Planning reinforcement of gas transportation networks with optimization methods

Jean ANDRE
R&D Division, Gaz de France, 361 Av. du Pdt Wilson, 93211 St Denis, France, jean-dr.andre@gazdefrance.com

Frédéric BONNANS
Ecole Polytechnique, Palaiseau, France, and INRIA, B.P. 105, 78153 Le Chesnay, France, frederic.bonnans@inria.fr

Laurent CORNIBERT
GRTgaz, Courcellor 1-2 Rue Curnonsky - 75017 Paris, France, laurent.cornibert@grtgaz.com

ABSTRACT We consider the problem of minimizing the investment's costs on an existing gas transmission network under the condition that the demands of all users are satisfied with high enough gas pressure. We have to choose, for each pipe, if an additional parallel pipe is to be used. The size of its diameter is to be taken among a discrete set of values. The paper presents new heuristics to solve this large integer NLP problem, based on two main steps. The first one identifies a subset of pipes needing reinforcement in a continuous relaxation of the problem. A generalized potential formulation of the gas transportation networks including valves and compressor stations is introduced to initialize the resolution of this step. The second one chooses diameters in this subset using specific Branch & Bound algorithms. Throughout this work, we also show the limits of the policy "The bigger, the better" which is not always true on specific networks (known as the "Pillay's Paradox").

KEYWORDS Gas Networks; Dimensionning; Nonconvex Optimization; Branch & Bound Algorithms; Continuous.

1 Introduction

The French high-pressure natural gas transmission system consists of 31589 kilometers of pipelines (between 80 and 1000 mm in diameter). The Regional Networks embedded in this system, are high pressure networks (between 20 and 68 bar) which take gas from the Main Network (with compressor stations) to city distribution networks. Regional structures irrigate a limited geographical area and feed a substantial number of customers. Hence they are complex structures, made up of several pipe sections of different diameters. Each section must be adapted to the various conditions of flow and pressure.

In order to cope with increasing forecasted demand in gas, gas transportation companies need to plan the reinforcement of these regional transportation networks. This is an important issue and its aim is to determine the cheapest set of diameters to be added each year, insuring that user's demands will be satisfied with high enough pressure.

In this paper, we focus on the specific problem where we consider:

- a unique maximal scenario of demand (one stage problem),
- regional transportation networks which only include pipes and regulators (no compressor stations).

In the first part of this paper, we briefly recall the modelling of the problem. Then, we describe the method we used to solve a relaxed problem which provides a first solution of the problem after discretization. We further detail the main steps of the Branch & Bound algorithm which allows to improve the first point in reasonable computation times. To conclude, we present numerical results of these methods on real networks.

2 Modelization of the problem

The set of nodes of the network is denoted by N. With each node i are associated a pressure P^i and the energy head $\pi^i := (P^i)^2$. The set A of arcs is partitioned in a union of the set A_{pipe} of pipes and the set A_{reg} of regulators. With each arc are associated a flow Q_a and diameter DD_a. The latter is the control variable of Doubling Diameter (since the network is to be reinforced in laying out new pipes in parallel which is called looping). The value of diameter, for each arc a, is to be chosen among a finite set $\{\Delta_a^k\}$ including 0, and whose largest element is denoted by $\overline{\Delta_a}$. Let M denote the node-arc incidence matrix, whose (column) partition for pipes and regulators are called M_{pipe} and M_{reg}, respectively.

The resulting model is as follows:

$$(P) \quad \begin{cases} \min_{(DD,Q,\pi)} \sum_{a \in A_{cana}} c_a(DD_a) \\ \text{(i)} \quad DD_a \in \{0, \Delta_a^1, ..., \Delta_a^k, ..., \overline{\Delta_a}\}, \quad \text{for all } \ a \in A_{pipe} \\ \text{(ii)} \quad \pi_a^i - \pi_a^j = C_a.(DI_a^s + DD_a^s)^{-5/s}.Q_a.|Q_a|, \quad \text{for all } \ a \in A_{pipe} \\ \text{(iii)} \quad \underline{\pi} \leq \pi \leq \overline{\pi} \\ \text{(iv)} \quad M_{reg}^T \pi \geq 0 \\ \text{(v)} \quad 0 \leq \underline{Q_a} \leq Q_a \leq \overline{Q_a}, \quad \text{for all } \ a \in A_{reg} \\ \text{(vi)} \quad MQ = b \end{cases} \tag{1}$$

