An optimal control approach for minimum-fuel deployment of multiple spacecraft formation flying

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Outline

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   - Space dynamics equations
   - Optimal control formulation

2. The solution approach
   - Pontryagin’s Maximum Principle
   - The continuation-smoothing method

3. Numerical results - A deployment in Low Earth Orbit
   - Statement of the test case
   - Deployment over seven days - Seven local solutions found
   - Balancing the fuel consumption

4. Conclusion and future prospects

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Problem statement

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Problem statement

Space dynamics equations

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Dynamics equations in cartesian coordinates

Two-body problem with perturbations and engine thrust

\[
\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{||\mathbf{r}||^3} + \gamma_{p_1} + \gamma_{p_2}
\]

- \(\mu\): the Earth’s gravitational constant
- \(\gamma_{p_1}\): natural perturbative acceleration (geopotential disturbances, lunar and solar third body gravities, atmospheric drag, solar radiation pressure,...)
- \(\gamma_{p_2} = \frac{\mathbf{F}}{m}\): perturbative acceleration caused by the thrust
- \(\mathbf{F}\): thrust vector of the engine
- \(m\): mass of the satellite
Orbital parameters (1/2)

Keplerian osculating elements

- $a$: semi-major axis
- $e$: eccentricity
- $i$: inclination
- $\omega$: argument of perigee
- $\Omega$: longitude of the ascending node
- $\nu$: true anomaly

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Orbital parameters (2/2) - Perturbative acceleration

Eccentric anomaly - Mean anomaly

- \( \cos(E) = \frac{\cos(v) + e}{1 + e \cos(v)} \)
- \( \sin(E) = \frac{\sqrt{1 - e^2 \sin(v)}}{1 + e \cos(v)} \)
- \( M = E - e \sin(E) \)

Perturbative acceleration in the \((\mathbf{T}, \mathbf{N}, \mathbf{W})\) local orbital frame

- \( \mathbf{\gamma}_p = \mathbf{T} \mathbf{T} + \mathbf{N} \mathbf{N} + \mathbf{W} \mathbf{W} \)
- \( \mathbf{T} = \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|} \), \( \mathbf{W} = \frac{\mathbf{r} \wedge \dot{\mathbf{r}}}{\|\mathbf{r} \wedge \dot{\mathbf{r}}\|} \), \( \mathbf{N} = \mathbf{W} \wedge \mathbf{T} \)
Gauss equations

\[
\begin{align*}
\dot{a}(t) &= \frac{2V}{n^2a} T \\
\dot{e}(t) &= \frac{1}{V} \left[ 2(e + \cos(v))T - \sin(v) \frac{r}{a}N \right] \\
\dot{i}(t) &= \frac{r \cos(\omega+v)}{na^2 \sqrt{1-e^2}} W \\
\dot{\Omega}(t) &= \frac{r \sin(\omega+v)}{na^2 \sin(i) \sqrt{1-e^2}} W \\
\dot{\omega}(t) &= \frac{1}{Ve} \left[ 2\sin(v)T + \frac{2e + (1+e^2)\cos(v)}{1+e \cos(v)} N \right] - \frac{r \cos(i) \sin(\omega+v)}{na^2 \sin(i) \sqrt{1-e^2}} W \\
\dot{M}(t) &= n - \frac{\sqrt{1-e^2}}{Ve} \left[ 2\sin(v) \left( 1 + \frac{e^2}{1+e \cos(v)} \right) T + \cos(v) \frac{1-e^2}{1+e \cos(v)} N \right] \\
\end{align*}
\]

with \(r = \frac{a(1-e^2)}{1+e \cos(v)}\), \(V = \sqrt{\mu \left( \frac{2}{r} - \frac{1}{a} \right)}\) and \(n = \sqrt{\frac{\mu}{a^3}}\)
Natural perturbations taken into account

Earth’s oblateness only - Atmospheric drag neglected

The non-sphericity of the Earth yields gravitational perturbations:

1. The Earth’s gravity field representation in cartesian coordinates is based on a spherical harmonic expansion.
2. The Earth’s oblateness term $J_2$ is the most important one after the central term.
3. The $J_2$ term corresponds to a certain expression of the perturbative accelerations $T$, $N$, and $W$.
4. First effect of $J_2$: short-period and long-period oscillations with zero mean on the orbital parameters.
5. Second effect: a secular effect, i.e. a linear drift, on $\Omega$ (rotation of the orbital plane), $\omega$ and $M$. 

