Understanding, Modeling and Managing Longevity Risk.

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OUTLINE OF THE TALK

1. CHARACTERISTICS OF LONGEVITY RISK
2. MODELING LONGEVITY RISK
3. TRANSFERRING LONGEVITY RISK
4. MODELING ISSUES FOR PRICING
INTRODUCTION

Improvements in longevity are bringing new issues and challenges at various levels: social, political, economic and regulatory.

Impacts on pensions:

- Many companies (especially in US) have closed the defined benefit retirement plans that they used to offer to their employees.
- In several countries, defined benefit pension plans have been continuously replaced with defined contribution plans.
- Some governments are about to increase the retirement age by 2 or 5 years

Influence of longevity on economy and dependence
The insurance industry is also facing some specific challenges related to longevity risk.

- Need of more and more capital to face this long-term risk
- Important to find a suitable and efficient way to cross-hedge or to transfer part of the longevity risk to reinsurers or to financial markets.
- Accurate longevity projections are delicate (prospective life tables)
- Modeling the embedded risk (such long term interest rate risk) remains challenging
Longevity Risk

Pure Longevity risk

- change of the average trend
- short-term oscillations around the average trend (risk of over-reactions)
- Heterogeneity and basis risk: the evolution of the policyholders mortality is usually different from that of the national population (selection effects).

Financial Risk

- Long term interest rate risk
- Counterparty risk
CHARACTERISTICS OF LONGEVITY RISK

Mortality analysis of a population or an insured portfolio depends on the available data and their reliability. These data are based on statistics coming from various national institutes (INSEE in France, Bureau of Census in the US, CMI in the UK, etc.), available through the Human Mortality Database.

Classical life tables are well-suited to quantify short-term mortality risk (death insurance), for time horizons from 1 to 5 years provided that no exceptional event occurs (such as pandemic or heat wave). BUT these tables are not relevant for long term longevity-based contracts like annuities or pensions, as mortality rates are changing over time and one must take this evolution into account.
HETEROGENEITY AND BASIS RISK

Difference between the national mortality data and the one from an insured portfolio.

- Insurance companies have much more detailed information
  - They know the exact ages at death and not only the year of death (time continuous data)
  - Cause of death are specified
  - Characteristics of the policyholders: socio-economic level, living conditions ...

- BUT
  - Limited size of their portfolios (in comparison to national populations: 700,000 individuals from 19 different insurance companies)
  - Small range of the observation period
Longevity patterns and longevity improvements are very different from one company portfolio to the other, and even for different countries.

Factors affecting the mortality

- socio-economic level (occupation, income, education...)
- gender
- living environment (pollution, nutritional standards, hygienic...)

Furthermore, insurers are tending to select individuals (given their health and medical history for example).

This heterogeneity is very important for longevity risk transfer, as basis risk may be too important for insurers to accept to use financial instruments based on national indices to hedge their longevity risk, as this hedge would be too imperfect.
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Motivations

Longevity modeling aims at providing solutions for important issues

- Mortality evolution in the future
- Individual mortality (depending on individual features)
  \[\Rightarrow\text{Mortality modeling}\]
- Demographic projections and retirement issue
- Insurance-life products
- Heterogeneity and basis risk
  \[\Rightarrow\text{Microscopic modeling for population evolution}\]
The general CBD model gives the dynamic of the annual mortality rate $q_t(x)$ at age $x$ during the year $t$:

$$\text{logit}q_t(x) = \kappa^1_t \beta^1_x \gamma^1_{t-x} + \cdots + \kappa^n_t \beta^n_x \gamma^n_{t-x}.$$ 

where la fonction logit est définie par $\text{logit}(x) = \ln \left( \frac{x}{1-x} \right)$.

Three types of parameters

- $\beta^i$ specific to age
- $\kappa^i$ specific to calendar year
- $\gamma^i$ cohort effect parameters
INDIVIDUAL MORTALITY MODEL

AIM: Reduce the basis risk by estimating the deviation of the "individual mortality" from the general mean mortality given by the mortality tables that only depend on age.

- Find individual characteristics (such as socio-economic level or income) that can explain mortality
- Take them into account in a stochastic mortality model

The mortality model by age and trait is calibrated on national mortality data and on specific data (with information on individual characteristics)
MARITAL STATUS INFLUENCE AT INITIAL DATE (MALES)

FIGURE: Logit of mortality rate for French males in 2007 with different marital status
**Figure**: Logit of mortality rate for French males in 2017 with different marital status.
MICROSCOPIC MODELS FOR POPULATION EVOLUTION

Let us consider an initial population (generally a sample of the whole population) that are characterized by their age and individual features.