with $c_a(DD_a)$, a stepwise objective function, $s = 5/2$ and a constant C_a depending on the length of arc a. The constraints (ii) are the Darcy-Weisbach pressure drop equations on pipes. The constraints (iv) represent the fall of pressure on regulators. The constraints (vi) represent Kirchhoff'law of flow conservation. A wide number of papers dealing with of design of pipe networks (of either water or gas) are using meta-heuristics as genetic algorithms, see [1, 2, 15, 16]. The second class of papers are talking about methods based on continuous relaxation. In [7] (1991), Hansen, Madsen and Nielsen use a successive linear programming with "trust region". Their algorithm directly handles the discrete choice of diameter but each step (where the variation of diameter is continuous) needs a linearization of the objective function and constraints as well as a procedure to adjust the diameter to satisfy the lower bound on pressures. In [3] (1996), DeWolf and Smeers deal only with the continuous variables of diameter. They mix in one objective function the problem of minimizing the purchase gas costs on supply nodes with the minimization of the cost of looping which leads them to apply nondifferential methods as bundle methods. In [17] (1996), Zhang and Zhu propose a initial model similar to the one presented in this paper. However, their relaxation of the discrete constraints is quite different by assuming that they can lay out several sizes of diameter along a same section. With that relaxation, they can obtain a discrete solution with a proportion associated at each size. To get a solution, they reformulate the problem as a bilevel program and use trust-region methods. We can also note the works of Osiadack and Gorecki [12] (1995) who apply a sequential quadratic method on the continuous problem where the variables of flowrates are deleted in considering the speed of the gas on each pipeline as a constant. The continuous solution is rounded to the nearest discrete diameter.

This paper, clearly belonging to the second class of papers, differs in two ways from the above references. First, we consider the reinforcement problem for already existing networks. This increases a lot the nonconvexity of the model and the size of the problem. Second, we use a combination of continuous relaxation and B & B algorithm.

The nonconvexity of program (P) is due to the discrete values of diameters and to the nonlinearity of the pressure loss equations.

In order to reduce the nonlinearity of the pressure drop equation, we make the following change of variables: instead of reinforcement diameters, we use "equivalent diameters", whose expression is[1]

[1]With two pipes in parallel (1 & 2) and the gas flowing in the same direction for both pipes, we can write $Q_a = ((\pi_a^i - \pi_a^j).C_a^{-1}.D_a^5)^{1/2}, a = 1, 2$. Substituting these expressions of flows in the following expression $Deq = (C_a.(\pi_a^i - \pi_a^j)^{-1}.(Q_1 + Q_2)^2)^{1/5}$, we obtain the equivalent diameter formula.

$Deq_a := (DI_a^s + DD_a^s)^{1/s}$. The program (P) can be rewritten as follows:

$$(P) \qquad \begin{cases} \min_{(Deq,Q,\pi)} \sum_{a \in A_{pipe}} \tau_a(Deq_a) \\ \text{(i)} \qquad Deq_a \in \{DI_a, Deq_a^1, ..., Deq_a^k, ..., \overline{Deq_a}\}, \quad \text{for all} \quad a \in A_{pipe} \\ \text{(ii)} \qquad \pi_a^i - \pi_a^j = C_a.Deq_a^{-5}.Q_a.|Q_a|, \quad \text{for all} \quad a \in A_{pipe} \\ \text{(1-iii) to (1-vi)} \end{cases} \tag{2}$$

with $\tau_a(Deq_a) = c_a((Deq_a^s - DI_a^s)^{1/s})$. The resulting program is (again) a mixed nonlinear, nonconvex program, having discrete variables Deq and continuous variables π and Q.

3 Obtaining a first solution:Continuous relaxation

3.1 Relaxed program

The continuous relaxation allows all Deq_a values to be selected within a certain range:

$$Deq_a \in [DI_a, \overline{Deq_a}], \quad \text{for all} \quad a \in A_{pipe}$$

We extend function τ_a to a function over $[DI_a, \overline{Deq_a}]$ denoted also τ_a, whose value over $]Deq_a^k, Deq_a^{k+1}]$ is $\tau_a(Deq_a^{k+1})$. This stepwise function of (P) $\tau_a(Deq_a)$ can be approximated by a continuous concave function:

$$\phi_a(Deq_a) = \alpha_a.(Deq_a^s - DI_a^s)^{1/s}$$

with $\alpha_a = c_a(\overline{DD_a})/\overline{DD_a}$ which corresponds to the slope of the linear approximation of the stepwise function when the initial diameter is equal to 0.

The continuous program which provides a first continuous solution $Deq^\star$ is the following:

$$(P_{rel}) \qquad \begin{cases} \min_{(Deq,Q,\pi)} \sum_{a \in A_{pipe}} \alpha_a.(Deq_a^s - DI_a^s)^{1/s} \\ Deq_a \in [DI_a, \overline{Deq_a}], \quad \text{for all} \quad a \in A_{pipe} \\ \text{(2-ii),(1-iii) to (1-vi)} \end{cases} \tag{3}$$

Figure 1 illustrates the continuous approximation on the discrete costs of two pipes (one with an initial diameter equal to 0 and another with an initial diameter equal to 500 mm).