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An optimal control approach for minimum-fuel deployment
Gauss equations with equinoctial elements and $J_2 (1/2)$

### State and control variables

Let us consider a formation of $n$ satellites

- $x_j$: equinoctial orbital parameters for satellite $S_j$ ($j = 1, \ldots, n$)

$$x_j = \begin{pmatrix} a_j \\ e_{x,j} = e_j \cos (\omega_j + \Omega_j) \\ e_{y,j} = e_j \sin (\omega_j + \Omega_j) \\ h_{x,j} = \tan (i_j/2) \cos (\Omega_j) \\ h_{y,j} = \tan (i_j/2) \sin (\Omega_j) \\ L_j = \omega_j + \Omega_j + \nu_j \end{pmatrix}$$

- $(a_j, e_j, i_j, \omega_j, \Omega_j, \nu_j)$: Keplerian osculating elements for $S_j$

- $m_j$: mass of satellite $S_j$

- $u_j$: normalized thrust vector for $S_j$ in the $(\overrightarrow{T}, \overrightarrow{N}, \overrightarrow{W})$ frame
Gauss equations with equinoctial elements and $J_2$ (2/2)

State equations in compact form

\[
\begin{align*}
\dot{x}_j(t) &= f(x_j(t)) + F_{\text{max}} g(x_j(t)) \frac{u_j(t)}{m_j(t)} \\
\dot{m}_j(t) &= -F_{\text{max}} \frac{\|u_j(t)\|}{g_0 I_{sp}} \\
t &\in [t_0, t_f]
\end{align*}
\]

- $F_{\text{max}}$: maximum thrust modulus of the $n$ engines
- $I_{sp}$: specific impulse of the $n$ engines
- $g_0$: acceleration due to gravity at sea level
- $t_0$ and $t_f$: fixed initial and final dates
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4 Conclusion and future prospects
The problem to solve

Minimum-fuel deployment

\[
\begin{align*}
\text{Min } J (u_1, \ldots, u_n) &= -\sum_{j=1}^{n} m_j(t_f) \\
\dot{x}_j(t) &= f(x_j(t)) + F_{\text{max}} g(x_j(t)) \frac{u_j(t)}{m_j(t)} \\
\dot{m}_j(t) &= -F_{\text{max}} \frac{\|u_j(t)\|}{g_0 l_{sp}} \\
\|u_j(t)\| &\leq 1 \quad t \in [t_0, t_f] \\
x_j(t_0) &= x_{j,0} \quad m_j(t_0) = m_0 \\
\psi_j (x_j(t_f)) &= 0 \\
\phi (x_1(t_f), \ldots, x_n(t_f)) &= 0
\end{align*}
\]

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An optimal control approach for minimum-fuel deployment
Terminal conditions (1/2)

Non coupling conditions in our application (\(n = 4\))

\[
\psi_j(x_j(t_f)) = \begin{bmatrix}
  a_j(t_f) - a_j,f \\
  e_{x,j}(t_f) - e_{x,j,f} \\
  e_{y,j}(t_f) - e_{y,j,f} \\
  h_{x,j}(t_f)^2 + h_{y,j}(t_f)^2 - \tan^2 \left( \frac{i_{j,f}}{2} \right)
\end{bmatrix} \quad (j = 1, \ldots, n)
\]

where \(a_{j,f}, e_{x,j,f}, e_{y,j,f},\) and \(i_{j,f}\) \((j = 1, \ldots, n)\) are given

Remark

\(e_{x,j,f} = e_{y,j,f} = 0\) and \(i_{j,f} = i_f\) \((j = 1, \ldots, n)\) in our application
Terminal conditions (2/2)

