Building a trophic model in a peri-urban agroecosystem

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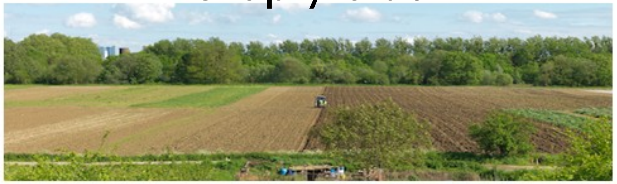
Aussois le 16 juin 2021

Study site : an agrosystem

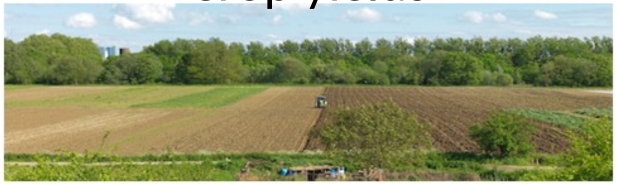
Crop yields



Study site : an agrosystem

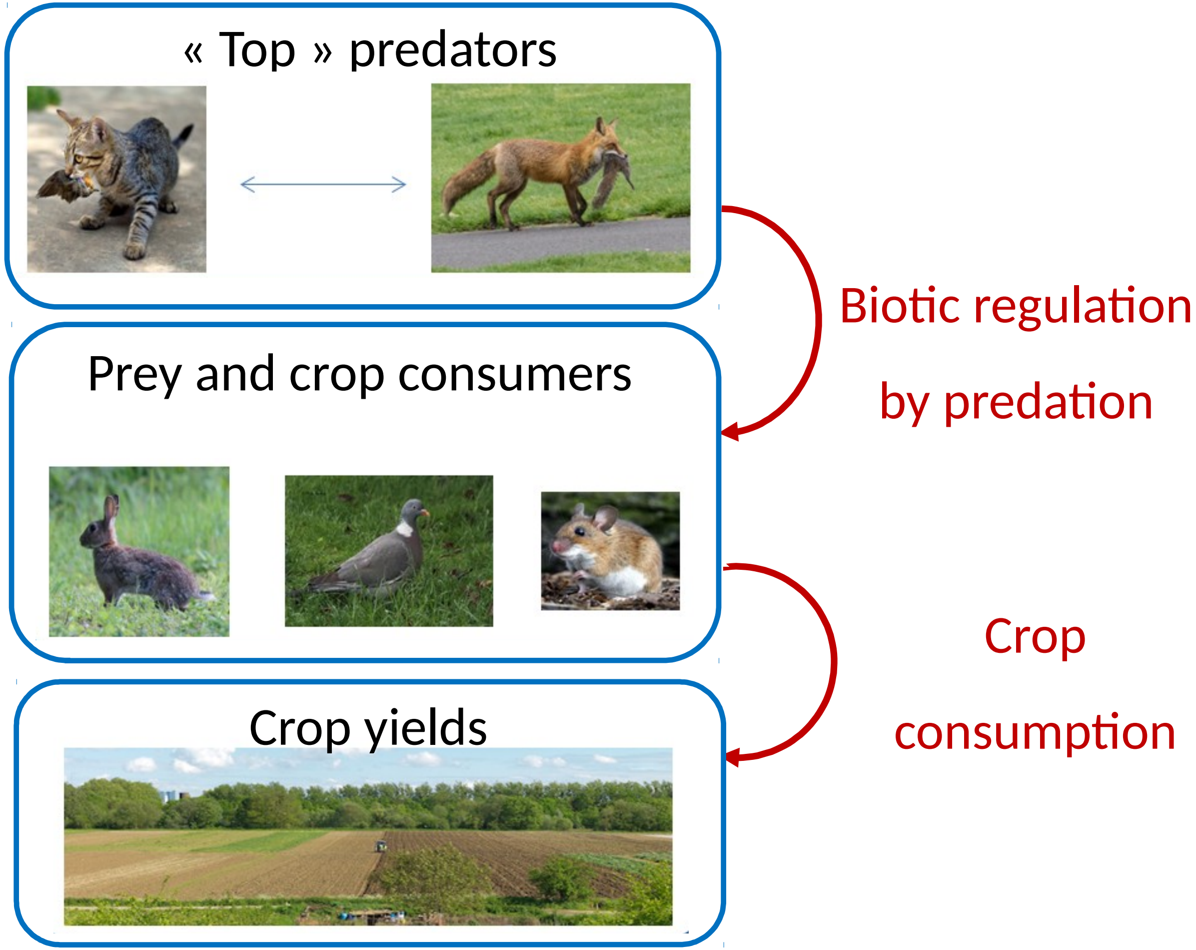
