Journée Idefix

Perfect transmission in periodic waveguides with localized defects

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Collaboration with T. Creuset[1] and Z. Moitier[1]

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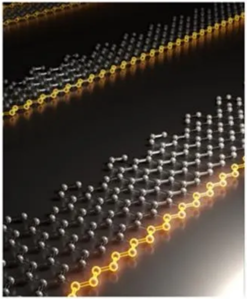
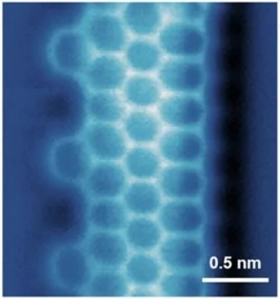
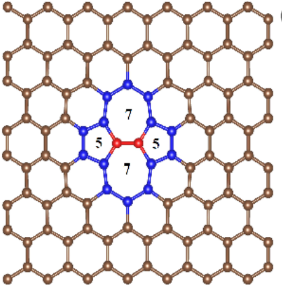
Introduction

- We are interested in periodic materials (graphene, photonic cristal, . . .).



- For certain bands of frequencies, waves can propagate in such media.

> - How to study scattering of waves by localized defects
> - How to design invisible defects

Outline of the talk

1. Wave propagation in periodic waveguides
2. Scattering by a defect
3. Perfect transmission
   - A first mechanism
   - Perfect transmission via the use of the Fano resonance
   - Study at the edges of the spectral bands

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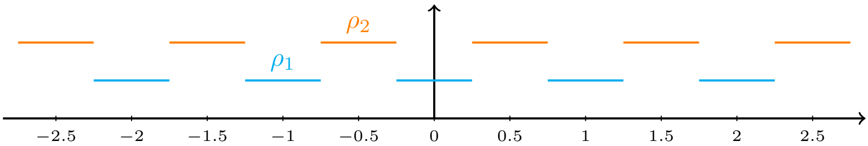
Setting



- For $\omega > 0$, consider the 1D problem

$$u'' + \rho\,\omega^2 u = 0 \quad \text{in } \mathbb{R}, \tag{$\mathscr{P}$}$$

with a piecewise constant ρ which is 1-periodic.

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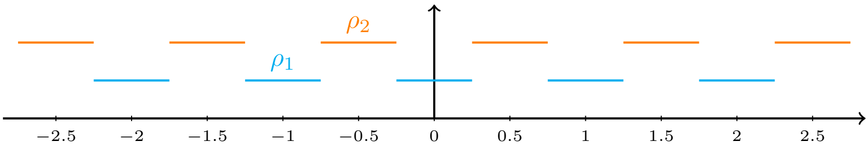

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$$u'' + \rho\,\omega^2 u = 0 \quad \text{in } \mathbb{R}, \qquad (\mathscr{P})$$

with a piecewise constant ρ which is 1-periodic.

- Denote by A the unbounded operator of $\mathrm{L}^2(\mathbb{R})$, endowed with the inner product $(u, v) \mapsto \int_{\mathbb{R}} \rho uv\,dx$, such that

$$Au = -\frac{1}{\rho}u'' \qquad \text{and} \qquad D(A) = \mathrm{H}^2(\mathbb{R}).$$

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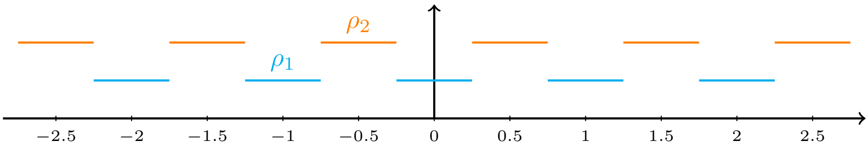

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Proposition. A is selfadjoint and positive.

The Floquet-Bloch transform

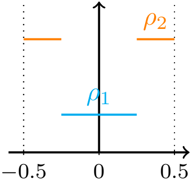
Definition. For $u \in \mathscr{D}(\mathbb{R})$, the Floquet-Bloch transform is defined by:

$$\forall x, \eta \in \mathbb{R}, \qquad U_\eta(x) := \sum_{n\in\mathbb{Z}} u(x+n)e^{i\eta n}.$$

► It converts the initial problem set in $\mathbb{R}$ into a family of spectral problems set in the bounded unit cell $I := (-1/2; 1/2)$ with quasi-periodic BCs:

For $\eta \in (-\pi; \pi]$,

$$\left|\begin{array}{rcl} \multicolumn{3}{l}{\text{Find } U_\eta \in \mathrm{H}^1(I) \text{ such that}} \\ -U_\eta'' &=& \lambda(\eta)\,\rho\,U_\eta \quad \text{in } I \\ U_\eta(-1/2) &=& e^{i\eta}U_\eta(1/2) \\ \partial_x U_\eta(-1/2) &=& e^{i\eta}\partial_x U_\eta(1/2). \end{array}\right.$$



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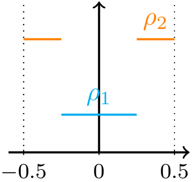
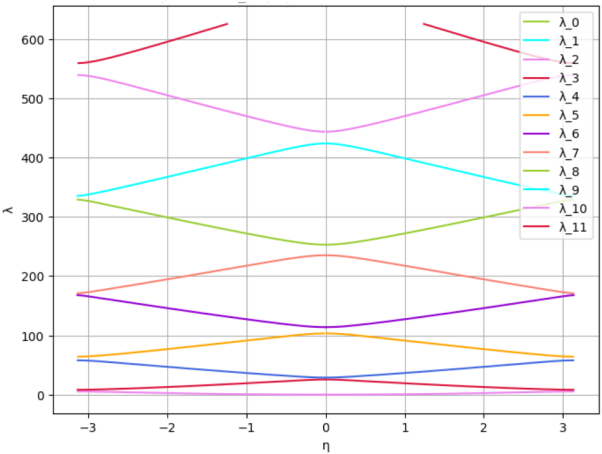


