Imaging a periodic waveguide from far field data

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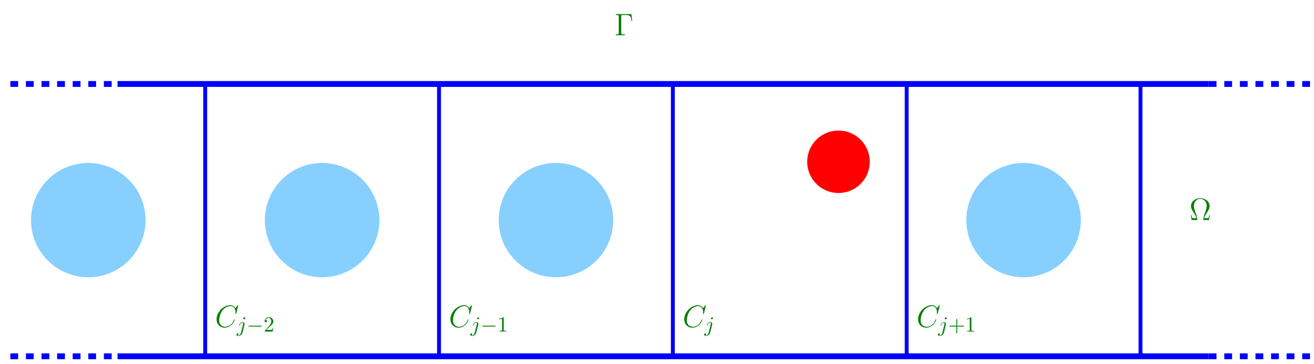
Statement of the problem

Geometry : a 2D waveguide with periodic real refractive index n_p



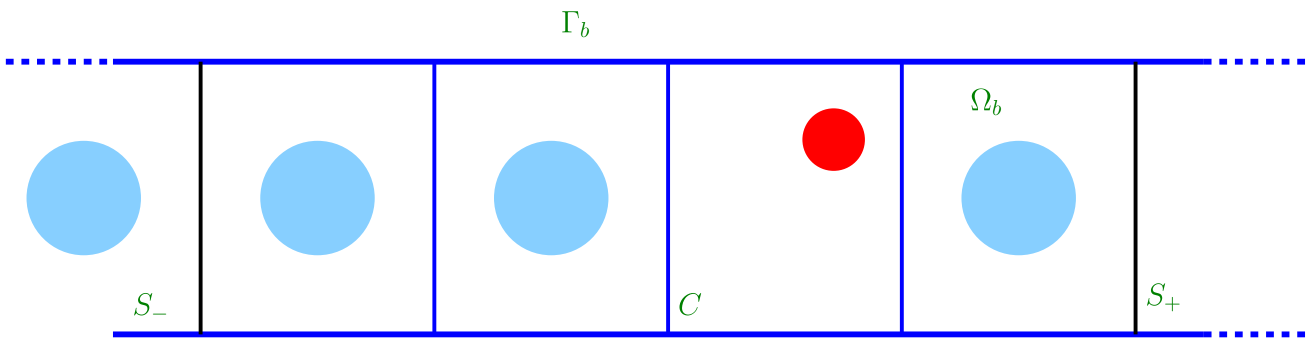
Objective : find the infected cell C_j and the support of the defect $q = n^2 - n_p^2$ in C_j from far field scattering data

Already some litterature on inverse scattering for periodic structures (T. Arens, A. Kirsch, A. Lechleiter, ...) : but the objective was to find the unknown periodic geometry of the structure from scattering data (no defect)

Outline of the talk

- **First part**
  1. the forward problem (briefly)
  2. the inverse problem with near field data
- **Second part**
  1. the inverse problem with far field data
  2. numerical experiments

The forward problem

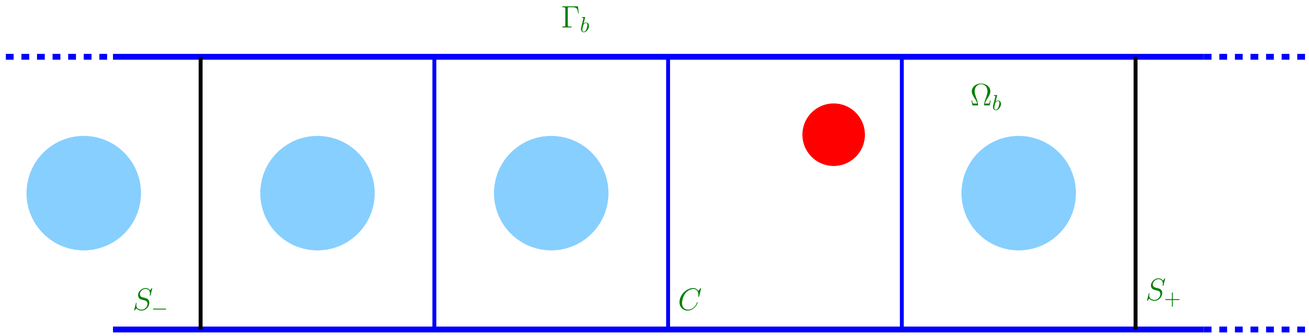


The forward problem : $u = u^s + u^i$

$$\begin{cases} (\Delta + \omega^2 n^2)u = 0 & \text{in} \quad \Omega_b \\ \partial_2 u = 0 & \text{on} \quad \Gamma_b \\ \pm\partial_1 u^s = T_\pm u^s & \text{on} \quad S_\pm \end{cases}$$

- $n_p \geq c > 0$, n_p is $1-$ periodic
- $n \geq c > 0$, contrast $q = n^2 - n_p^2$ with $\overline{D} := \text{supp}(q) \subset C$
- u^i solves $\Delta u^i + \omega^2 n_p^2 u^i = 0$ in Ω and $\partial_2 u^i = 0$ on Γ
- $T_\pm$: Dirichlet to Neumann operator on $S_\pm$

The forward problem



Homogeneous waveguide : the forward problem has a unique solution u in $H^2(\Omega_b)$ expect for at most a countable set of ω

Periodic waveguide :

- **Proposition** : assuming n_p is constant near the transverse sections of the cell, the forward problem is of Fredholm type
- **Conjecture** : the D-to-N operator is an analytic function of ω
- **Theorem** : the forward problem has a unique solution u in $H^2(\Omega_b)$ expect for at most a countable set of ω
  (proof : Fredholm analytical theorem provides uniqueness)

Factorization method (A. Kirsch)

