Optimal control of a gasoline-fueled car engine under pollution constraint

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November 29, 2002

Conférence CANUM 99, Ax-Bonascre (France), mai 1999

Abstract

The paper deals with the problem of optimal control of a gasoline-fueled car engine, that uses a new technology called of catalytic converters called "NOx trap". The aim is to minimize fuel consumption, with constraints on NO_x emissions. The model is an impulse optimal control problem that is solved by a dynamic programming type algorithm. We present the model, discuss the discretization scheme and analyse numerical results.

1 Context of the study

In recent years, a new technology of gasoline engines was introduced in order to decrease fuel consumption. This technology, called "lean-burn engine", uses an air-fuel mixture which contains less fuel than in a classical engine. The air-fuel mixture of an engine is characterized by a quantity called "Normalized Fuel-Air Ratio" (NFAR) given by:

$$R = \frac{\text{fuel mass / air mass}}{\text{fuel mass / air mass}_{\text{stoichiometric}}}.$$

Here, "stoichiometric" means that the combustion is complete. The value of R is close to 1 for a classical engine, and less than 1 for a lean-burn engine.

The drawback of lean-burn engines is high NOx (nitrogen oxide) harmful emissions, especially when R is far from 1. A possible remedy is to use a after-treatment system which contains a new type of catalytic converters (catalyst) called "NOx trap".

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1.1 Operation on a NOx trap catalyst

The NOx trap catalyst has two operation modes:

- Storage mode: in lean condition (NFAR ≤ 1), the catalyst stores some of the emitted NO_x. It operates as a "non-ideal tank": the higher the tored quantity, the lower the proportion of incoming pollutants that are stored. In addition, the tank size depends highly on temperature. The device does not work when temperature is too low or too high.

- Regeneration mode: in rich condition (NFAR set to some value greater than 1), the stored pollutants are partially eliminated. The regeneration is quite fast with respect to the storage process, although it can last several seconds.

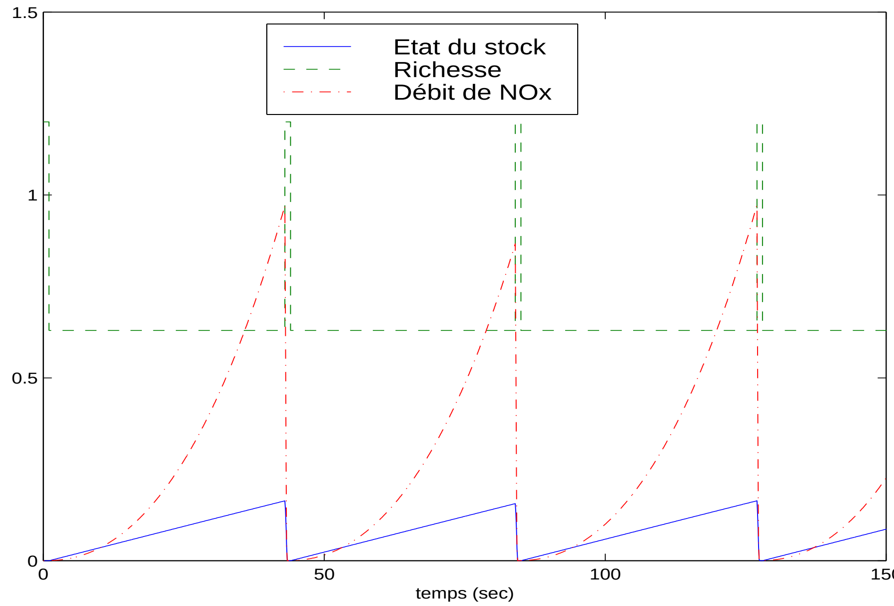


Figure 1: Typical behavior of the NOx Trap catalyst

1.2 Relative advantages of operation modes

- The lean mode allows a lower fuel consumption, but produces high NO_x emissions.
- The rich mode allows to empty the catalyst but needs a high fuel consumption.
- The stoichiometric mode does not allow lowering fuel consumption, but does not produce high NO_x emissions.

The problem is to find the optimal strategy which minimizes the fuel consumption while respecting the standard value of NO_x emissions. ¿From the above discussion, we can infer that the stoichiometric mode should be used when the temperature of the catalyst is too low or too high. Otherwise one has to use the lean mode, untill the storage level is so high that the inciming pollutant is not stored enough. Then one has to use the rich mode.

Such a strategy, however, needs a careful tuning for: (i) the value of R in the lean mode, (ii) choosing at what times the rich mode should be activated, (iii) choosing the duration of the rich mode when activated.

For a stationary behavior (constant values of N and C), this tuning is already a nontrivial operation since it has to be done for each possible value of engine revolution N and torque C. Note that the engine controller may have effects on the temperature of the engine, and hence of the controller.

In addition, for a real (non stationary) driving profile, the storage process is slow enough for interacting with changes of N and C.

1.3 Mathematical tools

When controlling classical thermic motors, the long term dynamics are negligible in the sense that that there is no need to forecast what will happen in the future. Here the situation is quite different, since the knowledge of future is used for choosing when making the catalyst empty. In real conditions, the future is quite uncertain. However, in order to have an idea of the improvement brought by this new technology, it makes sense to assume the future to be perfectly known and to compute the corresponding (deterministic) optimal control strategy.

