Mathematical modeling of age-structured natural populations

- Age-structured natural populations can be modeled using stochastic individual-based models (Fournier and Méléard, 2004; Tran, 2008; ...).
- A large class of structured population models, including:
  - interactions,
  - non-stationary phenomena (shocks, varying environments, ...),
  - changing individual characteristics.
- **Advantages**:
  - Theoretical framework: pathwise representations, large-population approximations by partial differential equations, ...
  - Exact efficient simulation (R library `IBMPopSim`, Giorgi and Lemaire) : allows the computation of population indicator (health, viability, ...)

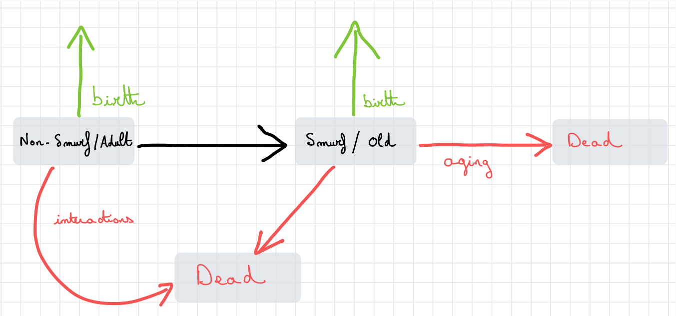
First goal: build model of aging in natural populations and explore parameter spaces/ compatibility with experimental measurements.

Modelling aging in the wild

Individuals $I = (\tau^b, \tau^d, x)$ characterized by their

- date of birth τ^b (age $a(t) = t - \tau^b$)
- date of death τ^d (∞ if alive)
- smurf phenotype $x = NS$ or S.

Events



A first model

Transition rates

$$\mathbb{P}\big(\text{event } e \text{ occurs to an individual } I \in (t, t + ṭ] | \mathcal{F}_t\big) \simeq \lambda^e_t(I, Z_t)ṭ.$$

- Demographic rates for $I = (\tau^b, \tau^d, x) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \{NS, S\}$:
  - Death rate:

$$\lambda^d(t, I, Z_t) = d\mathbf{1}_{\{x=S\}} + c_d N^{NS}_t.$$

  - Birth rate:

$$\lambda^b(t, I) = b_{NS}\mathbf{1}_{\{x=NS\}} + b_S\mathbf{1}_{\{x=S\}}, \quad b_S << b_{NS}.$$

- Transition NS to S: *Do interactions accelerate ageing?*

$$\lambda^s(t, I, Z_t) = k_S(a(t)) + c_s N^{NS}_t.$$

- Arrivals in the population at constant rate μ.

Numerical results

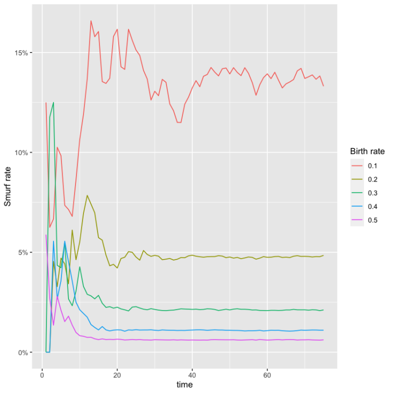


Figure 1: Smurf rate

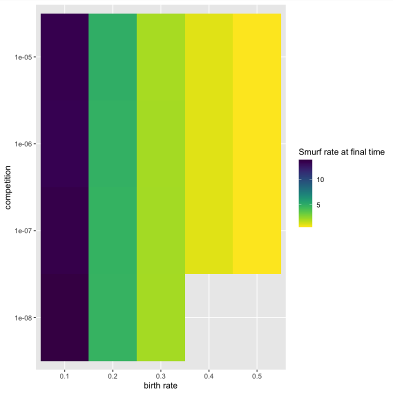


Figure 2: Smurf rate (birth rate vs c_d)

- Parameters: $c_s = 0$ (no acceleration of ageing due to environmental stress), $\mu = 9,\ b_S = 0,\ c_d = 10^{-6}$ (Figure 1).

Numerical results: impact of c_s

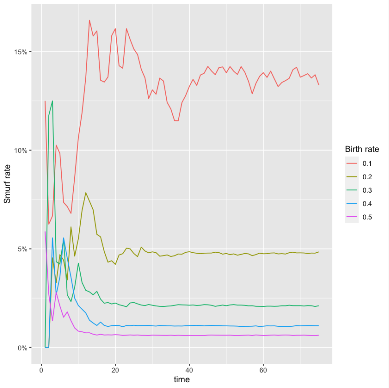


Figure 3: Smurf rate $c_s = 0$

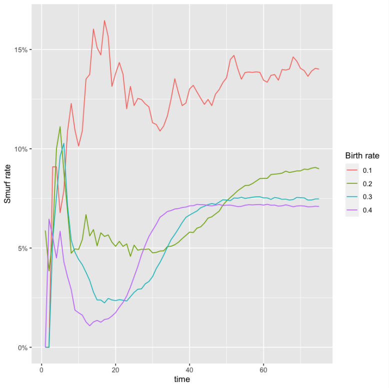


Figure 4: Smurf rate $c_s = 10^{-7}$

Parameters: $c_d = 10^{-6}$, $\mu = 9$.

