Invisibility in acoustic waveguides

Lucas Chesnel[1]

Coll. with L. Audibert[1], A. Bera[1,2], A.-S. Bonnet-BenDhia[2], J. Heleine[3], S.A. Nazarov[4], J. Taskinen[5]

[1]Idefix team, EDF/Ensta Paris/Inria, France
[2]Poems team, Ensta Paris/Inria, France
[3]IMT, Univ. Paul Sabatier, France
[4]FMM, St. Petersburg State University, Russia
[5]Univ. of Helsinki, Finland

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General setting 1/2

- We are interested in the propagation of waves in acoustic waveguides.

- We consider a simple but universal model

$$
\left|
\begin{array}{rcl}
\multicolumn{3}{l}{\text{Find } v = v_i + v_s \text{ such that } v_s \text{ is outgoing}} \\
\Delta v + \omega^2 v & = & 0 \quad \text{in } \Omega, \\
\partial_n v & = & 0 \quad \text{on } \partial\Omega
\end{array}
\right.
$$

(relevant also in optics, for microwaves, in water-waves theory, ...).

- At low frequency ω, this problem admits the solution

$$v = \left| \begin{array}{ll} e^{i\omega x} + R\, e^{-i\omega x} + \dots & \text{for } x < 0 \\ \qquad\quad T\, e^{i\omega x} \ \ + \dots & \text{for } x > 0 \end{array} \right.$$

$$|R|^2 + |T|^2 = 1.$$

Cons. of energy

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- Without obstacle, $v = e^{i\omega x}$ so that $(R, T) = (0, 1)$.

- With an obstacle, in general $(R, T) \neq (0, 1)$.

General setting 2/2

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Invisibility problem

How to find geometries such that $R = 0$, $T = 1$ (as if there were no obstacle)?

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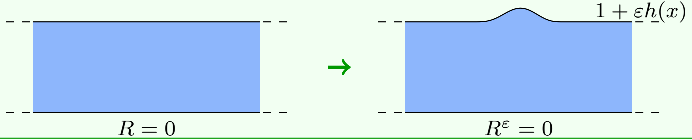
Invisibility problem

How to find geometries such that $R = 0$, $T = 1$ (as if there were no obstacle)?

Difficulty: R, T have a non explicit and non linear dependence wrt the geometry and ω.

First idea: perturbative approach

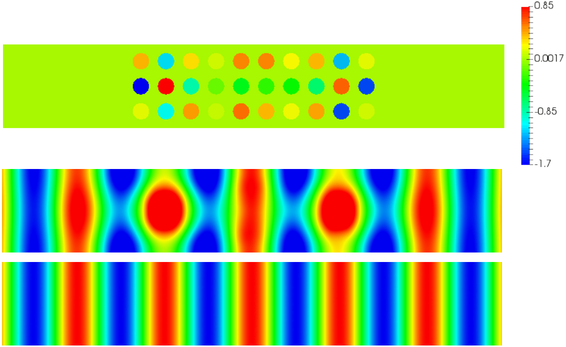
- In the PhD of A. Bera, we developed perturbative techniques based on variants of the implicit functions theorem to construct invisible obstacles.



Index of the penetrable obstacle

$\Re e\left(v(\boldsymbol{x})e^{-i\omega t}\right)$

$\Re e\left(v_i(\boldsymbol{x})e^{-i\omega t}\right)$



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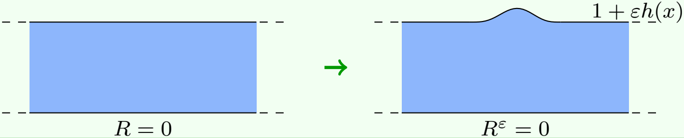

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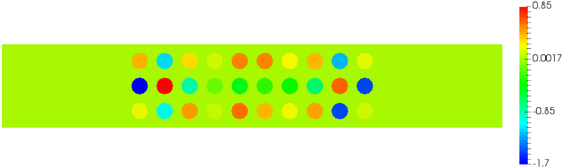

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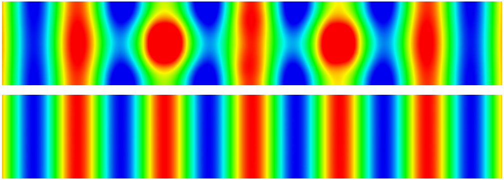
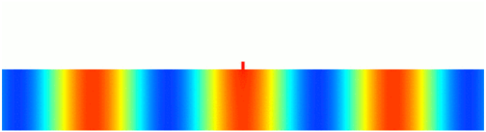
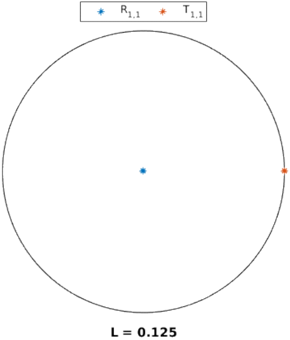


$\rightarrow$ We can create invisible objects. How to hide **given obstacles**?