► For all $\eta \in (-\pi; \pi]$, the corresponding eigenvalue problem admits a sequence of real positive eigenvalues

$$0 \leq \lambda_1(\eta) \leq \lambda_2(\eta) \leq \ldots.$$

Dispersion curves



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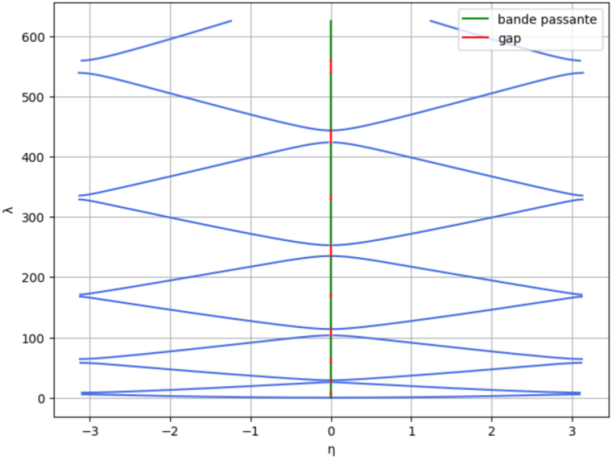

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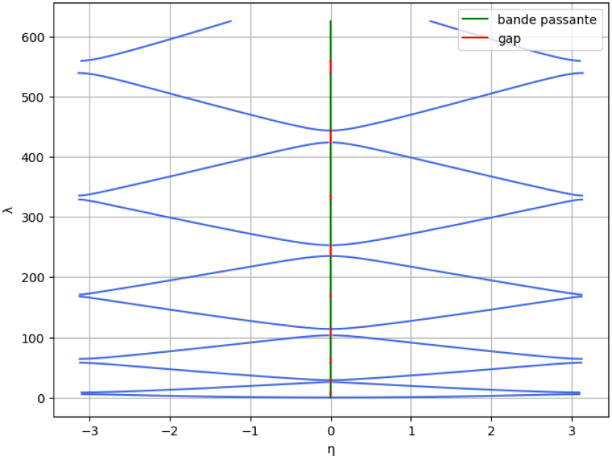

Definition.

- For $p \geq 1$, $\eta \mapsto \lambda_p(\eta)$ is the p-th dispersion curves.

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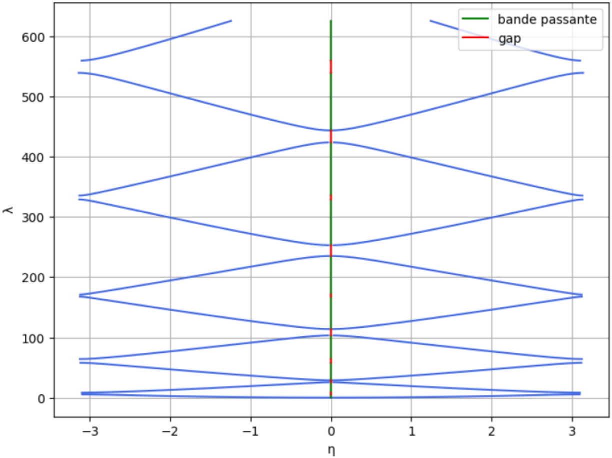


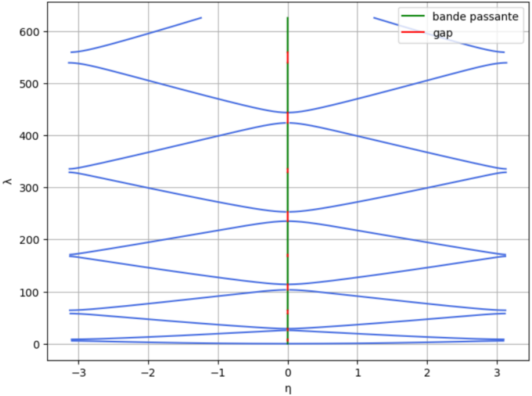
Definition.

- For $p \geq 1$, $\eta \mapsto \lambda_p(\eta)$ is the p-th dispersion curves.
- For $p \geq 1$, set $I_p := \overline{\lambda_p(-\pi;\pi)}$. We call I_p the p-th spectral band.

The Floquet theorem

Theorem. The spectrum of the operator A, denoted by $\sigma(A)$, is such that

$$\sigma(A) = \sigma_{\text{ess}}(A) = \bigcup_{p \in \mathbb{N}^*} I_p.$$



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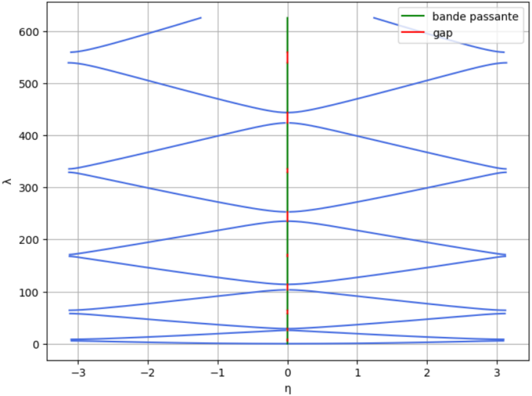


- This band/gap structure for $\sigma(A)$ is typical of periodic materials. It is used to create filters.