2 System modeling

In this section we set the mathematical model of the catalyst converter. We do not enter into details of nonlinear functions that appear in engine modeling of converter dynamics, since there expression is complicated. We prefer to use a compact notation that gives a global idea of the study. The structure of the model is given in figure 2.

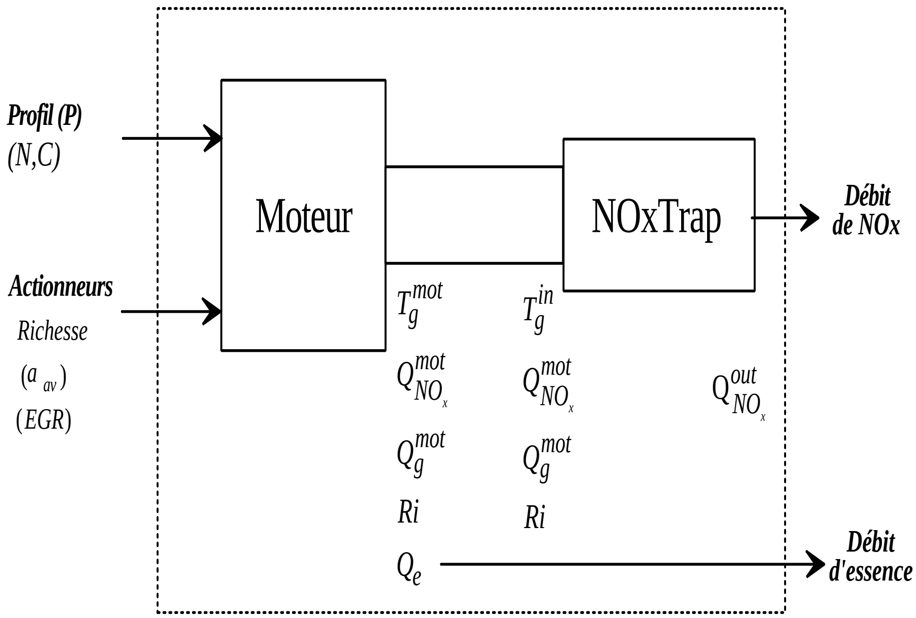


Figure 2: Structure of the model

Here are its main features:

- The driving profile $\mathcal{P}$ is given, and described by the variation versus time of the engine revolution N and its torque (some European normalized cycle for example).

- The control variables are R (NFAR) for the lean mode, and the initial times and duration of stoichiometric and rich modes.

- The engine is described by a nonlinear static model:

$$\left\{\begin{array}{lcl} T_g^{\text{mot}} & = & F_1(N,C,R), \\ Q_{NO_x}^{\text{mot}} & = & F_2(N,C,R), \\ Q_g^{\text{mot}} & = & F_3(N,C,R), \\ Q_e & = & F_4(N,C,R). \end{array}\right. \tag{1}$$

- The exhaust pipe is described by a non-linear dynamic model:

$$\dot{T}_g^{\text{in}} = G(T_g^{\text{in}}, T_g^{\text{mot}}). \tag{2}$$

- The catalyst is described by a nonlinear dynamic model with two state variables, M_s, mass of stored NO_x, and M_e, mass of exhausted NO_x:

$$\left\{\begin{array}{lcl} \dot{M}_s & = & f_1(\cdots), \\ \dot{M}_e & = & f_2(\cdots). \end{array}\right. \tag{3}$$

Although the model reduces to a single differential system, each operation mode is described by a specific dynamics:

$$
\begin{array}{ll}
\underline{R<1}: & \left\{\begin{array}{lcl} \dot{M}_s & = & \varphi_1(M_s, T_g^{\text{in}}, Q_g^{\text{mot}}, Q_{NO_x}^{\text{mot}}), \\ \dot{M}_e & = & Q_{NO_x}^{\text{mot}} - \varphi_1(\Theta, T_g^{\text{in}}, Q_g^{\text{mot}}, Q_{NO_x}^{\text{mot}}), \end{array}\right. \\
\underline{R=1}: & \left\{\begin{array}{lcl} \dot{M}_s & = & 0, \\ \dot{M}_e & = & 0, \end{array}\right. \\
\underline{R>1}: & \left\{\begin{array}{lcl} \dot{M}_s & = & \varphi_2(\Theta, T_g^{\text{in}}, Q_g^{\text{mot}}, Q_{NO_x}^{\text{mot}}), \\ \dot{M}_e & = & 0. \end{array}\right.
\end{array}
\tag{4}
$$

Note that $\varphi_2(\cdot)$ has nonpositive values, whereas $\varphi_1(\cdot)$ may have positive or negative values, but is always less than $Q_{NO_x}^{\text{mot}}$. Therefore the derivative of M_e is always nonnegative, whereas the one of M_s can change of sign.

3 Optimization problem formulation

The total system (engine + exhaust pipe + catalyst) can be described by:

- 2 state variables : $x_1 = M_s$ and $x_2 = M_e$,
- 1 control variable : $u = R$,
- 2 output variables : $y_1 = Q_e$ and $y_2 = M_e$.

