Application of convex lexicographical optimization to the balance of GRTgaz gas grid

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ABSTRACT. Shippers are daily users of the French gas grid. Differences between planned and realized amount of gas tend to unbalance the grid. To restore the balance, GRTgaz computes every day the amounts of gas transiting on the grid, injecting or withdrawing from the storages, balancing tolerances use rates and, finally, buying or selling amounts of gas to shippers (associated with penalties). To minimize billed penalties to shippers, GRTgaz uses these balancing facilities in a certain order. The problem to solve is a four stages lexicographical optimization. Constraints consist in flows equations and bound on variables. The cost function to be minimized at each stage is convex quadratic, sum of squares of slack variables in the flow equations. The Lagrange multiplier may be interpreted as a pressure; flows try to make equal pressure over the network. In the subset of nodes with zero pressure, called the zero pressure subset, a careful formulation of the higher stages problems is necessary in order to guarantee the robustness of computations. For one year, a tool implementing this method has been running each day and has been permitting to bill penalties perfectly well (the financial stake is very significant). Some numerical results are displayed.

KEYWORDS. Grid balance, lexicographical optimization, duality theory, convex optimization.

INTRODUCTION

Before the opening of the French gas market, the grid operator had to deal with one shipper. When the shipper had an excess or a lack of gas, the imbalance was absorbed by the grid operator. Now several gas shippers are acting on the French grid. Every day, GRTgaz (the grid operator) has to share the shippers' gas on its grid in order to balance it. When the grid cannot be balanced, the shippers responsible for the imbalance have to pay penalties. The shippers are offered by GRTgaz the possibility to subscribe to a daily balancing service that reduces their imbalances. The problem of computing the reduced shippers' imbalances, described in the next section, is a multi-criteria problem. The other European grid operators offer different balancing services (see the gas balancing alert system on the UK gas grid [8], the cumulated imbalance system in Belgium [6] and the Dutch "OLB" service [7]). Our method is original. Our problem is modelled as a lexicographic optimisation problem: for references on such problems see [3], the overview in [2] (including a duality theory and the link to sensitivity), and the theory of piecewise lexicographic programming [4,5]. Note also that lexicographic optimization may be viewed as the limit of the minimization of a weighted combination of the costs with weights equal to different powers of a small parameter "epsilon"; basically, for the case of two cost functions, this is what Danskin's theorem says (see e.g. Theorem 4.13 in [1]). The numerical minimization of such a composite cost, however, does not look promising from the numerical point of view. This option has been preferred to a single objective function model. Indeed, the choice of the weights of the criteria in a multi-criteria objective function is a sensitive problem by itself. Lexicographical modeling allows us to ignore this problem. Furthermore it is more readable, because each criteria is clearly isolated from the others. This is a key point because the balancing method had to be validated by the Commission de Régulation de l'Energie (regulator for the French electricity and gas markets) and understood by the shippers. The robustness of the method was also a crucial point because the algorithm is used every day, especially to bill the shippers for their imbalances. An important economical point is thus at stake.

The first approach to solve lexicographical problems is to solve the first problem, minimizing the first criteria. Then the optimized variables are fixed to their optimal values. After that, the second problem is solved with fixed variables, and so on. Despite this method often works, cases happen where it leads to numerical problems due to numerical approximation in fixing the variables. To avoid fixing variables, a solution is to write the optimality system. That ensures that the cost function of the next step problem is minimized over the solution set of the current step. In this way we avoid numerical difficulties. Parts 2 to 5 of this paper describe our problem, give the optimality systems and explain how to solve them. Part 6 gives performances features and numerical illustrations.

1. PROBLEM DESCRIPTION

GRTgaz' grid is used by several shippers. The grid is divided into "balancing zones". Each day, each shipper must be balanced on each zone: all the gas brought by the shipper to a zone must be consumed or put in a storage. If a shipper has brought too much gas to a zone or has a lack of gas, he has to pay penalties. However shippers can subscribe to a balancing service to help them reduce their penalties. This service is made up of two parts. The first one is a bigger or smaller storage use than what the shipper had scheduled. For each storage, this use is bounded by flexibilities around the scheduled quantity (flexibilities are contractually set). The second part is the use of balancing tolerances, defined for each balancing zone. The contract between the shipper and the grid operator sets the size of balancing tolerances. Flexibilities of the storages must be used prior to balancing tolerances, in order to absorb the potential gas lack/excess. If these mechanisms are not sufficient, penalties are due by the shipper on every zone where there is an imbalance. At last, arcs between zones must be used in order to maximize the share of the gas between these zones. A quadratic criteria is then used. To balance the grid, recourse to the offered facilities is minimized in this order: first the penalties, then the use of the balancing tolerances followed by the storage flexibilities and at last the use of the between-zones links (this use is also limited by contractual terms). This problem corresponds to a lexicographical optimization. In the next part, we briefly introduce the lexicographical optimization. Then we give the model formulation.

