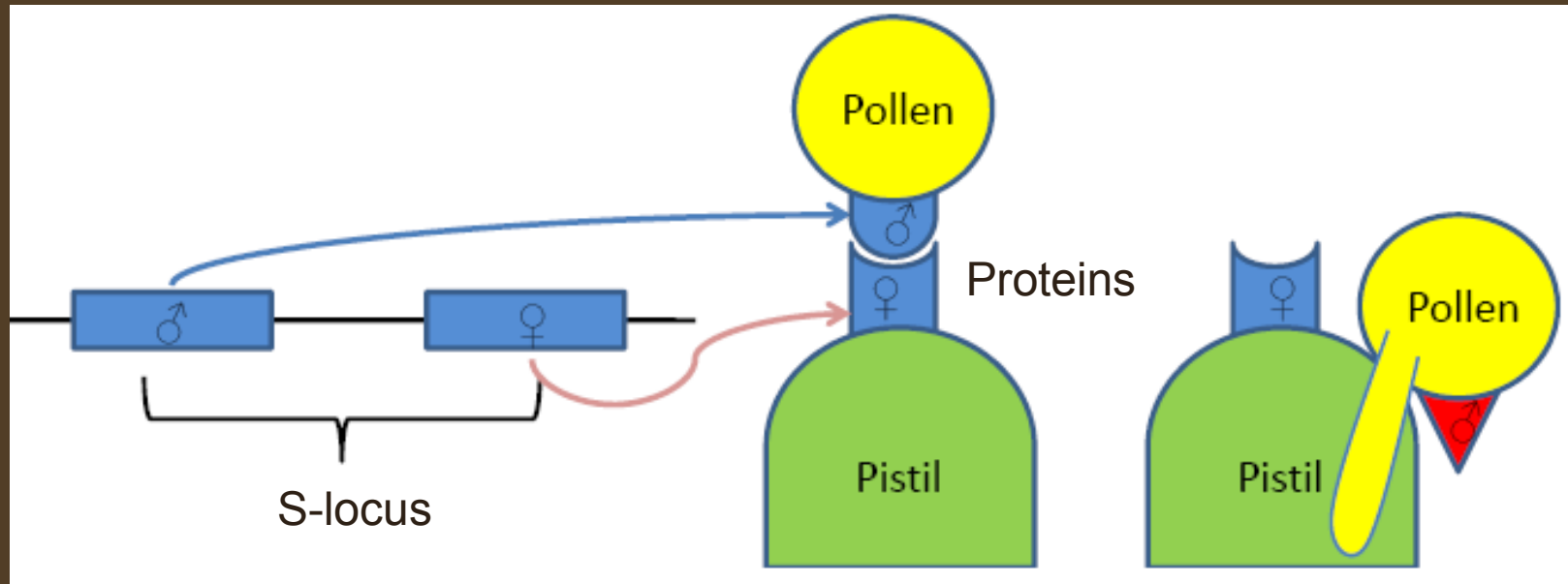
Camille Gervais, Vincent Castric, Adrienne Ressayre, Sylvain Billiard

Université Lille 1 / INRA Moulon - France

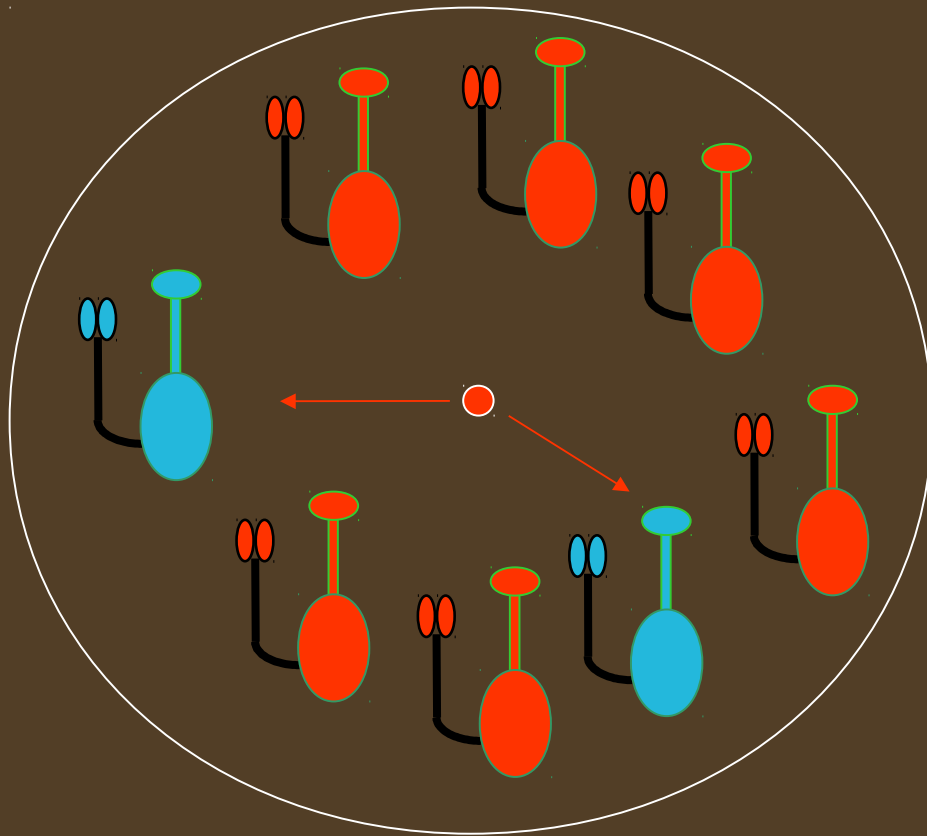


- Present in dozens Angiosperms families  
(e.g. Solanaceae, Brassicaceae, ...)

# Self-Incompatibility in plants



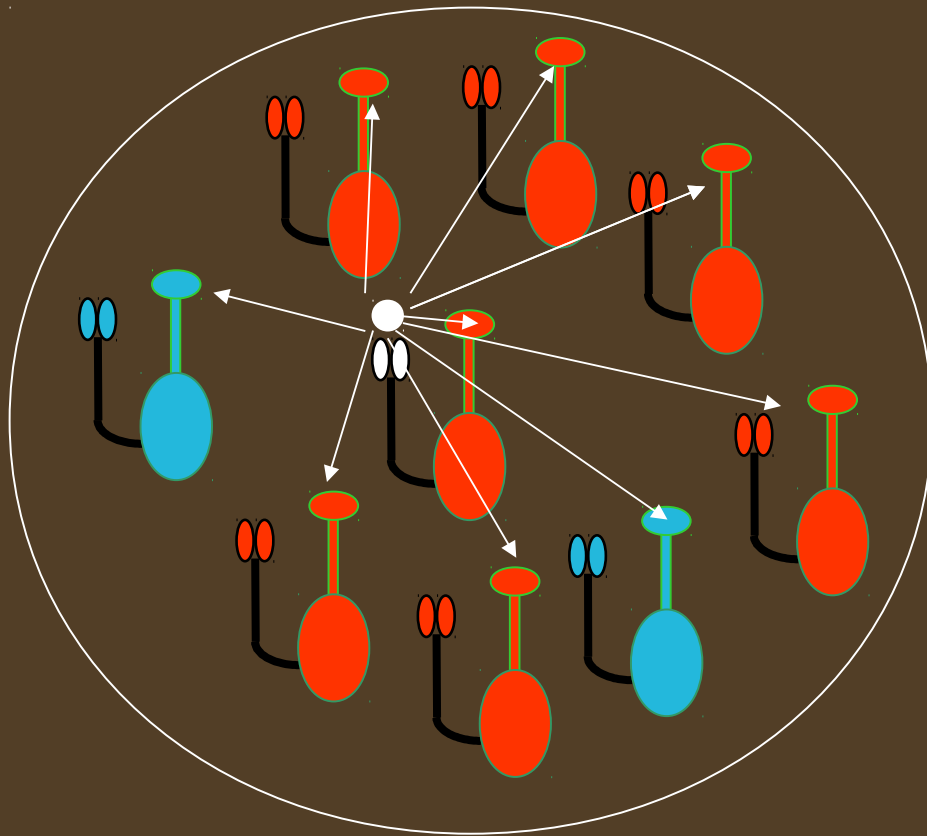
- Molecular recognition that prevents selfing and some cross-fertilization
- Up to 200 SI haplotypes thanks to the advantage of the rare (Wright 1939)  
(or negative frequency dependent-selection)