Numerical results

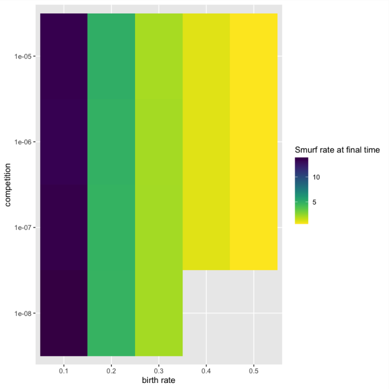


Figure 5: Smurf rate (birth rate vs c_d, $c_s = 0$)

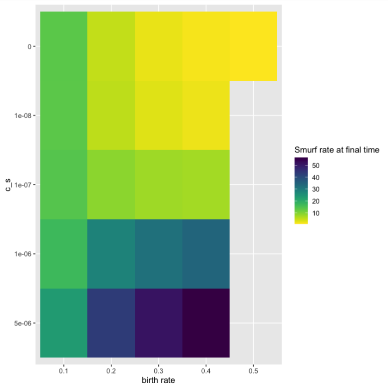


Figure 6: Smurf rate (birth rate vs c_s, $c_d = 10^{-6}$)

Large population approximation

Generalisation and large population approximation

- $n_{NS}(a,t)$: population density of non smurf of age a at time t.
- $n_S(u,t)$: population density of smurf of "smurf age" u at time t.

$$
\begin{cases}
(\partial_a + \partial_t) n_{NS}(a,t) = -c_d(N_{NS}(t), N_S(t)) n_{NS}(a,t) \\
\qquad\qquad\qquad\qquad -\big(k_s(a) + c_s(N_{NS}(t), N_S(t))\big) n_{NS}(a,t), \\
(\partial_a + \partial_t) n_S(u,t) = (-k_d(u) - \tilde{c}_d(N_{NS}(t), N_S(t))) n_S(u,t), \\
n_{NS}(0,t) = \int_0^\infty \big(b_{NS}(a) n_{NS}(a,t) + b_S(a) n_S(a,t)\big) da, \\
n_S(0,t) = \int_0^\infty \big(k_s(a) + c_s(N_{NS}(t), N_S(t))\big) n_{NS}(a,t) da
\end{cases}
$$

- With, $N_x(t) = \int_0^\infty n_x(a,t) da$, $x = NS$ or S.

Study of existence, steady state and stablity (L. Breuil)

Non parametric inference of individual transiton and death rate

A first question How can we estimate "individual" rates functions k_S and k_d?

- Going back to lab data:
  - Individual data Cohort of 1159 of females drosophila melanogaster (age of 10 days) in individual vials.
  - Observations: $(\tau^i_{NS}, \tau^i_S)_{i=1,\dots,m}$, i.i.d.

$$\mathbb{P}(\tau^{NS} \geq t) = e^{-\int_0^t k_S(u)\,\mathrm{d}u}, \quad \mathbb{P}(\tau^S \geq a) = e^{-\int_0^a k_d(u) du}$$

Non-parametric kernel estimation using **associated kernels**

- Associates kernels overcome boundary bias estimation issue of standard kernels.
- Asymptotic expansion of MISE, CLT, adaptive bandwith choice.

Nonparametric hazard rate estimation with associated kernels and minimax bandwidth choice, Breuil, K., preprint (2025)

Smurf transition rate and death rate

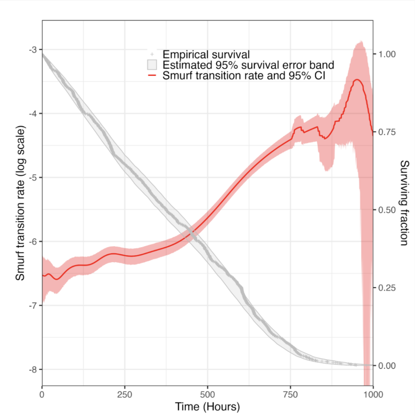


Figure 7: Smurf transition rate

Figure 8: Death rate once smurf

Two-phase ageing model : a comprehensive statistical analysis, Breuil, Doumic, K. and M. Rera, in preparation.

Perspectives

Coming back to natural populations (with overlapping generations)

- Inference with population data instead of individual data.
- Impact of interactions/competition on aging:
  - Study of drosophila population in simulated/controlled natural environment (ECOTRON).
  - Inference and statistical test for impact of environmental stress on aging.
- Estimation of demographic indicators with observations of the number (or proportion) of Smurf/Non-Smurf individuals over time?

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Thank you!