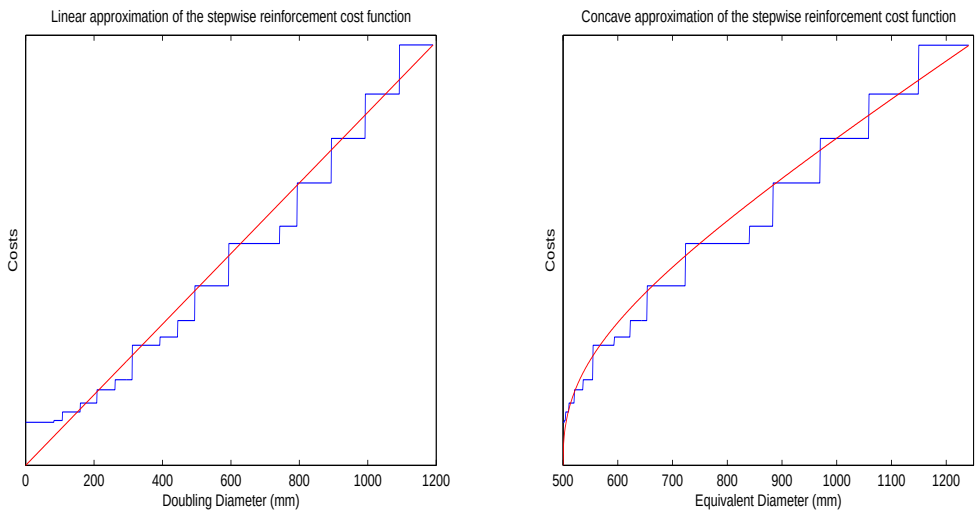

Figure 1: Relaxed objective functions

With the concave approximation, we can observe the decrease of marginal costs according to the increase of the diameter.

To get a first discrete solution, we round the relaxed diameter $Deq^\star$ up to the nearest discrete diameter.

3.2 Convex case

This relaxed program is convex if both following assumptions hold:

- *with a design problem* which is a particular case of the reinforcement of an existing network. We consider the initial network as an unbuild network whose layout is known and replace the initial diameter with 0 to solve the design problem. In this case, the objective funcion of (P_{rel}) is:
$$\phi(Deq, Q, \pi) = \sum_{a \in A_{pipe}} \alpha_a . Deq_a$$
With design problems, the objective function becomes linear.

- *with tree-networks* whose graph has no loop. The values of flows can be deduced from the flow equations, supplies and demands. Denoting by $\eta_a := \beta_a . L_a . Q_a . |Q_a|$ the (known) contribution of flows to the pressure drop equations, we can write the latter as:
$$\pi_a^i - \pi_a^j = \frac{\eta_a}{Deq_a^5}, \quad \text{for all } \ a \in A_{pipe} \tag{4}$$

Using the change of variables $\delta_a ; = 1/Deq_a^5$, we may rewrite program (P_{rel}) as the following convex program:

$$(P_{rel}) \qquad \begin{cases} \min_{(\delta,\pi)} \sum_{a \in A_{pipe}} \alpha_a . \delta_a^{-1/5} \\ \delta_a \in [\frac{1}{\overline{Deq_a}^5}, +\infty[, \pi_a^i - \pi_a^j = \eta_a . \delta_a, \quad \text{for all } \ a \in A_{pipe} \\ \text{(2-ii),(1-iii) to (1-vi)} \end{cases} \tag{5}$$

The convexity of (P_{rel}) follows from the linearity of constraints and the convexity of the cost function (whose domain is $\mathbb{R}_+$).

3.3 Search of an initial point

3.3.1 Search of a feasible point

The "bigger is better" principle Doubling diameters allows to reduce pressure losses. The following principle of monotonic behaviour is therefore often assumed: "the more we reinforce a network, the better the constraints (of maximal pressure for suppliers and minimal pressure for consumers) are satisfied". There are significant counter examples to this principle, see the "Pillay's paradox" [14]. Yet we consider it to be true for most practical applications. As a consequence, in order to obtain a feasible point of problem (P), it suffices to set all diameters to their maximal values and solve the resulting flow problem. Let $K_a = C_a / (\overline{Deq_a})^5$.

Simple potential formulation Once the diameters are set, we have to determine flows and pressures associated to these diameters. On a network only made up of pipes, we have at our disposal the simple potential formulation already used by Maugis ([10]), De Wolf & Smeers ([4]) and Zhang & Zhu ([17]) whose we recall the main features.

The simple potential formulation allows to meet exactly the pressure drop equation on the pipes thanks to a convex program aiming to minimize the potential energy of the network:

$$(P_{pot}) \qquad \begin{cases} \min_{(Q)} \sum_{a \in A_{pipe}} F_a(Q_a) \\ MQ = b \end{cases} \tag{6}$$

with the potential function: $F_a(Q_a) = \frac{1}{3} . K_a . |Q_a|^3$.

