Modèle pour une population d'arbres : structure phénotypique et plasticité

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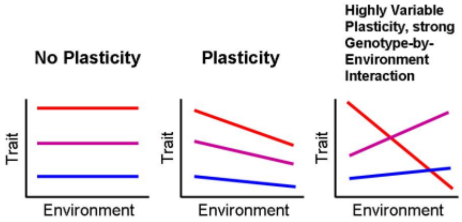
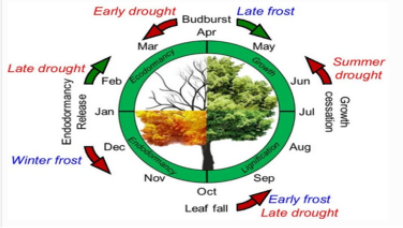
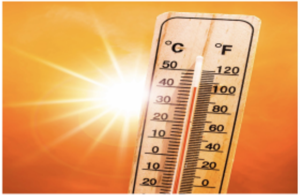




Charrier G. et al (2021) Annals of Forest Science



[illegible]

- [illegible] $x \in \mathbb{R}$ [illegible] $y \in \mathbb{R}$.

[illegible]

$$m_{k}(s; x; y)$$

- [illegible]

k : [illegible] $\in \mathbb{N}$
s : [illegible] $\in [0; 1]$
x : [illegible] $\in \mathbb{R}$
y : [illegible] $\in \mathbb{R}$

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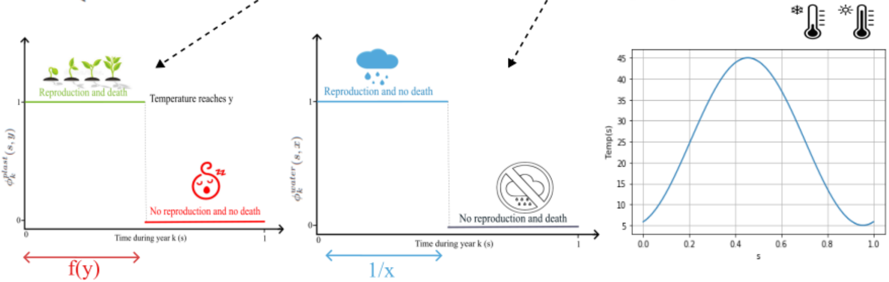
The full model is, for $\varepsilon > 0$,

$$\begin{cases} \partial_s m_k(s,x,y) = -\varepsilon \phi_k^{plast}(s,y)\gamma T_k^0(s)\left(1-\phi_k^{water}(s,x)\right) m_k(s,x,y), \\ \qquad\qquad\qquad\qquad \text{for } (k,s,x,y) \in \mathbb{N} \times [0,1] \times \mathbb{R}_+ \times \mathbb{R}^2, \end{cases}$$



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$$
\begin{cases}
\partial_s m_k(s;x;y) = -\gamma_k^{plast}(s;y)\, T_k^0(s)\,(1-\gamma_k^{water}(s;x))\, m_k(s;x;y); \\
\qquad \text{for } (k;s;x;y) \in \mathbb{N} \times [0;1] \times \mathbb{R}_+ \\
m_k(0;x;y) = m_{k-1}(1;x;y) + \dfrac{1-\iint m_{k-1}(1;\hat{x};\hat{y})\, d\hat{x}\, d\hat{y}}{\iint p_{k-1}(\hat{x};\hat{y})\, d\hat{x}\, d\hat{y}}\, p_{k-1}(x;y); \\
\qquad \text{for } (k;x;y) \in \mathbb{N} \times \mathbb{R}^2;
\end{cases}
$$

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$$
\begin{cases}
\partial_s m_k(s;x;y) = \; '' \; {}_k^{plast}(s;y) \; T_k^0(s)\,(1 \quad {}_k^{water}(s;x))\, m_k(s;x;y); \\
\qquad\qquad\qquad\qquad \text{for } (k;s;x;y) \; 2 \; \mathsf{N} \quad [0;1] \quad \mathsf{R}_+ \quad \mathsf{R}^2; \\
m_k(0;x;y) = m_{k \;\; 1}(1;x;y) + \dfrac{1 \;\; \iint m_{k-1}(1;\hat{x};\hat{y})\, d\hat{x}\, d\hat{y}}{\iint p_{k-1}(\hat{x};\hat{y})\, d\hat{x}\, d\hat{y}} p_{k \;\; 1}(x;y); \\
\qquad\qquad\qquad\qquad \text{for } (k;x;y) \; 2 \; \mathsf{N} \quad \mathsf{R}^2; \\
p_k(x;y) = \quad {}^2_k(x) \quad {}^2_y(y) \\
\quad + \sum_{k \;\; 1} \mathrm{RRRR} \; {}^2_x \; x \quad \dfrac{x_1 + x_2}{2} \quad {}^2_y \; y \quad \dfrac{y_1+y_2}{2} \quad x_1^+ \, m_k(s;x_1;y_1) \; {}_k^{plast}(s;y_1) \\
\qquad {}_k^{water}(s;x_1) \dfrac{m_k(s;x_2;y_2) \;\; {}_k^{plast}(s;y_2) \;\; {}_k^{water}(s;x_2)}{+\iint m_k(s;\hat{x};\hat{y}) \;\; {}_k^{plast}(s;\hat{y}) \;\; {}_k^{water}(s;\hat{x})\, d\hat{x}\, d\hat{y}} \, dx_1\, dx_2\, dy_1\, dy_2\, ds; \\
\qquad\qquad\qquad\qquad \text{for } (k;x;y) \; 2 \; \mathsf{N} \quad \mathsf{R}^2; \\
m_0(1;x;y) := m^0(x;y);\; p_0(x;y) = p^0; \;\text{ for } (x;y) \; 2 \; \mathsf{R}^2;
\end{cases}
$$