Prey and crop consumers

Crop yields

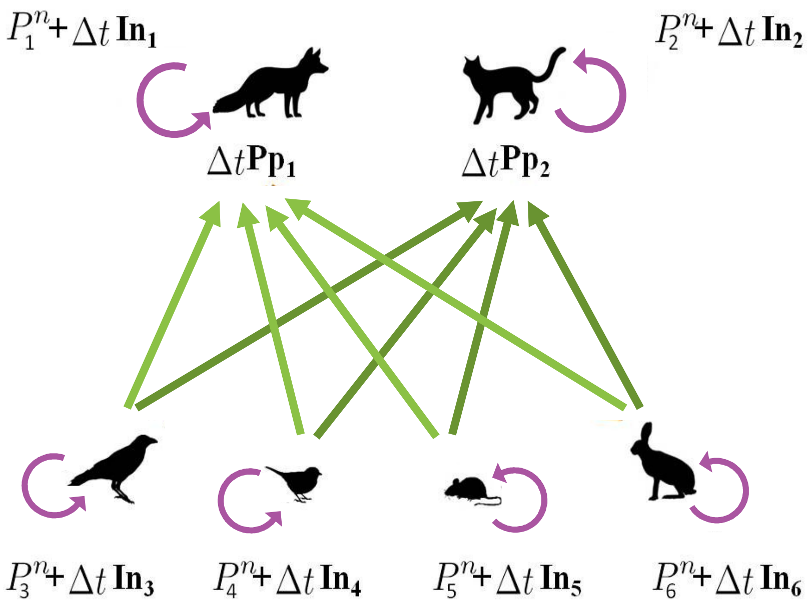


Crop consumption

Study site : an agrosystem



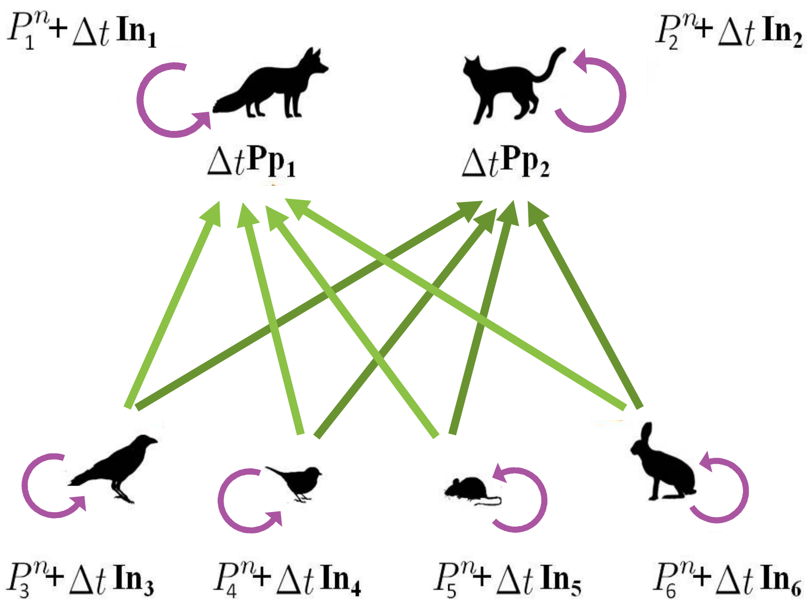
Studied trophic web



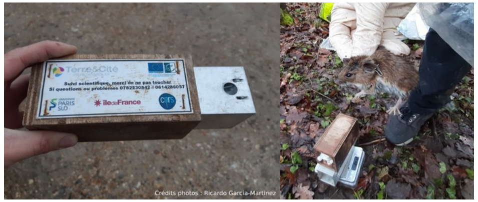




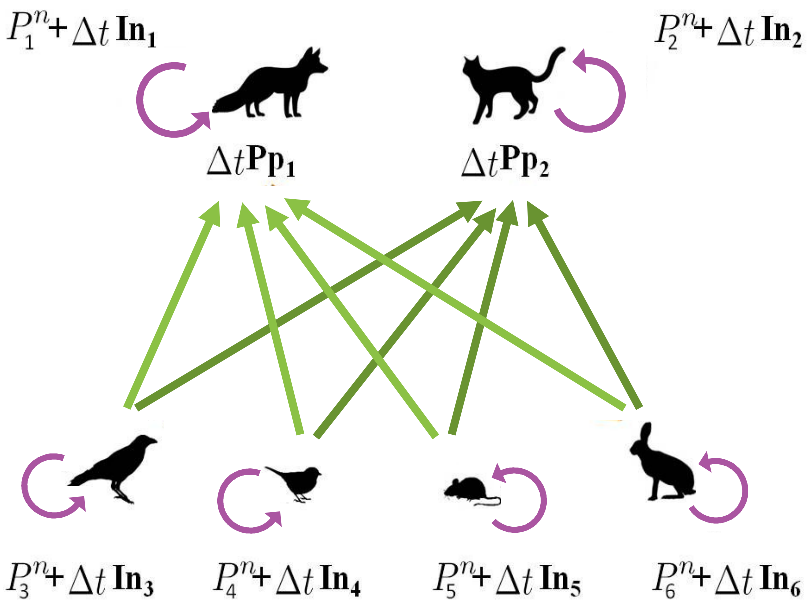

Studied trophic web



Faunistic counts :