Red phenotype is common

→ - potential mates

→ unfavored



White is self-compatible and compatible with anyone

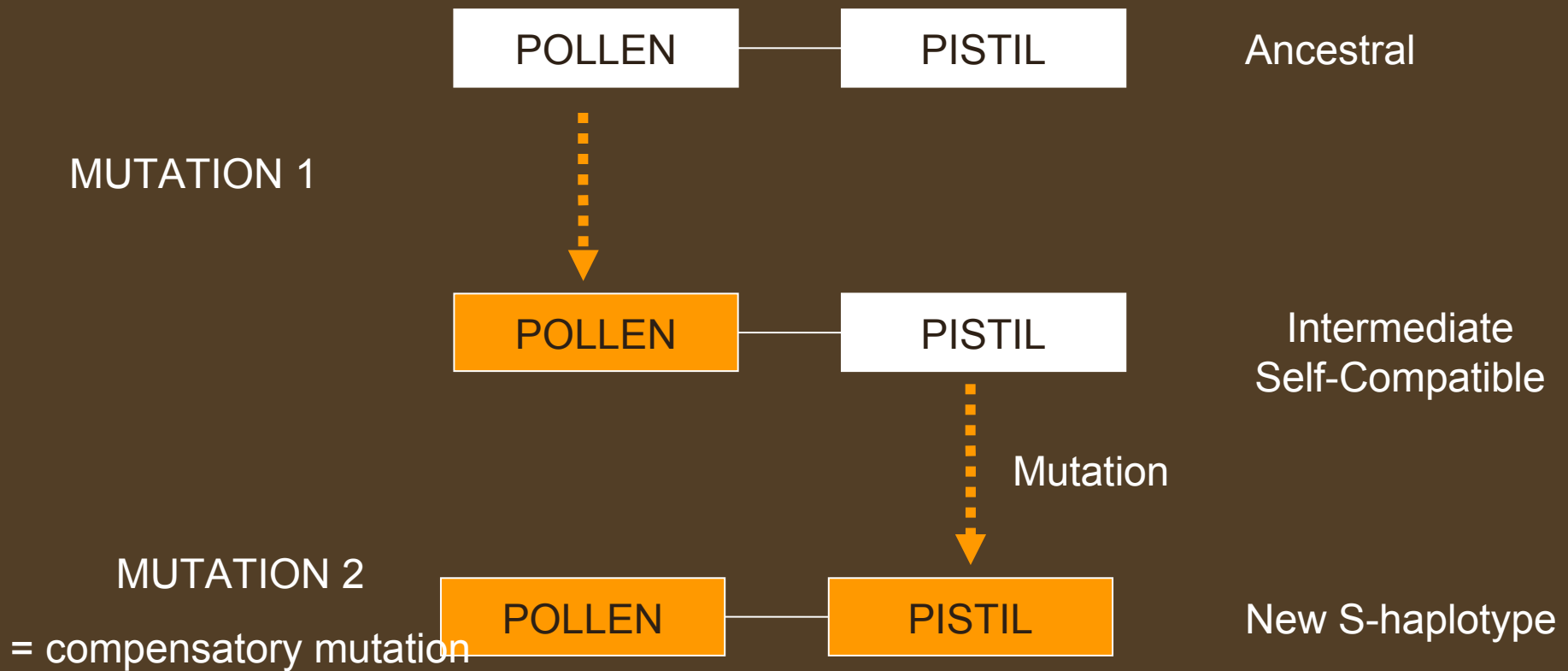
# The problem of S diversification

- Easy to understand how such a large diversity is maintained (Wright 1939)
- Hard to understand how it can be generated (sir R.A. Fisher 1961)