Coupling conditions in our application

\[ \phi(x_1(t_f),...,x_n(t_f)) = \begin{bmatrix} \phi^1(x_1(t_f),x_2(t_f)) \\ \vdots \\ \phi^{n-1}(x_{n-1}(t_f),x_n(t_f)) \end{bmatrix} \]

with, for \( j=1,...,n-1, \)

\[ \phi^j(x_j(t_f),x_{j+1}(t_f)) = \begin{bmatrix} h_{x,j}(t_f) h_{x,j+1}(t_f) + h_{y,j}(t_f) h_{y,j+1}(t_f) - \tan^2\left(\frac{i_f}{2}\right) \cos(\delta\Omega_j,f) \\ h_{x,j}(t_f) h_{y,j+1}(t_f) - h_{y,j}(t_f) h_{x,j+1}(t_f) - \tan^2\left(\frac{i_f}{2}\right) \sin(\delta\Omega_j,f) \\ \tan\left(\frac{L_{j+1}(t_f) - L_j(t_f)}{2}\right) - \tan\left(\frac{\delta L_j,f}{2}\right) \end{bmatrix} \]

Remark

\[ \phi^j(x_j(t_f),x_{j+1}(t_f)) = 0 \implies \begin{bmatrix} \Omega_{j+1}(t_f) - \Omega_j(t_f) = \delta\Omega_j,f \\ L_{j+1}(t_f) - L_j(t_f) = \delta L_j,f \text{ modulo } 2\pi \end{bmatrix} \]
Additional constraint - Balance of the fuel consumption

A nonlinear terminal inequality constraint

\[ \theta \left( m_1(t_f), \ldots, m_n(t_f) \right) = \frac{- \sum_{j=1}^{n} \zeta_j \log \zeta_j}{\log(n)} \geq \bar{\theta} \]

where \( \bar{\theta} \in [0, 1] \) and

\[ \zeta_j = \frac{m_j(t_f)}{\sum_{k=1}^{n} m_k(t_f)} \quad (j = 1, \ldots, n) \]

\( \bar{\theta} = 0 \): no constraint

\( \bar{\theta} = 1 \): perfect balance of the consumption
The solution approach

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The solution approach
Pontryagin’s Maximum Principle

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Conclusion and future prospects

Bang-off-bang controls

Optimal controls

\[ u_j^*(t) = \arg\min_{\|w\| \leq 1} \left[ -\frac{p_{m_j}(t)}{g_0 I_{sp}} \|w\| + p_{x_j}(t)^T g(x_j(t)) \frac{w}{m_j(t)} \right], \quad t \in [t_0, t_f] \]

Let \( \rho_j(t) = F_{\text{max}} \left( \frac{\|g(x_j(t))^T p_{x_j}(t)\|}{m_j(t)} + \frac{p_{m_j}(t)}{g_0 I_{sp}} \right) \). If \( g(x_j(t))^T p_{x_j}(t) \neq 0,1 \),

\[ u_j^*(t) = -\beta_j(t) \frac{g(x_j(t))^T p_{x_j}(t)}{\|g(x_j(t))^T p_{x_j}(t)\|}, \quad \text{with} \quad \beta_j(t) = \left\{ \begin{array}{ll}
0 & \text{if } \rho_j(t) < 0 \\
1 & \text{if } \rho_j(t) > 0 \\
w \in [0,1] & \text{if } \rho_j(t) = 0
\end{array} \right. \]

If \( g(x_j(t))^T p_{x_j}(t) = 0,1 \), \( u_j^*(t) \) is such that

\[ \|u_j^*(t)\| = \beta_j(t) \]
Numerical difficulties

Problem 1: limitation of the deployment duration in LEO
Short-period oscillations due to $J_2$ on the osculating parameters
$\implies$ small integration stepsize and time consumption

Problem 2: difficulties for the adaptive integration scheme
Ordinary Differential Equations with discontinuous right-hand sides
$\implies$ the integration scheme hardly reaches the desired accuracy

Problem 3: difficulties for Newton’s method
Nonsmooth shooting function with singular Jacobian matrix
$\implies$ very small convergence radius for Newton’s method

