Optimal control techniques based on infection age for the study of the COVID-19 epidemic

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Confinement control in the SIR setting:

\[
\begin{align*}
\dot{S}(t) &= -\delta(1-u(t))S(t)I(t) \\
\dot{I}(t) &= \delta(1-u(t))S(t)I(t) - (\eta + \mu)I(t) \\
\dot{R}(t) &= \mu I(t)
\end{align*}
\]

(1)

with \(u(t) \in [0, 1]\) confinement control, and \(\delta > 0\) infection coefficient, \(\eta > 0\) death rate, \(\mu > 0\) recovery rate.
Infection age; related differential calculs

For non hospitalized $z(t, a, b)$ and hospitalized population $h(t, a, b)$

$a$ class of population (strong/weak)

$b \in [0, B]$ infection age;

For $\varepsilon > 0$ small:

$$z(t + \varepsilon, a, b + \varepsilon) \approx z(t, a, b) - \varepsilon \nu(a, b)z(t, a, b)$$ \hspace{1cm} (2)

Corresponding differential equation

$$z_t(t, a, b) + z_b(t, a, b) = -\nu(a, b)z(t, a, b)$$ \hspace{1cm} (3)
Full model

States: $y$ susceptible, $z$ infected non hospitalized, $h$ hospitalized, $\tilde{y}$ recovered.

\[
\begin{align*}
\dot{y}(t,a) &= -\delta(a)(1-u(t,a))Z(t)y(t,a) \\
(z_t + z_b)(t,a,b) &= -\nu(a,b)z(t,a,b) \\
z(t,a,0) &= \delta(a)(1-u(t))Z(t)y(t,a) \\
(h_t + h_b)(t,a,b) &= \nu(a,b)z(t,a,b) - (\eta(a,b) + \gamma(a,b)E(t))h(t,a,b) \\
h(t,a,0) &= 0 \\
\dot{\tilde{y}}(t,a) &= z(t,a,B) + h(t,a,B)
\end{align*}
\]
Details of dynamics

\[ Z(t) = \int_0^A \int_0^B e(a,b)z(t,a,b)dadb, \quad (5) \]

with \( e(a,b) \) transmission factor

hospitalized patients do not contribute to the transmission

\( \nu(a,b) \) hospitalization coefficient,

\( E(t) \in [0, 1] \) is the hospital saturation estimate, given by

\[ E(t) := \frac{(H(t) - C)_+}{H(t) + C}; \quad H(t) := \int_0^A \int_0^B h(t,a,b)dadb, \quad (6) \]

where \( C > 0 \) nominal capacity.
Only hospitalized patients die, with death rate

\[ d(t, a, b) := \eta(a, b) + \gamma(a, b)E(t), \quad (7) \]

where \( \eta(a, b) \geq 0 \) is the minimal death rate and \( \gamma(a, b) \) sensitivity of the death rate w.r.t. the hospital saturation estimate.
Cost function

\[ J(M, u, D_T) := p_M M + p_u c(u) + p_D D_T, \]  
where the penalty coefficients \( p_M, p_u \) and \( p_D \) are nonnegative. 

\( D_T \), death toll; \( M \) hospital peak value, subject to

\[ H(t) \leq M, \quad \forall t \in [0, T]. \]  

In addition, in our model we include confinement duration constraints for each class, more precisely:

\[ \int_0^T u(t, a) \, dt \leq M(a), \quad \text{for a.a. } a \in (0, A). \]
Figure 1: Minimal hospital peak value: \( p_M = 10, \quad p_u = 0.005, \quad p_D = 1, \quad C = 0.01, \quad T = 260. \)