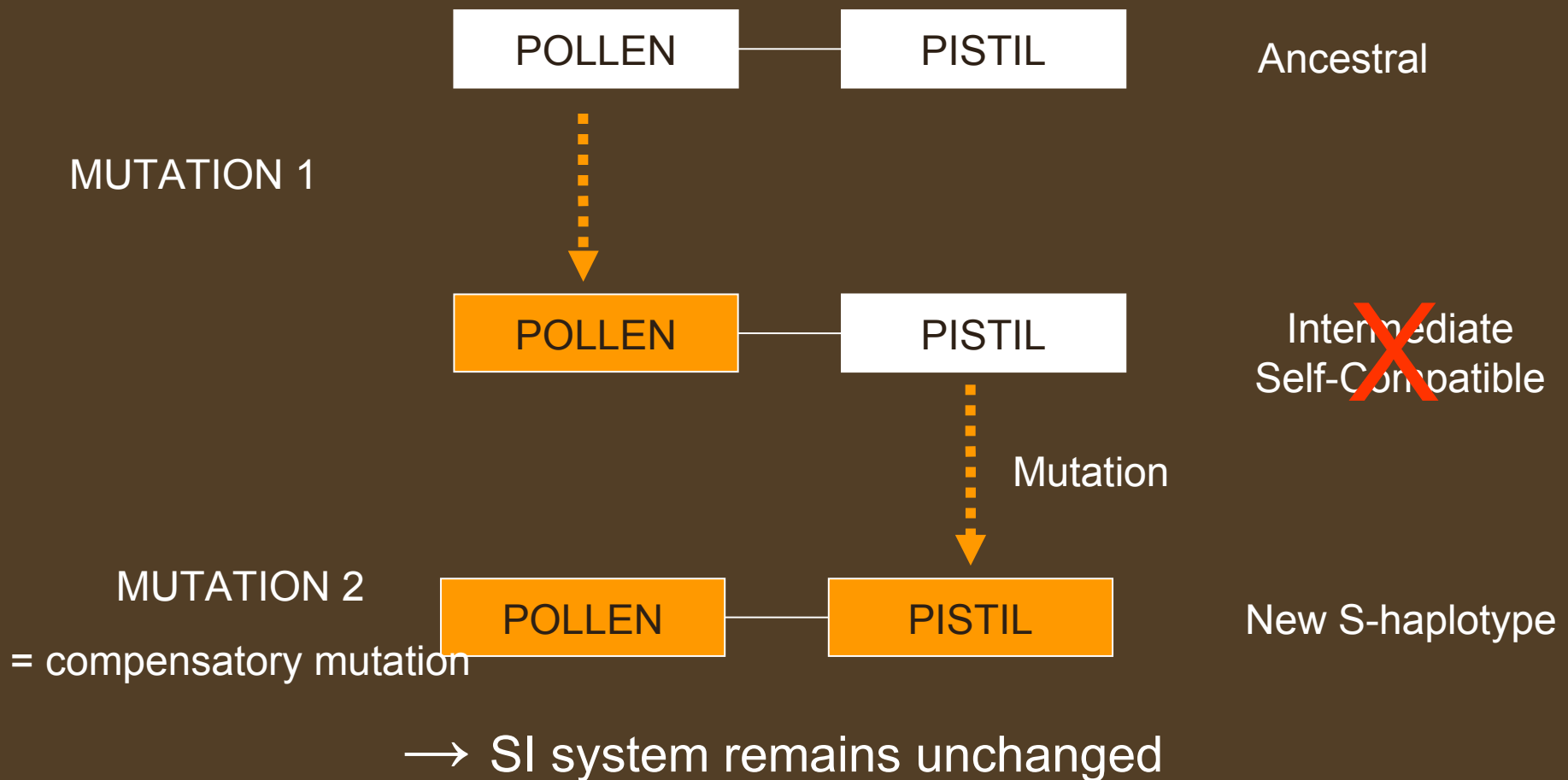


Self-Compatible haplotype can invade → SI Loss → Coevolution is necessary

# Uyenoyama et al. (2001)'s model

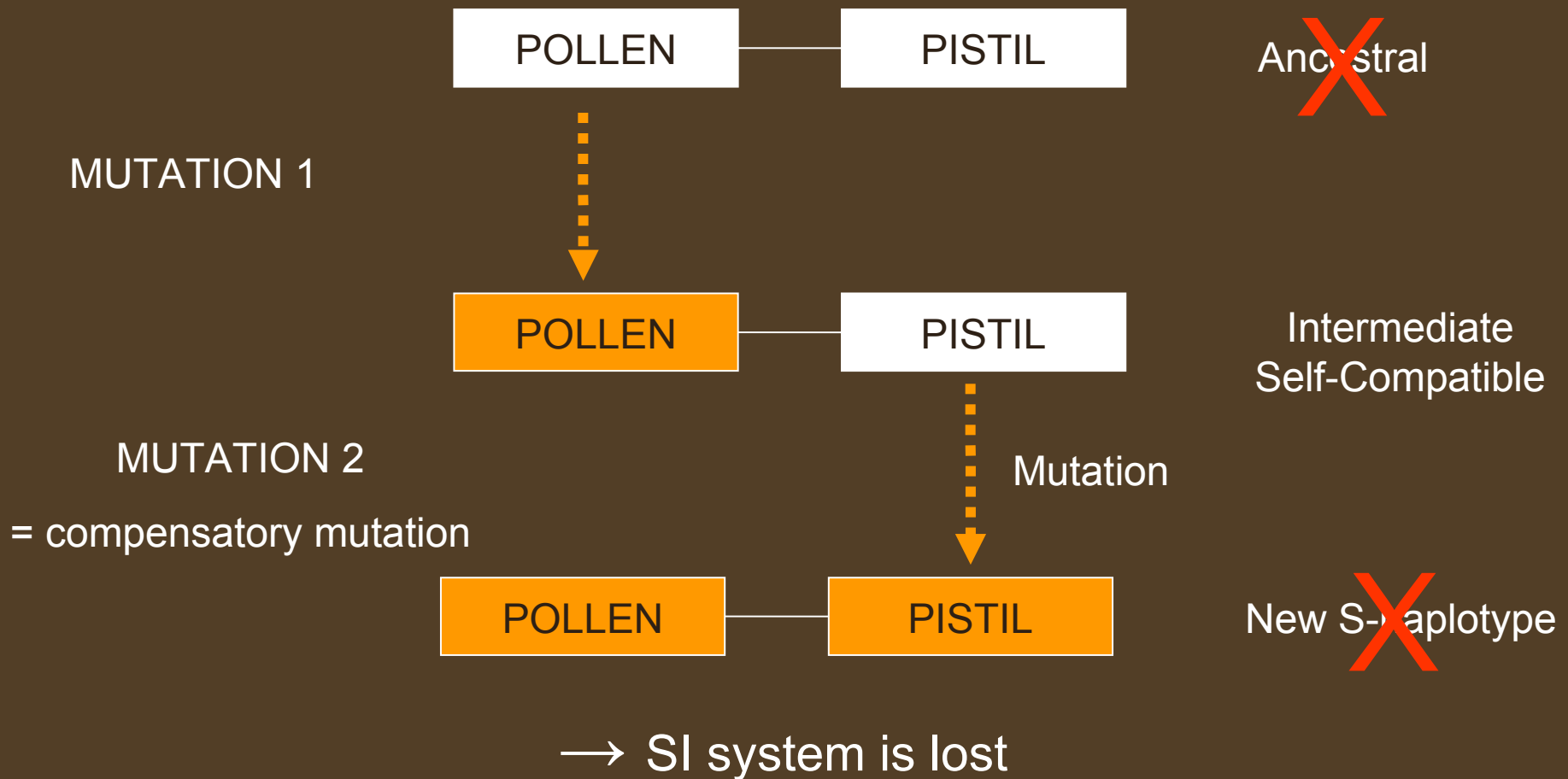


# Uyenoyama et al. (2001)'s model





# Uyenoyama et al. (2001)'s model



# The model (*based on Uyenoyama et al.2001*)

## ASSUMPTIONS

-Unstructured population with  $S$  diploid individuals

- Bipartite  $S$ -locus

POLLEN

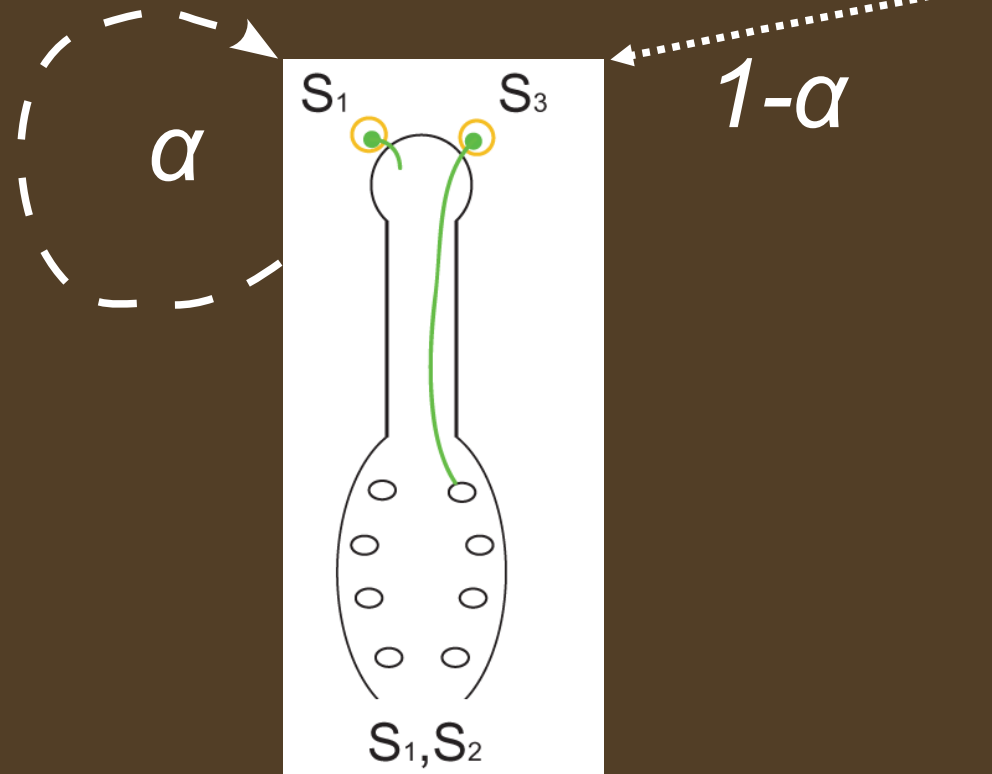
PISTIL

- Inbreeding depression  $\delta$  = decreased viability of selfed offspring

- Self-pollination rate  $\alpha$

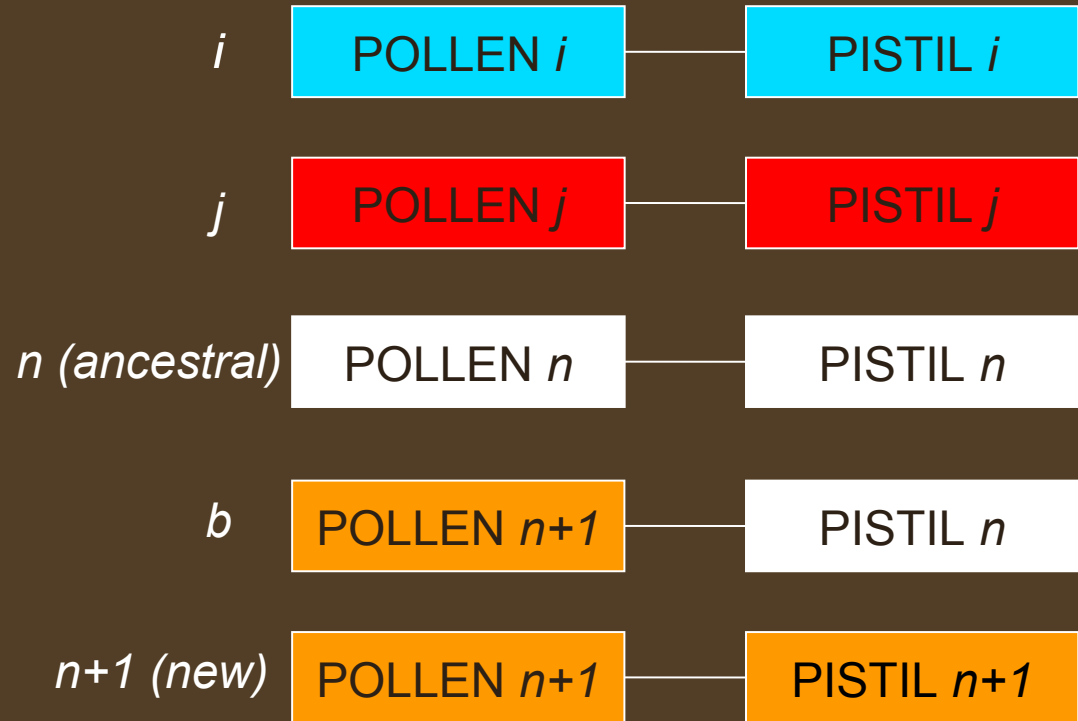
-Initial number of  $S$ -haplotypes  $n$

-Max number of different alleles  $K$



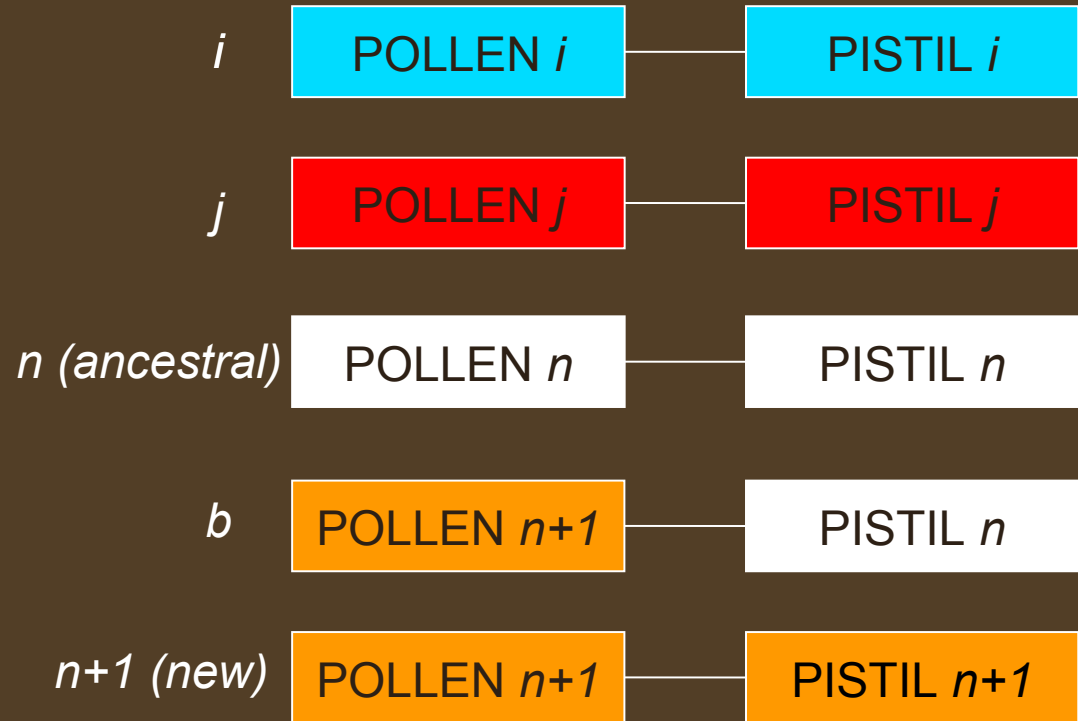
# Description

$$X^t = \begin{pmatrix} x_{ij}^t \\ x_{bb}^t \\ x_{bi}^t \\ x_{ni}^t \\ x_{bn+1}^t \\ x_{nn+1}^t \\ x_{in+1}^t \\ \dots \end{pmatrix}$$