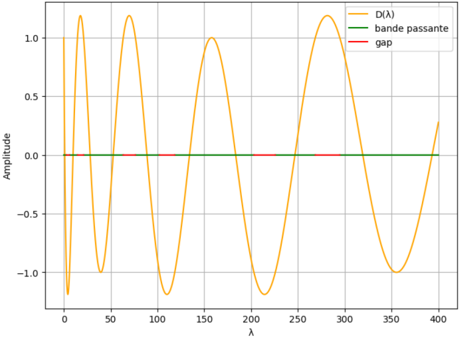
Characterization of spectral bands

- For this 1D problem, we can make explicit calculus. Set

$$D(\omega) := \cos\left(\frac{\sqrt{\rho_1\omega}}{2}\right)\cos\left(\frac{\sqrt{\rho_2\omega}}{2}\right) - \frac{1}{2}\left(\frac{\rho_1}{\rho_2} + \frac{\rho_2}{\rho_1}\right)\sin\left(\frac{\sqrt{\rho_1\omega}}{2}\right)\sin\left(\frac{\sqrt{\rho_2\omega}}{2}\right).$$

Theorem. Propagating modes exist ($\omega^2 \in \sigma_{\text{ess}}(A)$) $\Leftrightarrow \exists \eta \in (-\pi;\pi]$ satisfying

$$D(\omega) = \cos(\eta).$$



$D(\cdot)$ wrt $\lambda = \omega^2$ for $\sqrt{\rho_2}/\sqrt{\rho_1} = 2$.

Energy flux in the spectral band

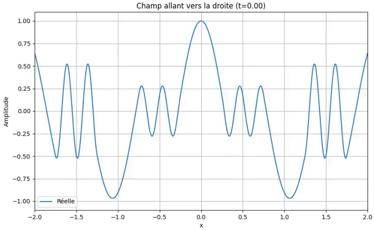
Definition. Let u be a solution of $(\mathscr{P})$. Its energy flux is

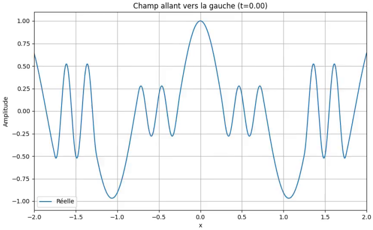
$$\phi(u) := \Im m\left(u'(x)\overline{u(x)}\right)$$

(independent of x).

$\rightarrow$ In the following, we shall always work with ω^2 in one of the I_p.

- Denote by w_+/w_- the wave propagating to the right/left ($\pm\phi(w_\pm) > 0$).





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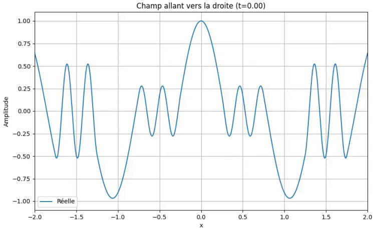
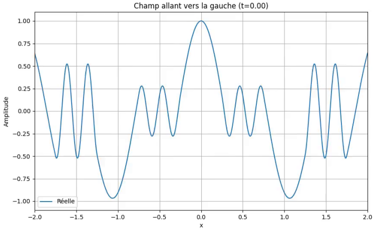
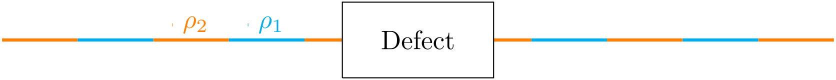




Remark. In 1D, one has **only one** propagating mode per spectral band.

Outline of the talk

Scattering by a defect



- We study the scattering problem

Find $u \in \mathrm{H}^2_{\mathrm{loc}}(\mathbb{R})$ and $R, T \in \mathbb{C}^2$ such that

$$\begin{aligned} u'' + \omega^2 \rho u &= 0 && \text{in } \mathbb{R} \\ u(x) &= w_+(x) + R w_-(x) && \text{after the defect} \\ u(x) &= T w_+(x) && \text{before the defect.} \end{aligned}$$

Scattering by a defect

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$$\left|\begin{array}{lcll} \multicolumn{4}{l}{\text{Find } u \in \mathrm{H}^2_{\mathrm{loc}}(\mathbb{R}) \text{ and } R, T \in \mathbb{C}^2 \text{ such that}} \\ u'' + \omega^2 \rho u & = & 0 & \text{in } \mathbb{R} \\ u(x) & = & w_+(x) + R w_-(x) & \text{after the defect} \\ u(x) & = & T w_+(x) & \text{before the defect.} \end{array}\right.$$