The state equation (dynamics) are as follows:

$$
(\mathcal{M}) \qquad \left\{\begin{array}{lcl} \dot{x}_1 & = & f_1(t, x_1, x_2, u), \\ \dot{x}_2 & = & f_2(t, x_1, x_2, u), \\ y_1 & = & g(t, u). \\ y_2 & = & x_2. \end{array}\right.
$$

The optimization problem is

$$
(P) \qquad \min_u \int_0^T g(t,u)\, dt \quad \text{subject to} \quad (\mathcal{M}) \quad \text{and} \quad x_2 \leq M.
$$

Note that $x_2 \geq 0$. Also, $x_1 \geq 0$ and upper bounds on x_1 can be easily computed using the storage model. Hence the state variables have both natural lower and upper bounds.

Problem (P) enters in the framework of the theory of optimal control. In addition, since there are only two bounded state variables, and a finite horizon, it is straightforward to solve it by a dynamic programming type algorithm. But before that we will slightly change the formulation of the problem, in order to take into account some specific aspects of the regeneration: (i) The decision of purging is followed by a delay, due to the distance between the engine and catalyst converter. (ii) The decision is to regenerate during a certain duration, and hence is not local in time. (iii) The duration of regeneration is small with respect to the duration of storage (the engine dynamics is neglected). (iv) Given storage and duration, the storage after regeneration can be readily computed.

For these reasons, it makes sense to consider the regeneration decision as an impulse control. This means that the decision is to make a jump from one value of state to another. The associated delay is the sum of an offset delay and of duration.

4 Numerical analysis

4.1 Abstract impulse control problems

Consider the following abstract framework for impulse control problems, where decisions of impulse take place at times $\{\theta_i\}$, $i = 1, \cdots, N$, and the duration of each impulse is a function $\nu(\cdot, \cdot)$ of the current state and variation of state ξ_i. It is convenient to denote the duration of ith impulse as

$$\nu_i := \nu(y_x(\theta_i), \xi_i). \tag{5}$$

The state equation is

$$\begin{aligned} \dot{y}_x(t) &= f(t, y_x(t), u(t)),\ t \in (\theta_i + \nu_i, \theta_{i+1}), \\ y_x(\theta_i + \nu_i) &= y_x(\theta_i) + \xi_i,\ i = 0, \cdots, N, \\ y_x(s) &= x. \end{aligned}$$

Here x is the initial state at initial time $s \in [0, T]$, $\theta_0 = s$ by definition, and $\{\theta_i\}$, $i = 1, \cdots, N$, is a (possibly infinite) increasing sequence of stopping times. The criterion to be minimized is

$$V(x, s, u, \theta, \xi) := \int_0^T \ell(t, y_x(t), u(t)) e^{-\lambda t} dt + \sum_{i=1}^{N} c(\xi_i) e^{-\lambda \theta_i} dt + \varphi(y_x(T)).$$

It includes a running cost ℓ, an impulse cost $c(\cdot)$, and an actualization coefficient $\lambda \geq 0$. We may include restrictions on the jump impulses ξ_i by assuming that $c(\cdot)$ has value $+\infty$ outside its effective domain

$$\operatorname{dom}(c) := \{\xi \in \mathbb{R}^n;\ c(\xi) \in \mathbb{R}\}. \tag{6}$$

These restrictions make sense in our application. In addition we have the state constraint

$$y_x(t) \in \operatorname{cl}\Omega, \quad t \in [s, T]. \tag{7}$$

Here Ω is an open subset of $\mathbb{R}^n$ whose closure is denoted $\operatorname{cl}\Omega$. Consider the problem, denoted $(P_{s,x})$, of minimizing the criterion over all possible controls, for an initial state x at time s. The value of the problem is

$$V(x, s) := \inf\{V(x, s, u, \theta, \xi); \quad (u, \theta, \xi) \text{ such that (7) holds}\}. \tag{8}$$

4.2 The Hamilton-Jacobi-Bellman equation

Consider the operator that with a function $v : \Omega \times [0,T] \to \mathbb{R}$ (a candidate for being equal to the value function $V(x,s)$) associates the optimal value after an impulsion:

$$(Mv)(x,s) := \inf_{\xi \in \mathbb{R}^n} \{v(x+\xi, s+\nu(x,\xi)) + c(\xi)\}. \tag{9}$$

Obviously

$$V(x,s) \leq (MV)(x,s), \quad \forall\, (x,s) \in \Omega \times [0,T], \tag{10}$$

with equality for those (x,s) where the optimal strategy is to take an impulse control. Consider the pseudo-Hamiltonian function

$$H(t,x,u,p) := \ell(t,x,u) + p \cdot f(t,x,u). \tag{11}$$

By the usual argument dealing with optimal control problems without impulse, we have that

$$\lambda V - V_t - \inf_{u \in U} H(t,x,u,V_x) \leq 0, \tag{12}$$

with equality if the strategy of making no impulse at point x and time s is optimal. Combining with (10) and the discussion following it, we obtain, at least formally, the Hamilton-Jacobi-Bellman equation for the value function:

$$\begin{cases} \max(\lambda V - V_t - \inf_{u\in U} H(t,x,u,V_x), V - MV) = 0, & (x,t) \in \mathbb{R}^n \times [0,T], \\ v(x,T) = \varphi(x). \end{cases} \tag{13}$$

Under appropriate technical conditions on the data, it can be proved using the techniques of [2, 4] that $V(x,s)$ is solution of (13) in the so-called viscosity sense. In order to have a well-posed equation (i.e. uniqueness of solution) we should add some boundary conditions on $\partial\Omega$, see [1, 2, 4]. We will not enter into the abstract theory, but rather discuss them later for the specific application considered in this paper.

