Robust Optimal Control for Nonlinear Dynamic Systems

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Joint work with Peter Kuehl, Boris Houska, Andreas Ilzhoefer

INRIA-Rocquencourt,
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Overview

- Dynamic Optimization Example: Control of Batch Reactors
- How to Solve Dynamic Optimization Problems? (recalled)
- Two Challenging Applications:
  - Robust Open-Loop Control of Batch Reactor
  - Periodic and Robust Optimization for „Flying Windmills“
Overview

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Control of Exothermic Batch Reactors

Cooperation between Heidelberg University and Warsaw University of Technology

Work of Peter Kühl (H.G. Bock, Heidelberg) with A. Milewska, E. Molga (Warsaw)
Esterification of 2-Butanol (B) by propionic anhydride (A): exothermic reaction, fed-batch reactor with cooling jacket

**Aim:** complete conversion of B, *avoid explosion!*

**Control:** dosing rate of A

\[ A + B \xrightarrow{K} C + D \]
Safety Risk: Thermal Runaways

Try to avoid by requiring upper bounds on
• reactor temperature $T_R$, and hypothetical
• adiabatic temperature $S$ that would result if all A reacts with B
\[ \dot{n}_A = u - r V \]
\[ \dot{n}_B = -r V \]
\[ \dot{n}_C = r V \]

\[ (C_{p,I} + C_p) \dot{T}_R = r H V - q_{\text{dil}} - U \Omega (T_R - T_J) - \alpha(T_R - T_a) - u c p_A (T_R - T_d), \]

(1)

\[ \rho_i = 1000 M_i \left( P_i Q_i \left( 1 - \left( \frac{T_R}{T_{c,i}} \right)^c \right) \right)^{-1}, \quad i = A, B, C, D \]
\[ c p_i = a_i + b_i T_R + c_i T_R^2 + d_i T_R^3, \quad i = A, B, C, D \]
\[ C_p = \sum_{i=A,B,C,D} c p_i n_i \]
\[ C_{p,I} = C_{p,I1} + \frac{C_{p,I2} - C_{p,I1}}{V_2 - V_1} (V - V_1) \]
\[ V = 1000 \left( \frac{n_A M_A}{\rho_A} + \frac{n_B M_B}{\rho_B} + \frac{n_C M_C}{\rho_C} + \frac{n_D M_D}{\rho_D} \right) \]
\[ \Omega = \Omega_{\text{min}} + 4 \frac{V - V_{\text{min}}}{1000 d} \]

(2)
Dynamic Optimization Problem for Batch Reactor

Constrained optimal control problem:

\[
\begin{align*}
\min_u & \quad \int_0^{t_f} n_B(\tau)^2 \, d\tau \\
\text{subject to} & \quad (1), (2) \\
& \quad 0 \text{ mol/s} \leq u(t) \leq 0.005 \text{ mol/s} \\
& \quad \int_{t_0=0}^{t_f} u(\tau) \, d\tau = 6.89396 \text{ mol} \\
& \quad T_R(t) \leq 343.15 \text{ K} \\
& \quad S(t) \leq 363.15 \text{ K},
\end{align*}
\]

\[S(t) = T_R(t) + \min(n_A, n_B) \frac{H_A}{\rho c_p V}\]

minimize remaining B
subject to dosing rate and temperature constraints

Generic optimal control problem:

\[
\begin{align*}
\min_{x(\cdot), u(\cdot)} & \quad \int_0^T L(x(t), u(t)) \, dt + E(x(T)) \\
\text{subject to} & \quad x(0) - x_0 = 0, \\
& \quad \dot{x}(t) - f(x(t), u(t)) = 0, \quad t \in [0, T], \\
& \quad h(x(t), u(t)) \geq 0, \quad t \in [0, T], \\
& \quad r(x(T)) \geq 0,
\end{align*}
\]
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  - Robust Open-Loop Control of Batch Reactor
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Recall: Direct Multiple Shooting [Bock, Plitt 1984]

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=0}^{N-1} l_i(s_i, q_i) + E(s_N) \\
\text{subject to} & \quad s_0 - x_0 = 0, & \text{(initial value)} \\
& \quad s_{i+1} - x_i(t_{i+1}; s_i, q_i) = 0, & i = 0, \ldots, N-1, \text{(continuity)} \\
& \quad h(s_i, q_i) \geq 0, & i = 0, \ldots, N, \text{(discretized path constraints)} \\
& \quad r(s_N) \geq 0. & \text{(terminal constraints)}
\end{align*}
\]
Solution of Peter’s Batch Reactor Problem

- **Dosing Rate**:
  - Units: mol/s
  - Graph shows dosing rate over time in seconds.