2. LEXICOGRAPHICAL OPTIMIZATION

The lexicographical order in $\Re^p$ is as follows. The approximation order of y and $z \in \Re^p$ is written:

$$I(y,z) := \min\{i; y_i \neq z_i\}. \tag{2.1}$$

We write that y lexicographically precedes z, and we write $y \leq_{lex} z$ if

$$y = z \text{ or } y_j < z_j, \text{ with } j = I(y,z). \tag{2.2}$$

Let X be an ordinary set and f a mapping from X to $\Re^p$. The lexicographical minimization problem

Find $\bar{x} \in X$ verifying $f(\bar{x}) \leq_{lex} f(x)$, for each $x \in X$,

written

$$\underset{x}{Min}\, f(x)\ ; \quad x \in X \qquad (LEX)$$

is equivalent to the problems stream (optimization of scalar criteria):

$$Min\, f_1(x)\ ; \quad x \in X \qquad (L_1)$$

$$Min\, f_i(x)\ ; \quad x \in S(L_{i-1}) \qquad (L_i)$$

for $i = 2, \ldots$, where $S(L_{i-1})$ is the solution set of the problem (L_{i-1}).

3. MODEL FORMULATION

The balancing problem applied to GRTgaz gas grid is written as follows. Given a grid. Let $i \in I$ be a zone and I be the set of the zones. For each zone i, let b_i be the shipper's initial imbalance (given), D_i the final imbalance imputed to the shipper on the zone, EBC_i the use of its balancing tolerance, DPM_i the recourse to the storage of the zone. Moreover, being i and j two zones, let $QJR_{i,j}$ be the gas transit between i and j.

For each point i, the following balance equation must be respected:

$$D_i + EBC_i - DPM_i + \sum_{j \in Le_i} QJR_{i,j} - \sum_{j \in Ar_i} QJR_{i,j} = b_i \qquad (3.3)$$

with b_i given, Le_i the set of arcs leaving i, Ar_i the set of arcs arriving at i.

D_i are unbounded variables. EBC_i, DPM_i and $QJR_{i,j}$ are variables bounded as follow:

$$\underline{EBC_i} \leq EBC_i \leq \overline{EBC_i}$$

$$\underline{DPM_i} \leq DPM_i \leq \overline{DPM_i} \qquad (3.4)$$

$$0 \leq QJR_{i,j} \leq \overline{QJR_{i,j}}$$

where the bounds $\underline{EBC_i}, \overline{EBC_i}, \underline{DPM_i}, \overline{DPM_i}, \underline{QJR_{i,j}}, \overline{QJR_{i,j}}$ are the given contractual min/max bounds on the balancing tolerances, the storages and the arcs.

The problem is to lexicographically minimize the criteria

$$f(D, EBC, DPM, QJR) = \frac{1}{2}\left(\sum_{i \in I} C_{1i} D_i^2, \sum_{i \in I} C_{2i} EBC_i^2, \sum_{i \in I} C_{3i} DPM_i^2, \sum_{i,j \in I} C_{4i,j} QJR_{i,j}^2 \right).$$

where constants C_{1i}, C_{2i}, C_{3i} *and* $C_{4i,j}$ are given: they are imbalance repartition coefficients calculated by GRTgaz. They allow GRTgaz to share the imbalances according to zones storages capacities and arcs capacities. Precisely, $C_{1i} = C_{2i} = 1 / \overline{EBC_i}$, $C_{3i} = 1$ and $C_{4i,j} = 1 / \overline{QJR_{i,j}}$.

The cost function for each stage is separable and quadratic convex. By induction, using that the set of the solutions of a convex problem is convex, we deduce that each problem of the stream is convex. Moreover, the strict convexity of each cost function with respect to vectors D, EBC, DPM, QJR, respectively, implies that each of these variables will be uniquely determined when solving the corresponding stage.