Studied trophic web



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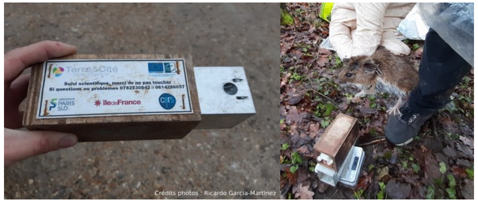
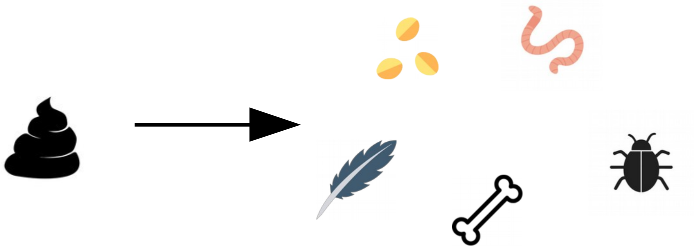
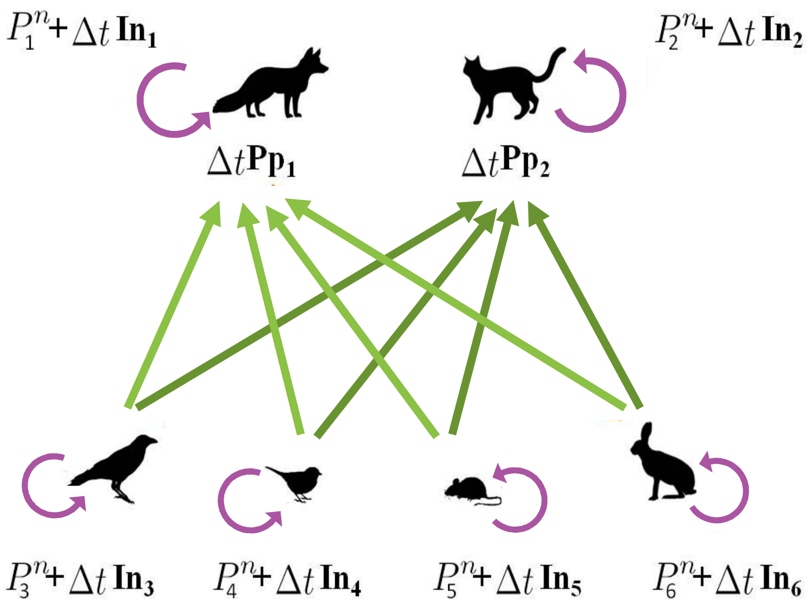


Diet analysis :



Studied trophic web





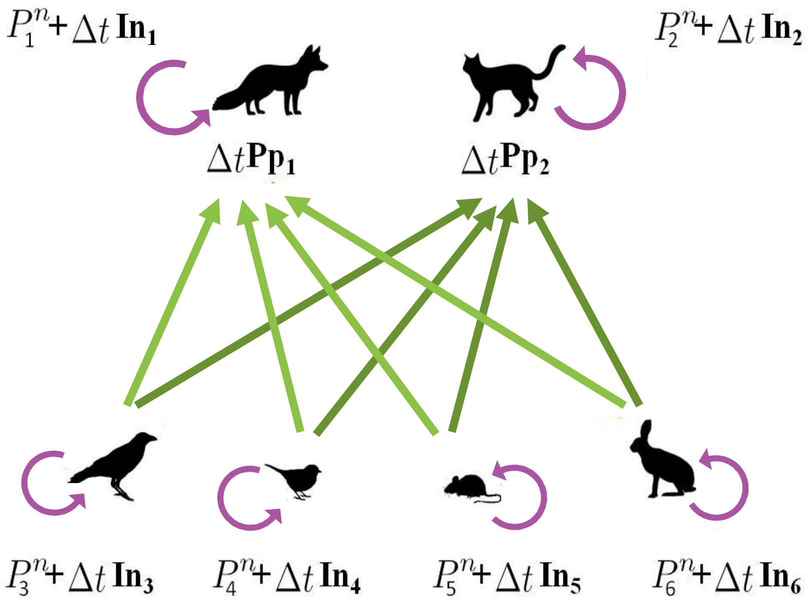



Objectives :

- Building a model of this trophic web with field data (completed with litterature data)
- Exploring scenarios with this model

The model

General caracteristics

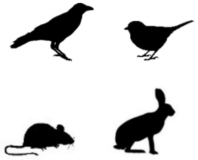


- Deterministic model
- Density of biomass (energetic considerations)
- Discret model (daily time step)
- Seasonal variations in trophic interactions (winter, spring, summer, automn)

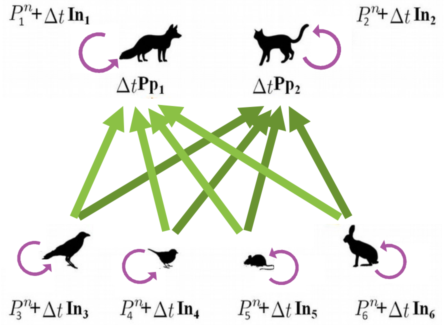
The model

General caracteristics : equations

Prey equation :



$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.\alpha_{max_i}.(1-\frac{P_i^t}{K_{max_i}}) - \Delta t.\sum_{j=3}^{6}[P_j^t.\phi_{jis}]$$



