Robust seismic velocity change estimation using ambient noise recordings

E.Daskalakis[1], **C. Evangelidis**[2], **J. Garnier**[3], **N. Melis**[2], **G. Papanicolaou**[4], **C. Tsogka**[1]

[1] Department of Mathematics and Applied Mathematics, University of Crete and IACM/FORTH, Heraklion, Greece
[2] Institute of Geodynamics, National Observatory of Athens, Athens, Greece
[3] Laboratoire de Probabilités et Modèles Aléatoires & Laboratoire Jacques-Louis Lions, Université Paris VII, Paris, France.
[4] Mathematics Department, Stanford University, Stanford, USA

Abstract

In this work, we consider the problem of imaging velocity changes in a medium using Cross-Correlations of noise recordings. In many applications in practice the noisy sources are not stationary in time and we have to process the measurements accordingly in order to "correct" for seasonal variations in the frequency content of the noise sources. We develop a simple signal processing treatment and illustrate its performance with numerical simulations, as well as, with real data from the Volcano of Santorini during a seismic unrest in 2011-12.

Cross Correlation function and SNR

The Cross-Correlation (CC) function is defined:

$$CC(x_1, x_2, \tau) = \frac{1}{T}\int_0^T u(x_1, t+\tau)u(x_2, t)dt \qquad (1)$$

Where $u(x_1, t)$ and $u(x_2, t)$ are the recordings of two sensors at positions x_1 and x_2. For T large enough we expect that the derivative of the cross-correlation between two sensors will converge to the symmetrized Green's function between them.

The Signal to Noise Ratio (SNR) of the cross-correlation increases with the $\sqrt{T}$ as expected from the theory but it also depends on the season as illustrated in Figure 1.

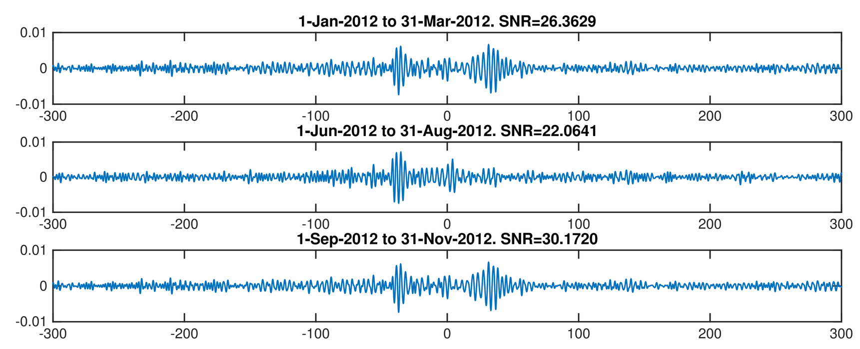

Figure 1: *Cross Correlation for three different seasons of the year.*

Estimating dv/v.

We are interested in estimating relative changes in the velocity of the medium (dv/v) using cross-correlations of noisy recordings. We first construct two CC functions, the reference CC, CC_{ref}, which is computed using all the available data and the current CC, CC_{cur}, which is computed by averaging the daily CC function over a few days around the current day we want to conduct the measurement on. The two CC functions are used by Stretching Method and the Moving Window Cross Spectral (MWCS) method to estimate velocity changes (dv/v).

- In the **Stretching Method** (SM), we search for the coefficient ε that maximises $C(\varepsilon)$ defined as:

$$C(\varepsilon) = \frac{\int CC_{\rm cur}(t(1+\varepsilon))CC_{\rm ref}(t)dt}{\sqrt{(\int CC_{\rm cur}(t(1+\varepsilon))^2 dt)(\int CC_{\rm ref}(t)^2 dt)}}. \qquad (2)$$

- The MWCS method on the other hand calculates time delays (δt_i) using the phase of the cross-spectrum of CC_{ref} and CC_{cur} in different time windows. The velocity change measurement follows from the relationship $dv/v = -\delta t/t$.

The problem of seasonal variations

Ambient noise may not be stationary in time. Indeed it has been observed that seasonal weather patterns can affect the frequency content of the noise source as illustrated in Figure 2. Those variations as suggested in [5] may affect the results of the stretching method and lead to "false" seasonal variations in the measurements of dv/v that are not really due to hydrological and/or thermoelastic variations of the medium as estimated in [3].

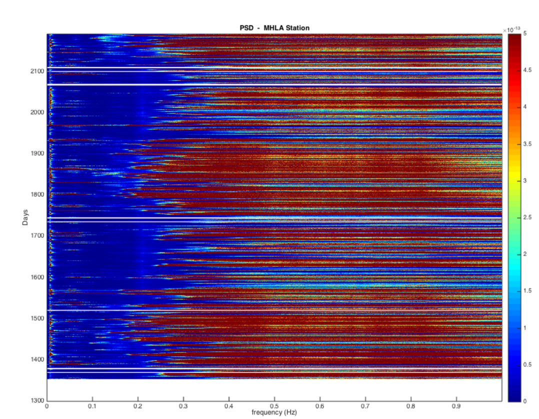

Figure 2: *Power Spectral Density of the data recorded on a station located on Milos island in the Aegean.*

Removing the effect of seasonal variations.