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- R/T are the reflection/transmission coef.. For $\omega^2 \in \sigma_{\mathrm{ess}}(A)$, one has

$$|R|^2 + |T|^2 = 1 \qquad \text{(conservation of energy).}$$

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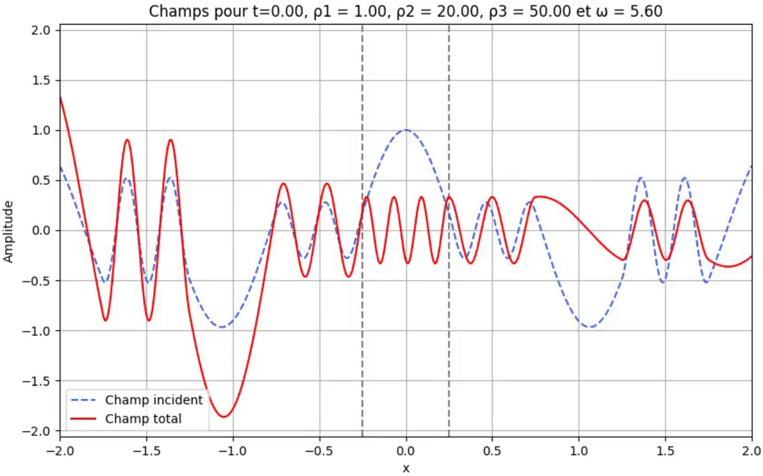
DEFINITION. Defect is said | non reflecting if $R = 0 \quad (|T| = 1)$
perfectly invisible if $T = 1 \quad (R = 0)$.

Numerical illustration

- $t \mapsto \Re e\left(u(x)e^{-i\omega t}\right)$ in a generic case $\rightarrow$ Here $\left| \begin{array}{l} R = 0.90 + 0.16i \\ T = 0.07 - 0.40i. \end{array} \right.$



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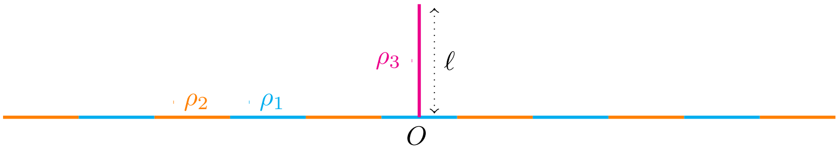
1. Wave propagation in periodic waveguides
2. Scattering by a defect

How to get $T = 1$?

3. Perfect transmission
   - A first mechanism
   - Perfect transmission via the use of the Fano resonance
   - Study at the edges of the spectral bands

Setting

- A ligament of length $\ell > 0$ is added at O. We denote by Γ the new geometry.



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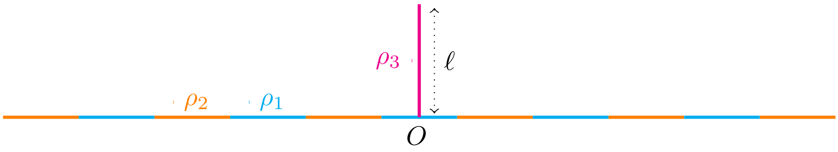

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\text{Kirchhoff:} & \multicolumn{3}{l}{\left| \begin{array}{l} u(0^-,0) = u(0^+,0) = u(0,0^+) \\ \partial_x u(0^-,0) = \partial_x u(0^+,0) + \partial_y u(0,0^+). \end{array} \right.}
\end{array}
\right.
$$

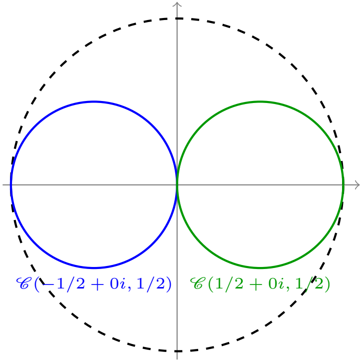
Remark

- Due to the **Kirchhoff** transmissions conditions, one has the constraint

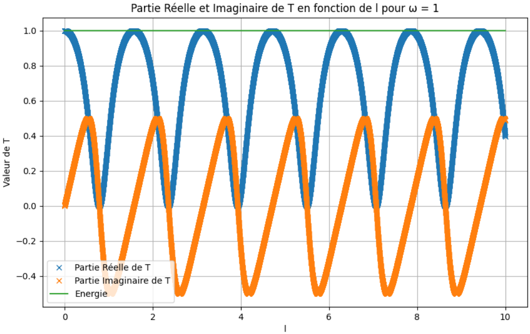
$$1 + R = T.$$

- With the energy conservation ($|R|^2 + |T|^2 = 1$), this implies:

THEOREM. R, T belong to the **circles** $\mathscr{C}(-1/2 + 0i, 1/2)$, $\mathscr{C}(1/2 + 0i, 1/2)$ of the complex plane.



Behavior of $\ell \mapsto R(\ell)$ and $\ell \mapsto T(\ell)$



THEOREM. The maps $\ell \mapsto R(\ell), \ell \mapsto T(\ell)$ are periodic and onto in $\mathscr{C}(-1/2+0i, 1/2)$, $\mathscr{C}(1/2+0i, 1/2)$.

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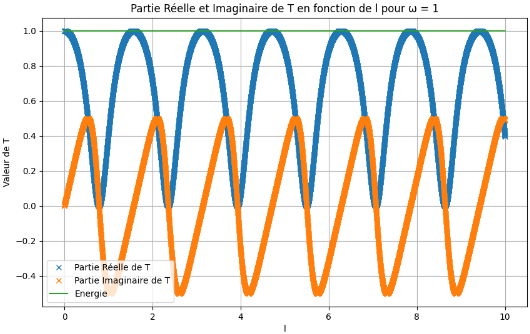

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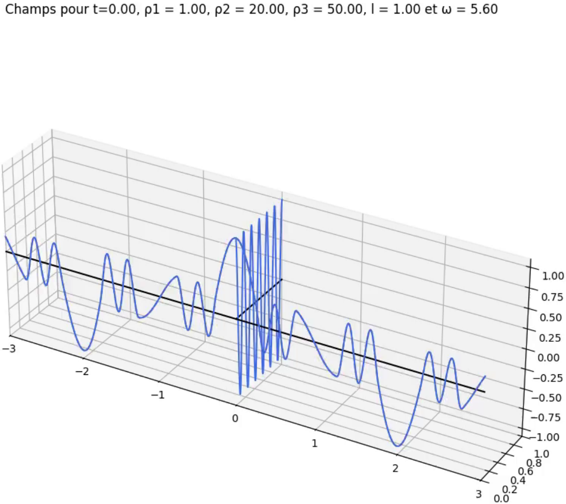
One has $T = 1$ (perfect trans.) for a periodic sequence of values of ℓ.