A solution for the last two problems
Smoothing the control law
Problem statement

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Optimal control formulation

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An optimal control approach for minimum-fuel deployment
Penalty - barrier approach

\[
\begin{align*}
\text{Min } J_\epsilon (u_1, \ldots , u_n) &= -\sum_{j=1}^{n} \left\{ m_j(t_f) + \epsilon \int_{t_0}^{t_f} F(\|u_j(t)\|) \, dt \right\} \\
\dot{x}_j(t) &= f(x_j(t)) + F_{\text{max}} \cdot g(x_j(t)) \frac{u_j(t)}{m_j(t)} \\
\dot{m}_j(t) &= -F_{\text{max}} \frac{\|u_j(t)\|}{g_0 \cdot l_{sp}} \\
\|u_j(t)\| &\leq 1 \quad t \in [t_0, t_f] \\
x_j(t_0) &= x_{j,0} \quad m_j(t_0) = m_0 \\
\psi_j (x_j(t_f)) &= 0 \\
\phi (x_1(t_f), \ldots , x_n(t_f)) &= 0
\end{align*}
\]
The penalty approach

\[ F^1(w) = w(1 - w) \geq 0, \quad \forall w \in [0,1] \]

- If \( g(x_j(t))^T p_{x_j}(t) \neq 0, 1 \), \( u_{*,j}^*(t) = -\beta_{\epsilon,j}(t) \frac{g(x_j(t))^T p_{x_j}(t)}{\|g(x_j(t))^T p_{x_j}(t)\|}, \) with

\[ \beta_{\epsilon,j}(t) = \begin{cases} 
0 & \text{if } \rho_j(t) \leq -\epsilon \\
1 & \text{if } \rho_j(t) \geq \epsilon \\
\frac{1}{2} + \frac{\rho_j(t)}{2\epsilon} & \text{if } \rho_j(t) \in [-\epsilon, \epsilon] 
\end{cases} \]

- If \( g(x_j(t))^T p_{x_j}(t) = 0, 1 \), \( u_{*,j}^*(t) \) is such that

\[ \|u_{*,j}^*(t)\| = \beta_{\epsilon,j}(t) \]
Principles of the method (3/3)

The logarithmic barrier approach

\[ F^2(w) = \log(w) + \log(1-w) \quad \forall \ w \in ]0,1[ \]

- If \( g(x_j(t))^T p_{x_j}(t) \neq 0_{3,1} \), \( u^*_{\epsilon,j}(t) = -\beta_{\epsilon,j}(t) \frac{g(x_j(t))^T p_{x_j}(t)}{\|g(x_j(t))^T p_{x_j}(t)\|} \), with

\[ \beta_{\epsilon,j}(t) = \frac{2\epsilon}{2\epsilon - \rho_j(t) + \sqrt{\rho_j(t)^2 + 4\epsilon^2}} \]

- If \( g(x_j(t))^T p_{x_j}(t) = 0_{3,1} \), \( u^*_{\epsilon,j}(t) \) is such that

\[ \|u^*_{\epsilon,j}(t)\| = \beta_{\epsilon,j}(t) \]
Continuation procedure - Convergence results

Continuation procedure

\[ \epsilon_1 \geq \epsilon_2 \geq \cdots \geq \epsilon_N \text{ until } \left| J_{\epsilon_{k+1}}(u^{*}_{\epsilon_{k+1}}) - J_{\epsilon_k}(u^{*}_{\epsilon_k}) \right| \leq (\epsilon_k - \epsilon_{k+1}) \eta \]

Proposition 1

\[ J_{\epsilon_1}(u^{*}_{\epsilon_1}) \geq J_{\epsilon_2}(u^{*}_{\epsilon_2}) \geq \cdots \geq J_{\epsilon_N}(u^{*}_{\epsilon_N}) \geq J(u^{*}) \geq J(u^{*}_{\epsilon_k}) \quad k = 1 \ldots N \]