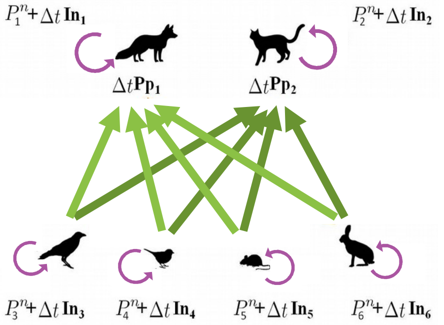
Predator equation :

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

- P_i^t : the total biomass density of the population i at time t ;
- Δt : the duration of one time step ;

The model

General caracteristics : equations



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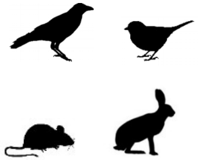

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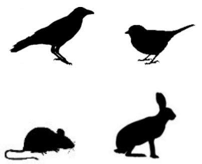
- P_i^t : the total biomass density of the population i at time t ;
- Δt : the duration of one time step ;
- α_{max_i} : intrinsic growth rate of the prey group i ;
- K_{max_i} : charge capacity of the prey group i ;
- $GM_{alt_{is}}$: rate of daily gained biomass by the population of predator i due to the consumption of alternative food during season s ;
- BMR_i : rate of daily lost biomass by the population of predator i due to its base metabolic rate ;
- R_i : rate of daily gained biomass by the population of predator i due to the reproduction and the growths of litters ;

The model

General caracteristics : equations

Prey equation :



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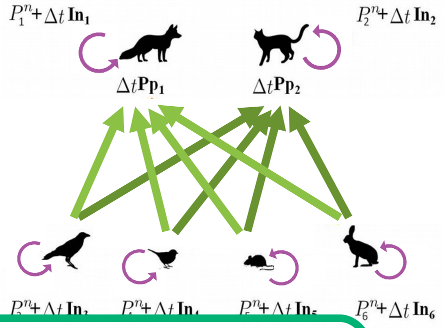

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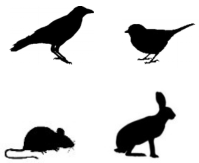
- R_i : rate of daily gained biomass by the population of predator i due to the reproduction and the growths of litters ;
- EM_{ijs} : mean proportion of consumed biomass on one item of prey j by predator i in season s ;
- GM_{ij} : conversion rate of 1kg of eaten prey j in 1kg of predator i during season s ;
- ϕ_{ijs} : daily biomass of the prey group j captured by 1 kg of predator i during season s.

The model

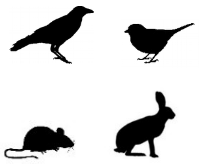
Parameter estimation : α_{max_i}



$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.\alpha_{max_i}.(1 - \frac{P_i^t}{K_{max_i}}) - \Delta t.\sum_{j=3}^{6}[P_j^t.\phi_{jis}]$$

The model

Parameter estimation : α_{max_i}

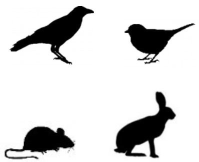


$$P_i^{t+1} = P_i^t + \Delta t . P_i^t . \alpha_{max_i} . (1 - \frac{P_i^t}{K_{max_i}}) - \Delta t . \sum_{j=3}^{6} [P_j^t . \phi_{jis}]$$

- Maximum **geometric** growth rate (no predation, unlimited resources)

The model

Parameter estimation : α_{max_i}



$$P_i^{t+1} = P_i^t + \Delta t . P_i^t \boxed{\alpha_{max_i}} . (1 - \frac{P_i^t}{K_{max_i}}) - \Delta t . \sum_{j=3}^{6} [P_j^t . \phi_{jis}]$$

- Maximum **geometric** growth rate (no predation, unlimited resources)
- May be approximated with Cole's equation (Cole 1954, Fagan 2010) :

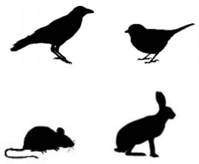
$$e^{-r_{max_i}} + \overline{m_i} e^{-r_{max_i} \beta_i} - \overline{m_i} e^{-r_{max_i} (\gamma_i + 1)} = 1 \rightarrow e^{-r_{max_i}}$$

$$\boxed{\alpha_{max_i} = r_{max_i} - 1}$$

- r_{max_i} : the maximum **exponential** growth rate of a mean prey i ;
- β_i : the age at first reproduction of a mean prey i ;
- γ_i : the age at last reproduction of a mean prey i ;
- $\overline{m_i}$: the average number of female offspring produced per female of prey i per year.

The model

Parameter estimation : K_{max_i}



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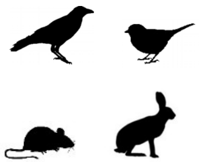
Difficult to get...