# What may happen...

$$X^t = \begin{pmatrix} x_{ij}^t \\ x_{bb}^t \\ x_{bi}^t \\ x_{ni}^t \\ x_{bn+1}^t \\ x_{nn+1}^t \\ x_{in+1}^t \\ \dots \end{pmatrix}$$



$$X^{t+1} \sim \frac{1}{S} M (X^t + R(X^t, \delta, \alpha) + U(X^t, u), S)$$

Reproduction  
+ Viability selection

Recurrent  
Mutation

$$X^{t+1} \sim \frac{1}{S} M (X^t + R(X^t, \delta, \alpha) + U(X^t, u), S)$$

$$u = 0$$

$$S \rightarrow \infty$$

$$X^t = \begin{pmatrix} x_{ij}^t \\ x_{bb}^t \\ x_{bi}^t \\ x_{ni}^t \\ x_{bn+1}^t \\ x_{nn+1}^t \\ x_{in+1}^t \end{pmatrix}$$

$R(X^t, \delta, \alpha) =$

$$\begin{aligned}
 & \frac{1}{W} \left( x_{ij} \frac{(n-3)p}{N_{ij}} + x_{bi}(1-s_{bi}) \frac{(n-2)p}{2N_{bi}} + x_{ni} \frac{(n-2)p}{2N_{ni}} + x_{in+1} \frac{(n-2)p}{2N_{in+1}} \right) \\
 & \frac{1}{W} \left( x_{bb} \left( s_{bb}(1-\delta) + (1-s_{bb}) \frac{p_b}{N_{bb}} \right) + \frac{x_{bn}}{2} \left( s_{bn}(1-\delta) + (1-s_{bn}) \frac{p_b}{N_{bn}} \right) + \right. \\
 & \left. \frac{x_{bi}}{2} \left( s_{bi}(1-\delta) + (1-s_{bi}) \frac{p_b}{N_{bi}} \right) \right) x'_{bn} = \frac{1}{W} \left( \frac{x_{bn}}{2} \left( s_{bn}(1-\delta) + (1-s_{bn}) \frac{p_b}{N_{bn}} \right) + x_{ni} \frac{p_b}{2N_{ni}} \right) \\
 & \frac{1}{W} \left( x_{ij} \frac{p_b}{N_{ij}} + x_{bb}(1-s_{bb}) \frac{(n-1)p}{N_{bb}} + x_{bn}(1-s_{bn}) \frac{(n-1)p}{2N_{bn}} \right. \\
 & \quad \left. + \frac{x_{bi}}{2} \left( s_{bi}(1-\delta) + (1-s_{bi}) \frac{(p_b + (n-2)p)}{N_{bi}} \right) + x_{ni} \frac{p_b}{2N_{ni}} + x_{bn+1} \frac{(n-1)p}{2N_{bn+1}} \right) \\
 & \frac{1}{W} \left( x_{ij} \frac{p_n}{N_{ij}} + x_{bn}(1-s_{bn}) \frac{(n-1)p}{2N_{bn}} + x_{ni} \frac{(n-2)p}{2N_{ni}} + x_{nn+1} \frac{(n-1)p}{2N_{nn+1}} + x_{in+1} \frac{p_n}{2N_{in+1}} \right) \\
 & \frac{1}{W} \left( x_{bb}(1-s_{bb}) \frac{p_{n+1}}{N_{bb}} + x_{bn}(1-s_{bn}) \frac{p_{n+1}}{2N_{bn}} + x_{bi}(1-s_{bi}) \frac{p_{n+1}}{2N_{bi}} \right) \\
 & \frac{1}{W} \left( x_{bn}(1-s_{bn}) \frac{p_{n+1}}{2N_{bn}} + x_{ni} \frac{p_{n+1}}{2N_{ni}} + x_{in+1} \frac{p_n}{2N_{in+1}} \right) \\
 & \frac{1}{W} \left( x_{ij} \frac{p_{n+1}}{N_{ij}} + x_{bi}(1-s_{bi}) \frac{p_{n+1}}{2N_{bi}} + x_{ni} \frac{p_{n+1}}{2N_{ni}} + x_{bn+1} \frac{(n-1)p}{2N_{bn+1}} + x_{nn+1} \frac{(n-1)p}{2N_{nn+1}} \right. \\
 & \quad \left. + x_{in+1} \frac{(n-2)p}{2N_{in+1}} \right),
 \end{aligned}$$

$$x'_{bi} = \frac{1}{\bar{W}} \left( x_{ij} \frac{p_b}{N_{ij}} + x_{bb}(1 - s_{bb}) \frac{(n-1)p}{N_{bb}} + x_{bn}(1 - s_{bn}) \frac{(n-1)p}{2N_{bn}} \right. \\ \left. + \frac{x_{bi}}{2} \left( s_{bi}(1 - \delta) + (1 - s_{bi}) \frac{(p_b + (n-2)p)}{N_{bi}} \right) + x_{ni} \frac{p_b}{2N_{ni}} + x_{bn+1} \frac{(n-1)p}{2N_{bn+1}} \right)$$

$x_{uv}$  frequency of genotype  $uv$

$S_{uv}$  selfing rate of genotype  $uv$

$N_{uv}$  proportion of compatible pollen received by genotype  $uv$

$p_b$  frequency of allele  $b$

$p$  frequency of allele  $i$

$W$  mean fitness (normalization term because of selection)



# 4 interesting equilibria

$$X^t = \begin{pmatrix} x_{ij}^t \\ x_{bb}^t \\ x_{bi}^t \\ x_{ni}^t \\ x_{bn+1}^t \\ x_{nn+1}^t \\ x_{in+1}^t \end{pmatrix}$$

$$\hat{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{X} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{X} = \begin{pmatrix} \hat{x}_{ij} \\ \hat{x}_{bb} \\ \hat{x}_{bi} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

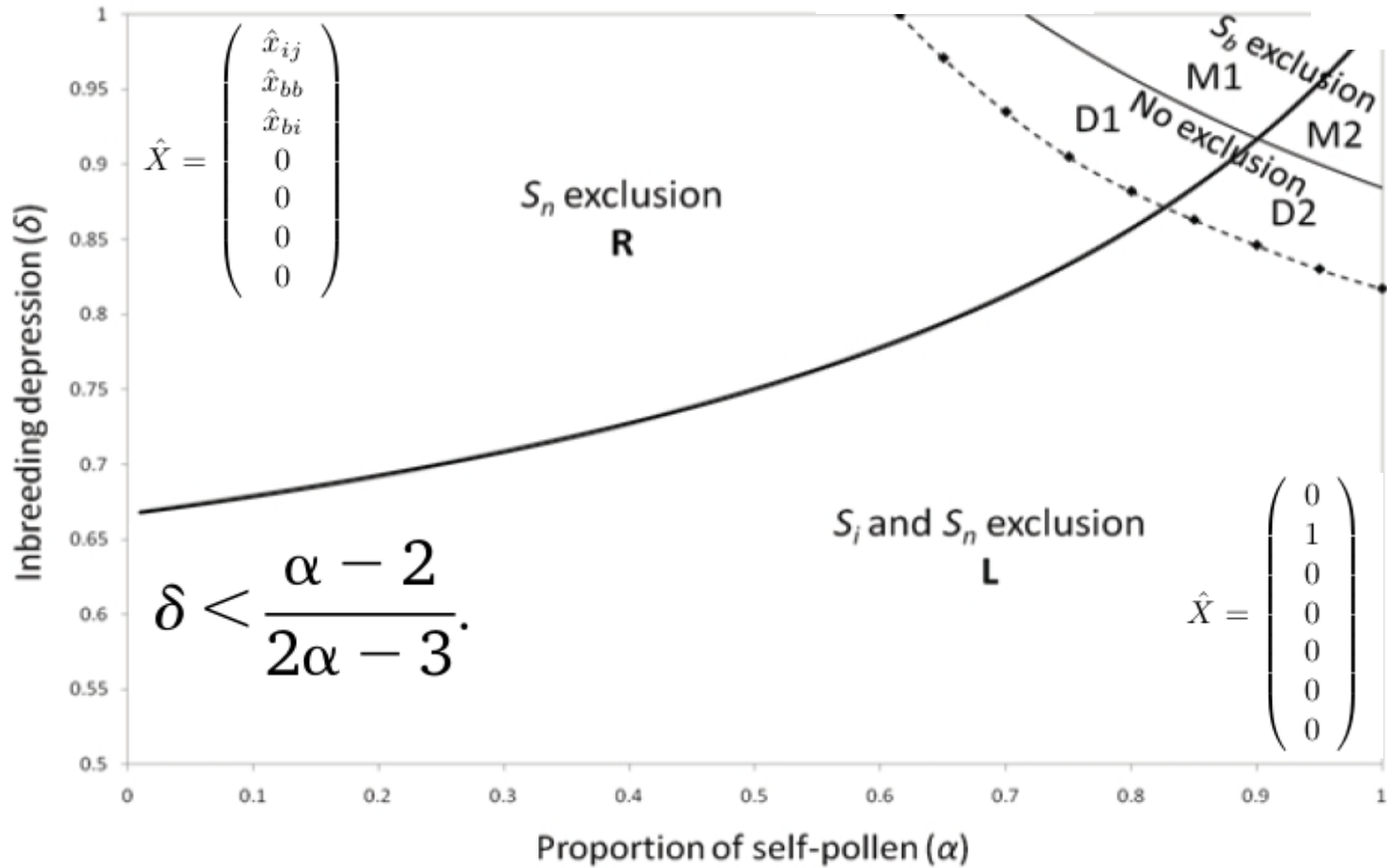
$$\hat{X} = \begin{pmatrix} \hat{x}_{ij} \\ \hat{x}_{bb} \\ \hat{x}_{bi} \\ \hat{x}_{ni} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$n = 5$

