Bats Monitoring: A Classification Procedure of Bats Behaviors based on Hawkes Processes

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Ecological problematic and motivation

Two behaviors:

- **commuting** mode;
- **foraging** mode.

Goal: predicting the majority behavior of bats at sites throughout France.

- discriminate the **foraging** behavior from the **commuting** behavior.

Motivations:

- contribute to address spatial ecology issues;
- automate decision-making with few input variables.

Data: time of echolocation calls of **differents species** recorded as part of **Vigie-Chiro** participatory project.

- we focus on the **Common Pipistrelle**.

VIGIECHIRO

Echolocation: used by bats for **foraging** and **commuting**.

Behavioral characterization: via the way bats emit calls (see Griffin *et al.* (1960)).

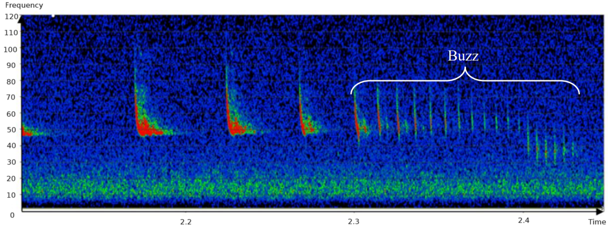


Figure: Sonogram containing a feeding buzz.

- consider the temporal distribution of the calls.
- sequence of calls $(T_\ell)_{\ell \geq 1}$ as a realization of a point process N.

Modeling the sequence of calls

Point processes: model the occurrence of random events over time.

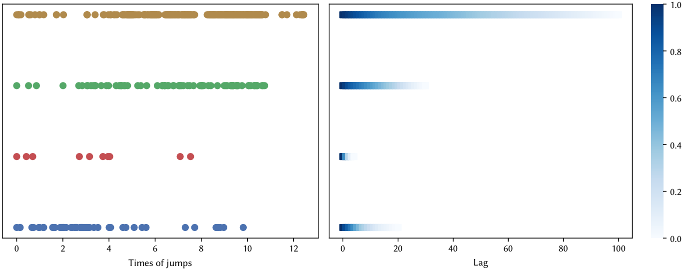


Figure: Left: the start times of echolocation calls sequences, right: autocorrelation as a function of the lag for four nights.

- presence of strong temporal dependence in data.

Data modeling with Hawkes processes

The linear exponential Hawkes process : a point process N with conditional intensity function (see Daley and Vere-Jones (2003)):

$$\lambda_\theta(t) := \mu + \int_0^t \alpha\beta e^{-\beta(t-s)} dN_s = \mu + \sum_{T_\ell < t} \alpha\beta e^{-\beta(t-T_\ell)},$$

where :
- $(T_\ell)_{\ell \geq 1}$ the **time jumps** of the process;
- $\theta \in \Theta = \{\mu > 0, 0 \leq \alpha < 1, \ \beta \geq 0\}$
- $\mu \longrightarrow$ **exogenous intensity**;
- $\alpha \longrightarrow$ **arrival intensity**;
- $\beta \longrightarrow$ **rate of the decay**.

Modelisation: **the start time** of a call correspond to a **jump** of the Hawkes process.

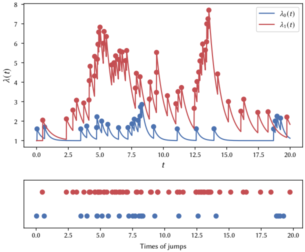
Classification model

Let $\mathcal{D}_n^L = \left\{(\mathcal{T}_T^1, Y^1), \ldots, (\mathcal{T}_T^n, Y^n)\right\}$ be a sample of i.i.d. observations such that:

- **Label:** $Y \sim \mathcal{B}\left(p^*\right), \; Y \in \{0, 1\}$;
- **Feature:** $\mathcal{T}_T = \left(T_1, \ldots, T_{N_T}\right)$ of intensity $\lambda_{\theta^*_Y}(t)$ on $[0, T]$ with $\theta^*_Y \in \Theta$.

Goal: learn a decision rule g from $\mathcal{D}_n^L$ such that $g(\mathcal{T}_T)$ is a prediction of the label Y.

▶ given a new unlabeled feature $\mathcal{T}_T^{n+1}$, our guess for Y^{n+1} is $g\left(\mathcal{T}_T^{n+1}\right)$.