Inverse problem with near field data : we measure on $\hat{S} = S_- \cup S_+$ the scattered field $\tilde{u}^s(\cdot, y)$ associated to the incident field $u^i = \overline{G(\cdot, y)}$ on $\hat{S}$: find D

- **Near field operator** : $\tilde{N} : L^2(\hat{S}) \to L^2(\hat{S})$

$$(\tilde{N}h)(x) := \int_{\hat{S}} \tilde{u}^s(x, y)h(y)\, ds(y), \quad x \in \hat{S}$$

- **Self-adjoint operator** : $\tilde{N}_\sharp = |\mathrm{Re}\tilde{N}| + |\mathrm{Im}\tilde{N}|$

- **Characterization of** D (with assumption that $q \geq c > 0$ or $q \leq -c$ with $c > 0$):

$$z \in D \Leftrightarrow G(\cdot, z)|_{\hat{S}} \in R(\tilde{N}_\sharp^{\frac{1}{2}})$$

Justification

First step (factorization of near field) : $\tilde{N} = H^* T H$

- **Reflectivity operator** $T : L^2(D) \to L^2(D)$

$$(Tf)(x) = \omega^2 \mathrm{sgn}(q(x)) \left(f(x) + \sqrt{|q(x)|}\, v(x) \right), \quad x \in D$$

where $\mathrm{sgn}(q) = q/|q|$ and v solves

$$\begin{cases} -(\Delta v + \omega^2 n^2 v) = \omega^2 (q/\sqrt{|q|}) f & \text{in} \quad \Omega_b \\ \partial_2 v = 0 & \text{on} \quad \Gamma_b \\ \pm \partial_1 v = T_\pm v & \text{on} \quad S_\pm \end{cases}$$

- **Herglotz operator** $H : L^2(\hat{S}) \to L^2(D)$

$$(Hh)(x) = \sqrt{|q(x)|} \int_{\hat{S}} \overline{G(x,y)}\, h(y)\, ds(y), \quad x \in D$$

Justification (cont.)

Second step (range test) : $z \in D \Leftrightarrow G(\cdot, z)|_{\hat{S}} \in R(H^*)$

Third step (fondamental theorem) :

Consider Hilbert spaces $X \subset U \subset X^*$ (dense inclusion) and $V = V^*$, operators $F : V \to V$, $H : V \to X$ and $T : X \to X^*$ with $F = H^* T H$

Assumptions :

1. H is compact and injective
2. $\mathrm{Re}T = T_0 + T_1$, T_0 self-adjoint coercive, T_1 compact
3. $\langle (\mathrm{Im}T)\phi, \phi \rangle \geq 0$, for all $\phi \in X$
4. T is injective

Statement : for $F_\sharp = |\mathrm{Re}F| + |\mathrm{Im}F|$, then $\mathrm{R}(H^*) = \mathrm{R}(F_\sharp^{\frac{1}{2}})$

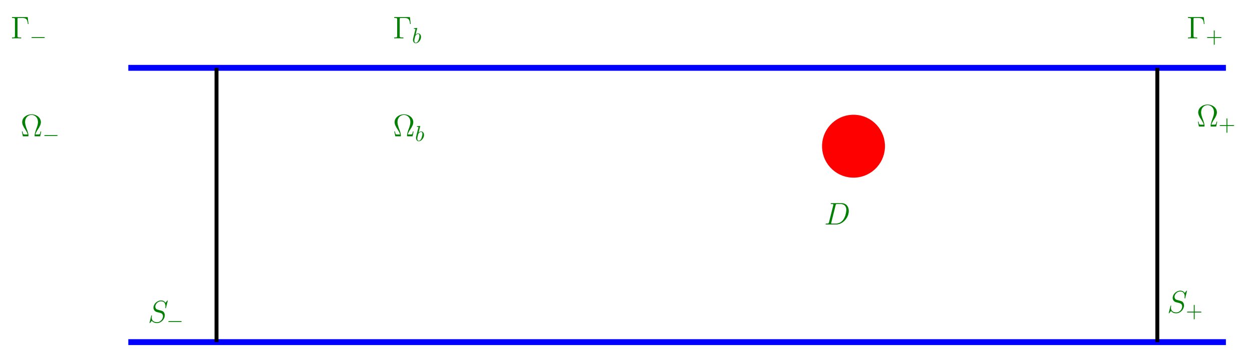
Proof of injectivity of H

Since $\overline{H} = \sqrt{|q|}J$, it suffices to prove injectivity of $J : L^2(\hat{S}) \to L^2(D)$

$$(Jh)(x) = \int_{S_-} G(x,y)h_-(y)\,ds(y) + \int_{S_+} G(x,y)h_+(y)\,ds(y) \quad x \in D$$

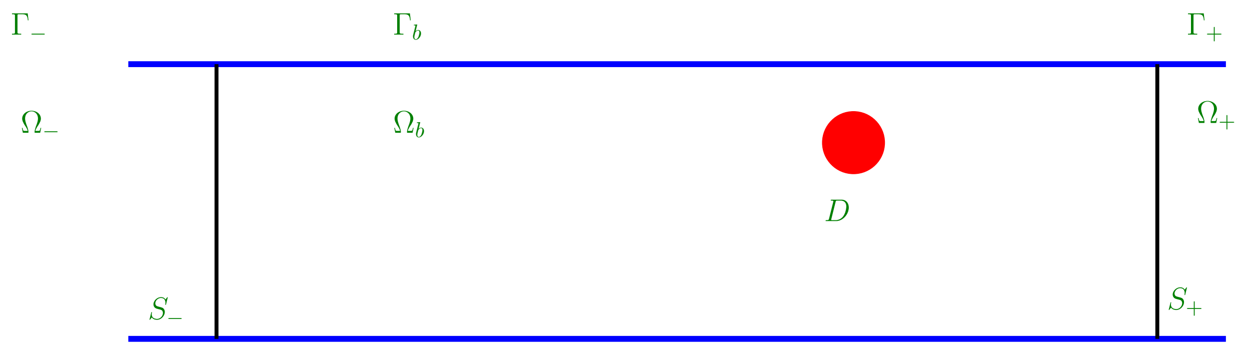
$v := Jh$ is the unique solution of the transmission problem

$$\begin{cases} -(\Delta + \omega^2 n_p^2)v = 0 & \text{in} & \Omega_- \cup \Omega_b \cup \Omega_+ \\ \partial_2 v = 0 & \text{on} & \Gamma_- \cup \Gamma_b \cup \Gamma_+ \\ [v]_\pm = 0 \quad [\partial_1 v]_\pm = -h_\pm & \text{on} & S_\pm \\ & + \quad \text{Radiation Condition} & \end{cases}$$



Proof of injectivity of H (cont.)