Numerical illustration

- $t \mapsto \Re e\left(u(x)e^{-i\omega t}\right)$ in a case where $T = 1$.



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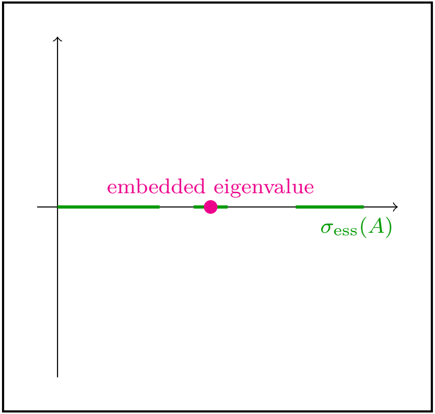
$T = 1$ via the use of the Fano resonance

General idea:

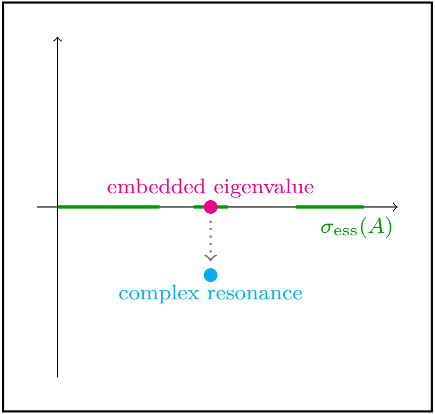
- Start with a setting with an eigenvalue embedded in $\sigma_{\text{ess}}(A)$;



$T = 1$ via the use of the Fano resonance

General idea:

- Start with a setting with an eigenvalue embedded in $\sigma_{\text{ess}}(A)$;
- Perturb slightly the geometry so that the eigenvalue becomes a complex resonance;



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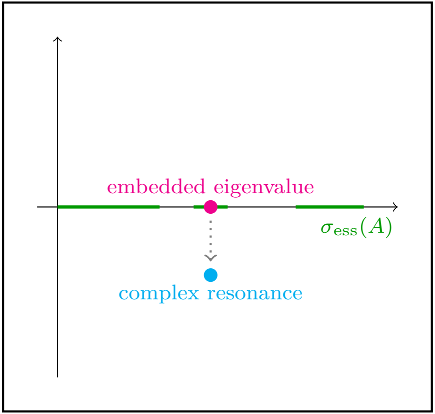

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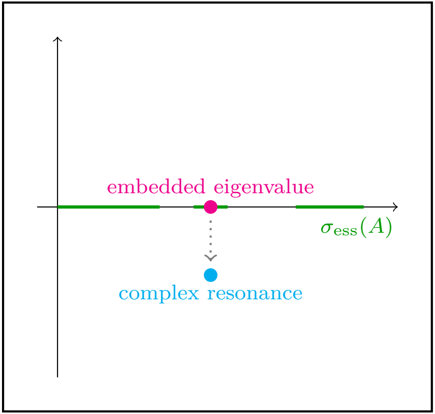
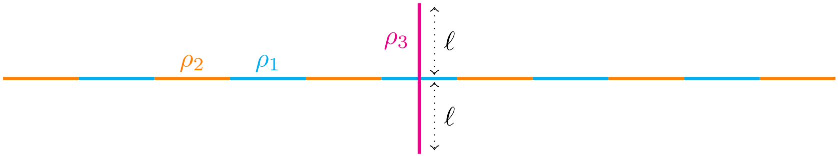
- Perturb slightly the geometry so that the eigenvalue becomes a complex resonance;
- For real ω in a neighborhood of this resonance, $\omega \mapsto R(\omega)$ and $\omega \mapsto T(\omega)$ have a rapid variation;
- With a constraint for R, T as before, one obtains $T(\omega_\star) = 1$ for a certain $\omega_\star$.



Embedded eigenvalue

- For $\rho_3 > 0$ given, we add a **second ligament** of length $\ell > 0$ at O. Denote by Ω this new waveguide.



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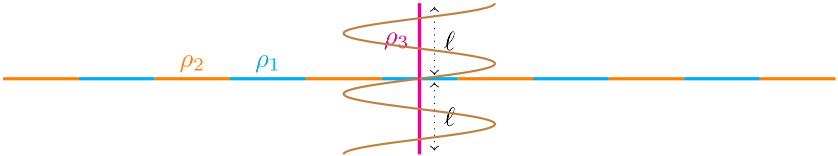

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PROPOSITION. In this geometry, trapped modes exist for $\omega \in \frac{\pi}{2\sqrt{\rho_3}\ell} + \frac{\pi}{\sqrt{\rho_3}\ell}\mathbb{Z}$.