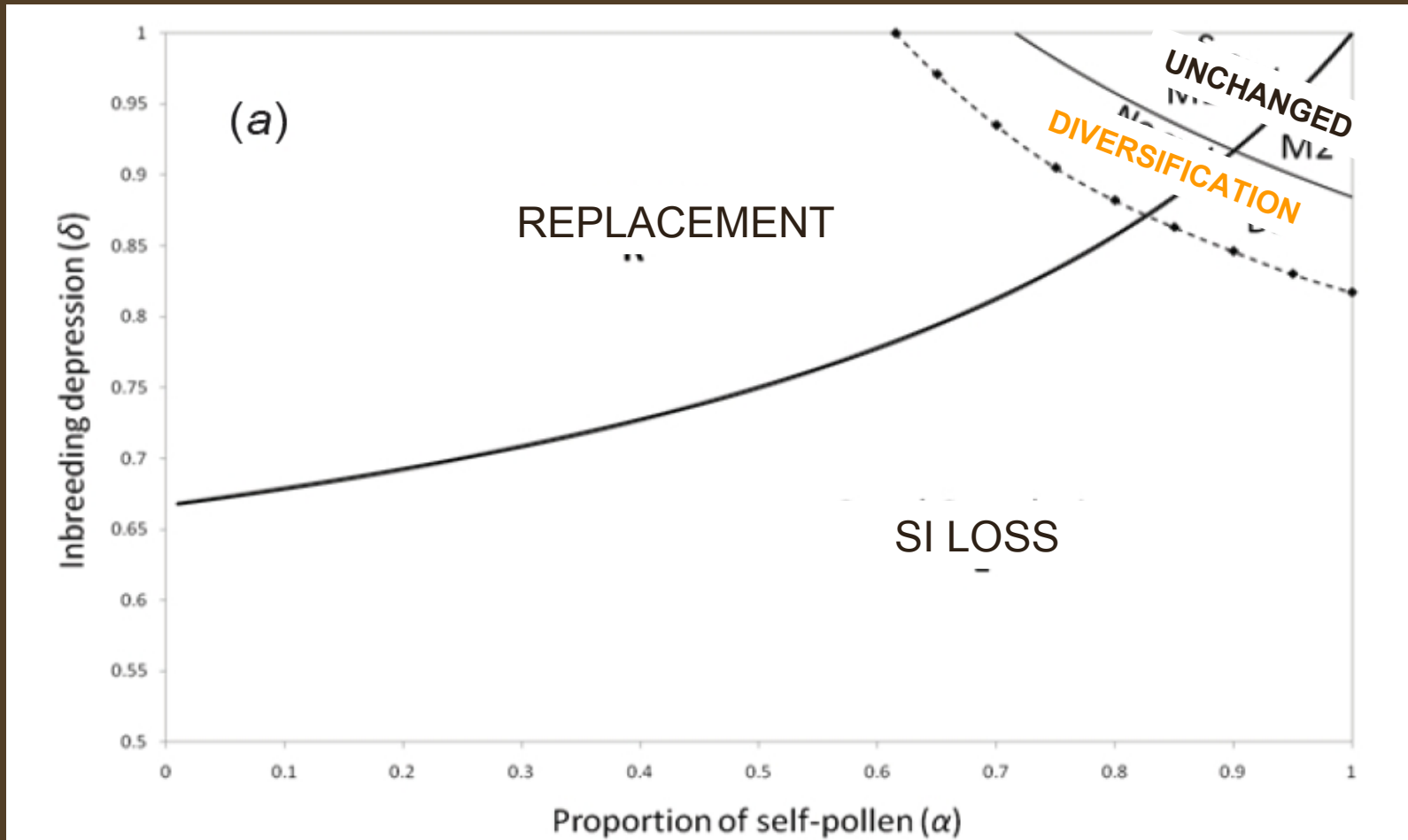
$$\hat{X} = \begin{pmatrix} \hat{x}_{ij} \\ \hat{x}_{bb} \\ \hat{x}_{bi} \\ \hat{x}_{ni} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

deterministic outcomes

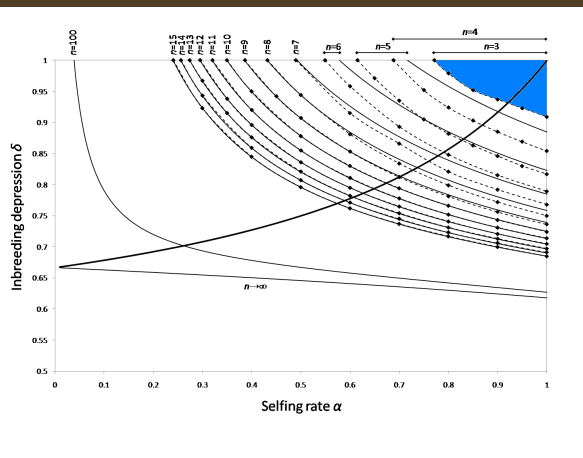
$$\hat{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