- **2-Butanol (B)**:
  - Units: mol
  - Graph shows concentration of 2-butanol over time in seconds.

- **Reactor Temperature**:
  - Units: °C
  - Graph shows reactor temperature over time in seconds.

- **Adiabatic Temperature**:
  - Units: °C
  - Graph shows adiabatic temperature over time in seconds.

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Experimental Results for Batch Reactor

- Mettler-Toledo test reactor R1
- batch time: 1 h
- end volume: ca. 2 l

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Experimental Results for Batch Reactor (Red)

Optimal dosing profile – Nominal case

- Dosing vs. time [s]
- Reactor temp. [°C]
- Acc. dosing [kg]
- Conversion [-]

Large model plant mismatch
Safety critical!

Blue: Simulation
Red: Experiments

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Experimental Results for Batch Reactor (Red)

- Optimal dosing profile – Nominal case

  - Dosing [kg/s]
  - Reactor temp. [°C]
  - Acc. dosing [kg]
  - Conversion [-]

  - Time [s]

  - Large model plant mismatch
  - Safety critical!

How can we make Peter’s and Aleksandra’s work safer?

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Robust Worst Case Formulation

\[
\min_{u \in \mathcal{U}} \ J[x(u, p), u]
\]

s.t. \[ h_i(x, u, p) \leq 0, \quad i = 1 \ldots n, \]
\[ \tilde{h}(x, u, p) \leq 0, \]

Make sure safety critical constraints are satisfied for \textbf{all possible} parameters \( p \)!

\[
\min_{u \in \mathcal{U}} \ J[x(u, \bar{p}), u]
\]

s.t. \[ \max_{\|p - \bar{p}\|_{2, \Sigma} \leq \gamma_i} h_i(x, u, p) \leq 0, \quad i = 1 \ldots n, \]
\[ \tilde{h}(x, u, \bar{p}) \leq 0. \]

Semi-infinite optimization problem, difficult to tackle...

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Fortunately, it is easy to show that up to first order:

\[
\max_{\|p - \bar{p}\|_2, \Sigma - 1 \leq \gamma_i} h_i(x, u, p) \approx h_i(x, u, \bar{p}) + \gamma_i \left\| \frac{d}{dp} h_i(x, u, \bar{p}) \right\|_{2, \Sigma}
\]

So we can approximate robust problem by:

\[
\begin{align*}
\min_{u \in \mathcal{U}} & \quad J[x(u, \bar{p}), u] \\
\text{s.t.} & \quad h_i(x, u, \bar{p}) + \gamma_i \left\| \frac{d}{dp} h_i(x, u, \bar{p}) \right\|_{2, \Sigma} \leq 0, \quad i = 1, \ldots, n_r, \\
& \quad \tilde{h}(x, u, \bar{p}) \leq 0.
\end{align*}
\]

Intelligent safety margins (influenced by controls)
Numerical Issues for Robust Approach

- for optimization, need further derivatives of $\frac{d}{dp} h_i(x, u, \bar{p})$
- treat second order derivatives by internal numerical differentiation in ODE/DAE solver
- implemented in MUSCOD-II Robust -Framework [C. Kirches]
- use homotopy: start with nominal solution, increase $\gamma_i$ slowly, employ warm starts

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### Estimated Parameter Uncertainties for Test Reactor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard deviation</th>
<th>gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{jacket}}$</td>
<td>0.3 K</td>
<td>3.0</td>
</tr>
<tr>
<td>$m_{\text{catalyst}}$</td>
<td>0.5 g (~10 %)</td>
<td>3.0</td>
</tr>
<tr>
<td>$U_A$</td>
<td>10.0 W/(m$^2$ K) (~10 %)</td>
<td>2.0</td>
</tr>
<tr>
<td>$u_{\text{offset}}$</td>
<td>$5.0 \times 10^{-5}$ kg/s (~10 % of upper bound)</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Robust Open Loop Control Experiments

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Robust Optimization Result and Experimental Test

Optimal dosing profile – Robust case

- Dosing [kg/s]
- Reactor temp. [°C]
- Acc. dosing [kg]
- Conversion [-]

Safety margin

Perturbed Scenarios (Simulated)

Blue: Simulation
Red: Experiments

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Different solution structure. Model plant mismatch and runaway risk considerably reduced. Complete conversion.