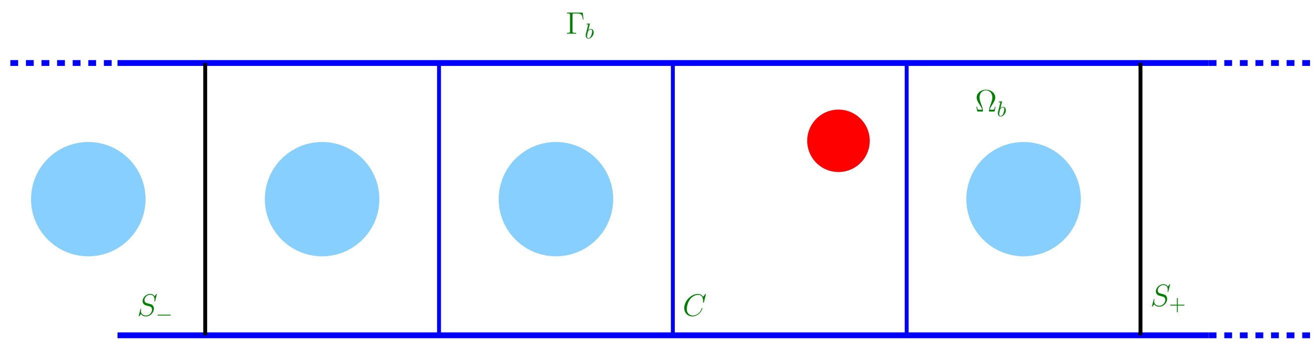
Assume $v = 0$ in D



- Since $(\Delta + \omega^2 n_p^2)v = 0$ in Ω_b, unique continuation implies $v = 0$ in Ω_b, in particular $v_- = 0$ on S_+ and v_+ on S_-
- Using jump relations $[v]_\pm = 0$ on $S_\pm$, we obtain $v_+ = 0$ on S_+ and $v_- = 0$ on S_-
- By using D-t-N operators T_+ and T_-, $v = 0$ in Ω_+ and $v = 0$ in Ω_-
- Using jump relations $[\partial_1 v]_\pm = -h_\pm$, we obtain $h_+ = 0$ and $h_- = 0$

We conclude $h = 0$ in $\hat{S}$: the proof is completed

Near field/far field data



The **inverse problem** with **near field data** : we measure on $\hat{S} := S_- \cup S_+$ the **scattered field** $\tilde{u}^s(\cdot, y)/u^s(\cdot, y)$ associated with the **incident field** $u^i = \overline{G(\cdot, y)}/G(\cdot, y)$ on $\hat{S}$: find the support D of the defect

What happens if $\hat{S}$ (support of the data) is far away from D (defect) $\longrightarrow$ **far field data** ?

Factorization method with near field data

Conjugated point source: we measure on $\hat{S}$ the scattered field $\tilde{u}^s(\cdot, y)$ associated to the incident field $u^i = \overline{G(\cdot, y)}$ on $\hat{S}$: find D

- **Near field operator**: $\tilde{N} : L^2(\hat{S}) \to L^2(\hat{S})$

$$(\tilde{N}h)(x) := \int_{\hat{S}} \tilde{u}^s(x, y)h(y)\, ds(y), \quad x \in \hat{S}$$

- **Factorization**: $\tilde{N} = H^* T H$
- **Self-adjoint operator**: $\tilde{N}_\sharp = |\mathrm{Re}\tilde{N}| + |\mathrm{Im}\tilde{N}|$
- **Characterization of** D (with assumption that $q \geq c > 0$ or $q \leq -c$ with $c > 0$):

$$z \in D \Leftrightarrow G(\cdot, z)|_{\hat{S}} \in R(\tilde{N}_\sharp^{\frac{1}{2}})$$

Linear Sampling Method with near field data

Point source : we measure on $\hat{S}$ the scattered field $u^s(\cdot, y)$ associated to the incident field $u^i = G(\cdot, y)$ on $\hat{S}$: find D

- **Near field operator** : $N : L^2(\hat{S}) \to L^2(\hat{S})$

$$(Nh)(x) := \int_{\hat{S}} u^s(x, y)h(y)\, ds(y), \quad x \in \hat{S}$$

- **Factorization**: $N = H^* T \overline{H}$
- **Half-characterization of** D (with assumption that $q \geq c > 0$ or $q \leq -c$ with $c > 0$):

$$z \in D \Leftarrow G(\cdot, z)|_{\hat{S}} \in R(N)$$

Sampling Methods with near field data

For z in the **sampling grid**,

Factorization Method: "solve"

$$\tilde{N}_{\sharp}^{\frac{1}{2}} h = G(\cdot, z)|_{\hat{S}}$$

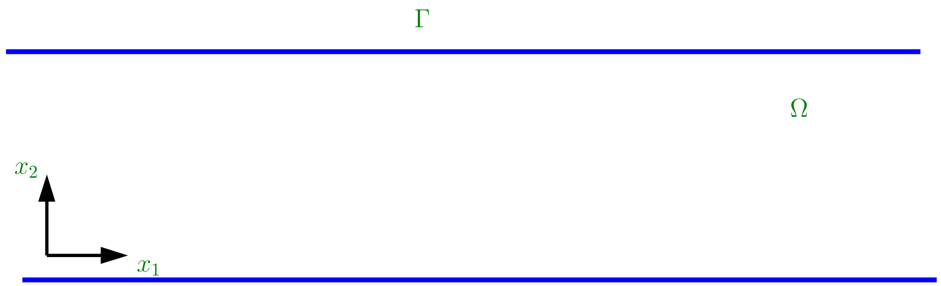
Linear Sampling Method: "solve"

$$N\, h = G(\cdot, z)|_{\hat{S}}$$

Indicator function of defect D :

$$\Psi(z) = 1/||h(z)||_{L^2(\hat{S})}$$

Homogeneous waveguide: the guided modes



- The guided modes : find u such that

$$\begin{cases} \Delta u + \omega^2 u = 0 & \text{dans} \quad \Omega \\ \partial_\nu u = 0 & \text{sur} \quad \Gamma \end{cases}$$

- θ_p and λ_p $(p > 0)$: Neumann eigenfunctions and eigenvalues of the 1D operator $-\Delta$ in transverse section S