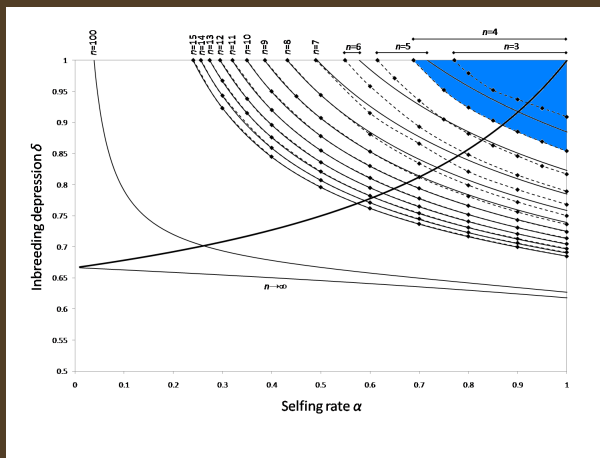
$n = 5$



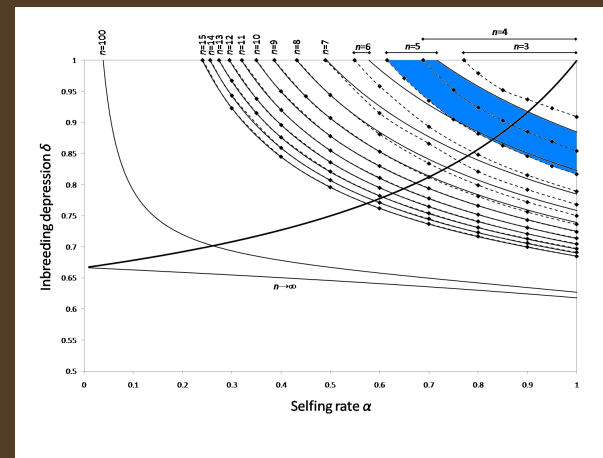
$n = 3$



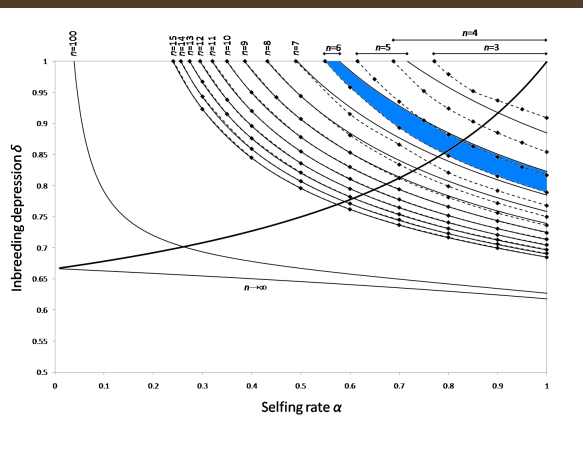
$n = 4$



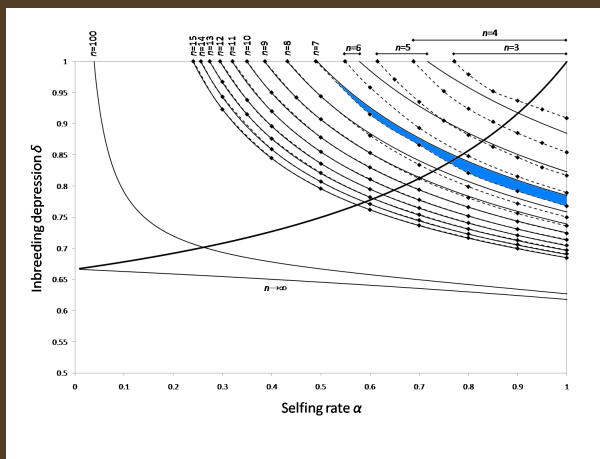
$n = 5$



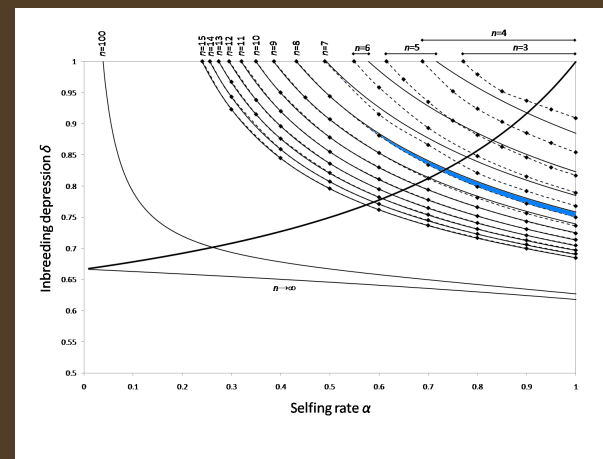
$n = 6$



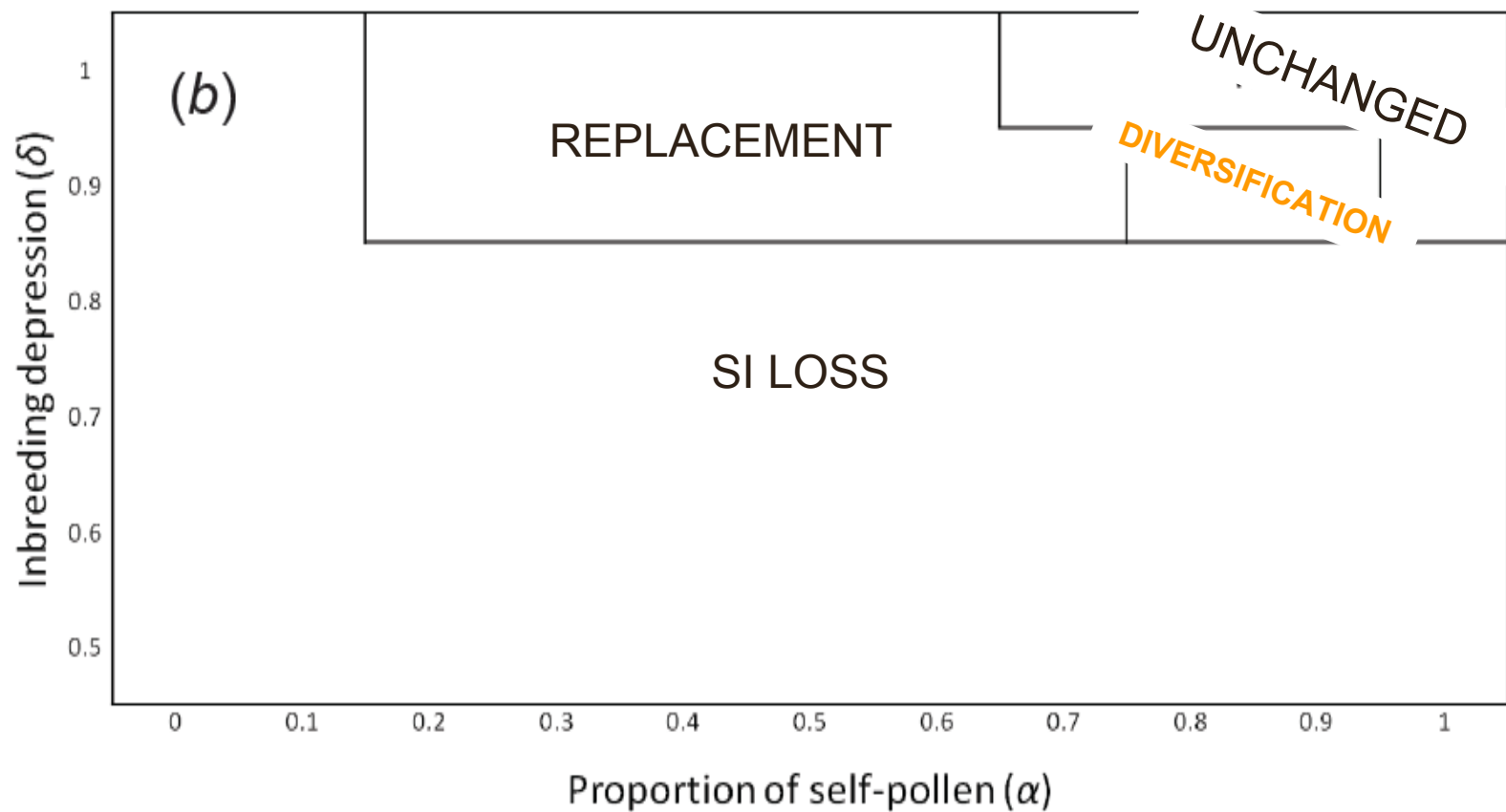
$n = 7$



$n = 8$



$$N = 5, K=20, u=5 \cdot 10^{-7}, S=5000$$



# Diversification probability in finite populations

100 replicates

$n = 5$

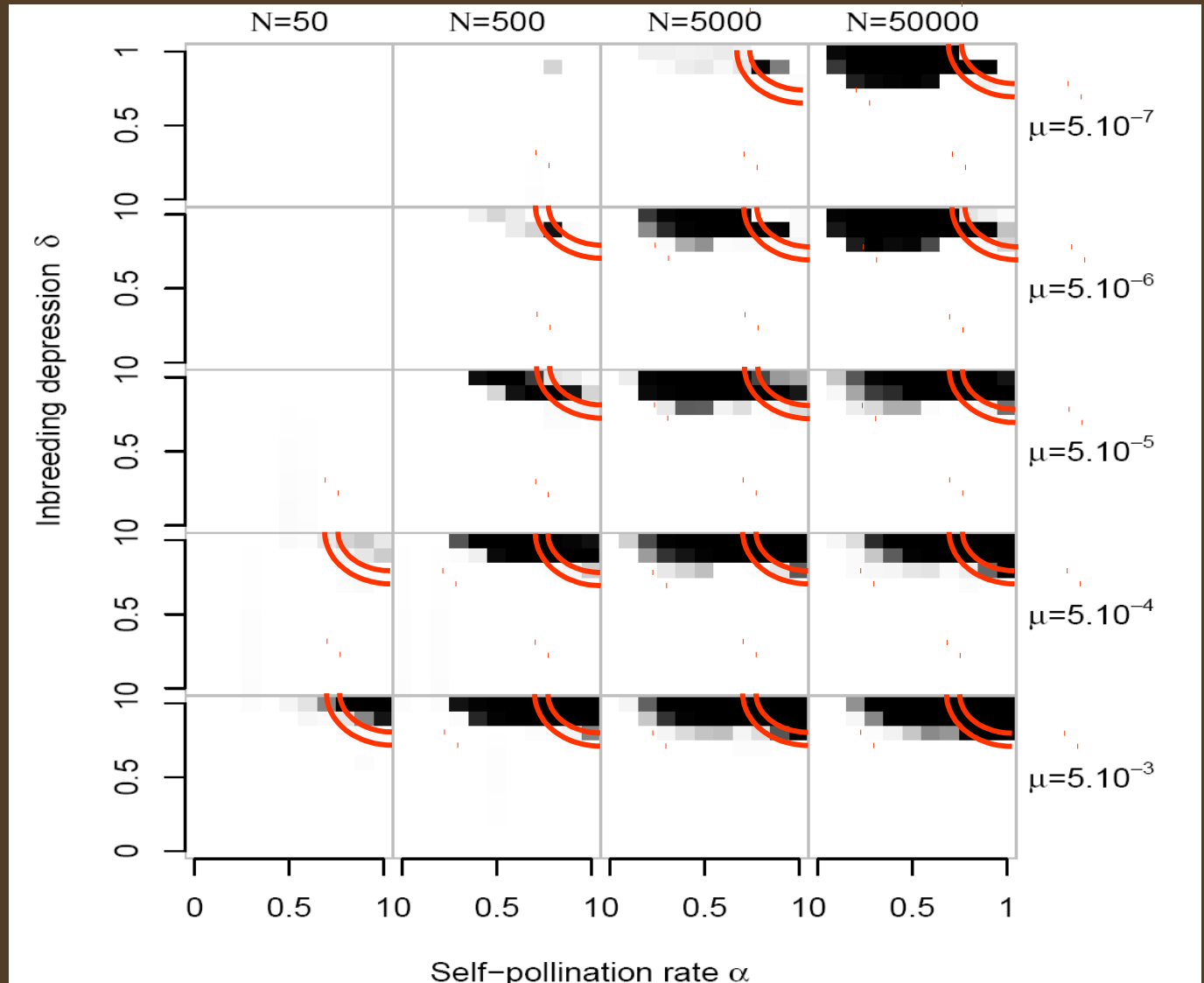
100%  
diversification

( $n > 5$ )