Proposition 2

\[ \lim_{\epsilon \to 0} J_{\epsilon}(u^{*}_{\epsilon}) = J(u^{*}) \quad \text{and} \quad \lim_{\epsilon \to 0} J(u^{*}_{\epsilon}) = J(u^{*}) \]

Reference

R. Epenoy, R. Bertrand: "New smoothing techniques for solving bang-bang optimal control problems - Numerical results and statistical interpretation" Optimal Control Applications and Methods, Vol. 23, No. 4, pp. 171-197, 2002
Numerical results - A deployment in Low Earth Orbit

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Statement of the test case

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An optimal control approach for minimum-fuel deployment
Initial and final conditions (1/2)

Initial configuration

- $a_0 = 7029.48$ km
- $e_0 = 1.27 \times 10^{-3}$
- $i_0 = 98.08$ deg
- $\omega_0 = 214.51$ deg
- $\Omega_0 = 209.80$ deg
- $m_j(t_0) = 120$ kg ($j = 1, \ldots, 4$)
- $v_2(t_0) - v_1(t_0) = 0.0163$ deg
- $v_1(t_0) - v_3(t_0) = 0.0163$ deg
- $v_3(t_0) - v_4(t_0) = 0.0163$ deg

Engines characteristics

$F_{\text{max}} = 4$ N and $l_{sp} = 210$ s
Initial and final conditions (2/2)

Final configuration

- \( a_{j,f} = 7031 \text{ km} (j = 1, \ldots, 4) \)
- \( e_{j,f} = 0 (j = 1, \ldots, 4) \)
- \( i_{j,f} = 98.08 \text{ deg} (j = 1, \ldots, 4) \)
- \( \delta\Omega_{1,f} = 0.0 \text{ deg} \)
- \( \delta\Omega_{2,f} = +0.47 \text{ deg} \)
- \( \delta\Omega_{3,f} = 0.0 \text{ deg} \)
- \( \delta L_{1,f} = +0.41 \text{ deg} \)
- \( \delta L_{2,f} = -0.14 \text{ deg} \)
- \( \delta L_{3,f} = -0.41 \text{ deg} \)
- \( d = 50 \text{ km} \)
- \( d' \approx 100 \text{ km} \)
Problem statement

- Space dynamics equations
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Conclusion and future prospects
The seven solutions

**Consumption and distribution features**

<table>
<thead>
<tr>
<th>sol</th>
<th>c (kg)</th>
<th>(1 − θ₀)</th>
<th>Δm_{max} (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.62</td>
<td>2.0 × 10⁻⁶</td>
<td>5.83 × 10⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>8.84</td>
<td>1.7 × 10⁻⁵</td>
<td>1.97</td>
</tr>
<tr>
<td>3</td>
<td>8.54</td>
<td>7.5 × 10⁻⁵</td>
<td>4.06</td>
</tr>
<tr>
<td>4</td>
<td>7.34</td>
<td>1.4 × 10⁻¹¹</td>
<td>1.61 × 10⁻³</td>
</tr>
<tr>
<td>5</td>
<td>7.12</td>
<td>1.5 × 10⁻⁷</td>
<td>1.60 × 10⁻¹</td>
</tr>
<tr>
<td>6</td>
<td>7.05</td>
<td>1.2 × 10⁻⁷</td>
<td>1.38 × 10⁻¹</td>
</tr>
<tr>
<td>7</td>
<td>6.58</td>
<td>4.7 × 10⁻⁸</td>
<td>8.61 × 10⁻²</td>
</tr>
</tbody>
</table>

\[ c = \sum_{j=1}^{n} (m_0 - m_j(t_f)), \quad \Delta m_{\text{max}} = \max_{1 \leq k < j \leq n} |m_j(t_f) - m_k(t_f)| \]

and \[ \theta_0 = \hat{\theta} (m_1(t_f), \ldots, m_n(t_f)) \]
The best solution obtained - Solution n° 7 - 27 maneuvers

Differential parameters between $S_2$ and $S_3$ vs time (argument of true latitude: $\alpha = \omega + \nu$)