The variables of squared pressures correspond to the dual variables of this program. Denoting π^i the dual variables (which can also be called potentials) associated to the flow balance equations at each node, the optimality conditions of the Lagrangian are written:

$$K_a.Q_a|Q_a| - \pi_a^i + \pi_a^j = 0, \quad \text{for all} \quad a \in A_{pipe}$$

Generalized Potential Formulation To simplify the presentation, we introduce a general variation (loss or gain) load law and consider later some particular cases. It is easier to introduce a generalized potential formulation:

$$\min_Q \sum_{a \in A} G_a(Q_a); \quad \text{such as} \quad M.Q = b. \tag{P_b}$$

where the potential function $G_a(Q_a)$ is convex, lower semi continuous.

Problem P_b is said to be stable (at point b) if, when b' is close to b (of sum equal to zero), then the problem $P_{b'}$ is feasible.

Lemma 3.1 *We assume that problem P_b is stable. Then any solution Q of this problem is characterized by the existence of Lagrange multipliers Π (called potentials) where*

$$\partial G_a(Q_a) - \pi_a^i + \pi_a^j \ni 0, \quad a \in A. \tag{7}$$

Proof: The cost function being strictly convex and continuously differentiable, and the constraints being linear, the existence of Lagrange multipliers is a standard result from convex analysis (see e.g. Thm 2.168 in [13]).

Under these assumptions, we know that potentials are uniquely determined, up to a positive constant.

The *valve* giving the constraint $Q_a \geq 0$ is equivalent to : $G_a(Q_a) = 0$, if $Q_a \geq 0, +\infty$ if not. We have $G_a(Q_a) = 0$ if $Q_a > 0$, and $G_a(0) =]-\infty, 0]$. The relation (7) is therefore equivalent to

$$Q_a \geq 0, \quad \pi_a^j \geq \pi_a^i, \quad \pi_a^j = \pi_a^i \quad \text{if } Q_a > 0 \tag{8}$$

which simulates well the behaviour of a valve.

Regulators and compressor stations allow to adjust pressures. We can introduce a "virtual" fit of pressure with the function: $G_a(Q_a) = \gamma_a Q_a$, which allows to impose the differential of pressure: $\gamma_a + \pi_a^j - \pi_a^i = 0$. If $Q_a \geq 0$, we deal with a compressor station if $\gamma_a < 0$ and with a regulator if $\gamma_a > 0$.

More generally, we will have *compression* when $G_a(Q_a)$ is nondecreasing on $\mathbb{R}_-$ and nonincreasing on $\mathbb{R}_+$ (the subdifferential appearing in (7) is then negative) and *expansion* if $G_a(Q_a)$ is nonincreasing on $\mathbb{R}_-$ and nondecreasing on $\mathbb{R}_+$.

Generalized potential formulation applied to regulators Including the regulators, we use the generalized potential formulation as follows:

$$(P_{pot-reg}) \qquad \begin{cases} \min_{(Q)} \sum_{a \in A_{pipe}} \frac{1}{3}.K_a.|Q_a|^3 \\ \text{(1-v),(1-vi)} \end{cases} \tag{9}$$

The behaviour of such regulators is the one of a valve (setting a minimum value of the flow) with a regulator constraining the flow to be below $\overline{Q_a}$.

The generalized potential formulation can also include the mandatory pressure loss through the regulators:

$$P_a^i \geq P_a^j + \Delta P_a^{fat}, \quad \text{for all} \quad a \in A_{reg} \tag{10}$$

with ΔP_a^{fat} is the minimal pressure drop (in bar) which has to be always applied on this element. It deals with the determination of the parameter $\gamma_a > 0$ which allows to meet the requirement (10). In

particular, if we choose $\gamma_a = \Delta P_a^{fat}.(\overline{P_a^i} + \overline{P_a^j})$, then the constraint (10) is satisfied. The program to be solved is the following:

$$(P_{pot-reg}) \qquad \begin{cases} \min_{(Q)} \sum_{a \in A_{pipe}} \frac{1}{3}.K_a.|Q_a|^3 + \sum_{a \in A_{reg}} (\Delta P_a^{fat}.(\overline{P_a^i} + \overline{P_a^j})).Q_a \\ \text{(1-v),(1-vi)} \end{cases} \tag{11}$$

Recovering bound constraints on pressures The main drawback of the potential formulation is to provide a set of pressures satisfying the pressure loss equations, but not the bounds on the pressures. The reason is that we cannot control the range of variation of the dual variables of the potential formulation.

We introduce then a new program aiming to minimize the slack variable T corresponding to the maximum of the gaps between the actual pressures and the pressure bounds in order to recover feasibility:

$$(P_{feas}) \qquad \begin{cases} \min_{(Q,\pi,T)} T \\ K_a.Q_a|Q_a| - \pi_a^i + \pi_a^j = 0, \forall a \in A_{pipe} \\ \pi^i - \overline{\pi^i} \leq T \\ \underline{\pi^i} - \pi^i \leq T, \forall i \in N \\ T \geq 0 \\ \text{(1-iv) to (1-vi)} \end{cases} \tag{12}$$

A feasible point of this program (P_{feas}) is provided by the generalized potential formulation.