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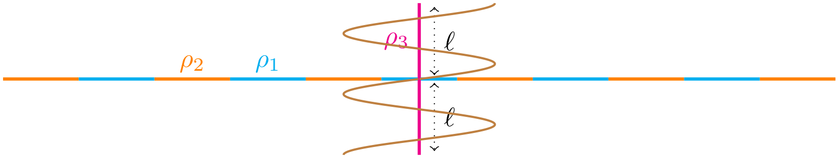

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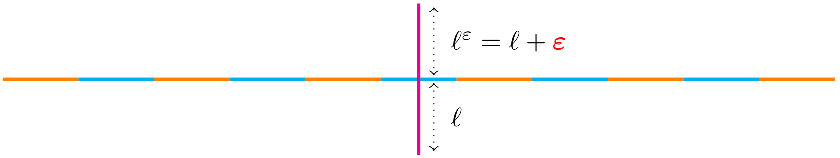
Proposition. In this geometry, trapped modes exist for $\omega \in \frac{\pi}{2\sqrt{\rho_3}\ell} + \frac{\pi}{\sqrt{\rho_3}\ell}\mathbb{Z}$.

Varying ℓ, one can have an eigenvalue anywhere in $\sigma_{\text{ess}}(A)$, say for $\omega = \omega_0$.

Symmetry breaking

We break the symmetry by changing the length of **one** ligament.



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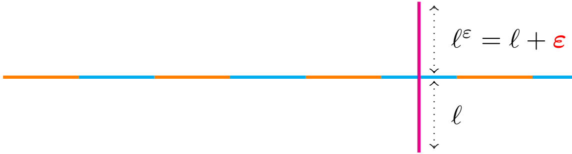

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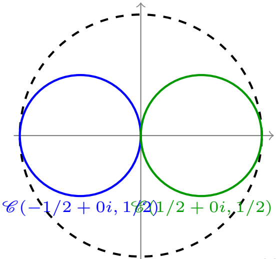
- As before:

  Kirchhoff transmission conditions

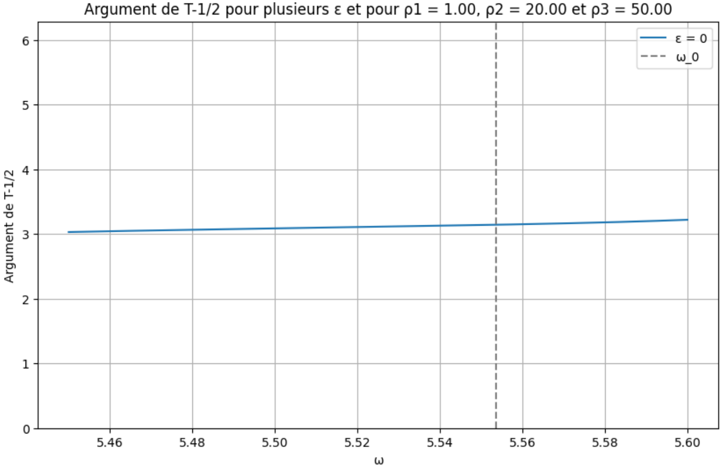
$\Rightarrow$ $1 + R^\varepsilon = T^\varepsilon$

$\Rightarrow$ $R, T \in \mathscr{C}(-1/2 + 0i, 1/2), \mathscr{C}(1/2 + 0i, 1/2)$.



Behavior of $\omega \mapsto T^\varepsilon(\omega)$

- Below we display $\omega \mapsto \arg(T^\varepsilon(\omega) - 1/2)$ around ω_0 for various ε.



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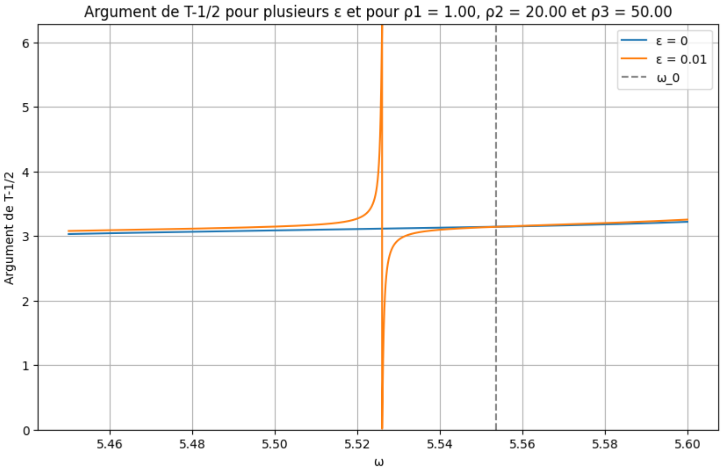

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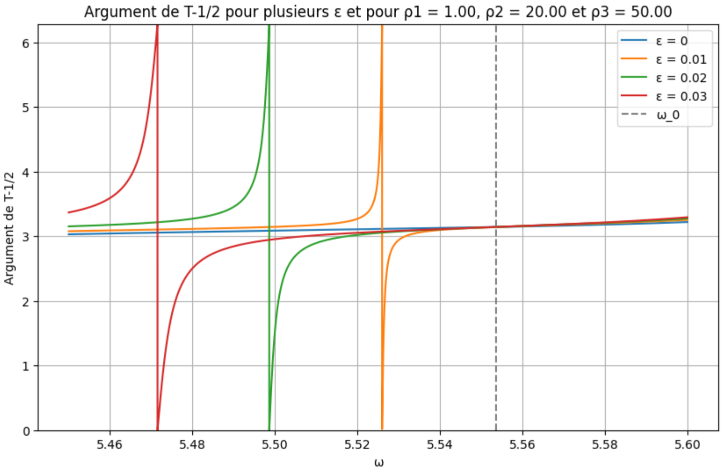
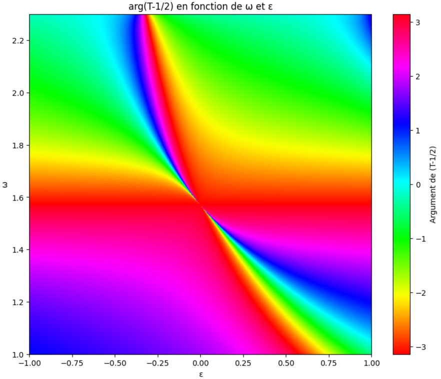