Hypothesis : K_{max_i} proportional to α_{max_i}

The model

Parameter estimation : K_{max_i}



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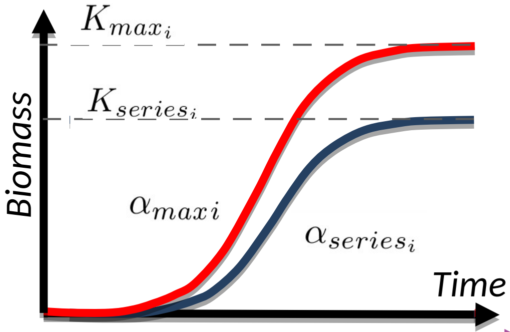
Difficult to get...
Hypothesis : K_{max_i} proportional to α_{max_i}

Apo44_Myo46_1v8Ha
25
45
26
99
27
38
11
43
182
3
129
132
96
66
224
31
126
46
118
93
83
71
38

Package PVAClone

α_{series_i}

K_{series_i}



$$K_{max_i} = \frac{K_{series_i} . \alpha_{maxi}}{\alpha_{series_i}}$$

The model

Parameter estimation : BMR_i

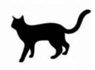
$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

Kleiber’s allometric relationship (Kleiber 1961)
Mean fox biomass (Artois & Le Gall 1988)
Mean cat biomass (Database Amniot)

The model

Parameter estimation : R_i

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

Outside a reproduction period

$$R_i = 0$$

During a reproduction period

$$R_i = \frac{SR_i.LS_i}{GP_i}$$

Where :

- SR_i : sex-ratio of predator i
- LS_i : mean litter size of predator i
- GP_i growth period of predator i (from fecondation to adult size)

 1 reproduction period per year

 2 reproduction period per year

Artois & Le Gall 1988, Meia 2016
Spotte 2014

The model

Parameter estimation : $GM_{alt_{is}}$

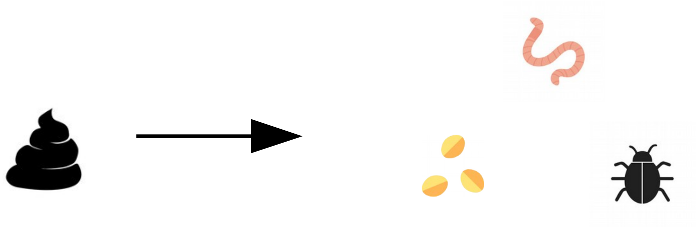

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + \boxed{GM_{alt_{is}}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

The model

Parameter estimation : $GM_{alt_{is}}$

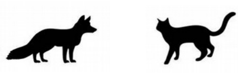
$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

Collect and analysis of scats



The model

Parameter estimation : $GM_{alt_{is}}$



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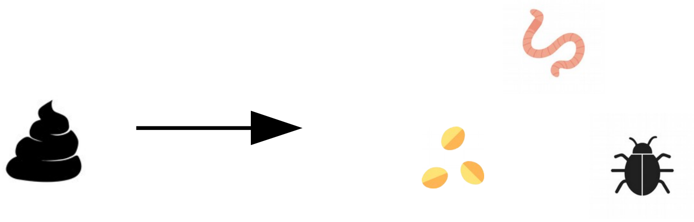

Conversion thanks to energetic coefficient of meat, fruit and insects

The model

Parameter estimation : $GM_{alt_{is}}$

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

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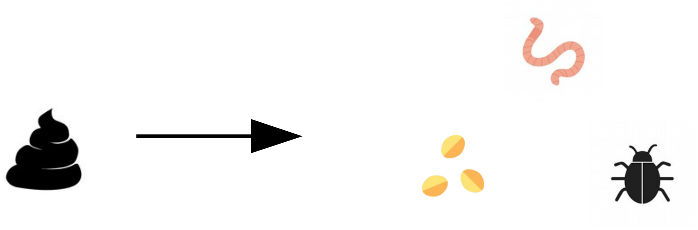

Conversion thanks to energetic coefficient of meat, fruit and insects

GM_{alt_i} different for each season

The model

Parameter estimation : EM_{ijs} and GM_{ij}

 

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

- Proportion of a prey item which is eaten
- Conversion thanks to energetic coefficient of meat, fruit and insects

The model

Parameter estimation : ϕ_{ijs}

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

The model

Parameter estimation : ϕ_{ijs}

 

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

$$\phi_{ijs} = \frac{a_{ijs}.(P_j^t)^2}{1 + \sum_{k=3}^{6}[a_{iks}.b_{iks}.(P_k^t)^2]}$$

(Holling functional response type III)

a_{ijs} : successful attack rate of predator i on the prey group j during its search time in season s

b_{ijs} : handling and digestion time of an individual of the group prey j by a predator i in season s

The model

Parameter estimation : ϕ_{ijs}

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

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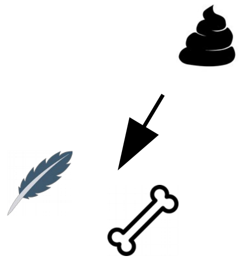
Faunistic counts



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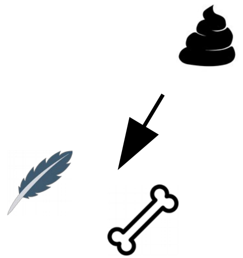

?

The model

Parameter estimation : ϕ_{ijs}

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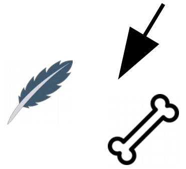
Faunistic counts



?