4.3 Discretization

We make several assumptions. The actualization term λ is supposed to be equal to 0, since the extension of the results to the general case is easy, and this simplifies the formulas. We assume that, as is the case in our application, Ω is the set obtained by taking into account bound constraints on the state variables. We may also assume without loss of generality that the lower bounds are 0, and denote as $x_1^M, \cdot, x_n^M$ the upper bounds. Given integers $N_1, \cdot, N_n$ greater than 1, let $h_i := x_i^M / N_i$ be the space steps, and denote by $h_0 = T/N_0$ the time step.

Finally denote by k and j the time and space indexes, respectively. It is convenient to denote

$$t_k := kh_0, \quad x_j := (j_1 h_1, \cdots, j_n h_n).$$

We wish to compute a function v^k defined over the space and time grid, such that v_j^k is an approximation of $V(t_k, x_j)$. It is convenient to denote v_j^k as $v(x_j, t_k)$. Therefore we want to have, in a sense to be more precise later,

$$v_j^k = v(x_j, t_k) \approx V(x_j, t_k). \tag{14}$$

Given x and $u \in U$, we consider the upwind numerical scheme applied to the differential equation $\dot{x} = f(t, x, u)$, (integrating from $t = T$ to $t = 0$, since we have a backward equation), and then maximize with respect to $u \in U$ for each time step. We need the following notations. We define the sign function for a vector, $\sigma : \mathbb{R}^n \to \{-1, +1\}^n$, by the relation

$$\forall x \in \mathbb{R}^n; \;\; \forall i \in \{1, \cdots, n\} \;\; \sigma(x)_i = 1 \text{ if } x_i \geq 0, \;\; -1 \text{ if not.}$$

Denote by e_i the ith element of natural basis of $\mathbb{R}^n$. With a "sign" $\varsigma \in \{-1, +1\}^n$, and coordinates (x, t), we associate a subset of U and a finite difference discretization of the spatial gradient D^ς, as follows:

$$U^\varsigma(x) := \{u \in U; \;\; \sigma(f(t, x, u)) = \varsigma\}, \tag{15}$$

$$(D^\varsigma v(x, t))_i := \begin{cases} \dfrac{v(t, x + h_i e_i, u) - v(t, x, u)}{h_i} & \text{if } \;\; \varsigma_i = 1, \\ \dfrac{v(t, x, u) - v(t, x - h_i e_i, u)}{h_i} & \text{if } \;\; \varsigma_i = -1. \end{cases} \tag{16}$$

Therefore if $u \in U^\varsigma(x)$, we have that the ith component of the spatial gradient of $v(x, t)$ is decentered to the right if $f_i(t, x, u) \geq 0$, and to the left otherwise. (Remember that the algorithm goes backward; hence this is really where the information comes from.) Consider an abstract discretized algorithm of the form

$$\begin{cases} \text{(i)} \;\; v^\flat(x_j, t_{k-1}) = v(x_j, t_k) + h_0 \displaystyle\inf_{\substack{\varsigma \in \{-1,+1\}^n \\ u \in U^\varsigma(x)}} H(t_k, x_j, u, D^\varsigma v(x_j, t_k)), \\ \text{(ii)} \;\; v(x_j, t_{k-1}) = \min\left(v^\flat(x_j, t_{k-1}), (Mv^\flat)(x_j, t_k)\right), \\ n = 1, \cdot, N_0, \\ \text{(iii)} \;\; v_j^{N_0} = \varphi(x_j). \end{cases} \tag{17}$$

That is, the algorithm computes the value without impulsion, the one with impulsion, and keeps the best value. The value without impulsion is itself the infimum over all values obtained by a finite difference discretization of the HJB equation, using an upwind scheme.

Remark 1 *(a) Given x, the set $U^\varsigma(x)$ may be empty for some of the ς, but not for all. Since by convention the infimum over an empty set is $+\infty$, this means that the above minimum is in fact taken among the signs ς for which the associated set $U^\varsigma(x)$ is not empty. By compactness of U, we have that $v^\flat(x_j, t_{k-1})$ whenever $v(\cdot, t_k)$ has finite values. (b) Note that step (ii) in* (17) *is explicit. (c) Since $v(\cdot, t_k)$ is defined only at points of grid, it implicitly uses some interpolation scheme.*

The scheme is said to be monotonous if $v(\cdot, t_{k-1})$ is a nondecreasing function of $v(\cdot, t_k)$.

It follows from the analysis of [3] that if the scheme is monotonous, then it is convergent. Note that step (ii) in (17) is monotonous, provided, as we assume in the sequel, that the interpolation scheme is itself monotonous. In that case, monotonicity in (14) occurs iff the first argument in the minimum is monotonous. A sufficient condition is given by the following classical CFL condition:

$$h_0 \sum_{i=1}^{n} \frac{\|f_i(x,u)\|_\infty}{h_i} \leq 1. \tag{18}$$

4.4 Boundary condition

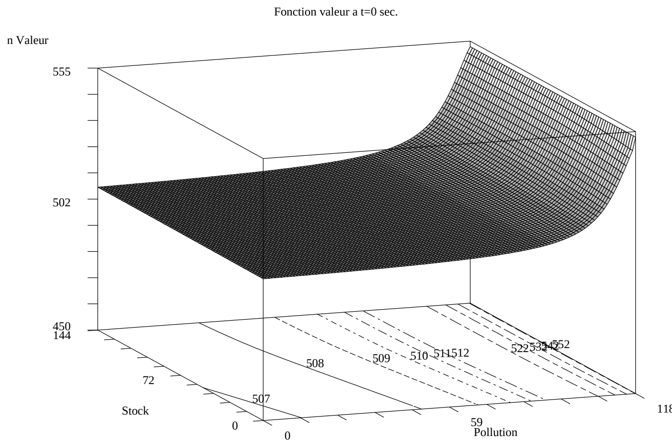
In order to be well defined, the upwind scheme needs boundary conditions for incoming boundaries (i.e. the part of the boundary where some of the characteristics start at the boundary). We discuss this point only for our specific example.