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Comparison **Nominal** and **Robust** Optimization

Different solution structure. Model plant mismatch and runaway risk considerably reduced. Complete conversion.

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Due to high speed, wing tips are most efficient part of wing.

High torques at wings and mast limit size and height of wind turbines.

But best winds are in high altitudes!
Due to high speed, wing tips are *most efficient* part of wing.

- High torques at wings and mast limit size and height of wind turbines.
- But best winds are in high altitudes!

*Could we construct a wind turbine with only wing tips and generator?*
Crosswind Kite Power (Loyd 1980)

- use *kite* with *high lift-to-drag-ratio*
- *use strong line*, but no mast and basement
- *automatic control* keeps kites looping

**But where could a *generator* be driven?**
New Power Generating Cycle

New cycle consists of two phases:

- **Power generation phase:**
  - add *slow downwind motion* by prolonging line (1/3 of wind speed)
  - *generator at ground* produces power due to large pulling force
New Power Generating Cycle

New cycle consists of two phases:

- **Power generation phase:**
  - add *slow downwind motion* by prolonging line (1/3 of wind speed)
  - *generator at ground* produces power due to large pulling force

- **Retraction phase:**
  - change kite’s angle of attack to *reduce pulling force*
  - pull back line

Cycle produces same average power as wind turbine of same wing size, but much larger units possible

*independently patented by Ockels, Ippolito/Milanese, D.*

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Can stack kites, can use on sea
Periodic Optimal Control (with Boris Houska)

Have to regard also cable elasticity

ODE Model with 12 states and 3 controls

- Differential states:
  \[ x := (r_0, r, \phi, \theta, \dot{r}_0, \dot{r}, \dot{\phi}, \dot{\theta}, n, \Psi, C_L, W)^T \]
- Controls:
  \[ u := (\ddot{r}_0, \dot{\Psi}, \dot{C}_L)^T \]

Control inputs:
- line length
- roll angle (as for toy kites)
- lift coefficient (pitch angle)
### Some Kite Parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass of the kite area</td>
<td>( m_k )</td>
<td>850 kg</td>
</tr>
<tr>
<td>area</td>
<td>( A )</td>
<td>500 m(^2)</td>
</tr>
<tr>
<td>volume</td>
<td>( V )</td>
<td>720 m(^3)</td>
</tr>
<tr>
<td>pure drag reference wind</td>
<td>( c_{D,0} )</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>( w_0 )</td>
<td>10 m/s</td>
</tr>
<tr>
<td>gravit. const. air density</td>
<td>( g )</td>
<td>9.81 ( \frac{m}{s^2} )</td>
</tr>
<tr>
<td></td>
<td>( \rho )</td>
<td>1.23 ( \frac{kg}{m^3} )</td>
</tr>
<tr>
<td>cable density</td>
<td>( \rho_c )</td>
<td>1450 ( \frac{kg}{m^3} )</td>
</tr>
<tr>
<td>cable friction</td>
<td>( c_{D,C} )</td>
<td>1.0</td>
</tr>
<tr>
<td>internal friction</td>
<td>( b_0 )</td>
<td>10(^5) ( \frac{kg}{sm^2} )</td>
</tr>
<tr>
<td>elastic modulus</td>
<td>( E )</td>
<td>1.5 ( \times ) 10(^{11}) Pa</td>
</tr>
</tbody>
</table>

E.g. 10 m x 50 m, like Boeing wing, but much lighter material.

Standard wind velocity for nominal power of wind turbines.
Solution of Periodic Optimization Problem

Maximize mean power production:

\[
\overline{P} := \frac{1}{T} W(T) := \frac{1}{T} \int_{0}^{T} F_c \dot{r}_0 dt
\]

by varying line thickness, period duration, controls, subject to periodicity and other constraints:

\[
\begin{align*}
\text{maximize} & \quad \overline{P}(x(T), T) \\
\text{subject to:} & \\
\forall t \in [0, T]: & \quad \dot{x}(t) = f(x(t), u(t), d_c) \\
\forall t \in [0, T]: & \quad 0 \geq \eta(x(t), u(t), d_c) \\
& \quad 0 = \chi(x(0), x(T))
\end{align*}
\]
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Solution of Periodic Optimization Problem