4. FIRST PROBLEM ANALYSIS

4.1. Optimality system

In this section, we analyze the problem:

$$Min\frac{1}{2}\sum_{i\in I} C_{1i} D_i^2 \quad \text{under (3.3) – (3.4).}$$

Let λ_i be the Lagrange multiplier associated with the balance equation associated to the zone i and $s_x \geq 0$, $t_x \geq 0$ those associated with the upper and lower bounds of x variable. The problem is convex and all its constraints are linear. So, a Lagrange multiplier exists and at the optimality, we can exhibit the optimality system constituted by primal constraints (3.3) – (3.4) and dual constraints (for each *i*):

$$C_i D_i - \lambda_i = 0 \tag{4.5}$$

$$-\lambda_i + t_{EBC_i} - s_{EBC_i} = 0, \tag{4.6}$$

$$-\lambda_i + t_{DPM_i} - s_{DPM_i} = 0, \tag{4.7}$$

and for each flow from *i* to *j*:

$$\lambda_j - \lambda_i + t_{QJR_{i,j}} - s_{QJR_{i,j}} = 0. \tag{4.8}$$

Finally, we write the slackness conditions (for each *i*):

$$t_{EBC_i}(EBC_i - \overline{EBC_i}) = 0; \qquad \mathrm{s}_{EBC_i}(EBC_i - \underline{EBC_i}) = 0, \tag{4.9}$$

$$t_{DPM_i}(DPM_i - \overline{DPM_i}) = 0; \qquad \mathrm{s}_{DPM_i}(DPM_i - \underline{DPM_i}) = 0, \tag{4.10}$$

$$t_{QJR_{i,j}}(QJR_{i,j} - \overline{QJR_{i,j}}) = 0; \qquad \mathrm{s}_{QJR_{i,j}} QJR_{i,j} = 0. \tag{4.11}$$

We can consider the problem from another point of view by seeing λ_i as pressures. The flows $QJR_{i,j}$ allows the gas to communicate between two storages. They tend to balance pressures, what is represented by (4.8). Amounts EBC_i and DPM_i tend to bring pressures to 0, what is translated by (4.6) and (4.7).

4.2. Interpretation

Nodes of the graph linked by an arc are said to be adjacent. A connected subset of nodes of the graph with equal pressure in each node is called a communicating subnetwork. Communicating subnetworks are a partition of the set of nodes of the graph. A communicating subnetwork with nonzero (resp. zero) pressure is said saturated (resp. non saturated). By (4.8), flows between nodes of different communicating subnetworks are set to a bound. Therefore, once the pressure associated with one of the four stages is known, the subsequent stages split into independent subproblems for each communicating subnetwork. We therefore restrict the discussion to the analysis of a single class.

4.3. Characterization of the solution of a class

Let $\overline{\lambda} \in \Re$ be the common pressure of the class. D_i can be eliminated in the balance equation. That leads to:

$$C_{1i}^{-1}\overline{\lambda} + EBC_i - DPM_i + \sum_{j \in Le_i} QJR_{i,j} - \sum_{j \in Ar_i} QJR_{i,j} = b_i \qquad (4.12)$$

Knowing that Lagrange multipliers associated with the bounds of the flows are equal to zero, the only equations regarding flows to be satisfied are:

$$0 \le QJR_{i,j} \le \overline{QJR_{i,j}}\ . \qquad (4.13)$$

Regarding variables EBC_i and DPM_i, two cases have to be analyzed:

a) if the pressure is non zero, variables *EBC*$_i$ and *DPM*$_i$ are fixed to a known bound and so can be eliminated.

b) if the pressure equals zero, $\overline{\lambda}$ can be replaced by 0 in the balance equation (4.12). In any case, the bounds constraints have to be taken into account:

$$\begin{aligned} &\underline{EBC_i} \le EBC_i \le \overline{EBC_i} \\ &\underline{DPM_i} \le DPM_i \le \overline{DPM_i} \end{aligned} \qquad (4.14)$$

5. STREAM

We can now discuss the treatment of the other problems.

a) The easiest case is the one of a nonzero pressure: *EBC*$_i$ and *DPM*$_i$ are then equal to one of their bound, depending on the sign of the Lagrange multiplier. So, we can directly minimize the last criteria, under constraints (4.12) and (4.13), being aware that $\overline{\lambda}$ is an unknown.

b) In the case of a null pressure class, the second criteria has to be minimized under constraints (4.12) to (4.14). Here, $\overline{\lambda}$ equals zero and then can be eliminated. Let λ^2 be the Lagrange multiplier associated with the balance constraint. Relations analogous to (4.8) are written:

$$\lambda_j^2 - \lambda_i^2 + t_{QJR_{i,j}} - s_{QJR_{i,j}} = 0. \qquad (5.15)$$

As for the first stage, we can put together points by under class of equivalence having the same second stage pressure λ^2. Flows between different points are fixed to bounds. The problem decomposes in two sub-cases:

b1) When $\overline{\lambda^2}$ is non zero, *DPM*$_i$ is at a bound and is eliminated. We can pursue by minimizing the fourth criteria. For the *EBC*$_i$ variables, the optimality equation is written as follow:

$$C_{2i}EBC_i - \overline{\lambda^2} + t_{EBC_i} - s_{EBC_i} = 0. \qquad (5.16)$$