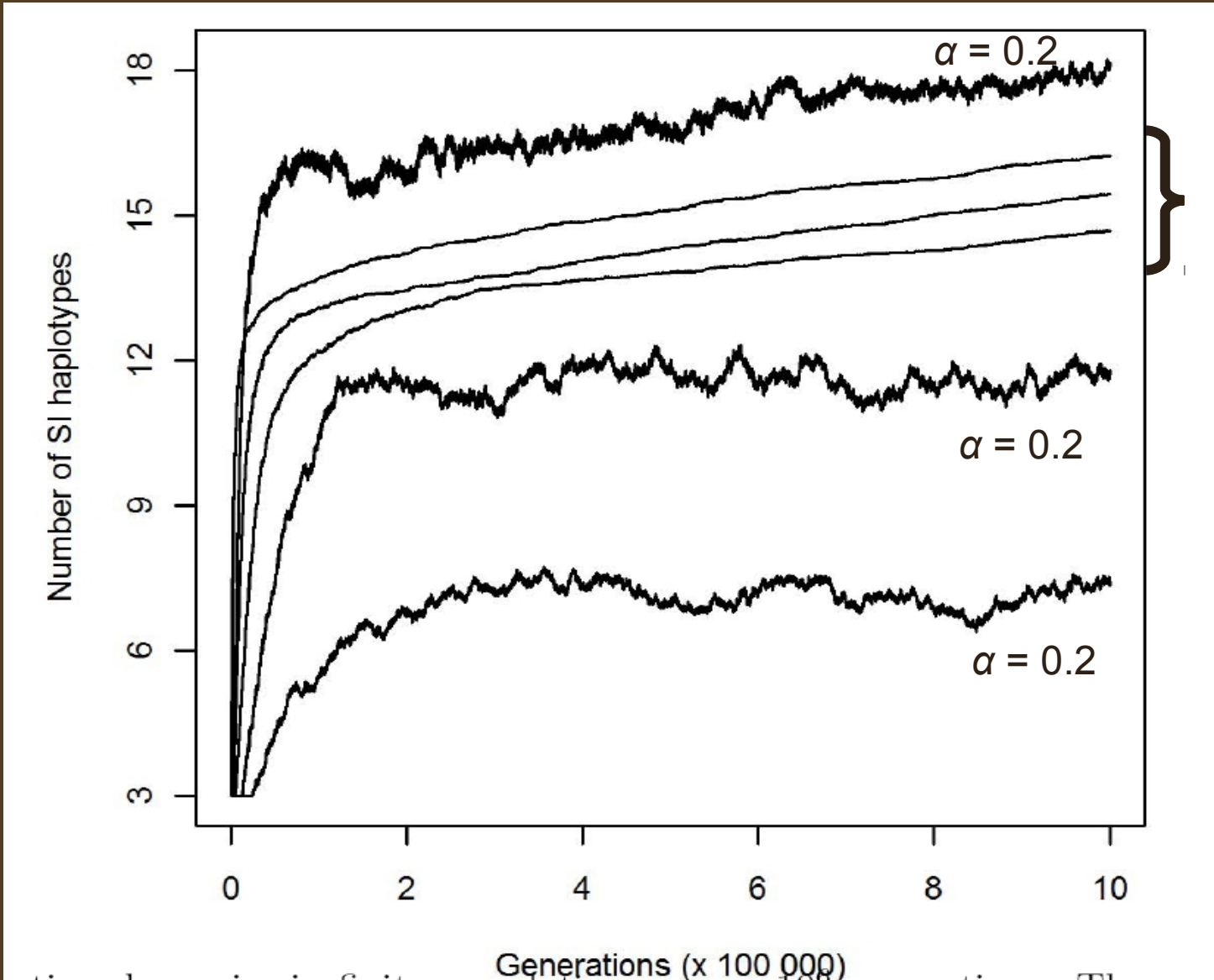


0%  
diversification

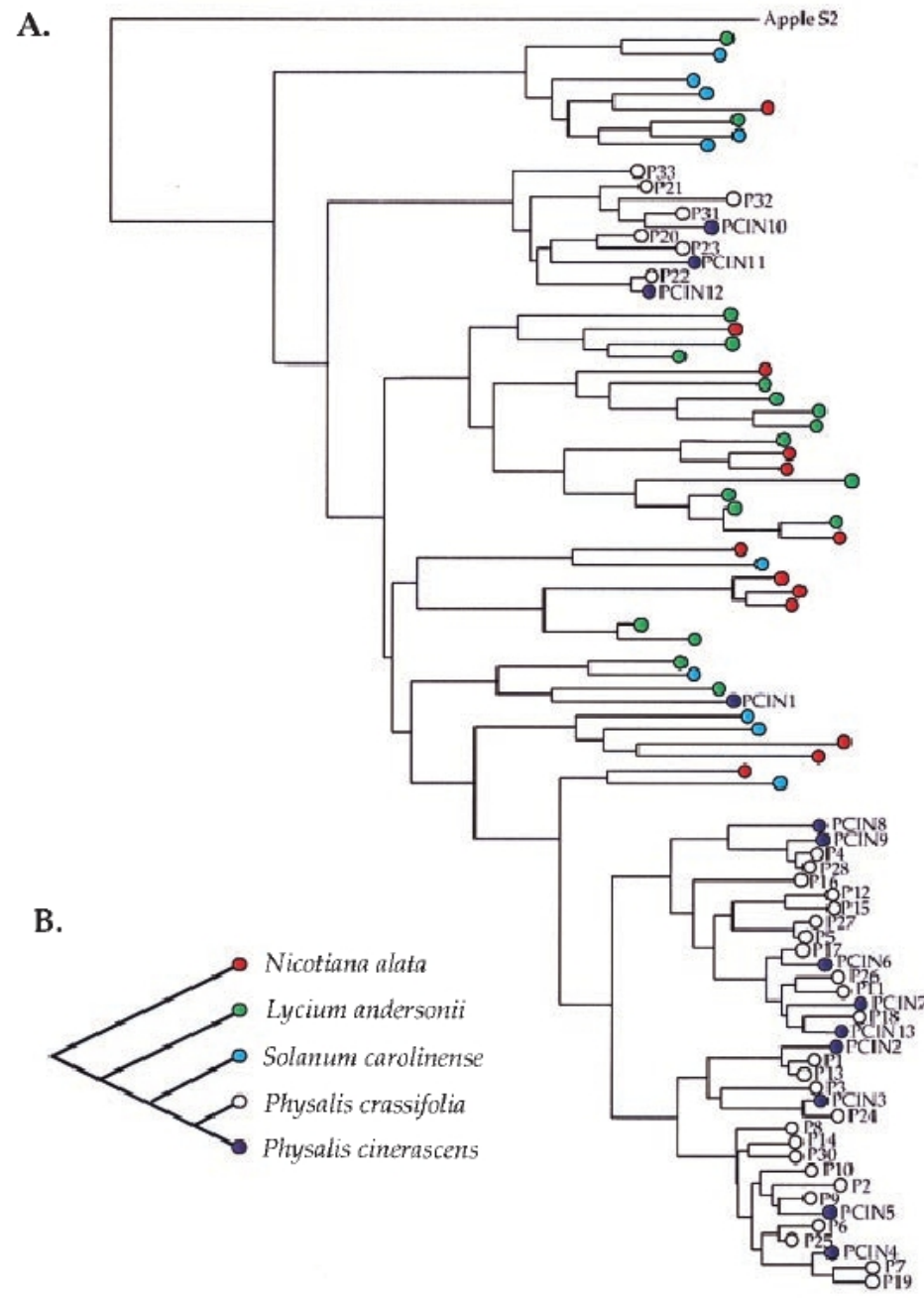
( $0 < n < 5$ )



# $n$ with time



e.g. In Solanaceae  
(Richman & Kohn 1999)