Digression: thin resonators

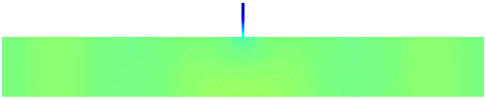
- Below we add a thin resonator to the ref. waveguide and vary its length.

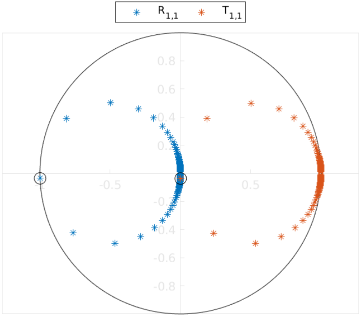




Digression: thin resonators

A thin ligament whose length is well tuned can stop wave propagation!





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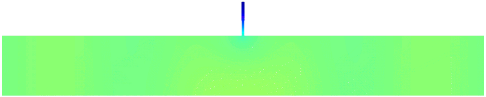
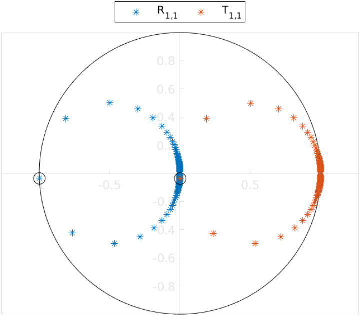




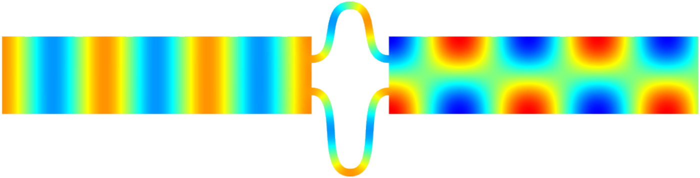
With thin resonators, we can act strongly on fields and control wave propagation. Let us use that for different applications.

Application 1: mode converter

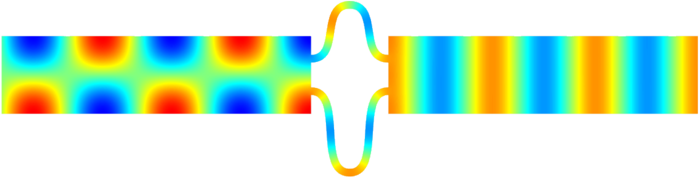
- In the post-doc of J. Heleine, we used these resonators to create modes converters.

- We work at higher ω so that two modes can propagate. Tuning the positions and lengths of the ligaments in the geometry below, we can ensure absence of reflection and mode conversion.

$\Re e\left(v_1(\boldsymbol{x})e^{-i\omega t}\right)$



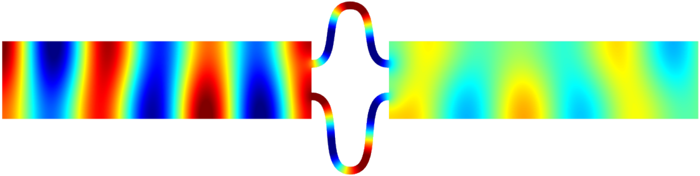
$\Re e\left(v_2(\boldsymbol{x})e^{-i\omega t}\right)$



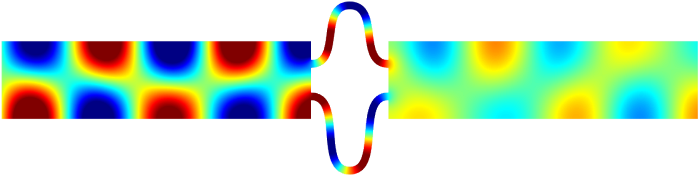
Application 1: mode converter

- Note that without particular tuning of the lengths of the ligaments, energy is mostly backscattered.