We can assume that it is possible to know if $t_{EBC_i} \ne 0$ or $s_{EBC_i} \ne 0$. In any case, we can eliminate *EBC*$_i$ which equals a bound if t_{EBC_i} or s_{EBC_i} is non zero and equals $C_{2i}^{-1}\overline{\lambda^2}$ otherwise. The balance equation (on each point of a sub class) is written (depending if the associated Lagrange multipliers are zero or not):

$$C_{2i}^{-1}\overline{\lambda^2} + \sum_{j\in Le_i} QJR_{i,j} - \sum_{j\in Ar_i} QJR_{i,j} = b_i^2 \tag{5.17}$$

or
$$\sum_{j\in Le_i} QJR_{i,j} - \sum_{j\in Ar_i} QJR_{i,j} = b_i^3 . \tag{5.18}$$

Here $b_i^2 = b_i + DPM_i$ and $b_i^3 = b_i + DPM_i - EBC_i$ are known. But $\overline{\lambda^2}$ is still a variable.

b2) On a sub class where the second stage pressure equals zero, we have to minimize the third criteria with the following balance equation:

$$-DPM_i + \sum_{j\in Le_i} QJR_{i,j} - \sum_{j\in Ar_i} QJR_{i,j} = b_i^4 . \tag{5.19}$$

Point out that $b_i^4 = b_i - EBC_i$ is known because EBC_i is either zero, either equals to a bound.

Let λ^3 be the Lagrange multiplier associated with the balance equation. We have to treat sub-sub classes of constant sub sub pressures λ^3. The optimality equation for the DPM_i variables is written:

$$C_{2i} DPM_i + \overline{\lambda^3} + t_{DPM_i} - s_{DPMi} = 0. \tag{5.20}$$

Again, we deduce the elimination of DPM_i because it equals either $-C_{3i}^{-1}\overline{\lambda^3}$, either a bound. Depending on the case, for each point, the fourth stage balance equation can be written as:

$$C_{3i}^{-1}\overline{\lambda^3} + \sum_{j\in Le_i} QJR_{i,j} - \sum_{j\in Ar_i} QJR_{i,j} = b_i^5 \tag{5.21}$$

or
$$\sum_{j\in Le_i} QJR_{i,j} - \sum_{j\in Ar_i} QJR_{i,j} = b_i^6 . \tag{5.22}$$

Finally $\overline{\lambda^3}$ is a variable except if it is null.

5.1. Synthesis of the method

For each of the four stages, a convex quadratic program has to be solved, and the resulting communicating subnetwork has to be identified. For those with a nonzero pressure, all optimization variables are set to a given bound, and hence, the optimization process is finished. For those with a zero pressure, then for the first stage, the corresponding variable D_i is also equal to 0 by (4.5). For the second and third stages, we have that either the optimization variable of the corresponding stage is on a bound (if the corresponding Lagrange multiplier is nonzero) or is to be computed by either (5.16) or (5.20). In this way we obtain a stable computation of the minimum of the cost function. A tool implementing this method has been developed. It is used every day by GRTgaz. Numerical results given in the next parts come from this tool.

6. NUMERICAL RESULTS

6.1 Performances

Each balancing problem has 32 variables and initially 6 constraints (the use of the optimality systems method can add constraints along the stages of the optimization).

A balancing problem is associated with one shipper, therefore every day there are as many problems solved as there are shippers on the grid. The algorithm has to run fast. The calculation takes on average 20 ms by shipper on a 2.80 GHz PC with 1 Go RAM.

Tests have shown that without the optimality systems methods, five balancing calculations would have failed in 2005. Knowing that a shipper's daily penalties can reach hundreds of thousands of euros, defaults of billing must absolutely be avoided.

6.2 Balancing examples

Here is an example of absorption of a shipper's imbalance. Figure 1 gives a view of the tool. This example illustrates the spread of the λ_i "pressures" and identifies classes and sub classes.