- Guided modes : $u_p^{\pm}(x_1, x_2) = \theta_p(x_2) e^{\pm i\beta_p x_1}$, $\beta_p = \sqrt{\omega^2 - \lambda_p}$ for $p \leq P$ (propagating modes) and $\beta_p = i\sqrt{\lambda_p - \omega^2}$ for $p > P$

- Assumption on ω : $\beta_p \neq 0$

Homogeneous waveguide: the Green function

- Fundamental solution :

$$G(x,y) = i\sum_{p=1}^{+\infty} \frac{e^{i\beta_p|x_1-y_1|}}{2\beta_p}\theta_p(x_2)\theta_p(y_2)$$

- Far field : for large x_1 and $\pm := \operatorname{sgn}(x_1 - y_1)$

$$G(x,y) = G^{\pm}_{\infty}(x,y) + \mathcal{O}(e^{-\alpha|x_1|}), \quad \alpha > 0$$

with

$$G^{\pm}_{\infty}(x,y) = i\sum_{p=1}^{P} \frac{e^{\pm i\beta_p(x_1-y_1)}}{2\beta_p}\theta_p(x_2)\theta_p(y_2)$$

Short expression :

$$G^{\pm}_{\infty}(x,y) = i\sum_{p=1}^{P} \frac{u_p^{\pm}(x)u_p^{\mp}(y)}{2\beta_p}$$

Periodic waveguide: the Floquet modes

- Unbounded operator $A(\xi)$ in the cell $C = (-\frac{1}{2}, \frac{1}{2}) \times (0, 1)$:

$$A(\xi) = -\frac{1}{n_p^2}\,\Delta \,:\, L^2(C, n_p^2\, dx_1 dx_2) \longrightarrow L^2(C, n_p^2\, dx_1 dx_2)$$

$$D(A(\xi)) = \{u \in H^1(C),\ \Delta u \in L^2(C),$$

$$\partial_2 u = 0 \ \text{ on } \ \partial C \cap \Gamma,\ u \in \mathrm{QP}_\xi(C)\}$$

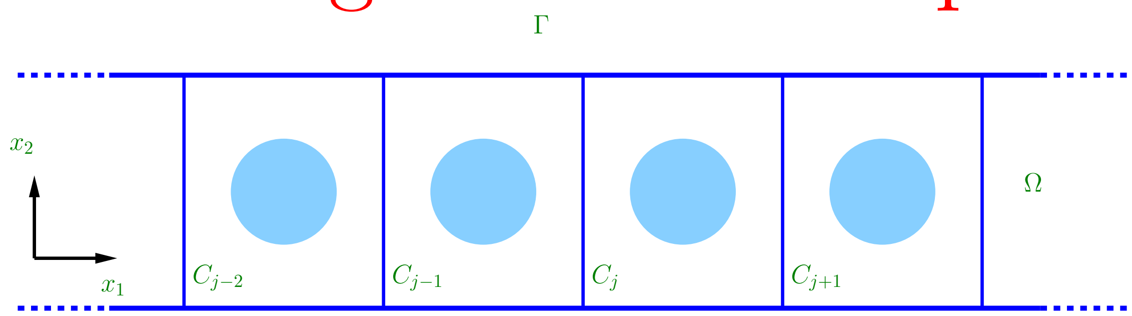
$\mathrm{QP}_\xi(C) :=$ ξ-quasiperiodic functions for $\xi \in (-\pi, \pi]$

$$u(1/2, x_2) = e^{i\xi} u(-1/2, x_2), \quad \partial_1 u(1/2, x_2) = e^{i\xi} \partial_1 u(-1/2, x_2)$$

- $A(\xi)$ is self-adjoint, positive and has compact resolvent : eigenvalues $\lambda_n(\xi)$, eigenfunctions $\phi_n(\cdot;\xi) \in H^2(C)$

$$0 \le \lambda_1(\xi) \le \lambda_2(\xi) \le \ ... \ \le \lambda_n(\xi) \longrightarrow +\infty$$

Periodic waveguide: the Floquet modes



- Find u s.t. for some $\xi \in (-\pi, \pi]$, $u \in \mathrm{QP}_\xi(C_j)$ for all $j \in \mathbb{Z}$ and

$$\begin{cases} (\Delta + \omega^2 n_p^2)u = 0 & \text{in} \quad \Omega \\ \partial_\nu u = 0 & \text{on} \quad \Gamma \end{cases}$$

For such u, we have $A(\xi)u = \omega^2 u$ and $u = \sum_n \alpha_n \phi_n(\cdot;\xi)$ in C

$$\Rightarrow \quad \sum_n \alpha_n \lambda_n(\xi)\phi_n = \sum_n \alpha_n \omega^2 \phi_n \quad \Rightarrow \quad \alpha_n = 0 \quad \text{or} \quad \lambda_n(\xi) = \omega^2$$

- **Floquet modes** : for $(x, j) \in C \times \mathbb{Z}$

$$u_n(x_1 + j, x_2; \xi) = \phi_n(x_1, x_2; \xi)e^{ij\xi}, \quad \forall n \in I(\omega), \ \forall \xi \in \Xi_n(\omega)$$

with $I(\omega) = \{n, \ \exists \xi, \ \lambda_n(\xi) = \omega^2\}$ and $\Xi_n(\omega) = \{\xi, \ \lambda_n(\xi) = \omega^2\}$

Properties of the Floquet modes

- Symmetry of eigenvalues and eigenfunctions of $A(\xi)$:

$$\lambda_n(-\xi) = \lambda_n(\xi), \quad \phi_n(\cdot;-\xi) = \overline{\phi_n(\cdot;\xi)}$$