8 equations (couple prey-predators)
16 unknown variables

The model

Parameter estimation : ϕ_{ijs}

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

$$\phi_{ijs} = \frac{a_{ijs}.(P_j^t)^2}{1+\sum_{k=3}^{6}[a_{iks}.b_{iks}.(P_k^t)^2]} \xrightarrow{P_j^t\to+\infty} \frac{1}{b_{ijs}}$$

The model

Parameter estimation : ϕ_{ijs}

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

$$\phi_{ijs} = \frac{a_{ijs}.(P_j^t)^2}{1+\sum_{k=3}^{6}[a_{iks}.b_{iks}.(P_k^t)^2]} \xrightarrow{P_j^t \to +\infty} \frac{1}{b_{ijs}} \quad \Rightarrow \quad EM_{ijs}.\phi_{ijs} \xrightarrow{P_j^t \to +\infty} \frac{EM_{ijs}}{b_{ijs}}$$

The model

Parameter estimation : ϕ_{ijs}

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\boxed{\phi_{ijs}}]$$

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DFC_i : Daily food consumption of the predator i

Hypothesis :

$$EM_{ijs}.\phi_{ijs} \xrightarrow{P_j^t \to +\infty} DFC_i$$

The model

Parameter estimation : ϕ_{ijs}

 

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

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$$\Downarrow$$

DFC_i : Daily food consumption of the predator i

Hypothesis :

$$EM_{ijs}.\phi_{ijs} \xrightarrow{P_j^t \to +\infty} DFC_i \quad \Rightarrow \quad \boxed{b_{ijs} = \frac{EM_{ijs}}{DFC_i}}$$

The model

Parameter estimation : ϕ_{ijs}

$$P_i^{t+1} = P_i^t + \Delta t.P_i^t.(-BMR_i + R_i + GM_{alt_{is}}) + \Delta t.P_i^t.\sum_{j=3}^{6}[EM_{ijs}.GM_{ij}.\phi_{ijs}]$$

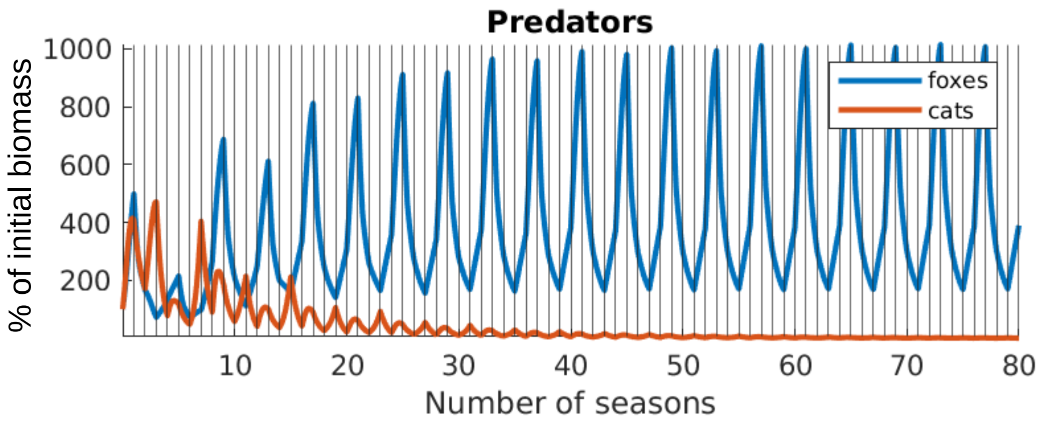
$$\phi_{ij} = \frac{a_{ij}.(P_j^t)^2}{1 + \sum_{k=3}^{6}[a_{ik}b_{ik}.(P_k^t)^2]}$$

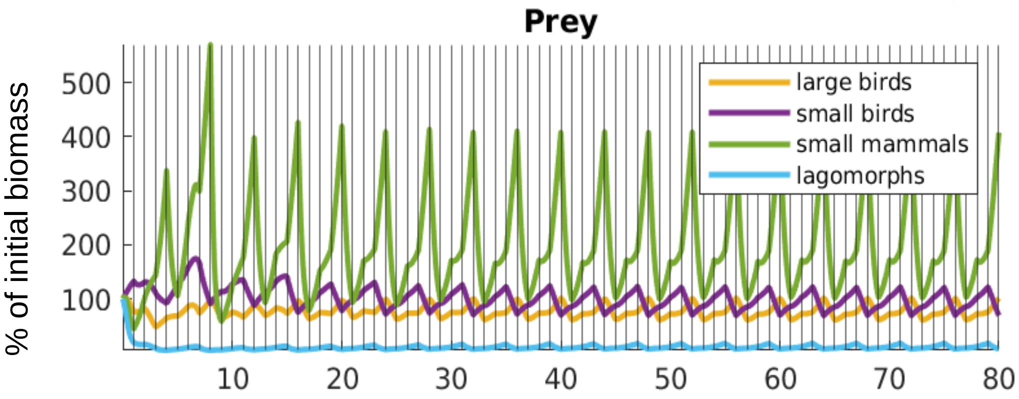
?