Then, if a feasible point exists for the initial relaxed program (P_{rel}), the variable T will be equal to 0 (thanks to the positivity constraint on T).

3.3.2 Determination of a "good" initial point

Once we get a point $x^1 = (\overline{Deq}, Q^1, \pi^1)$ fulfilling all constraints of program P_{rel}, we can search for a (local) optimum using a nonlinear solver.

Note that, this first feasible point x^1 is the worst possible, since it gives the highest reinforcement cost. We can assume that this point will be quite far from the global optimum. Thus, we try to improve this initialization by generating another feasible point x^2 from x^1.

In order to achieve this goal, we relax the program (P_{rel}) in (3) by minorizing its cost function by a linear one. The idea is to delete the nonlinearity in the goal and to approach the objective function from below to get near the optimum of the concave function. This idea has been freely adapted from the idea developped in the book by Horst & Tuy ([8]) for the minimization of concave function under linear contraints where they approach concave function with piecewise linear functions.

Hence, denoting X_{rel}, the feasible set of P_{rel}, $x^2 = (Deq^2, Q^2, \pi^2)$ is the local solution of the program below with x^1 as an initial point:

$$(P_{rel}^{(L)}) \qquad \begin{cases} \min_{(Deq,Q,\pi)} \sum_{\in A_{pipe}} \alpha_a.(Deq_a - DI_a) \\ (Deq, Q, \pi) \in X_{rel} \end{cases} \tag{13}$$

$(P_{rel}^{(L)})$ is a nonconvex program (since X_{rel} remains nonconvex) and is solved with a nonlinear solver as SNOPT [5]. The advantages of using SNOPT in this framework are detailed in part 5.

4 Branch & Bound algorithms

Maugis [10] already suggested the use of a Branch & Bound method for solving the design problem, although he gave little information about implementation or numerical efficiency.

The Branch & Bound algorithm will be used here to improve the first solution given by (P_{rel}). The exploration space of (P) is reduced to the pipes which have been proposed to be reinforced by (P_{rel}) i.e. $A^r_{pipe} = \{a \in A_{pipe} : Deq^*_a > 0\}$ with $|A^r_{pipe}| = nar$, the number of pipes to reinforce. We have nowadays to determine which are the optimal discrete diameters to lay out on these nar pipes among a list of commercially available sizes (considering $\Delta^0_a = 0$ as a commercial diameter). The program we propose to solve with a B&B algorithm is the following:

$$(P_{red}) \qquad \begin{cases} \min_{(DD,Q,\pi)} \sum_{a \in A_{pipe}} c_a(DD_a) \\ DD_a \in [\Delta^0_a, \Delta^1_a, ..., \Delta^k_a, ..., \overline{\Delta_a}], \text{for all } a \in A^r_{pipe} \\ DD_a = 0, \text{for all } a \in A_{pipe} \setminus A^r_{pipe} \\ \text{(1-ii) to (1-vi)} \end{cases} \tag{14}$$

4.1 General principles

Let DD^{ini} be the solution obtained after rounding the solution Deq^* up to the nearest commercial diameters:

$$DD^{ini}_a = \Delta^{ini}_a, \forall a \in A^r_{pipe}$$

We have at our disposal a first solution $DD^{max} = DD^{ini}$ which gives a maximal evaluation of the reinforcement cost c^{max}. The construction of the tree is based on two steps:

1. Split a set into smaller subsets (**Branch**) in order to make the exploration easier. For this aim, we will choose a separation variable DD_a, which can take each of the possible values $DD_a = \Delta^0_a$,$DD_a = \Delta^1_a$,$DD_a = \Delta^2_a$,...that are called the nodes of the tree. We have chosen to set *a priori* the exploration order *by sorting the pipes in decreasing costs*:

   $$\overline{c_{a_1}} \geq \overline{c_{a_2}} \geq ... \geq \overline{c_{a_{nareinf}}}$$

   with $\overline{c_a} = c_a(\overline{\Delta_a})$, the maximal reinforcement cost of the pipe a. This ranking has the advantage to make "important" choices high in the tree and, hence, to cut bigger part of the space of combinations. The exploration of the leaves will be done *in ascending order of the discrete diameters*:

   $$DD_a = \Delta^0_a < \Delta^1_a < ... < \overline{\Delta_a}$$

   The exploration beginning from the smallest diameters should lead quickly to "good" updates of the upper bound since the smallest diameters are the cheapest as well.

2. At a node of the tree, assess the consequences *a minima* on the cost of the choices done higher in the tree (**Bound**). Let denote $(A^r_{pipe})'$, the set of pipes on which the diameter is set and $(A^r_{pipe})''$, the set of pipes on which the diameter is undefined. At each evaluation of a subset, the minimal bound will be calculated with the resolution of a program of type (P_{feas}) with the diameter set in this way:

   $$\begin{cases} Deq_a = DI_a, \text{ for all } a \in A_{pipe} \setminus A^r_{pipe} \\ Deq_a = Deq^{k'}_a, \text{ for all } a \in (A^r_{pipe})' \\ Deq_a \in [DI_a, \overline{Deq_a}], \text{ for all } a \in (A^r_{pipe})'' \end{cases} \tag{15}$$

   Hence, the calculation of a minimal bound is actually a feasibility test when we set all the unfixed diameters to their maximal diameters. This choice is based on the assumption that the "Pillay's Paradox" is not verified i.e. if there is unfeasibility for this set of maximal diameters, then all combinations of lower diameters will be unfeasible.