- Assumption on ω : $\forall n \in I(\omega)$, $\forall \xi \in \Xi_n(\omega)$

$$\lambda_n(\xi) \text{ is simple and } \lambda_n'(\xi) \neq 0$$

- Group velocity :

$$V_n(\xi) = \frac{1}{2}\lambda_n^{-1/2}\lambda_n'(\xi)$$

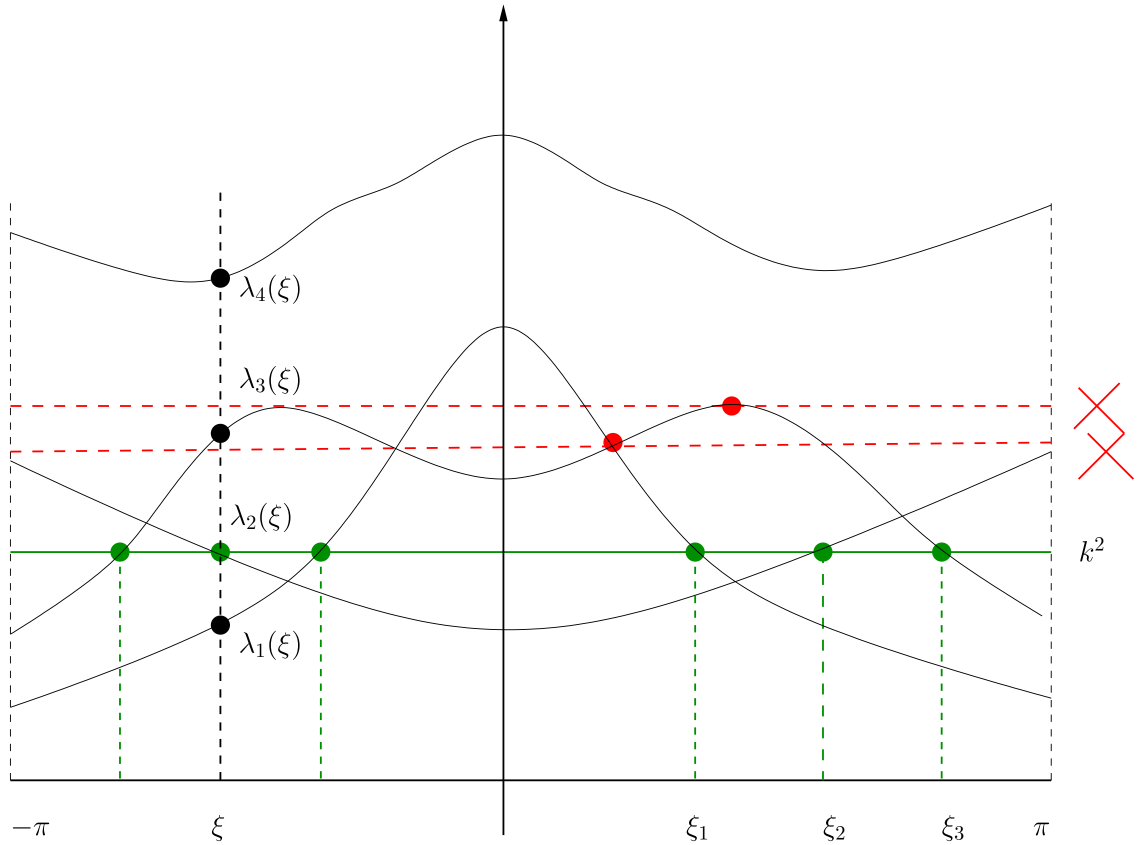
For $\lambda_n'(\xi) > 0$,

$$u_n^+(\cdot;\xi) := u_n(\cdot;\xi), \quad u_n^-(\cdot;\xi) := u_n(\cdot;-\xi)$$

- Symmetry of Floquet modes :

$$u_n^-(\cdot;\xi) = \overline{u_n^+(\cdot;\xi)}$$

Dispersion curves



$$I(\omega) = \{2\} \quad \Xi_2(\omega) = \{\pm\xi_1, \pm\xi_2, \pm\xi_3\}$$

Periodic waveguide: the Green function

The fund. sol. G is given, $\forall x, y \in C$ and $\forall p, q \in \mathbb{Z}$, by:

$$G(x_1+p, x_2; y_1+q, y_2) = \frac{1}{2\pi} \sum_{n \notin I(\omega)} \int_{-\pi}^{\pi} \frac{\phi_n(x;\xi)\overline{\phi_n(y;\xi)}}{\lambda_n(\xi)-\omega^2} e^{i(p-q)\xi}\, d\xi$$

$$+\frac{1}{2\pi} \sum_{n \in I(\omega)} \left\{ \text{p.v.} \int_{-\pi}^{\pi} \frac{\phi_n(x;\xi)\overline{\phi_n(y;\xi)}}{\lambda_n(\xi)-\omega^2} e^{i(p-q)\xi}\, d\xi \right.$$

$$\left. + i\pi \sum_{\xi \in \Xi_n(\omega)} \frac{\phi_n(x;\xi)\overline{\phi_n(y;\xi)}}{|\lambda_n'(\xi)|} e^{i(p-q)\xi} \right\}$$

Reciprocity:

$$\forall x, y \in \Omega, \quad G(x,y) = G(y,x)$$

Periodic waveguide: the Green function

Far field:

$\forall x, y \in C$, $\forall q \in \mathbb{Z}$, for large $p \in \mathbb{Z}$ and $\pm := \operatorname{sgn}(p-q)$

$$G(x_1 + p, x_2; y_1 + q, y_2) = G^{\pm}_{\infty}(x_1 + p, x_2; y_1 + q, y_2) + \mathcal{O}(e^{-\alpha|p|}), \quad \alpha > 0$$

with

$$G^{\pm}_{\infty}(x_1 + p, x_2; y_1 + q, y_2) = i \sum_{n \in I(\omega)} \sum_{\substack{\xi \in \Xi_n(\omega) \\ \pm\lambda'_n(\xi) > 0}} \frac{\phi_n(x;\xi)\overline{\phi_n(y;\xi)}}{|\lambda'_n(\xi)|} e^{i(p-q)\xi}$$

Short expression:

$$\forall x, y \in \Omega, \quad G^{\pm}_{\infty}(x; y) = i \sum_{n \in I(\omega)} \sum_{\substack{\xi \in \Xi_n(\omega) \\ \lambda'_n(\xi) > 0}} \frac{u^{\pm}_n(x;\xi) u^{\mp}_n(y;\xi)}{\lambda'_n(\xi)}$$

Far field approximation (LSM)

- Far field of the Green function G:

$$G_\infty^\pm(x,y) = i \sum_{n\in I(\omega)} \sum_{\xi\in\Xi_n(\omega),\lambda_n'(\xi)>0} \frac{u_n^\pm(x;\xi)u_n^\mp(y;\xi)}{\lambda_n'(\xi)}$$

- Far field of the scattered field associated with $G(\cdot,y)$:

$$u_\infty^\pm(x,y) = i \sum_{n\in I(\omega)} \sum_{\xi\in\Xi_n(\omega),\lambda_n'(\xi)>0} \frac{u_n^{s\pm}(x;\xi)u_n^\mp(y;\xi)}{\lambda_n'(\xi)}$$

where $u_n^{s\pm}(\cdot;\xi)$: scattered field associated with $u_n^\pm(\cdot;\xi)$