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The solutions of $m_{bs/"c}(0;x;y)$ *converge when* $" \ ! \ 0$ *to the solutions of :*

$$@_t n(t;x;y) = a(t;x;y)n(t;x;y) + \frac{\iint a(t;\hat{x};\hat{y})n(t;\hat{x};\hat{y})\,d\hat{x}\,d\hat{y}}{+ R[n](t)} \iiiint {}^{2}_{x}\ x \ \frac{x_1+x_2}{2} \ {}^{2}_{y}\ y \ \frac{y_1+y_2}{2} \ x_1^{+} n(t;x_1;y_1)$$

$$n(t;x_2;y_2)\ [n(t;\ ;\ )](t;x_1;y_1;x_2;y_2)\,dx_1\,dx_2\,dy_1\,dy_2 + {}^{2}_{x}(x) \ {}^{2}_{y}(y) \ ;$$

° \0}0

$$a(t;x;y) = \int_0^1 T\ (s)\ {}_{plast}(s;y)(1 \ \ {}_{water}(s;x))\,ds$$

9jL

$$[n(t;\ ;\ )](t;x_1;y_1;x_2;y_2) = \int_0^1 \frac{{}^{plast}(t;s;y_1)\ {}^{plast}(t;s;y_2)\ {}^{water}(t;s;x_1)\ {}^{water}(t;s;x_2)}{+ \iint n(t;\hat{x};\hat{y}) \int_0^1 {}^{water}(t;s;\hat{x})\ {}^{plast}(t;s;\hat{y})\,ds\,d\hat{x}\,d\hat{y}}\,ds:$$

[0«}_ §_H g9H}n Hnq_H hnL0eI L´j9h_H nT §\0 qnq«e9§_nj _¶0 9jL h09j q\0jn§´q_H §}9_§

¯ \0j ^2_x 9jL ^2_y 9}0 h9eeJ °0 \9®0

$$n(t; \; ; \;) \qquad {}^2_x(\quad X(t)) \quad {}^2_y(\quad Y(t))$$

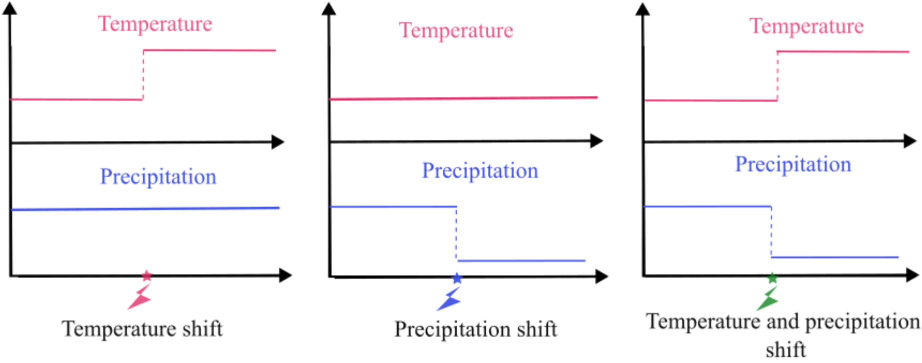
p}nqn _§_nj

¦ \0 h9H}n Hnq_H 0x«9§_nj _ §\0j

$$\frac{d}{dt} \begin{matrix} X \\ Y \end{matrix} = \begin{matrix} {}^2_x \left(-@_x a(t; X(t); Y(t)) + a(t; X(t); Y(t)) \frac{@_{x_1}}{} + \frac{a(t; X(t); Y(t))}{2X(t)} \right) \\ {}^2_y \left(-@_y a(t; X(t); Y(t)) + a(t; X(t); Y(t)) \frac{@_{y_1}}{} \right) \end{matrix}$$