   If the objective function of program (P_{feas}) is equal to zero, there is at least one point satisfying the set of constraints and the cost $c'_{min} = \sum_{a \in (A^r_{pipe})'} \tau_a(Deq^{k'}_a)$. If not, there is no feasible point and the cost c'_{min} takes the value $+\infty$.

4.2 Reduced Branch & Bound

We propose here to use the initial solution DD^{ini} not only to determine the set A^r_{pipe} but also to obtain an information on the optimal diameters to lay out on the arcs $a \in A^r_{pipe}$.

A neighbourhood study of the initial solution can be done using a Branch & Bound algorithm with a reduced number of discrete diameters around DD^{ini} in order to strongly reduced the size. These reduced Branch & Bound are presented below according to the increasing size of the space of combinations to explore :

1. **B&B 1**: for each pipe, we test if we lay out or not a reinforcement (binary choice) with the proposed diameter by DD^{ini} (2^{nar} combinations):

$$DD_a \in [\Delta^0_a, \Delta^{k_{ini}}_a], \quad \text{for all} \;\; a \in A^r_{pipe}$$

2. **B&B 2**: for each pipe, we test if we lay out or not the proposed diameter by DD^{ini} as well as the immediate smaller diameter in the commercial list (3^{nar} combinations):

$$DD_a \in [\Delta^0_a, \Delta^{k_{ini}-1}_a, \Delta^{k_{ini}}_a], \quad \text{for all} \;\; a \in A^r_{pipe}$$

3. **B&B 3**: for each pipe, we test the same choices as B& B 2 widening the choice to the nearest bigger diameter to the solution DD^{ini} (4^{nar} combinations):

$$DD_a \in [\Delta^0_a, \Delta^{k_{ini}-1}_a, \Delta^{k_{ini}}_a, \Delta^{k_{ini+1}}_a], \quad \text{for all} \;\; a \in A^r_{pipe}$$

5 Numerical Results

5.1 Use of SNOPT

The nonlinear solver used to solve the programs $P_{pot-reg}$,P_{feas}, $P_{rel}^{(L)}$ and P_{rel} is SNOPT 7 (developed by the Systems Optimization Laboratory of Stanford University) called through MATLAB within the TOMLAB 4.8 framework. SNOPT is a well-established sequential quadratic programming (SQP) code, which is designed to work with sparse data structures, so it can handle large-scale problems. SNOPT is highly effective for problems with a nonlinear objective function and large numbers of sparse linear constraints as Kirchhoff'laws of flow conservation on networks. However, the main weakness of these techniques used in an nonconvex framework is the need to provide an initial point that respects the nonconvex equations. Fortunatly, to solve P_{feas}, we bypass this weakness thanks to the initialization given by the potential formulation $P_{pot-reg}$ (which doesn't need any initial point since all the constraints are linear). Moreover, this weakness can be transformed as a strength in order to reach a better local optimum. Hence, as we consider that the solution of $P_{rel}^{(L)}$ is a good guess as an initial solution for P_{rel}, all this information is used in the resolution of P_{rel} with SNOPT. Further details on the algorithm can be found in [5, 6].

5.2 Study on a 2 pipe-network

In order to outline the nonlinearity and the nonconvexity of the cost function, a simple example has been set up. Let us consider the following network:

- two successive pipes of equal lengths and initial diameters (400 mm and 50 km),
- a supply node with a maximal pressure of 45 bar,
- two delivery nodes located one in the middle, the other at the end of the line (with an equal demand of 174102 m^3/h). The pressure on these nodes must be kept above 20 bar,

As 17 diameters are available between 0 and 1200 mm, the number of combinations to evaluate is 17x17=289 combinations. In this case, the enumeration is possible and gives an optimal cost of 32150 kEuros with a doubling diameter of 500 mm on arc 1 and of 300 mm on arc 2.