For $\varepsilon > 0$, $\omega \mapsto T^\varepsilon(\omega)$ has a rapid variation in a neighborhood of ω_0 (even faster as $\varepsilon > 0$ is small).

Perfect transmission

- Below we display $(\varepsilon, \omega) \mapsto \arg(T^{\varepsilon}(\omega) - 1/2)$.



Proposition. The map $(\varepsilon, \omega) \mapsto T^{\varepsilon}(\omega)$ is not continuous at $(0, \omega_0)$.

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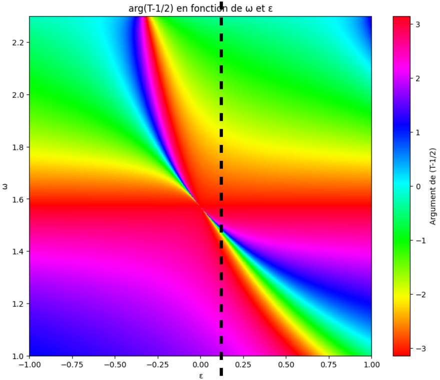

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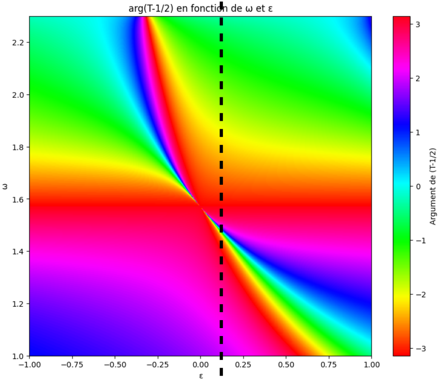

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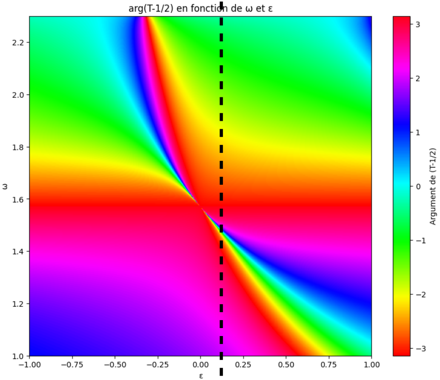

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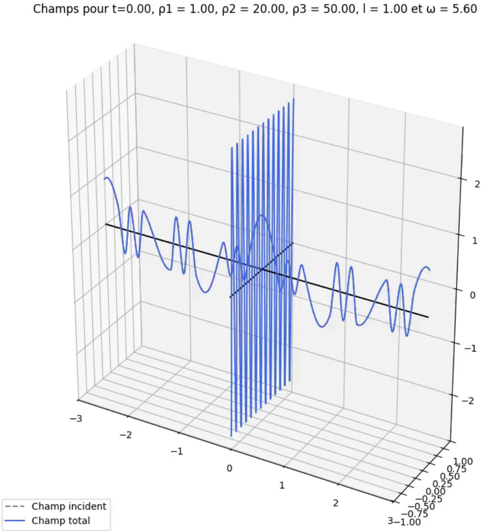
NOTE. When $\varepsilon > 0$ is very small, it becomes delicate to adjust ω...

Numerical illustration

- $t \mapsto \Re e\left(u(x)e^{-i\omega t}\right)$ in a case where $T = 1$.



Outline of the talk

1. Wave propagation in periodic waveguides
2. Scattering by a defect

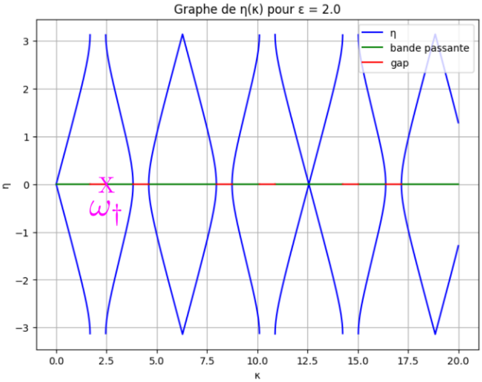
How to get $T = 1$?

3. Perfect transmission
   - A first mechanism
   - Perfect transmission via the use of the Fano resonance
   - Study at the edges of the spectral bands

Behavior at the edges of the spectral bands

- Consider a defect of index ($\rho_3 > 0$).

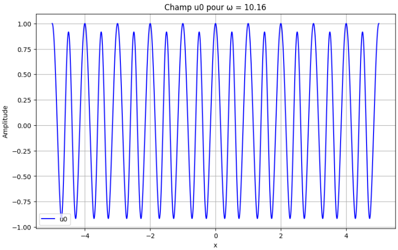
- Let $\omega_\dagger > 0$ be an edge of one of the spectral bands.