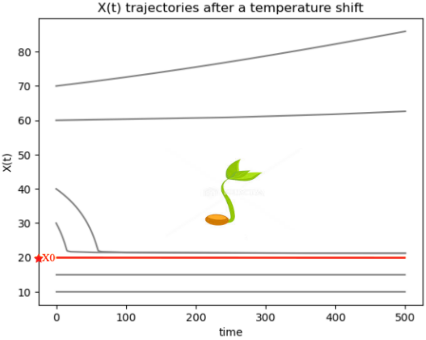
Ge_h9§0 H\9jX0 9 9@}«q§ _T§

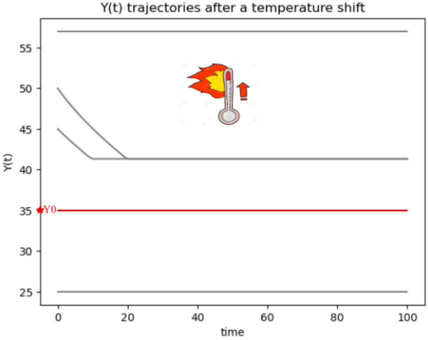
~§09L´] §9§0 ! 0j®_}nj h0j§9e _T§ ! 0®ne«§_nj9}´ n«§Hnh0



N¸OH§ nT 9 §0 h q0}9§«}0 _T§ nj §\0 0®ne«§_nj nT *X* 9jL *Y*

¦0 h q0}9§«}0 _T§) jn h n}§9e_§´) jn 9L9q§9§_nju

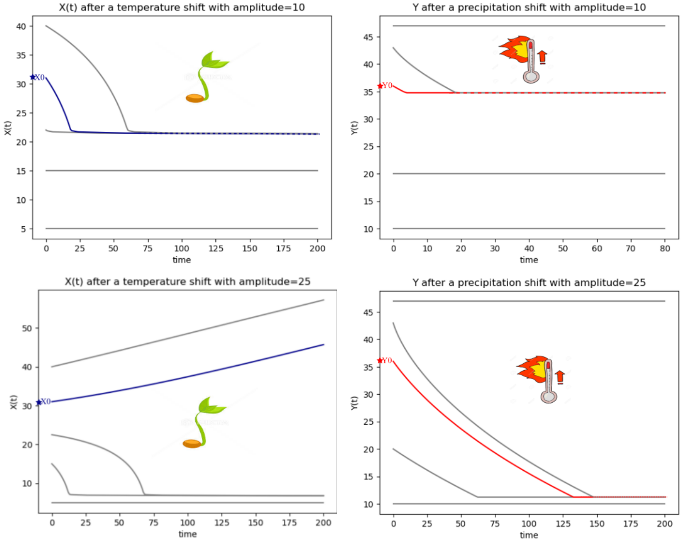




N¸OH§ nT 9 q}OH_q_§9§_nj _T§ nj §\O O®ne«§_nj nT *X* 9jL *Y*

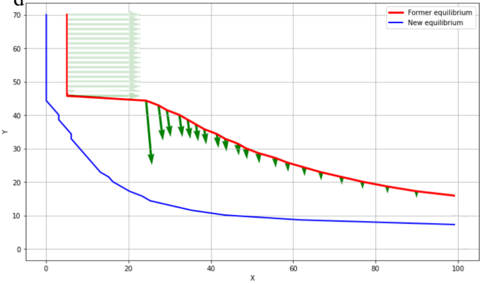
p}OH_q_§9§_nj _T§ *!* ~h9ee _T§ *)* 8L9q§9§_nj

! ?_X _T§ *)* G\9jX0 nT @_nenX_H9e j_H\0



Gn]0®ne«§_nj nT X 9jL Y 9T§0} 9 _T§

d



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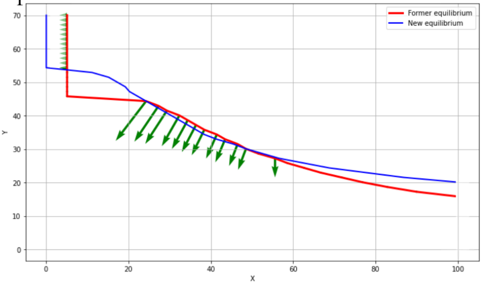

) Gn]0®ne«§_nj nT X 9jL Y h9´ \0eq §\0 9L9q§9§_nj §n §\0 j0° 0j®_}nj h0j§ 0®0j °\0j §\0 9hqe_§«L0 nT q}0H_q_§9§_nj _T§ _ _X\u

) 8j 9L9q§9§_nj §n 9 _T§ _j @n§\ §0hq0}9§«}0 9jL q}0H_q_§9§_nj L0q0jL nj §\0 @9e9jH0 @0§°00j §\0 §°n

GnjHe« _nj

- pe9 §_H_§´ h9´ q}0®0j§ hn}§9e_§´J @«§ _jL0} 0®ne«§_nj
- ¦0hq0}9§«}0 9jL °9§0} §}0 H9j \9®0 L_¸0}0j§ 9L9q§_®0 Hnj 0x«0jH0
- 8hqe0 _T§ H9j e09L §\0 q0H_0 §n 9 L_¸0}0j§ @_nenX_H9e j_H\0

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Thank you for your attention !