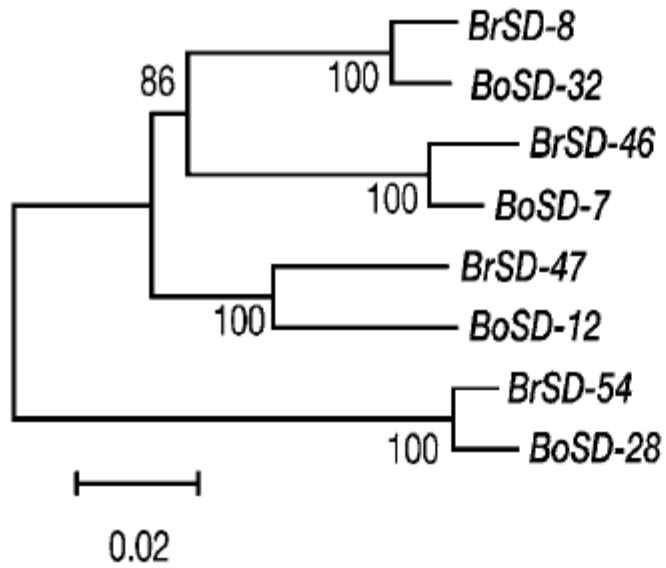




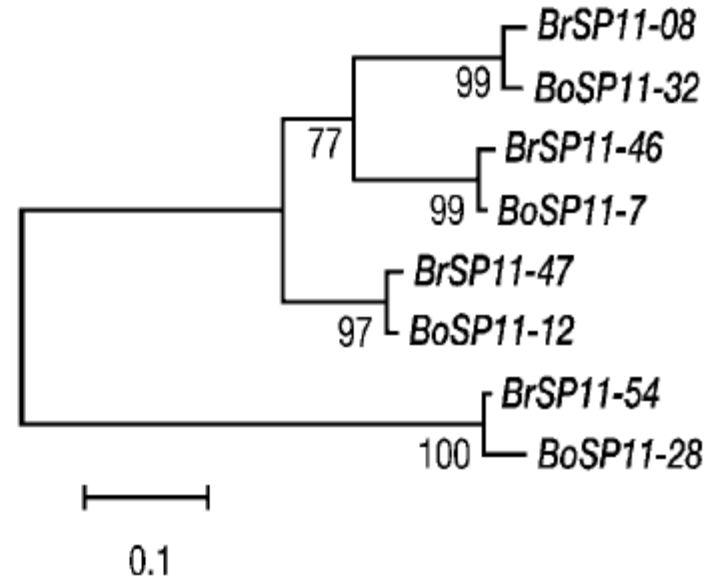
e.g. In Brassicaceae (*B. rapa*)  
(Takuno et al. 2007)



**C**



PISTIL

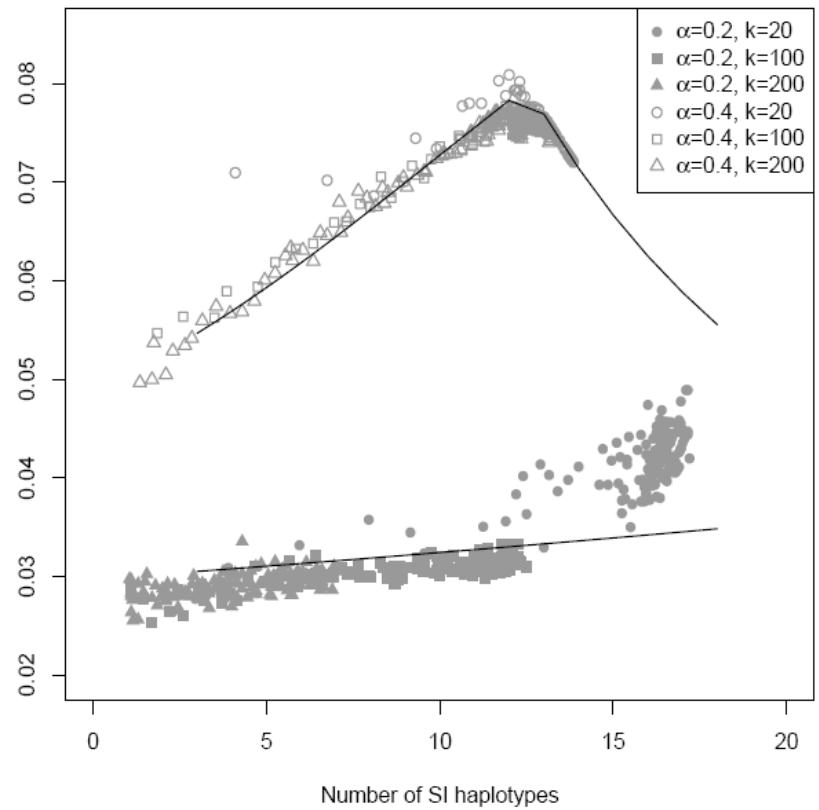
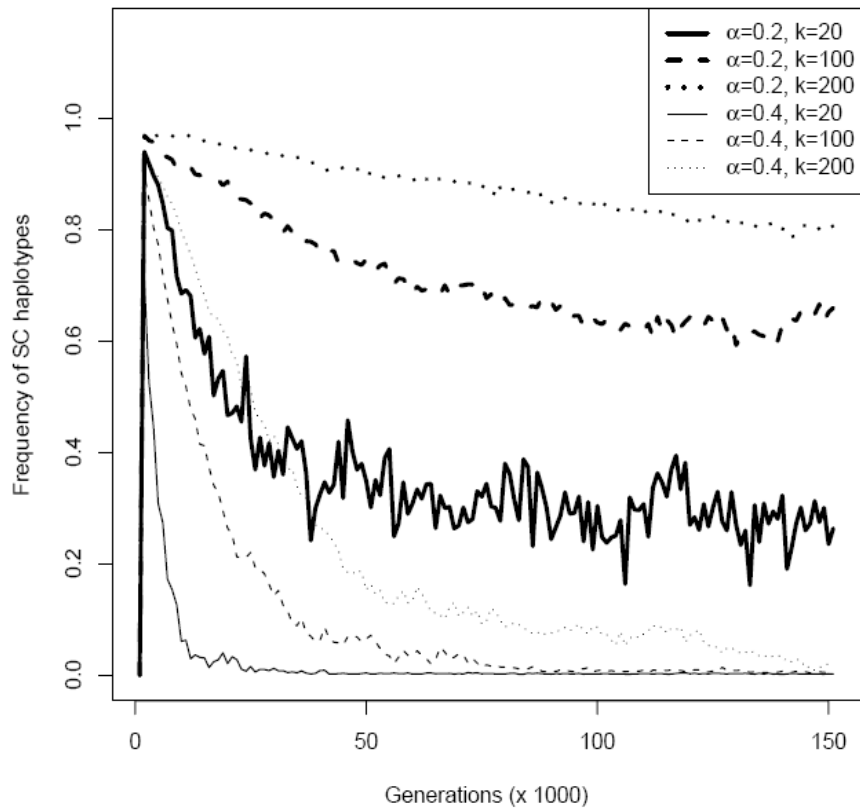


POLLEN

## Allele number

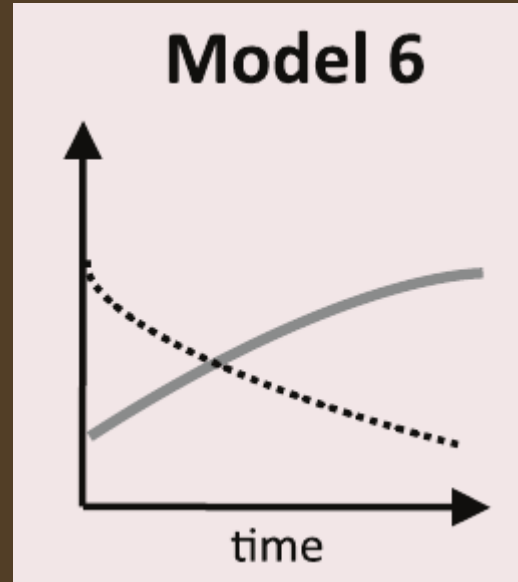
(Castric &amp; Vekemans 2004)

Species	$N_{\text{ind}}$	$n_{\text{alleles}}$
Gametophytic SI system		
<i>Crataegus monogyna</i>	13	17
<i>Lycium andersonii</i>	16	22
<i>Oenothera organensis</i>	67	34
<i>Papaver rhoeas</i>	51	31
<i>Phlox drummondii</i>	24	30
<i>Physalis cinerascens</i>	14	13
<i>Physalis crassifolia</i>	22	28
<i>Prunus lannesiana</i>	67	21
<i>Solanum carolinense</i>	24	12
<i>Sorbus aucuparia</i>	20	20
<i>Trifolium repens</i>	25	36
<i>Witheringia maculata</i>	12	10

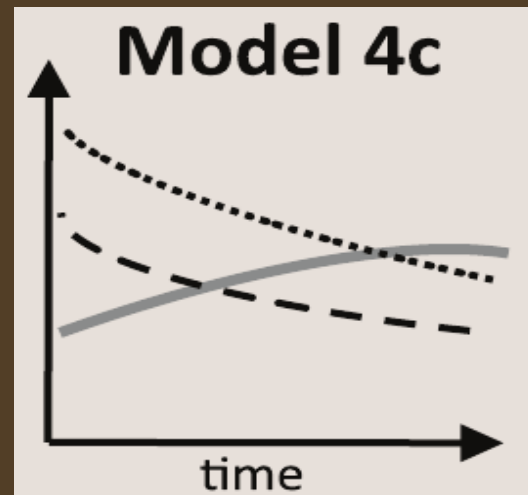


# Decaying diversification rate with increasing diversity: a general property?

Best model of speciation  
(Morlon et al. 2010)



Best model of diversification  
at the S-locus?



— log(diversity)  
..... speciation rate  $\lambda$   
- - - extinction rate  $\mu$

# Decaying diversification rate with increasing diversity: a general property?

Looking for analogs: speciation with coevolution?