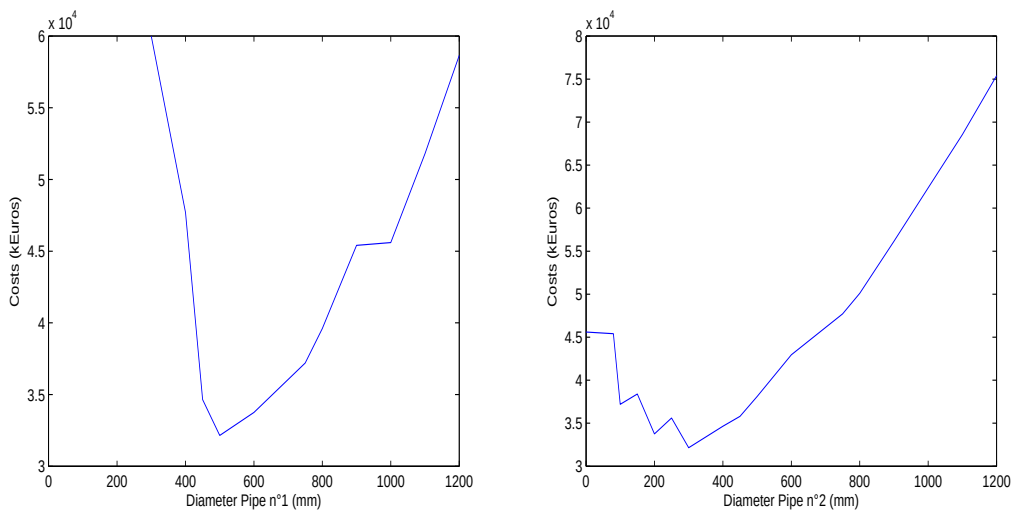

Figure 2: Discrete reinforcement cost on the 2 pipe-network

The continuous relaxation proposed in the part 3 allows us to determine a continuous local minimum.

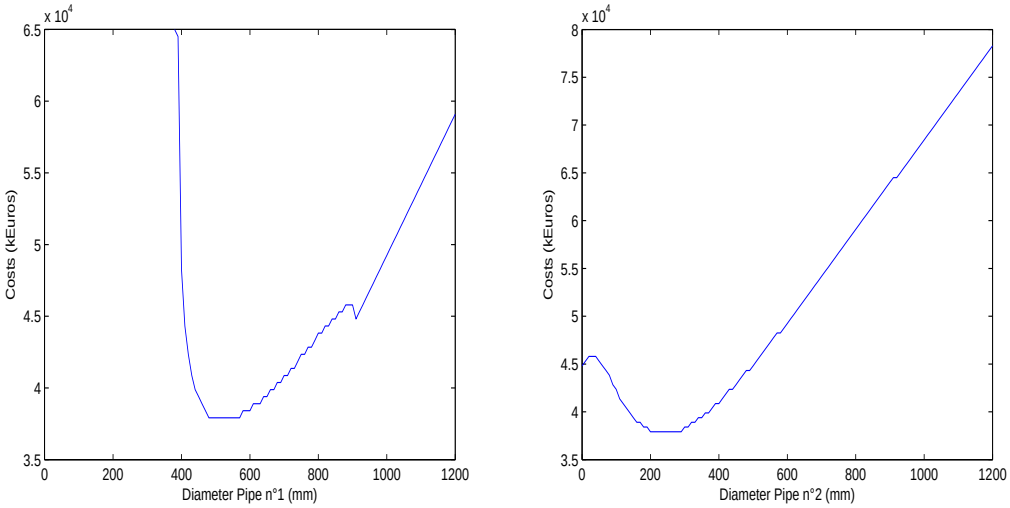

Figure 3: Continuous reinforcement cost on the 2 pipe-network

At the end of this step, taking the continuous minimum (533,255), the first discrete solution rounding these values up to the nearest commercial diameters (600,250) gives a cost of 35600 kEuros. The B&B 1 doesn't modify the solution since we cannot reduce to zero neither arc 1 nor arc 2 while keeping these diameters. The B&B 2 proposes to put a diameter of 200 mm on the arc 2 (instead of 250) what brings down the cost to 33750 kEuros. The B&B 3 reachs the best solution because it is necessary to increase the diameter of arc 2 from 250 to 300 mm and to decrease the diameter of arc 1 from 600 to 500 mm in the same time.

5.3 Results on real networks

We have tested the methods described in this paper on a wide number of regional networks based on high scenarios of demand (since taking account the growth of the consumption over the next 20 or 30 years). Figure 4 shows the topology of two of them.