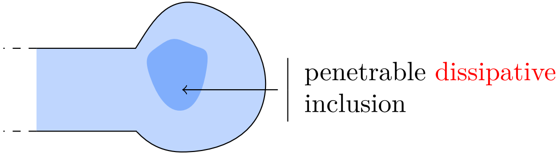
$\Re e\left(v_1(\boldsymbol{x})e^{-i\omega t}\right)$



$\Re e\left(v_2(\boldsymbol{x})e^{-i\omega t}\right)$

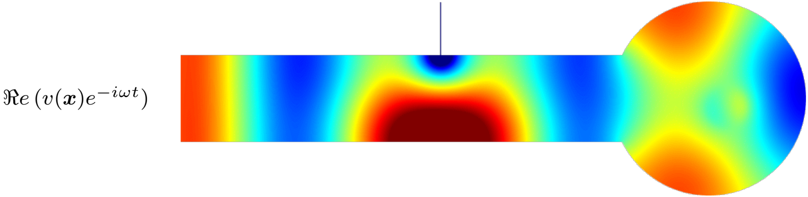


Application 2: energy absorber



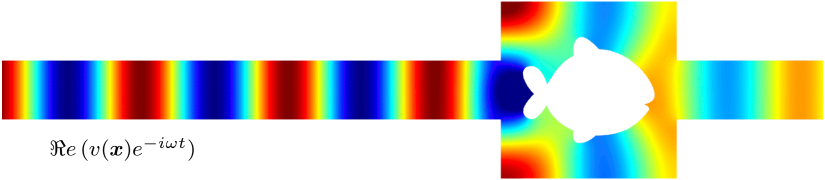
→ in general, a part of the energy of the incident wave is dispersed in the inclusion and another is reflected.

- Adding a well-tuned resonator, we showed that energy can be completely dispersed in the inclusion.

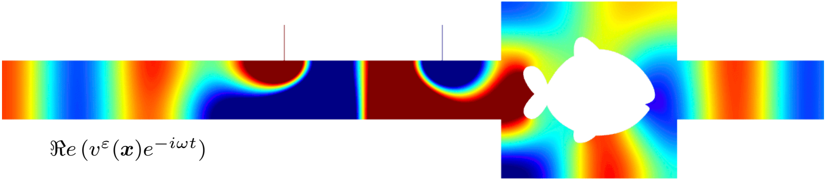


Invisibility

- Finally, we have showed that these resonators can be used to hide any given obstacle.



$\Re e \left(v(\boldsymbol{x}) e^{-i\omega t} \right)$

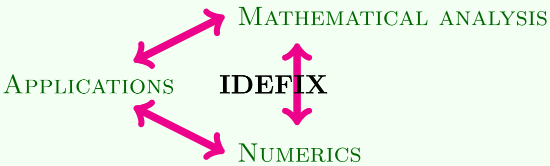


$\Re e \left(v^{\varepsilon}(\boldsymbol{x}) e^{-i\omega t} \right)$

$\Re e \left(e^{i\omega(x-t)} \right)$

Conclusion

- This is only to give an example of research at IDEFIX.
- What we want to stress here is the scheme



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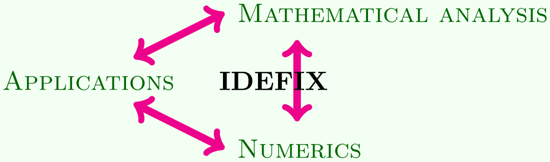
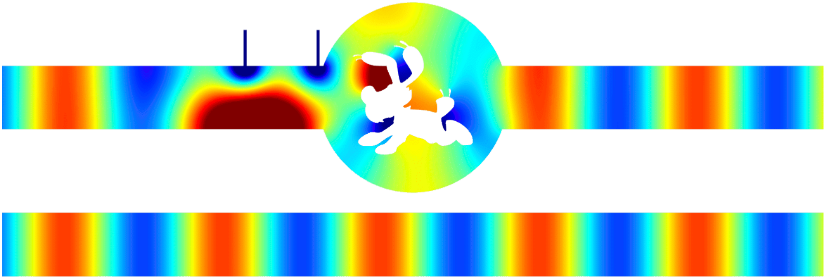


On going project with **EDF**

Can resonant ligaments be useful for ice detection in overflow pipes via acoustic waves?



Thank you for your attention!