8 equations
8 unknown variables

$$a_{ijs} = \frac{\phi_{ijs}}{(P_j^t)^2(1 - \sum_{k=3}^{6}[\phi_{iks}.b_{iks}])}$$

Populations trajectories

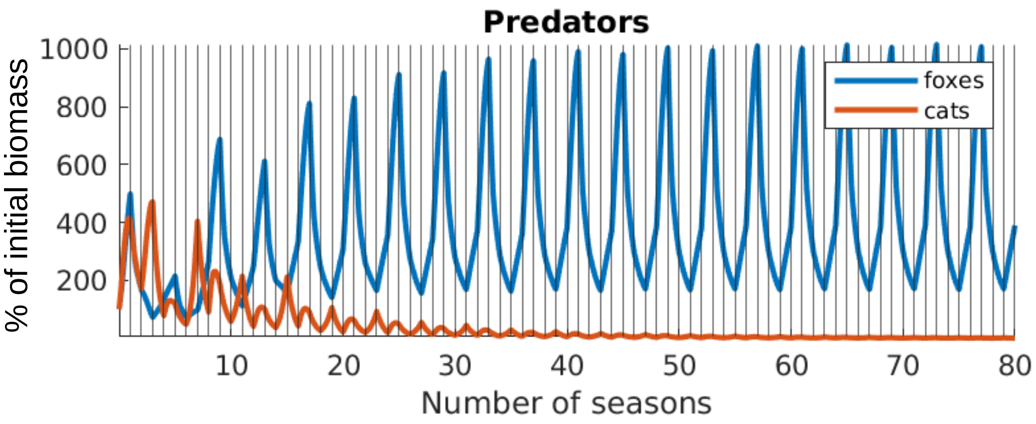


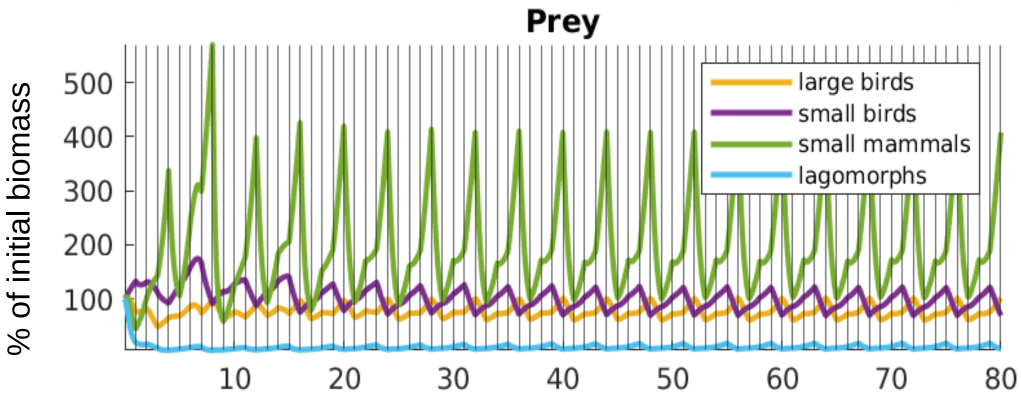


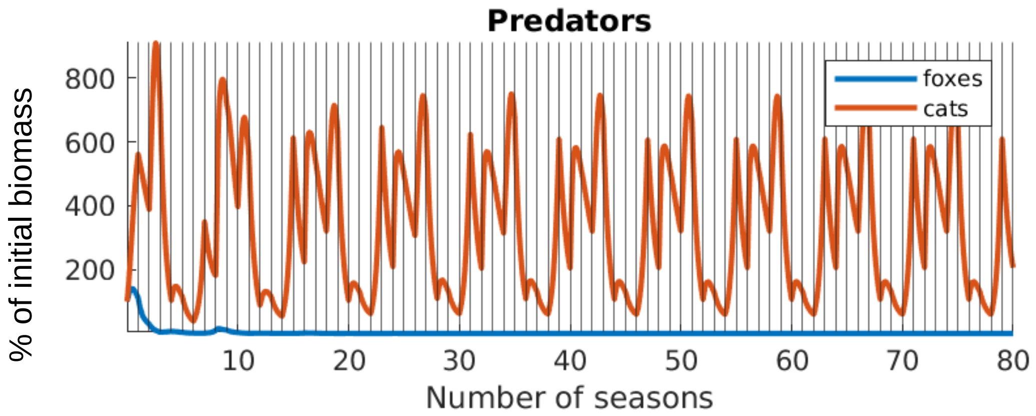
MATLAB

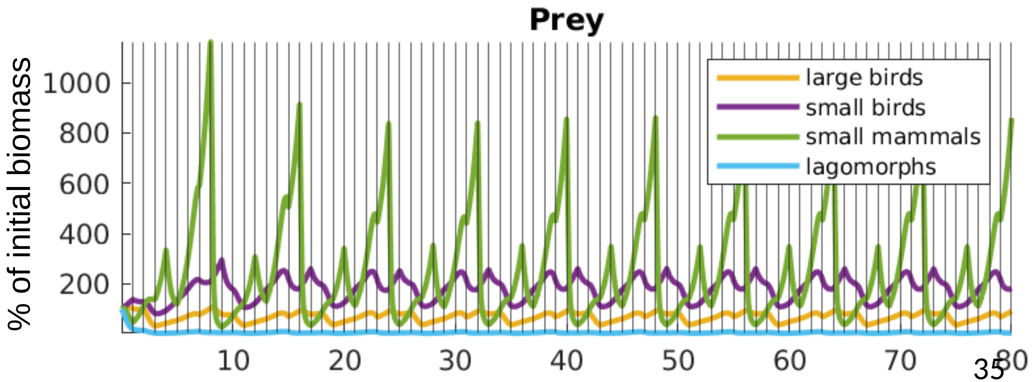
Populations trajectories

If the foxes no longer eat large birds : $a_{13s} = 0$



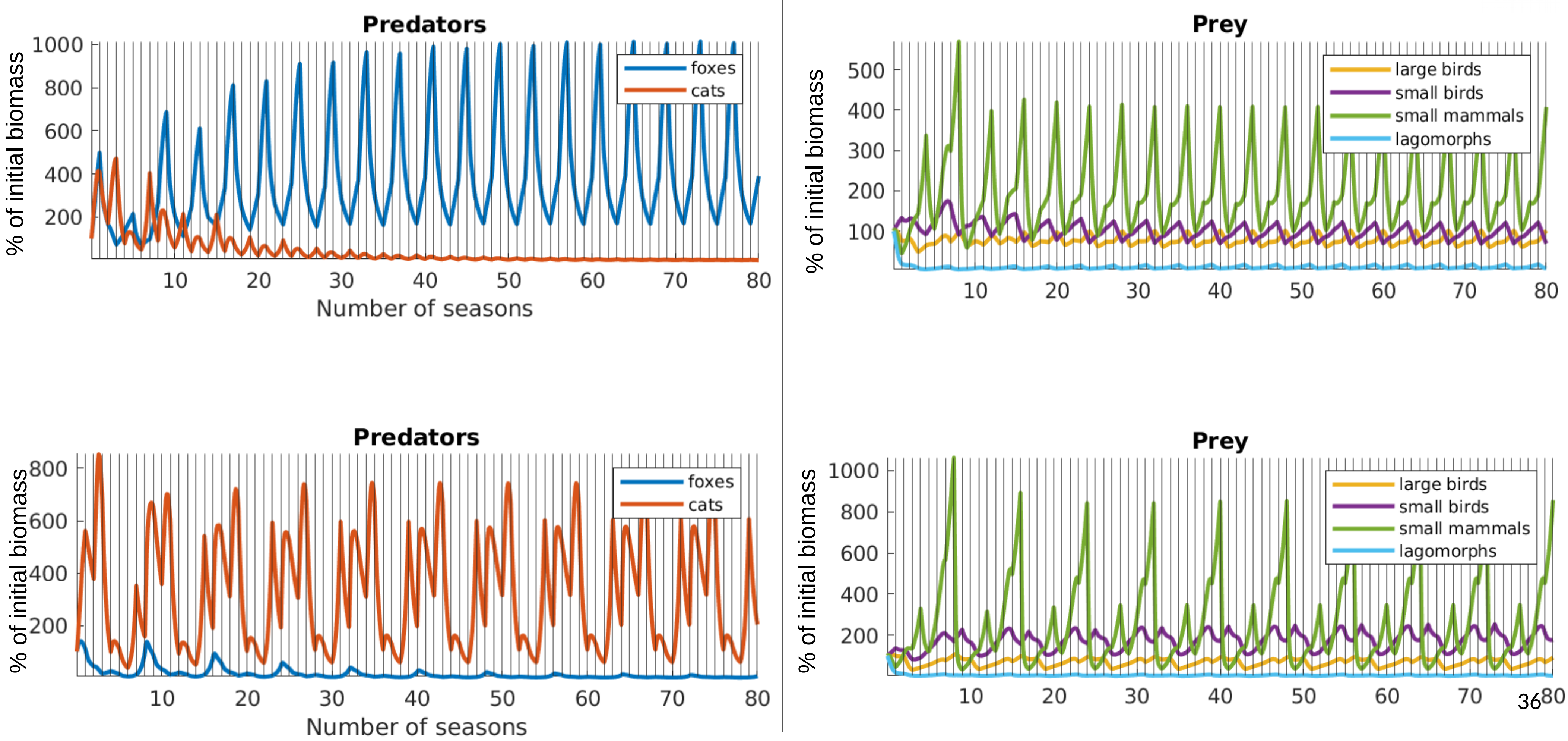






Populations trajectories

If the foxes no longer eat large birds during automn : $a_{131} = 0$



Conclusion

- The model works
- Some adjustments are still needed
- Making some brutal tests allow us to detect the importance of certain prey during certain seasons

Perspectives

- Sensibility analysis of the different model parameters using the next metrics :
  - Period length
  - Integral of the absolute value of one period
  - Oscillation size
  - Mean value at the end of each season

- Testing scenarios (cat kibbles, fox trapping and hunting,…)

- Any suggestions ?

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Thank you for your attention !