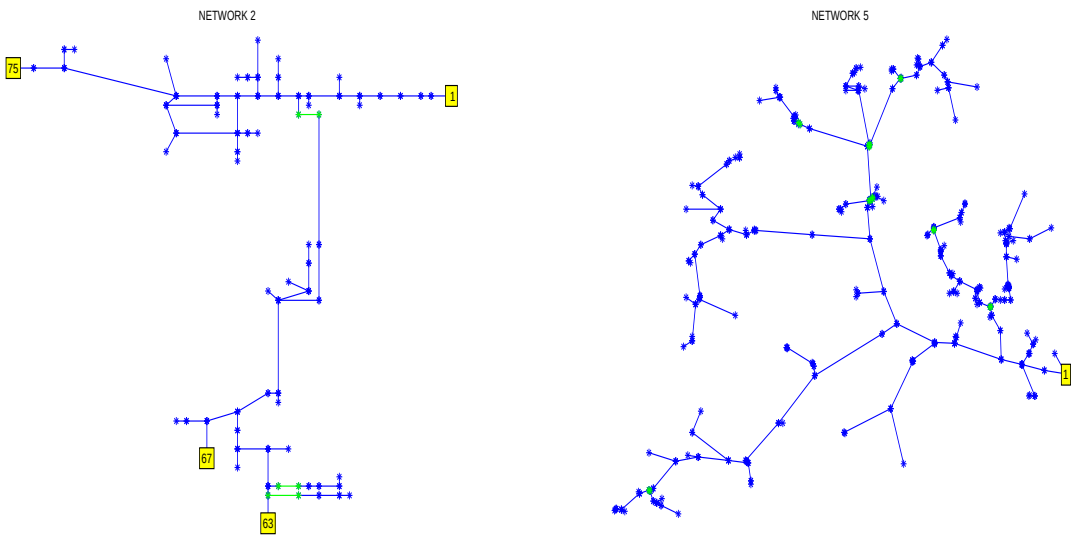

Figure 4: Examples of real networks

We present below the results on 9 networks in terms of cost and computation times with the following indicators for each network:

- features of the networks (L, the total length, na, the number of arcs and nn the number of nodes). The number of loops nl can be deduced from Euler's formula: $nl = na - nn + 1$ when nl is positive,
- nar, the number of selected arcs by (P_{rel})
- C_r^{ini}, the cost of the discretized solution of (P_{rel}) and the associated calculation times,
- Costs of solutions given by B&B 1,2,3 and the associated calculation times.

The results on real networks are sorted according to the number of selected pipes nar which are critical for the B & B calculation times. The computation times (obtained with an Intel processor Penthium 4 2.66 GHz) are given as a rough guide in order to assess the exponential growth of the times with regard to the size of the problem.

	Features			(P_{rel})+Rounding			B&B 1		B&B 2		B&B 3	
Networks	L (km)	nn	na	nar	C_r^{ini} (kE)	Times (s)	Costs (kE)	Times (s)	Costs (kE)	Times (s)	Costs (kE)	Times (s)
Net_{01}	91	17	16	2	2131	8	2131	0.5	2131	1	2131	1
Net_{02}	73	75	76	5	2484	4	2484	1	1835	6	1835	30
Net_{03}	90	71	70	7	2799	17	2776	2	2397	14	2389	213
Net_{04}	366	88	90	11	15497	5	9050	99	7795	1890	*	*
Net_{05}	718	243	250	11	27162	44	23265	23	20750	839	*	*
Net_{06}	248	38	37	12	18941	4	16547	5	16216	339	*	*
Net_{07}	137	30	29	16	16519	8	13618	25	13563	791	*	*
Net_{08}	129	55	54	17	29848	4	28772	3	27297	506	*	*
Net_{09}	1898	362	371	17	23697	1607	18789	1011	*	*	*	*

(*) means that the computation times have exceeded the maximal bound of 3600 s.

We can see on these results that:

- the selection of arcs to reinforce is rather small compared with the total number of pipelines. It means that the (local) optimal solution provided by the relaxation is focused on a small set of pipelines with high enough diameters. In other terms, it represents the economy of scale given by the concave cost function seen in part 3.1,
- for the selection of arcs $(P_{rel}) + Rounding$, we notice that the time to get a first solution is very small (whithin one minute) on most of the networks. However, the effectiveness of SNOPT is seriously affected above 300 arcs (which represent about one thousand variables including diameters, flowrates and pressures) and in particular on Net_{09}. This fact is consistent with the documentation of SNOPT [5] which notes that the speed is strongly reduced if the number of variables exceeds one or two thousands as long as we have a wide number of nonlinear constraints,
- the number of loops (between 0 and 10 depending on the network) has no impact on the computation times,
- the B & B algorithms can yield a lot of savings in comparison with the cost of the relaxation (up to 50% saved). These important gains can be partly explained by the overestimation given by the rounding of the continuous relaxation which creates unusefull capacities on the networks. In most cases, the costs of proposed solutions are improved at each time we widen the search including new diameter in the list but we cannot state that we have reached the global optimum of our problem (even though the costs remains equal for each B &B),

- for a same B&B, the CPU times are rather different for a network from another. These differences are directly tied to the role of the more expensive pipes on the networks (not easy to predict). The choice of diameters on these pipes can cut the search tree at very different levels, and hence, can have a weak or strong impact on the algorithm's efficiency. We can also see that the exponential growth of CPU times (exceeding one hour) are a real barrier to carry on our extension of the list of diameters and to get better solutions.

6 Conclusions & Discussions

The techniques presented in this paper are suitable to obtain a reinforcement cost and a proposal of diameters on existing regional networks of various sizes within reasonable computation times.

However, we had to reduce the number of proposed diameters to keep these computation times acceptable. A planned improvement is to include the whole commercial list in the Branch & Bound. For this goal, new lower bounds, more effective than the feasibility test, should be set up with the help of convex or Lagrangian relaxations.

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