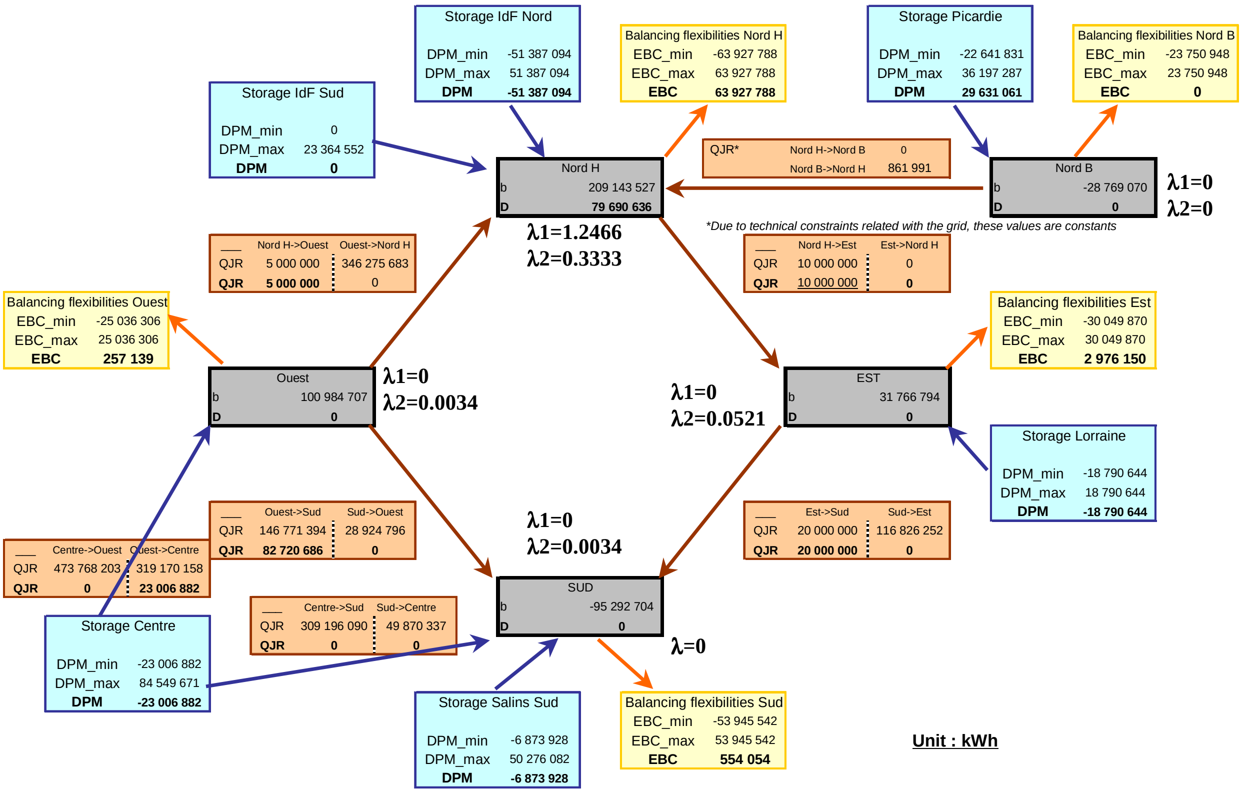

Figure 1. spread of the λ_i "pressures".

Zone Nord B is isolated because there is no transportation capacity arriving at this zone. Zone Nord H belongs to a saturated class. Zones Est, Sud and Ouest belong to the same non saturated class. Into this last class, Zone Ouest and Sud belong to the same 0.0034 sub pressure class. Zone Est belongs to another sub class.

7. CONCLUSION

Robustness and clarity of the model are useful, if not compulsory, features of industrial models, in particular for gas transportation. Balancing a gas grid is one of this kind. A method clearly explainable was found to solve it: the lexicographical optimization. The second feature was obtained by using a method based on an optimality systems approach: the Lagrange multiplier may be interpreted as a pressure; flows try to make equal pressure over the network. In the subset of nodes with zero pressure, called the zero pressure subset, a careful formulation of the higher stages problems is necessary in order to guarantee the robustness of computations. Tests have confirmed that such a method stabilizes the problem. The balancing tool is now used every day by GRTgaz and we estimate that about five balancing calculations a year would have failed without this method. That represents hundreds of thousands of euros.

In this period of markets opening, the European competition bodies and the national regulators pay a lot of attention to grid operators' behaviors. So it is very important that the tool is reliable because GRTgaz has to be equally fair towards all the shippers.

This robust lexicographical method can be very useful in the gas scope where multi-criteria optimization often occurs (optimization of a physical criteria, economical criteria…).

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