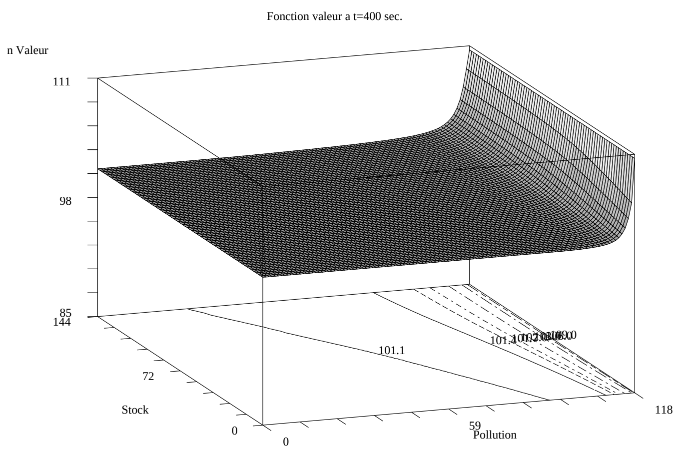
For the first state variable, the bounds are such that it can be guaranteed that no boundary is incoming. The second state variable is the total amount of NOx emitted since the beginning ($t = 0$) and hence decreases along a backward time trajectory. That is, the boundary corresponding to the maximum NOx allowed is incoming. Along this boundary the only possible strategy is to take the NFAR equal to 1. Therefore a simple preliminary computation allows to compute the value function on this boundary.

5 Numerical results

5.1 Constant-speed driving profile

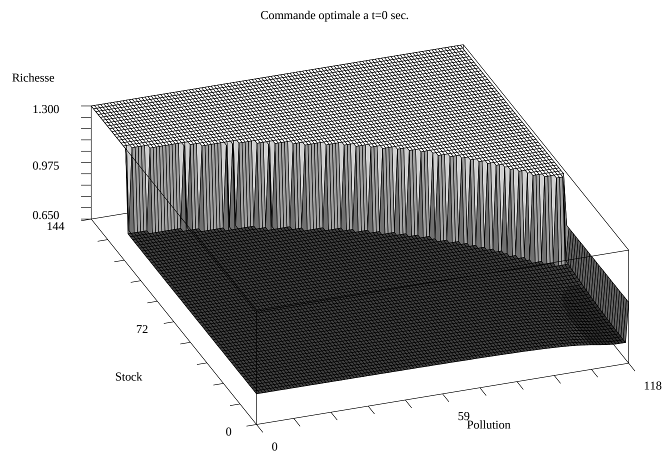
We have perform validation tests in constant-speed configuration, so that engine speed and torque are constant. We represent the value function, optimal control feedback, and optimal state and control along a given trajectory.

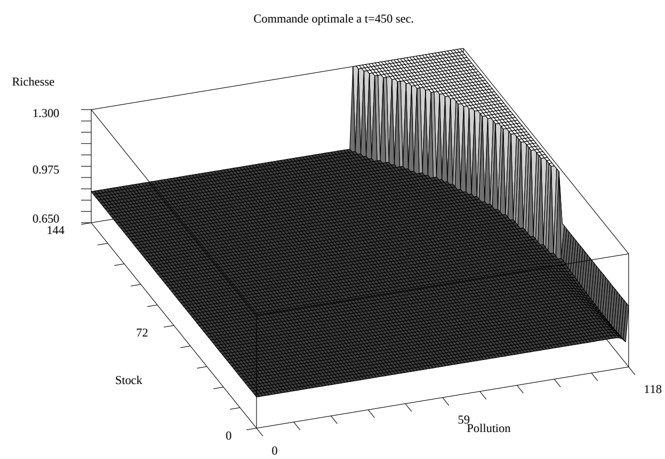




Figures 3 and 4: Value function, constant-speed driving profile

We represent the value fonction for the initial time and close to the final time. The value function is clearly increasing with stock and pollution, especially for a high level of pollution, which is the expected behavior.





Figures 5 and 6: Optimal feedback control, constant profile

On these figures we clearly see the domain of purge ($R = 1.3$) and the domain of low values for R. The purge occurs for high values of stock and pollution. Close to the final time, the cost of a purge will dominate its future savings, which is reflected in the second figure.