Quality of label prediction: measured by its missclassification risk

$$\mathcal{R}(g) := \mathbb{P}\left(g\left(\mathcal{T}_T^{n+1}\right) \neq Y^{n+1}\right).$$

Bayes rule and empirical risk minimization

Bayes rule: characterized by

$$g_{p^*,\boldsymbol{\theta}^*}(\mathcal{T}_T) = \mathbb{1}_{\left\{\eta_{p^*,\boldsymbol{\theta}^*}(\mathcal{T}_T) > \frac{1}{2}\right\}}$$

where $\eta_{p^*,\boldsymbol{\theta}^*}(\mathcal{T}_T) := \mathbb{P}(Y = 1 | \mathcal{T}_T) = \dfrac{p^* \exp\Big(F_{\theta_1^*}(\mathcal{T}_T)\Big)}{p^* \exp\Big(F_{\theta_1^*}(\mathcal{T}_T)\Big) + (1-p^*) \exp\Big(F_{\theta_0^*}(\mathcal{T}_T)\Big)}$

Empirical risk: based on $\mathcal{D}_n$ estimates $\hat{p} = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\{Y^i=1\}}$ and solve :

$$\hat{\boldsymbol{\theta}} \in \underset{\boldsymbol{\theta}\in\Theta^2}{\operatorname{argmin}} \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\{g_{\hat{p},\boldsymbol{\theta}}(\mathcal{T}_T^i) \neq Y^i\}}$$

- minimize this require to solve a non convex optimization problem.

ERM procedure

Convexification: replace the $0-1$ loss by a **convex surrogate** (see Zhang (2004)) and based on $\mathcal{D}_n$ solve instead :

$$\hat{\theta} \in \underset{\theta \in \Theta^2}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \left(Z^i - f_{\hat{p},\theta}(\mathcal{T}_T^i) \right)^2$$

where $Z^i = 2Y_i - 1$ and $f_{\hat{p},\theta}(\mathcal{T}_T) = 2\eta_{\hat{p},\theta}(\mathcal{T}_T) - 1$ with

$$\eta_{\hat{p},\theta}\left(\mathcal{T}_T\right) = \frac{\hat{p} \exp\left(F_{\theta_1}(\mathcal{T}_T)\right)}{\hat{p} \exp\left(F_{\theta_1}(\mathcal{T}_T)\right) + (1-\hat{p}) \exp\left(F_{\theta_0}(\mathcal{T}_T)\right)}$$

Classifier: $\hat{g}(\mathcal{T}_T) = \mathbb{1}_{\left\{\hat{f}(\mathcal{T}_T) \geq 0\right\}}$.

ERM procedure: provides estimates of (θ_0^*, θ_1^*).

- gives a model for the behavior within each class.

Model evaluation: by performing a goodness-of-fit test.

- using the **Time-Rescaling Theorem** (see Daley and Vere-Jones (2003)):

Theorem

Let $\Lambda(t) = \int_0^t \lambda(s)\, ds$ be the **compensator** of the process N. Then, a.s., the transformed sequence $\{\tau_j = \Lambda(T_j)\}$ is a realization of a unit-rate Poisson process if and only if the original sequence $\{T_j\}$ is a realization from the point process N.

Test H_0: "*the sequence of observations is a realization of the point process with intensity $\lambda_{\hat{\theta}_k}$*".

- test if $\{\Lambda_{\hat{\theta}_k}(T_{j+1}) - \Lambda_{\hat{\theta}_k}(T_j)\} \overset{\text{iid}}{\sim} \mathcal{E}(1)$

Simulation: by cluster representation using the branching properties of the self-exciting Hawkes process (see Hawkes and Oakes (1974)).

Panel of scenarios:

	Scenario 1	*Scenario 2*
μ_0	1.0	1.0
μ_1	1.0	0.5
α_0	0.2	0.0
α_1	0.7	0.5
β_0	3.0	0.0
β_1	1.5	1.5

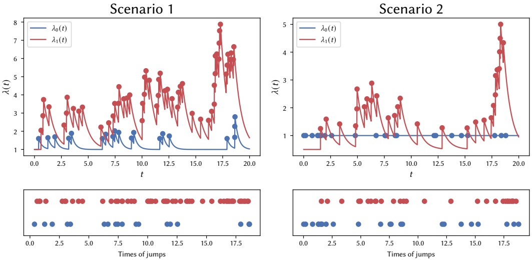


Table: Scenario panel used to study procedure performance.

Simulation scheme: 20 Monte-Carlo repetitions in each scenario.

In both scenarios: $n_{\text{train}} = 300$, $n_{\text{test}} = 1000$, $T = 20$, $p^* = 0.5$.

Empirical error rate: $\hat{\mathcal{R}}_n(g) := \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\left\{g(\mathcal{T}_T^{(i)}) \neq Y_i\right\}}$.

	Error Rate		
	Bayes	ERM	RF
Scenario 1	0.07 (0.00)	0.07 (0.00)	0.09 (0.01)
Scenario 2	0.17 (0.00)	0.17 (0.01)	0.30 (0.03)

Table: Empirical error averaged over 20 repetitions.