- Far field operator N_∞: kernel $u^s(x,y)$ replaced by kernel $u_\infty(x,y)$

$\longrightarrow$ **Far field formulation** for LSM: “solve”

$$N_\infty h = G_\infty(\cdot,z)|_{\hat{S}}$$

Far field approximation (Factorization Method)

Far field of the Green function G:

$$G_\infty^\pm(x,y) = i \sum_{n\in I(\omega)} \sum_{\xi\in\Xi_n(\omega),\lambda_n'(\xi)>0} \frac{u_n^\pm(x;\xi)u_n^\mp(y;\xi)}{\lambda_n'(\xi)}$$

Recall the symmetry of Floquet modes:

$$u_n^-(\cdot,\xi) = \overline{u_n^+(\cdot,\xi)}, \quad \forall n\in I(\omega), \quad \forall\xi\in\Xi_n(\omega), \quad \lambda_n'(\xi)>0$$

Far field of the conjugated Green function $\overline{G(\cdot,y)}$:

$$\overline{G_\infty^\pm(x,y)} = -i \sum_{n\in I(\omega)} \sum_{\xi\in\Xi_n(\omega),\lambda_n'(\xi)>0} \frac{u_n^\mp(x;\xi)u_n^\pm(y;\xi)}{\lambda_n'(\xi)}$$

Far field of the scattered field associated with $\overline{G(\cdot,y)}$:

$$\tilde{u}_\infty^\pm(x,y) = -i \sum_{n\in I(\omega)} \sum_{\xi\in\Xi_n(\omega),\lambda_n'(\xi)>0} \frac{u_n^{s\mp}(x;\xi)u_n^\pm(y;\xi)}{\lambda_n'(\xi)}$$

Kernel $\tilde{u}^s(x,y)$ of $\tilde{N}$ is replaced by kernel $\tilde{u}_\infty(x,y)$ of $\tilde{N}_\infty$ → **far field formulation of FM**: "solve" $N_{\infty,\sharp}^{1/2}h = G_\infty(\cdot,z)|_{\hat{S}}$

Projection (LSM)

Conclusion : in the far field formulation of LSM/FM, the data are the scattered fields $u_n^{s\pm}(\cdot;\xi)$ associated with the propagating Floquet modes.

$$(N_\infty h)(x) = i \sum_{n\in I(\omega)} \sum_{\substack{\xi\in\Xi_n(\omega)\\ \lambda_n'(\xi)>0}} \frac{u_n^{s+}(x;\xi)e^{iM\xi}}{\lambda_n'(\xi)} \int_0^1 \overline{\phi_n(-1/2,y_2;\xi)} h_-(y_2)\, ds(y_2)$$

$$+ i \sum_{n\in I(\omega)} \sum_{\substack{\xi\in\Xi_n(\omega)\\ \lambda_n'(\xi)>0}} \frac{u_n^{s-}(x;\xi)e^{iN\xi}}{\lambda_n'(\xi)} \int_0^1 \phi_n(1/2,y_2;\xi) h_+(y_2)\, ds(y_2).$$

Two complete basis $(\psi_m^{\pm})_{m>0}$ of $L^2(]0,1[)$:

$$u_n^{s+}(\cdot;\xi)|_{S_-} = \sum_{k>0} U_{nk}^{+-}(\xi)\psi_k^-, \quad u_n^{s+}(\cdot;\xi)|_{S_+} = \sum_{k>0} U_{nk}^{++}(\xi)\psi_k^+,$$

$$u_n^{s-}(\cdot;\xi)|_{S_-} = \sum_{k>0} U_{nk}^{--}(\xi)\psi_k^-, \quad u_n^{s-}(\cdot;\xi)|_{S_+} = \sum_{k>0} U_{nk}^{-+}(\xi)\psi_k^+.$$

Projection of left hand side (LSM)

$$(N_\infty h)|_{S_-} = i \sum_{k>0} [\cdot]_- \, \psi_k^- \qquad (N_\infty h)|_{S_+} = i \sum_{k>0} [\cdot]_+ \, \psi_k^+$$

$$[\cdot]_- = \sum_{n \in I(\omega)} \sum_{\substack{\xi \in \Xi_n(\omega) \\ \lambda_n'(\xi) > 0}} \left(\frac{U_{nk}^{+-}(\xi) e^{iM\xi} (\sum_{m,m'>0} \Phi_{nm}^-(\xi) M_{mm'}^- h_{m'}^-)}{\lambda_n'(\xi)} + \frac{U_{nk}^{--}(\xi) e^{iN\xi} (\sum_{m,m'>0} \Phi_{nm}^+(\xi) M_{mm'}^+ h_{m'}^+)}{\lambda_n'(\xi)} \right)$$

$$[\cdot]_+ = \sum_{n \in I(\omega)} \sum_{\substack{\xi \in \Xi_n(\omega) \\ \lambda_n'(\xi) > 0}} \left(\frac{U_{nk}^{++}(\xi) e^{iM\xi} (\sum_{m,m'>0} \Phi_{nm}^-(\xi) M_{mm'}^- h_{m'}^-)}{\lambda_n'(\xi)} + \frac{U_{nk}^{-+}(\xi) e^{iN\xi} (\sum_{m,m'>0} \Phi_{nm}^+(\xi) M_{mm'}^+ h_{m'}^+)}{\lambda_n'(\xi)} \right)$$

Projection of right hand side (LSM)

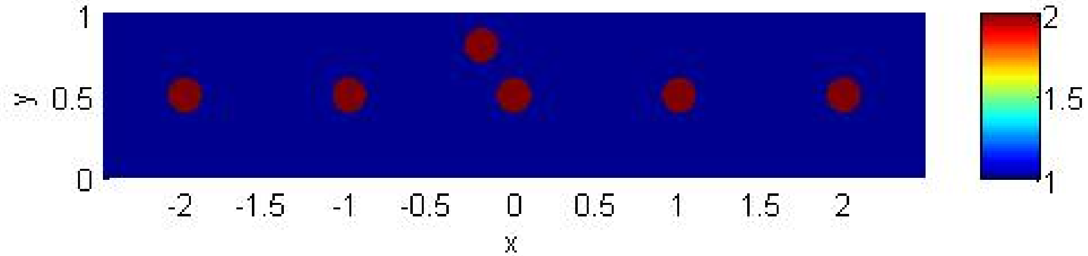
$$G_\infty(\cdot, z)|_{S_-} = i \sum_{k>0} \left[\sum_{n \in I(\omega)} \sum_{\substack{\xi \in \Xi_n(\omega) \\ \lambda_n'(\xi)>0}} \left(\frac{e^{i(M+q_z)\xi} \phi_n(z_1, z_2; \xi) \Phi_{nk}^-(\xi)}{\lambda_n'(\xi)} \right) \right] \psi_k^-$$

$$G_\infty(\cdot, z)|_{S_+} = i \sum_{k>0} \left[\sum_{n \in I(\omega)} \sum_{\substack{\xi \in \Xi_n(\omega) \\ \lambda_n'(\xi)>0}} \left(\frac{e^{i(N-q_z)\xi} \overline{\phi_n(z_1, z_2; \xi)} \Phi_{nk}^+(\xi)}{\lambda_n'(\xi)} \right) \right] \psi_k^+$$