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### Influence of $J_2$

#### Secular effect of $J_2$ in near-circular orbits ($e \approx 0$)

\[
\begin{align*}
\dot{a}(t) &= 0 \\
\dot{e}(t) &= 0 \\
\dot{i}(t) &= 0 \\
\dot{\omega}(t) &\approx \frac{3}{4} \frac{R^2}{a^{7/2}} \sqrt{\mu} J_2 (4 - 5 \sin(i)^2) \\
\dot{\Omega}(t) &\approx -\frac{3}{2} \frac{R^2}{a^{7/2}} \sqrt{\mu} J_2 \cos(i) \\
\dot{M}(t) &\approx n + \frac{3}{4} \frac{R^2}{a^{7/2}} \sqrt{\mu} J_2 (2 - 3 \sin(i)^2)
\end{align*}
\]

- $R$: the Earth’s equatorial radius
- $J_2$: the second zonal term of the Earth potential

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An optimal control approach for minimum-fuel deployment
The best solution obtained - Space dynamics interpretation

The strategy

Two combined effects for obtaining the targeted difference in $\Omega$ at $t = t_f$ ($\delta\Omega_{2,f} = 0.47 \text{ deg}$), between $S_2$ and $S_3$:

1. The differential drift in $\Omega$, due to $J_2$, that relates to an inclination difference between the orbits $\Delta i \approx +0.08 \text{ deg}$, caused by out-of-plane and opposed thrusts on both satellites.

2. The instantaneous effect on $\Omega$ due to the same thrusts, located at both beginning and end of the deployment.

In addition, a non significant semi-major axis difference $\Delta a \approx +90 \text{ m}$, resulting from very small tangential maneuvers creates the difference in true longitude $\delta L_{2,f} = -0.14 \text{ deg}$ at $t = t_f$.
An example of alternative solution - Solution n° 5

Differential parameters between $S_2$ and $S_3$ versus time

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An optimal control approach for minimum-fuel deployment
The alternative solution - Space dynamics interpretation

The strategy

Two combined effects for obtaining the targeted difference in $\Omega$ at $t = t_f$ ($\delta\Omega_{2,f} = 0.47$ deg), between $S_2$ and $S_3$:

1. The differential drift in $\Omega$, due to $J_2$, that relates to
   - A semi-major axis difference between the orbits $\Delta a \approx -45$ km, caused by tangential and opposed thrusts on both satellites
   - An inclination difference $\Delta i \approx +0.05$ deg, inferior to the value obtained for the best solution, due to out-of-plane thrusts

2. The instantaneous effect on $\Omega$ due to the thrusts

In addition, the semi-major axis difference enables to create the difference in true longitude $\delta L_{2,f} = -0.14$ deg at $t = t_f$
Evolution of the best solution versus $t_f$ (1/2)

Evolution of the $\Delta i$ between $S_2$ and $S_3$ versus $t_f$
Space dynamics interpretation

1. The maximum value of $\Delta i$ is obtained for $t_f = 16.5$ days.
2. The differential drift in $\Omega$ due to $J_2$ and caused by $\Delta i$ continuously grows with $t_f$.
3. The fuel consumption decreases as $t_f$ grows, which shows that the strategy benefits more and more from the $J_2$ effect.
4. For $t_f \leq 16.5$ days, the direct effect of the thrusts on $\Omega$ is predominant over the differential drift due to $J_2$ for obtaining $\delta \Omega_{2,f} = 0.47 \text{ deg}$ at $t = t_f$.
5. For $t_f = 16.5$ days, both effects are equal.
6. For $t_f \geq 16.5$ days, the differential drift effect is predominant.
7. $\lim_{t_f \to \infty} \Delta i = 0$.
Problem statement
- Space dynamics equations
- Optimal control formulation

The solution approach
- Pontryagin’s Maximum Principle
- The continuation-smoothing method

Numerical results - A deployment in Low Earth Orbit
- Statement of the test case
- Deployment over seven days - Seven local solutions found
- Balancing the fuel consumption

Conclusion and future prospects
Taking into account the inequality constraint

A continuation method

\[ \theta (m_1(t_f), \ldots, m_n(t_f)) \geq \bar{\theta} \]

is replaced by

\[ \theta (m_1(t_f), \ldots, m_n(t_f)) = \eta \]

with \( \eta \in [\theta_0, \bar{\theta}] \)