Individual events occur in the future (birth, death, migration, marriage,...) and change the population structure. These events are represented by random variables: their probability of occurrence depending on individual demographic rates.

Many individual events are taken into account in the model:

- Birth
- Death
- Migration
- Evolution of individual characteristics (marriage, education, spc)
**Micro/macro modeling**

Macroscopic models:
- Population evolution considering macroscopic information (aggregated data)
- Whole population
- Provide mean evolution

Microscopic models:
- Evolution at the individual scale using accurate information (Individual data)
- Individual demographic rates
- Sample of the population
- Interaction between persons
- Uncertainty $\Rightarrow$ Scenarii for the population evolution

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**Evolution Scenarios in 2097**

**Figure:** Evolution Scenarios of the size of the population in 2097 with a mortality scenario.
Figure: Age pyramid scenarii in 2097 with a mortality scenario
Figure: Age pyramid in 2097: mean scenario
Double stochasticity (mortality process and evolution process)
⇒ various scenarii

Mean scenario is realistic and "coherent" with INSEE projections

The microscopic model provides interesting macroscopic information. In the case of large population, the density of the population is approximated by a Stochastic Partial Differential Equation that generalizes demographical equations (McKendrick and Von Foerster model)

⇒ Identification of the different scenarii (extreme or not) generated by the model
RETIREMENT ISSUE

With a population dynamic model, it is possible to analyze what are the optimal policies concerning immigration and the age of retirement providing an acceptable demographic situation.

Considering a criterion based on "potential support ratio", the study of the population until 2050:

- Immigration cannot provide a solution to the retirement issue: the number of migrants would be greater than 600000 a year (unrealistic policy)

- HOWEVER, with a reform on the age of retirement, the number of migrants could decrease at about 200000 migrants a year (about the rate in the seventies in France).
**Basis Risk: Marital Status Influence (Males)**

**Figure:** Cash flows of a life annuities portfolio for 60 years old French male considered by central scenario for different marital status.
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CAN WE "BUY" MORTALITY RISK IN ORDER TO HEDGE LONGEVITY RISK?

Even if a certain mutualization between mortality and longevity risks obviously exists, it is very difficult to obtain a significant risk reduction between the two, because of their different natures

- mortality risk is a short-term risk (1 to 5-year maturity) with a catastrophic component (pandemic, heat wave, ...)
- longevity risk is a long-term risk with maturities ranging from 20 to 80 years and is mainly about changes in the trend

new regulations ⇒ increase of solvency requirements ⇒ need for capital
Insurance-Linked Securitization:

- complement the reinsurance industry in catastrophic risk management
- use of capital markets to transfer insurance risk

ILS market has been growing very fast over the last 15 years

- The non-life part of the ILS market is the most visible with the famous and highly successful cat-bonds
- The life part of the ILS market is the bigger in terms of volume of the transactions with an estimated outstanding of 35 to 40 billion USD
No ILS related to longevity risk has been completed yet BUT

- the estimation of the underlying public and private exposure on longevity risk is over 20 trillions USD
- The Life and Longevity markets association (LLMA) launched on February 2010 to promote trading of longevity risk (established by AXA, Deutsche Bank, JP Morgan, Legal & General, Pension Corporation, Prudential, RBS and Swiss Re).

Obstacles to develop capital markets’ solutions

- one-way exposure of investors: there is almost no natural buyers of longevity risk → creates a problem to generate demand
- basis risk: heterogeneity between full population mortality indices and those of individual pension funds and insurers, regional and socio-economic basis risk...
A good longevity index should be based on national data (for transparency) but be flexible enough as to reduce the basis risk for the original longevity risk bearer. Today, the existing indices are:

- Credit Suisse Longevity Index (December 2005) based upon national statistics for the US population, with some gender and age specific sub-indices.

- JP Morgan Index with LifeMetrics (March 2007) covers the US, England & Wales and the Netherlands and used national population data + future longevity modeling through various stochastic mortality models.

- Xpect Data by Deutsche Borse (March 2008) initially delivered monthly data on life expectancy for Germany, but now covers the Netherlands.
JP Morgan has developed some standardized longevity instruments called "q-forwards".

- contracts are based upon an index (the mortality rate or the survival rate, as quoted in LifeMetrics).
- The mechanisms of the q-forwards are quite simple: a pension fund hedging its longevity risk will expect to be paid by the counterpart of the forward if the mortality falls by more than expected.

  a pension fund = q-forward seller
  an investor = q-forward buyer
LONGEVITY SWAP TRANSACTIONS

Very recently, some longevity swap transactions have been completed: private transactions and their pricing remains confidential. Over the last year 2008, two particular longevity swaps have been arranged by JP Morgan

- A customized swap transaction (July 2008), notional amount of GBP 500 millions for 40 years. The UK life insurer pay fixed payments and receive floating payments which replicates the actual benefit payments made on a closed portfolio of retirement policies (no basis risk). At the same time, JP Morgan entered into smaller swaps with several investors who take the longevity risk at the end. The counterpart risk for this swap is important because of the long term maturity and the number of agents involved.