$\longrightarrow$ Many possible choices for the basis $(\psi_k^\pm)$

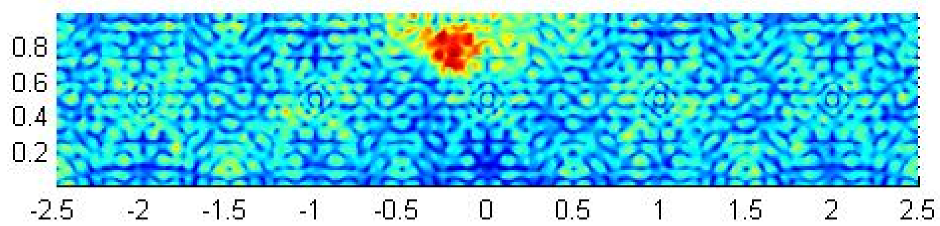
Some numerical experiments

Linear Sampling Method



12 Floquet modes

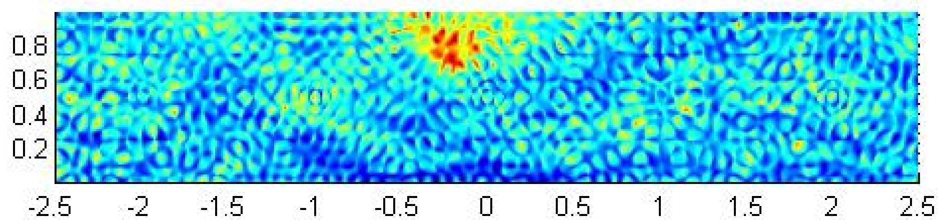
1% noise



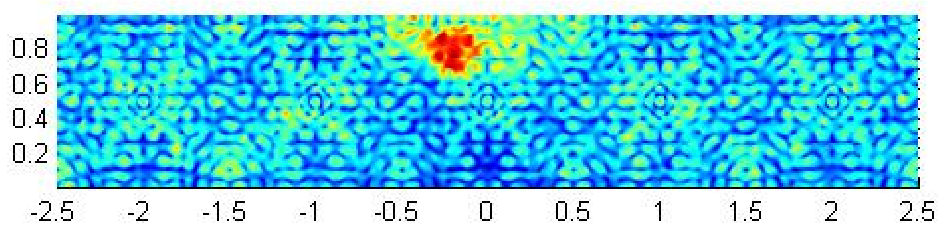
Some numerical experiments

Choice of projection basis :

Basis θ_p

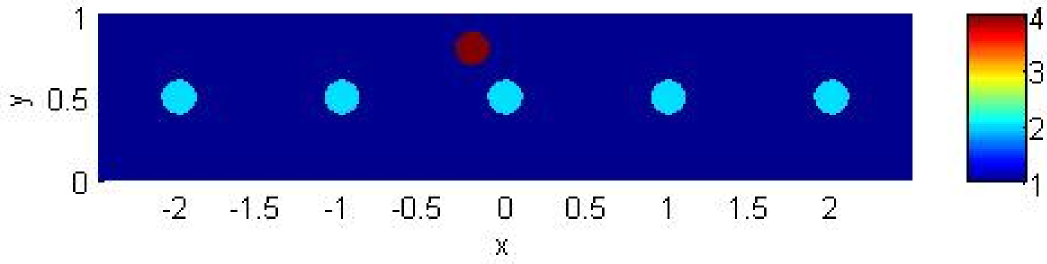


Traces of
Floquet modes



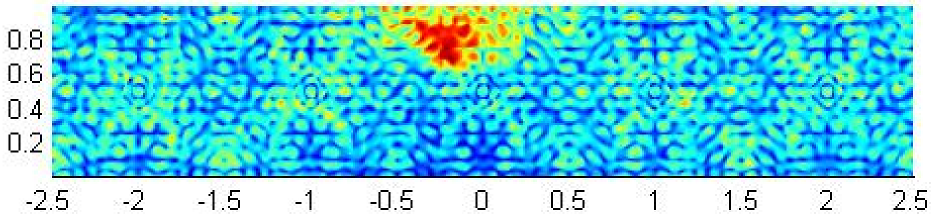
Some numerical experiments

Higher contrast :



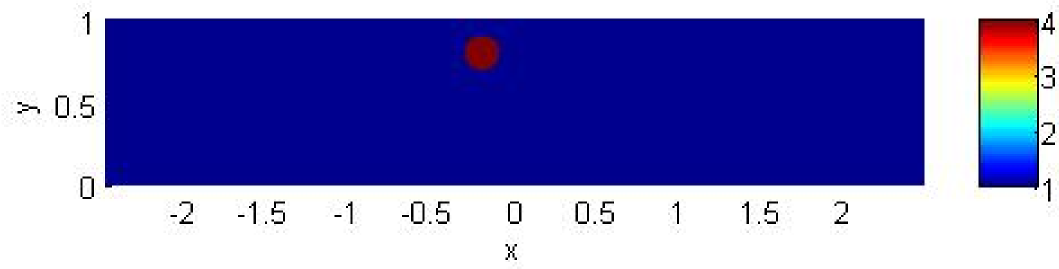
12 Floquet modes

1% noise



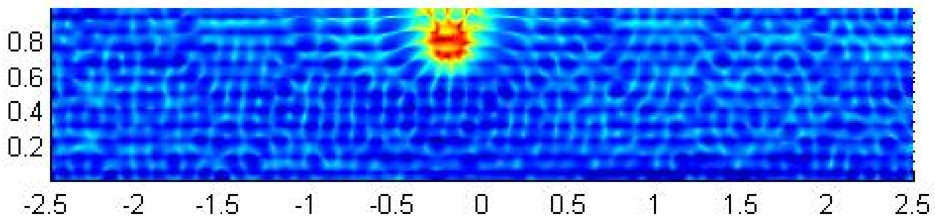
Some numerical experiments

Comparison with homogeneous waveguide :