\( \theta_0 \): value of \( \theta \) at a local minimum of the unconstrained problem

Optimality concerns - Nonconvex problem

If \( \theta_0 \) is sufficiently closed to \( \bar{\theta} \), we hope that the continuation process does not converge to a solution of the initial problem with constraint \( \theta (m_1(t_f), \ldots, m_n(t_f)) = \bar{\theta} \) that would not be optimal for our inequality constrained problem
Numerical results - An example

<table>
<thead>
<tr>
<th>sol</th>
<th>c (kg)</th>
<th>((1 - \theta_0))</th>
<th>(\Delta m_{\text{max}}) (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.62</td>
<td>2.0 \times 10^{-6}</td>
<td>5.83 \times 10^{-1}</td>
</tr>
<tr>
<td>2</td>
<td>8.84</td>
<td>1.7 \times 10^{-5}</td>
<td>1.97</td>
</tr>
<tr>
<td>3</td>
<td>8.54</td>
<td>7.5 \times 10^{-5}</td>
<td>4.06</td>
</tr>
<tr>
<td>4</td>
<td>7.34</td>
<td>1.4 \times 10^{-11}</td>
<td>1.61 \times 10^{-3}</td>
</tr>
<tr>
<td>5</td>
<td>7.12</td>
<td>1.5 \times 10^{-7}</td>
<td>1.60 \times 10^{-1}</td>
</tr>
<tr>
<td>6</td>
<td>7.05</td>
<td>1.2 \times 10^{-7}</td>
<td>1.38 \times 10^{-1}</td>
</tr>
<tr>
<td>7</td>
<td>6.58</td>
<td>4.7 \times 10^{-8}</td>
<td>8.61 \times 10^{-2}</td>
</tr>
<tr>
<td>32</td>
<td>8.72</td>
<td>1.7 \times 10^{-5}</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Solution sol\(_3\) is brought by continuation to a solution denoted sol\(_{32}\) with a similar fuel balance than sol\(_2\). The resulting solution is based on the same "strategy" than sol\(_3\).
Conclusion and future prospects

1. Problem statement
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   - Optimal control formulation

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4. Conclusion and future prospects
Conclusion

Concerning the method
Efficient extension of the continuation-smoothing technique to the multi-satellite framework

Local minima
A globalization technique allows to find out the problem’s local minima

Fuel balance
The method enables to accurately balance the fuel consumption among the satellites
Future prospects

Additional constraint - Collision avoidance

The collision risk between the satellites exists

⇒ necessity to take into account a collision avoidance constraint

⇒ nonconvex state constraint difficult to handle through PMP

Other type of applications

Deployment in a High Elliptical Orbit for example

⇒ necessity to take into account additional perturbative forces:
  gravitational influence of the Moon and the Sun, solar radiation
  pressure, ...

⇒ necessity to regenerate the costate equations with ADIFOR*

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*C. Bischof, A. Carle, G. Corliss, A. Griewank, P. Hovland: "ADIFOR - Generating Derivative Codes from Fortran Programs" *Scientific Progr.*, no. 1, pp. 1-29, 1992
Recent and forthcoming publications

Note technique CNES

J.-B. Thevenet, R. Epenoy: ”Trajectoires en consommation minimale pour le déploiement d’une formation de satellites - Techniques de commande optimale” Note Technique du CNES N° 151, Mars 2007 (in french)

EUCASS 2007

J.-B. Thevenet, R. Epenoy: ”An optimal control approach for minimum-fuel deployment of multiple spacecraft formation flying” To appear in the proceedings of the 2nd European Conference for Aerospace Sciences (EUCASS), July 1-6 2007, Brussels, Belgium

Journal of Guidance, Control and Dynamics

J.-B. Thevenet, R. Epenoy: ”Minimum-fuel deployment for formation flying of satellites - An optimal control approach” Submitted to Journal of Guidance, Control and Dynamics
Thank you for your attention