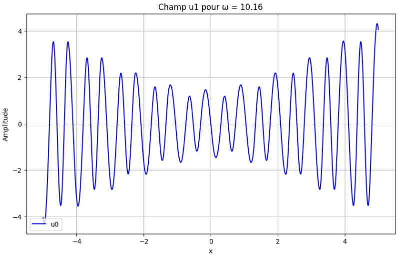


- For ω tending to $\omega_\dagger$, the two propagating modes degenerate and their flux of energy vanishes.

Waves packets

- At $\omega_\dagger$, there is a Jordan chain of length one with corresponding generalized eigenfunctions u_0, u_1.





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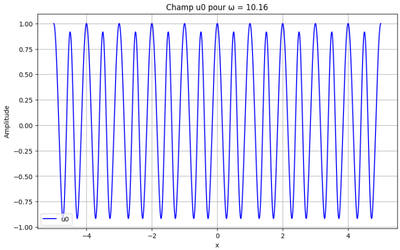

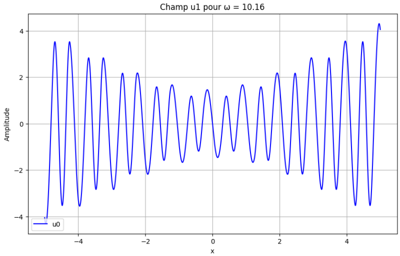

Define the waves packets $W_\pm := u_1 \pm i u_0$. We have $\pm\phi(W_\pm) > 0$.

- At $\omega_\dagger$, we have the new scattering solutions $U_\dagger^\pm$ such that

$$U_\dagger^+ = \left| \begin{array}{c} W_+ + \mathscr{R}^+ W_- \\ \mathscr{T}\, W_+ \end{array} \right. \qquad U_\dagger^- = \left| \begin{array}{cc} \mathscr{T}\, W_- & x \to -\infty \\ W_- + \mathscr{R}^- W_+ & x \to +\infty. \end{array} \right.$$

Scattering matrices

- For $\omega = \omega_\dagger + \delta$ with $\delta > 0$, we denote by

$$\mathbb{S} = \begin{pmatrix} R^+ & T \\ T & R^- \end{pmatrix}$$

the usual scattering matrix, $R^\pm, T$ being the scattering coef of $(\mathscr{P})$.

- For $\omega = \omega_\dagger$, we denote by

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Proposition. Both $\mathbb{S}$ and $\mathscr{S}_\dagger$ are symmetric unitary matrices.

$\Rightarrow$ the eigenvalues of $\mathbb{S}$, $\mathscr{S}_\dagger$ are located on the **unit circle**.

Limit of $\omega \mapsto \mathbb{S}(\omega)$ at $\omega_\dagger$

THEOREM. If -1 is

- not an eigenvalue of $\mathscr{S}_\dagger$ (generic case), then

$$\lim_{\delta \to 0^+} \mathbb{S}(\omega_\dagger + \delta) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \text{(zero transmission);}$$

- an eigenvalue of multiplicity one of $\mathscr{S}_\dagger$ then

$$\lim_{\delta \to 0^+} \mathbb{S}(\omega_\dagger + \delta) = \frac{1}{1+\alpha^2}\begin{pmatrix} \alpha^2 - 1 & 2\alpha \\ 2\alpha & \alpha^2 - 1 \end{pmatrix} \qquad (T = 1 \text{ can occur});$$

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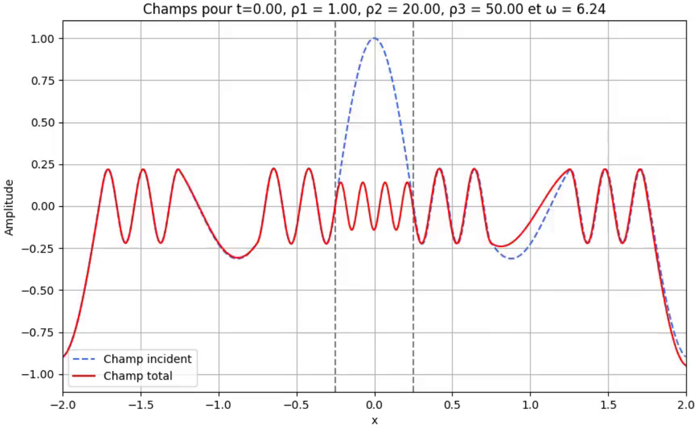
In general, at the edge of the spectral bands, $\boldsymbol{T}$ **tends to** 0.
However for **certain defects**, one can have $\boldsymbol{T} = \mathbf{1}$.

Numerical illustration

- $t \mapsto \Re e\left(u(x)e^{-i\omega t}\right)$, $\omega = \omega_\dagger + \delta$, in a case where $\dim\ker(\mathscr{S} + \mathrm{Id}) = 1$.



Outline of the talk

Conclusion

What we did

- We considered the scattering of waves in 1D periodic waveguides by local defects.
- We presented three mechanisms to get $T = 1$.
  - Tunning the length of long ligaments;
  - Playing with the Fano resonance;
  - Working at the edges of the spectral bands.

Conclusion

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- We considered the scattering of waves in 1D periodic waveguides by local defects.
- We presented three mechanisms to get $T = 1$.
  - Tunning the length of long ligaments;
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  - Working at the edges of the spectral bands.

Future work

- How to hide given obstacles?
- How to adapt these ideas in dimension $d \geq 2$?
- For a given defect, can we identify the frequencies such that $R = 0$ as the spectrum of some operator?

Thank you for your attention!