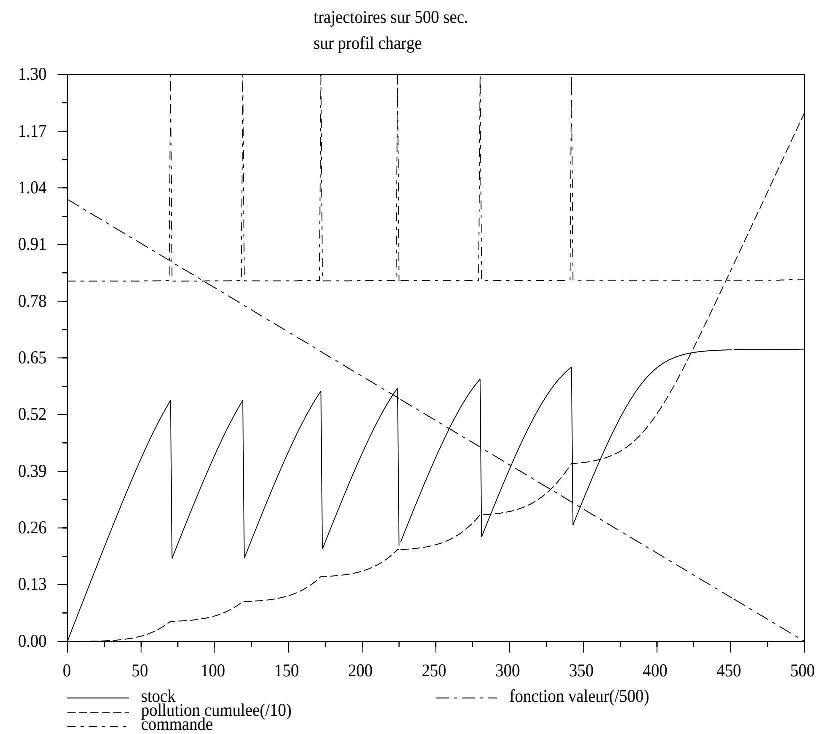


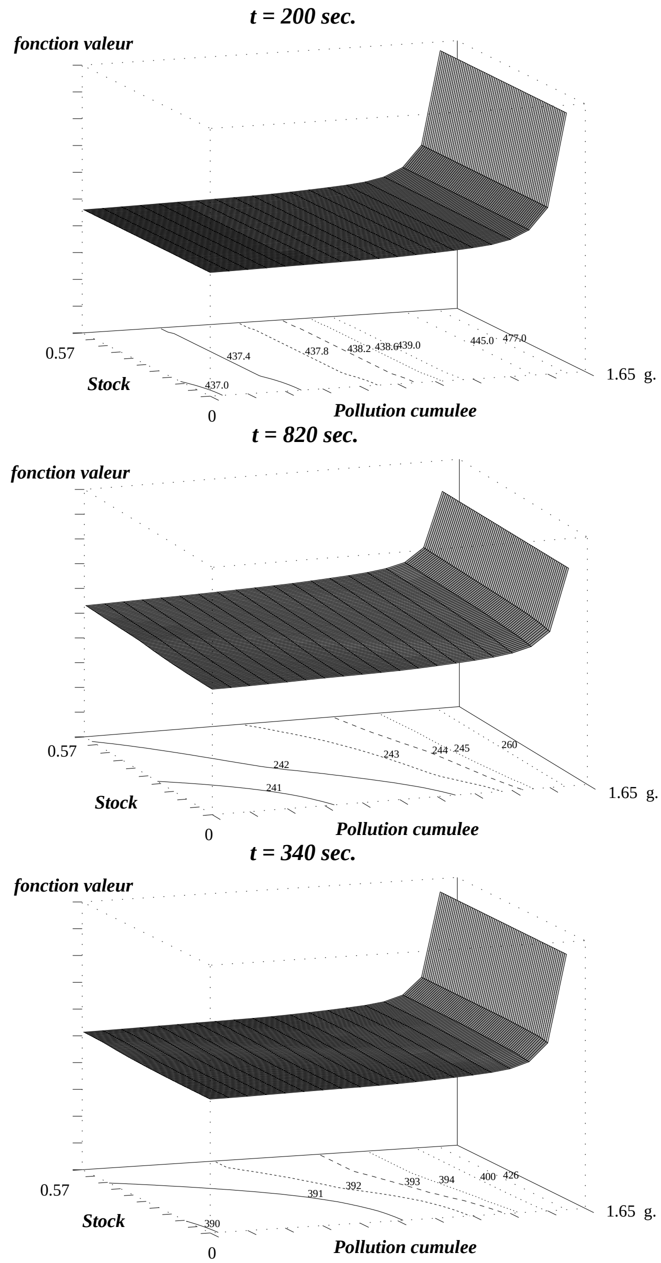
Figure 7: Optimal control vs time

In this case of a constant profile, we obtain the optimal control at any time a "threshold curve" which limits two zones, the first with storage control and the second with regeneration control. As was expected, the profile of optimal control vs. time seems to be cyclic with a constant period except close to the initial and final time. The behavior there is somewhat artificial since we have a given initial state and a constraint on the final state of charge.