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& \quad 0 = \chi(x(0), x(T))
\end{align*}
\]

\[ \overline{P} = 4.90 \text{ MW} \]

\[ d_c = 6.7 \text{ cm} \]

\[ T = 19.9 \text{s} \]
Visualization of Periodic Solution

\[ r \text{ [km]} \]

\[ r_{\text{max}} - r_{\text{min}} = 5 \text{ km} \]

\[ W \text{ [100 MJ]} \]

\( \Delta W = 0.18 \)

\[ C_L \text{ vs. } t \text{ [s]} \]
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Prototypes built by Partners in Torino and Delft

New cycle consists of two phases:

**UIrecord**

**Power generation phase:**
- add slow downwind motion by prolonging line (1/3 of wind speed)
- generator at ground produces power due to large pulling force

**UIrecord**

**Pull back phase:**
- change kite’s angle of attack to reduce pulling force
- pull back line

Cycle allows same power production as wind turbine of same size!
What about 'dancing' kites?
Optimization with 'dancing' kites: 14 MW possible

- 2 x 500 m² airfoils
- kevlar line 1500 m, diameter 8 cm
- wind speed 10 m/s
Question: could kite also fly without feedback?

Stability just by smart choice of open-loop controls?
Linearization of Poincare Map determines stability

„Monodromy matrix“ = linearization of Poincare Map.

Stability ⇔ Spectral radius smaller than one.

Cons of Spectral radius:
• Nonsmooth criterion difficult for optimization
• Uncertainty of parameters not taken into account
Lyapunov Lemma [Kalman 1960]:
Nonlinear system

\[
\dot{x}(t) = f(x(t), u(t), \delta w(t))
\]

is stable if and only if periodic Lyapunov Equation

\[
\dot{P}(t) = A(t)P(t) + P(t)A(t)^T + B(t)B(t)^T \\
\]
\[
P(0) = P(T)
\]

with

\[
A = \frac{\partial f}{\partial x} \\
B = \frac{\partial f}{\partial \delta w}
\]

has bounded solution.
Robust stability optimization problem

\[
\begin{align*}
\text{minimize} & \quad J[x_r(\cdot), P(\cdot), u(\cdot), p, T] \\
\text{subject to:} & \quad \dot{x}_r(t) = f(x_r(t), u(t), p, 0) \quad \dot{P}(t) = A(t)P(t) + P(t)A(t)^T \\
& \quad x_r(0) = x_r(T) + B(t)B(t)^T \quad P(0) = P(T) \\
& \quad 0 \geq h_i(x_r(t), u(t), p) + \gamma \sqrt{c_i(t)P(t)c_i(t)^T} \\
\end{align*}
\]

for all \( t \in [0, T]\), \( i \in \{1, \ldots, n_h\}\), \( A := \frac{\partial f}{\partial x} \), \( B := \frac{\partial f}{\partial u} \), \( c_i := \frac{\partial h_i}{\partial x} \)

Allows to robustly satisfy inequality constraints!
Orbit optimized for stability (using periodic Lyapunov eq.)

Long term simulation:

Kite does not touch ground

Open-loop stability only possible due to nonlinearity!

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Alternative: NMPC Control after turn of wind direction
Summary: Nonlinear Dynamic Optimization

Open-Loop Optimization
(prone to model-plant-mismatch)

Robust Open-Loop
(no sensor feedback needed, simple)

Model Predictive Control
(feedback by fast online optimization)
Two events of interest this year:

- Workshop on NMPC Software and Applications (NMPC-SOFAP), Loughborough, United Kingdom, April 19-20, 2007. (inv. speakers: Biegler, Findeisen, Kerrigan, Richal et, Schei)
- 13th Czech-French-German Conference on Optimization (CFG07), Heidelberg, Germany, September 17-21, 2007. (inv. speakers: Fletcher, Scherer, Trelat, Waechter, ...)

Traditionally strong in optimal control.

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cfg07.uni-hd.de
4 PhD Positions in Numerical Optimization:

- Sequential Convex Programming Algorithms for Nonlinear SDP
- Large Scale & PDE Constrained Real-Time Optimization Algorithms
- Fast Model Predictive Control Applications in Mechatronic Systems
- Shape Optimization of Mechanical Parts under Inertia Loading

(deadline: June 21, 2007)