- A standardized transaction (January 2008), notional amount of GBP 100 millions for 10 years. standardized longevity swap with the pension insurer Lucida, using LifeMetrics index for England and Wales as underlying index (basis risk for Lucida).
The challenge lies in developing transparency and liquidity without neglecting the hedging purposes of the instruments.

- essential challenge: designing suitable, efficient and attractive structures for both risk bearers and risk takers (as underlined by the failure of the EIB-BNP Paribas longevity bond in 2005).

- Emphasizing the importance of assessing counterpart risk, to secure transactions (even more critical when considering longevity risk, due to the long-term maturity)

- question of the pricing of risk transfer solutions
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Pricing methodologies

Characteristics of the pricing of any longevity "derivatives"

- it depends on the estimate of uncertain future mortality trends and the level of uncertainty.
- this risk induces a mortality risk premium that should be priced by the market.
- absence of any liquid traded longevity security → it is today impossible to rely on market data.
- Long term interest rate modeling
WHAT WILL BE A GOOD PRICING MEASURE FOR LONGEVITY?

longevity market is an immature market based on a non-financial risk ⇒ the arbitrage-free (or risk-neutral) pricing methodology is far from being applicable.

A good pricing measure for longevity must be

- equivalent to this historical probability measure.
- robust with respect to the statistical data.
- coherent with the prices of the liquid assets quoted in the market.
**Pricing Methods**

**Indifference pricing**: the maximum price that an agent is ready to pay is the price such that she is indifferent (from her preference point of view) between doing or not the transaction.

- not a linear pricing rule (the size of the transaction is important)
- provides a price range (difficult to compute) instead of a single price
- extension to a dynamic point of view (Barrieu and El Karoui (2009))
LONG-TERM INTEREST RATES

- Embedded long term interest rate risk in longevity-linked securities
  Because of the lack of liquidity for long horizon, the standard financial point of view cannot be easily extended.

- Abundant literature on the economic aspects of long-term policy-making

The economy is represented by the strategy of the representative agent. The derivation of the yield curve for far-distant maturities is induced from the maximization of the representative agent’s intertemporal utility function on the aggregate consumption:

$$\max_{c \geq 0} \int_{t \geq 0} e^{-\delta t} u(c_t) dt$$

where $c_t$ is the aggregate consumption, $u$ the agent’s utility function and $\delta$ her pure time preference parameter.
This leads to the so-called "Ramsey rule" (for deterministic rate and consumption)

\[
\frac{1}{t} \int_{0}^{t} r_s ds = \delta - \frac{1}{t} \ln \frac{u'(\hat{c}_t)}{u'(c_0)}.
\]

Extension of the Ramsey rule in a stochastic framework:

\[
R_0(t) := \frac{1}{t} \ln \mathbb{E}(e^{\int_{0}^{t} \tilde{r}_s ds}) = \delta - \frac{1}{t} \ln \frac{\mathbb{E}(u'(\hat{c}_t))}{u'(c_0)},
\]
**AIM** (I. Camilier Thesis): Extend the economic framework

- by a dynamic and stochastic point of view
- by taking into account the existence of a financial market

In a complete market

\[
B(0, t) = \exp(-\delta t) \frac{\mathbb{E}[u'(\hat{c}_t)]}{u'(c_0^*)},
\]

where \( B(0, t) = \mathbb{E}^* \left[ \exp \left( - \int_0^t \tilde{r}_s ds \right) \right] \) is the price at time 0 of a zero-coupon with maturity \( t \).

⇒ same relation as the Ramsey rule - with \( B(0, t) = \exp(-R_0(t)t) \).

In a incomplete market: the pricing probability is not universal and might depend on the maturity and the utility function, with impact on the yield curve.
CONCLUSION/PROJECTS

- Detection of changes in longevity trends (N. El Karoui, S. Loisel, C. Mazza)
- Microscopic modelling of population dynamic: an analysis of longevity risk (H. Bensusan, N. El Karoui)
- On inter-age correlations in stochastic mortality models (S. Loisel and Y. Salhi)
- Designing a longevity product (P. Barrieu, H. Bensusan, N. El Karoui, S. Loisel)
- Pricing longevity securities (N. El Karoui, S. Loisel, C. Hillairet, Y. Salhi)
- Growth optimal portfolio and Long term interest rate (I. Camilier, N. El Karoui, C. Hillairet)
- Variables annuities (S. Loisel, T.L. Nguyen)