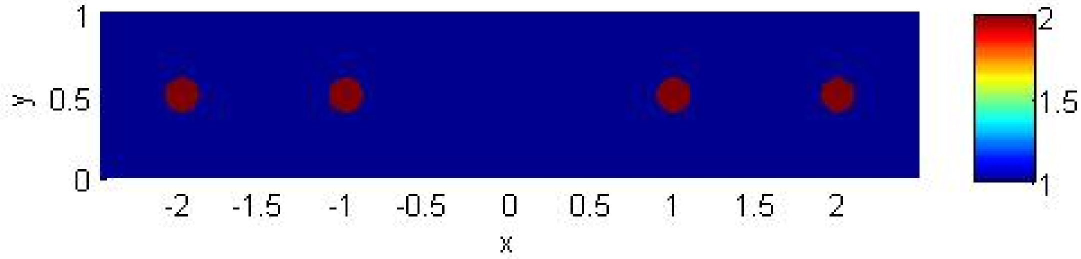


12 guided modes

1% noise

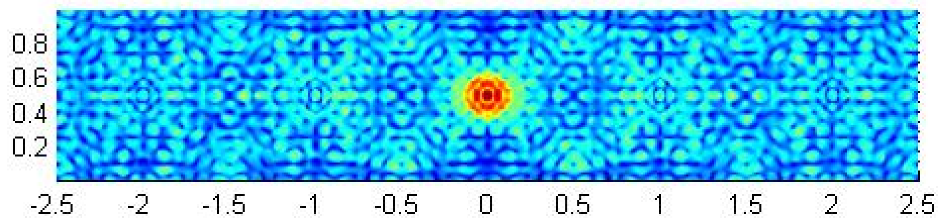


Some numerical experiments



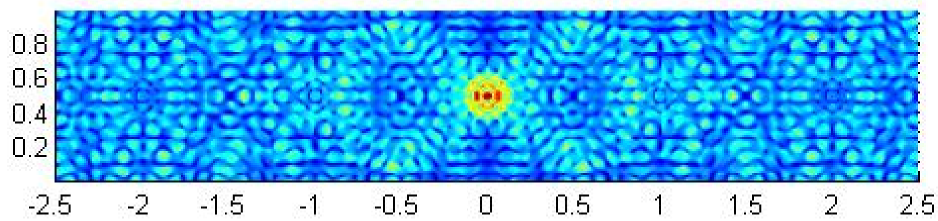
12 Floquet modes

1% noise

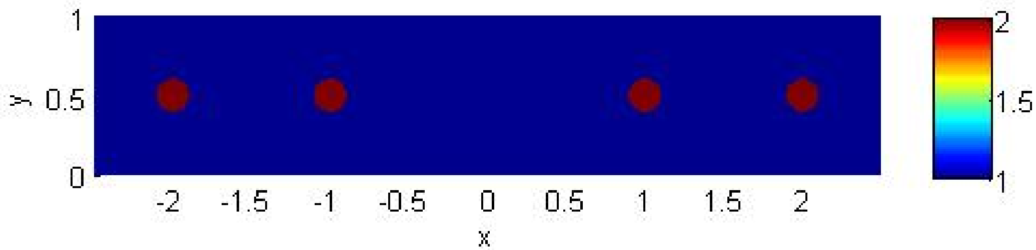


12 Floquet modes

10% noise

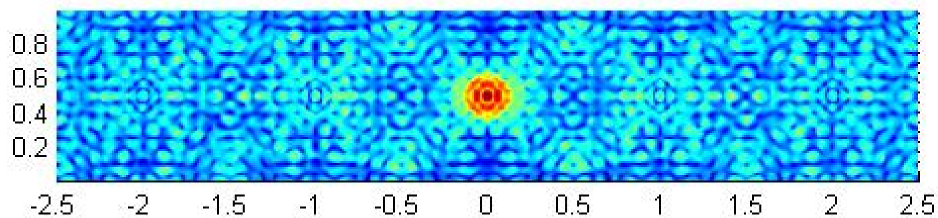


Some numerical experiments



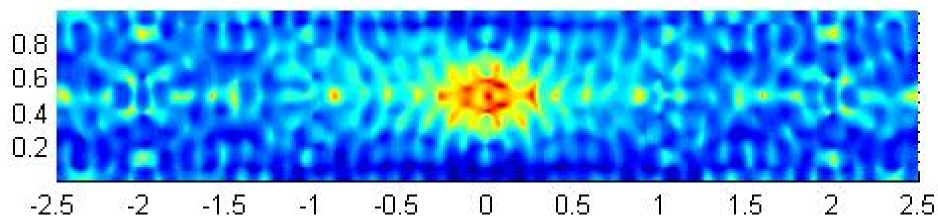
12 Floquet modes

1% noise



6 Floquet modes

1% noise



Some bibliography

1. **The linear sampling method in a waveguide: a modal formulation** (L.B. and E. Lunéville, 2008)

2. **Solutions of the time-harmonic wave equation in periodic waveguides: asymptotic behaviour and radiation condition** (S. Fliss and P. Joly, accepted)

3. **On the identification of defects in a periodic waveguide from far field data** (L.B. and S. Fliss, 2014)

4. Guided waves by electromagnetic gratings and non-uniqueness examples for the diffraction problem (A.-S. Bonnet-Ben Dhia, 1994)

5. Variational formulations for scattering in a three-dimensional acoustic waveguide (T. Arens, D. Gintides and A. Lechleiter, 2008)

Some bibliography (cont.)

6. Qualitative methods in inverse scattering theory (F. Cakoni and D. Colton, 2006)
7. The factorization method for inverse problems (A. Kirsch and N. Grinberg, 2008)
8. The factorization method is independent of transmission eigenvalues (A. Lechleiter, 2009)
9. The factorization method for an acoustic wave guide (A. Charalambopoulos, D. Ginides, K. Kiriaki and A. Kirsch, 2006)
10. Direct and inverse medium scattering in a three-dimensional homogeneous planar waveguide (T. Arens, D. Gintides and A. Lechleiter, 2011)

Some perspectives and open questions

- The forward scattering problem for periodic waveguides: an open question
- Bi-periodic structures (many applications): defining the far field is an open question
- Find a junction between two periodic half-waveguides
- Imaging a (periodic) waveguide in the time domain and with realistic data
- ...

Thank you for your attention !