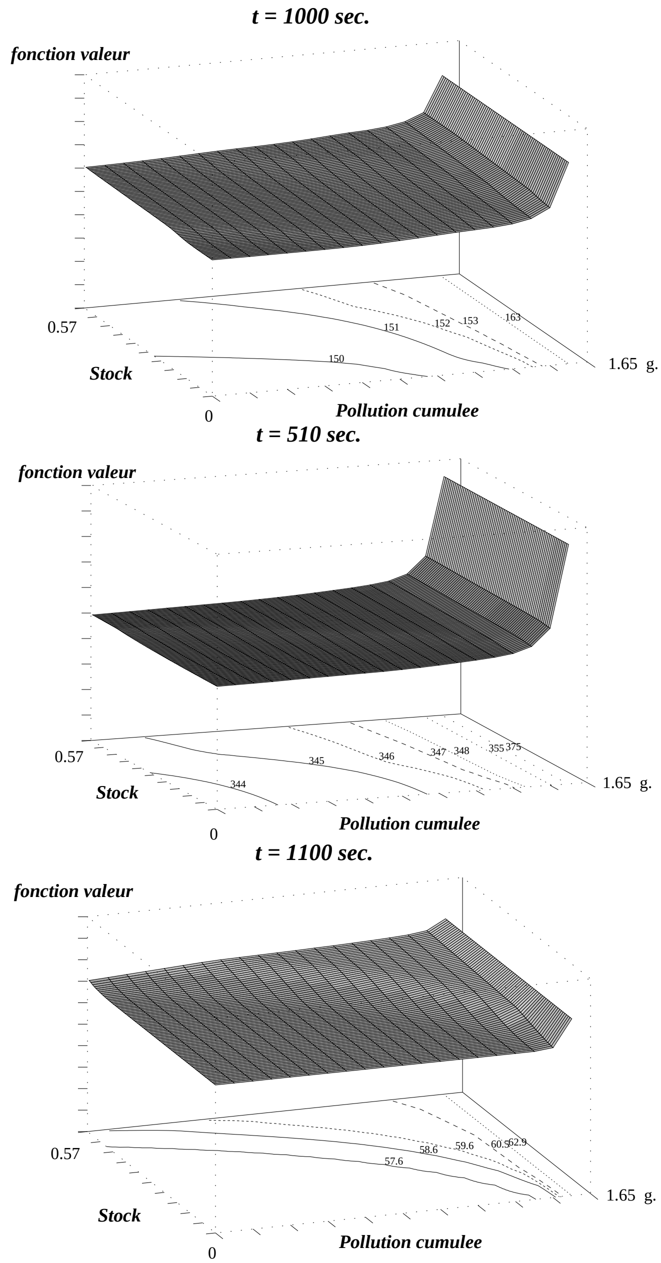
5.2 Normalized driving cycle

We have performed a second test with more realistic conditions, using input data corresponding to the normalized European cycle (MVEG) imposed by pollution standards.

The profile of the optimal control is more complicated compared with the constant-speed case. We can observe that the optimal control gives a stoichiometric control in the beginning and the end of the cycle. This result is logical since the efficiency of storage/regeneration mechanism is low in this zones. In fact, it is well-known that a NOx trap is efficient only in a limited temperature zone called "temperature window". In addition, the catalyst temperature is too low in the beginning of the cycle and too high in its end.

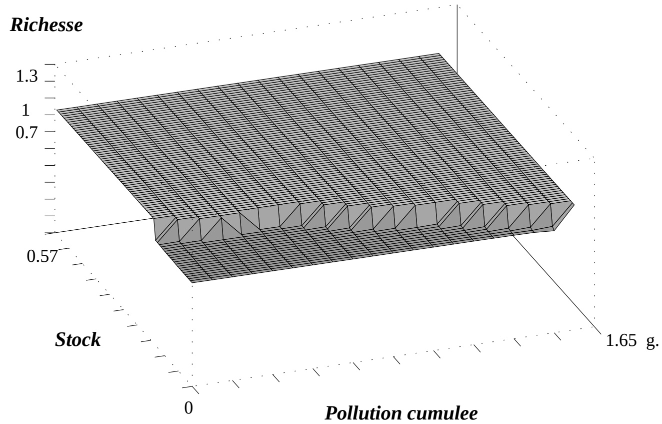


Figures 8 to 11: Value function, European cycle

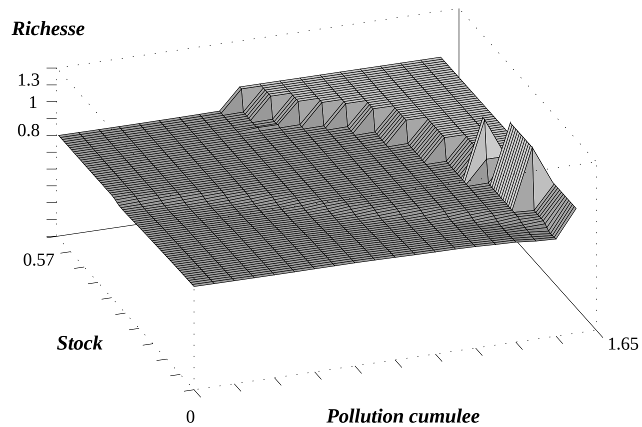


Figures 12 to 14: Value function, European cycle

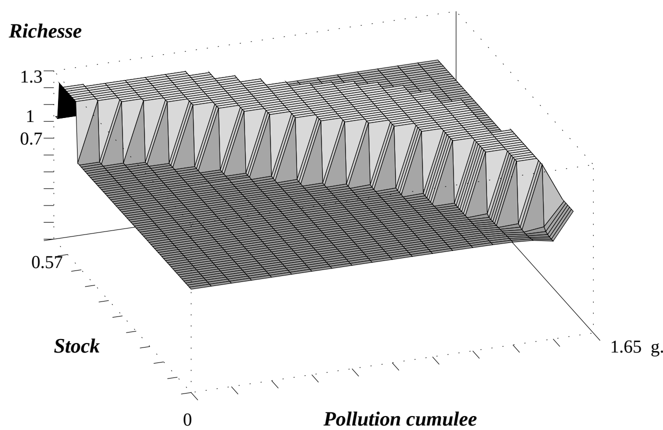
t = 200 sec.