Goodness-of-fit test: if $\hat{g}\left(\mathcal{T}^i\right) = k$ test if $\{\Lambda_{\hat{\theta}_k}(T^i_{j+1}) - \Lambda_{\hat{\theta}_k}(T^i_j)\} \overset{\text{iid}}{\sim} \mathcal{E}(1)$

		$\hat{g}(\mathcal{T})$	
		p-value	Acceptance Rate
Scenario 1	Class 0	0.51 (0.01)	0.96 (0.01)
	Class 1	0.51 (0.03)	0.95 (0.02)
Scenario 2	Class 0	0.41 (0.01)	0.89 (0.01)
	Class 1	0.41 (0.03)	0.90 (0.01)

Table: Mean p-values and acceptance rate for a 5% significance level test over 20 repetitions.

Real data

- Calls recorded over one night at 755 sites in France.

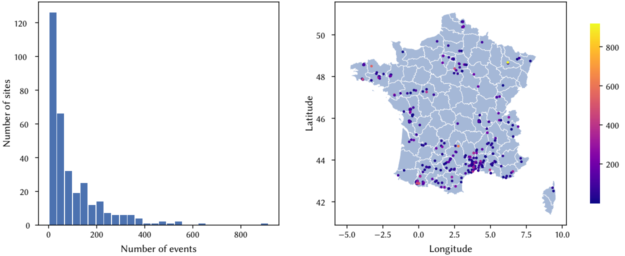


Figure: Each point on the map represents a site and its colour refers to the number of events in the temporal sequences.

- 332 labeled sites.
- 423 unlabeled sites.

Classification on labeled data

Assess the performance: by comparing with labels given by the metric.

Evaluation scheme: repeat 20 times:

- choose 75% for training and the remaining 25% for testing.

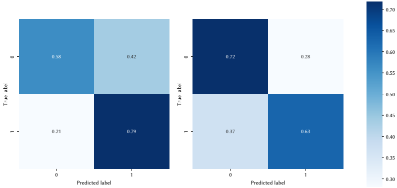


Figure: Confusion matrix of prediction on $\mathcal{D}^L_{n_{\text{test}}}$. Score: ERM: 68.13% (4.15), RF: 67.35% (2.21).

	$\hat{g}(\mathcal{T})$	
	p-value	Acceptance Rate
Class 0	0.26 (0.06)	0.66 (0.11)
Class 1	0.15 (0.03)	0.45 (0.07)

Table: Mean p-values and reject rate for a 5% significance level test on $\mathcal{D}^L_{n_{\text{test}}}$.

Prediction on intermediate sites: tricky since bats have mixed behavior.

Training: based on labeled data. ► $(\hat{\theta}_0, \hat{\theta}_1)$, $\hat{\eta}$, $\hat{g}$.

Goodness-of-fit test: on unlabeled data

	$\hat{g}(\mathcal{T})$	
	p-value	Acceptance Rate
Class 0	0.15	0.43
Class 1	0.21	0.49

Table: Mean p-values and acceptance rate for a 5% significance level test.

Discussion:

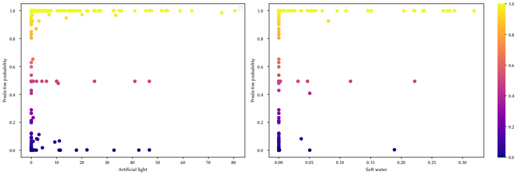


Figure: Predictive probability given by $\hat{g}$ on $\mathcal{D}_n^U$ as a function of environmental covariates.

Conclusion:

- validation of the procedure on synthetic data;
- Hawkes processes modeling: revelant for echolocation calls data;
- classification procedure: prediction and behavioral confidence index;
- provides a tool to ecologist for predicting bats behavior.

Bats Monitoring: A Classification Procedure of Bats Behaviors based on Hawkes Processes, C. Denis, C. Dion-Blanc, R.E. Lacoste, L. Sansonnet and Y. Bas (2023), The Journal of the Royal Statistical Society, Series C.

Ongoing work (1/2)

Future exploration: consider species with more marked majority behavior.

- data processing for the **Western Barbastelle**, **Daubenton's myotis**.

Model enrichment: by considering multivariate Hawkes processes.

- model **simultaneously** the call sequence of multiple species.
- incorporates the effects of cooperation and competition between species.

ERM-Lasso classification rule for Multivariate Hawkes Processes paths, C. Denis, C. Dion-Blanc, R.E. Lacoste and L. Sansonnet, HAL/arXiv.

- include inhibition interaction in the model.

Ongoing work (2/2)

Package development: user-friendly tools for simulation, estimation and classification of exponential multivariate Hawkes processes.

Features :

- learning for short-time path repetition data.
- suitable for large-scale networks (Lasso procedure).
- code source implemented in C++ for rapid computation.

Sparkle: a statistical learning toolkit for Hawkes process modeling in `Python`, R.E. Lacoste, soon on HAL/arXiv.

- ▶ Available soon on GitHub at **https://github.com/romain-e-lacoste**

Thank you for your attention!

Any questions?