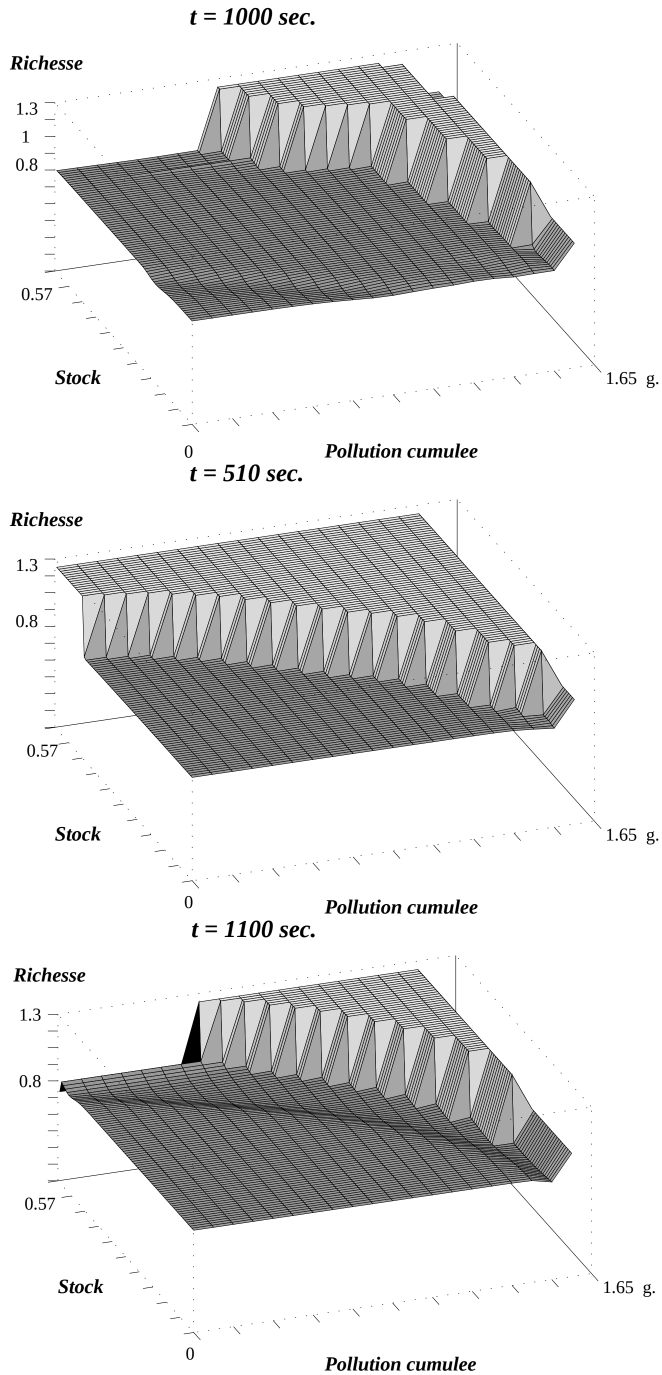


t = 820 sec.



t = 340 sec.





Figures 15 to 20: Optimal feedback control, European cycle

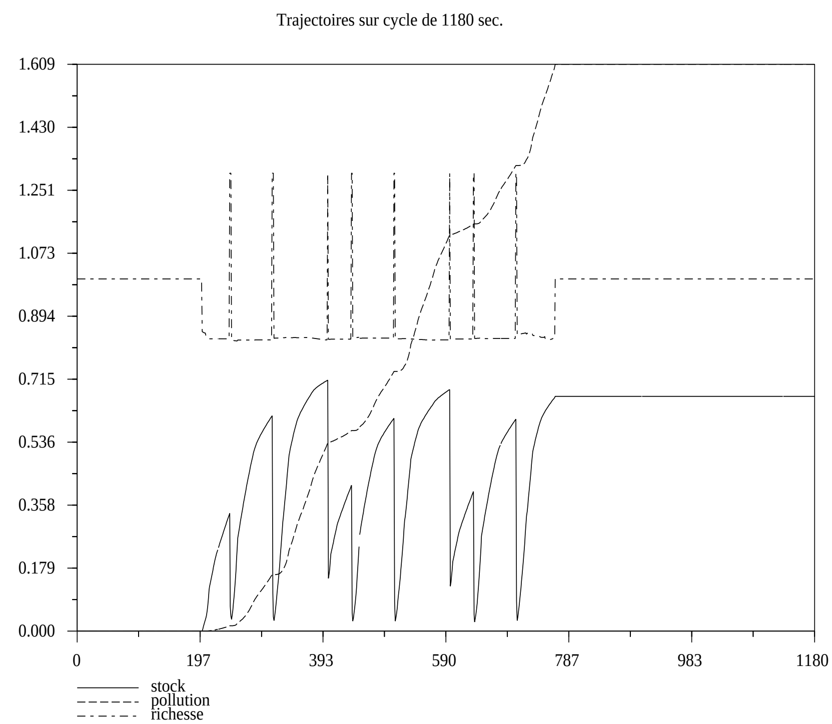


Figure 21: Optimal state and control, European cycle

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