Modelling the seasonal variations: The experiment
In our numerical model we consider the acoustic wave equation:

$$\frac{1}{c(\mathbf{x})^2}\frac{\partial^2 u}{\partial t^2}(t, \mathbf{x}) - \Delta_{\mathbf{x}} u(t, \mathbf{x}) = n(t, \mathbf{x}), \qquad (3)$$

where $n(t, \mathbf{x})$ models the noise sources. The solution of (3) in a homogeneous medium at a given point $\mathbf{x}$ can be written as,

$$u(t, \mathbf{x}) = \int\int G(t-s, \mathbf{x}, \mathbf{y})n(s, \mathbf{y})d\mathbf{y}ds, \qquad (4)$$

or equivalently in the frequency domain,

$$\hat{u}(\omega, \mathbf{x}) = \int \hat{G}(\omega, \mathbf{x}, \mathbf{y})\hat{n}(\omega, \mathbf{y})d\mathbf{y}. \qquad (5)$$

Here hat denotes the Fourier transform.
Modelling the seasonal variations: The sources
The noise sources are located on a circle of radius 25km as illustrated in Figure 3. We assume that the wave field is recorded at two receivers $\mathbf{x}_1 = (-5, 0)$km and $\mathbf{x}_2 = (5, 0)$km.

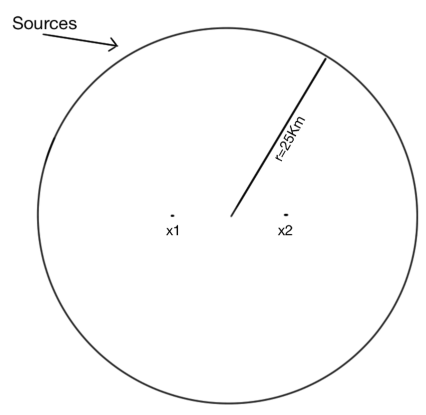

Figure 3: *Location of the noise sources and receivers at* $\mathbf{x}_1$ *and* $\mathbf{x}_2$

We assume that $n(t, \mathbf{x})$ s a zero-mean random process, stationary in time with a covariance function of the form

$$\langle n(t_1, \mathbf{y}_1), n(t_2, \mathbf{y}_2)\rangle = \Gamma(t_2 - t_1, \mathbf{y}_1)\delta(\mathbf{y}_2 - \mathbf{y}_1). \qquad (6)$$

Here $\langle\cdot\rangle$ stands for statistical averaging. The function $t \to \Gamma(t, \mathbf{y})$ is the time correlation function of the noise signals emitted by the noise sources at location $\mathbf{y}$. The function $\mathbf{y} \to \Gamma(0, \mathbf{y})$ characterizes the spatial support of the sources. In our case we assume that the sources are uniformly distributed on a circle $\mathcal{C}$ of radius of $R_{\mathcal{C}} = 25$km as illustrated in Figure 3:

$$\Gamma(t, \mathbf{y}) = \frac{1}{2\pi R_{\mathcal{C}}}\Gamma_0(t, \mathbf{y})\delta_{\mathcal{C}}(\mathbf{y}).$$

We also assume that we have two receivers at $\mathbf{x}_1 = (-5, 0)$km and $\mathbf{x}_2 = (5, 0)$km.
To model seasonal variations of the noise sources we introduce $\Gamma_0^j(t, \mathbf{y})$ the covariance function of the sources at day j. We take $N_s = 180$ point sources uniformly distributed on the circle of Figure 3 and then the equation (5) becomes

$$\hat{u}^j(\omega, \mathbf{x}) = \frac{1}{N_s}\sum_{i=1}^{N_s}\hat{G}^j(\omega, \mathbf{x}, \mathbf{y}_i)\hat{n}_i^j(\omega), \qquad (7)$$

where $\hat{n}_i^j(\omega)$ is the frequency content of the noise sources at $\mathbf{y}_i$ during day j, which is random such that $\left\langle \hat{n}_i^j(\omega)\right\rangle = 0$ and

$$\left\langle \hat{n}_i^j(\omega)\overline{\hat{n}_i^j}(\omega')\right\rangle = 2\pi\hat{\Gamma}_0^j(\omega, \mathbf{y}_i)\delta(\omega - \omega').$$

Modelling the seasonal variations: Variation models
Our model for the power spectral density of the noise sources is

$$\hat{\Gamma}_0^j(\omega, \mathbf{y}) = \hat{F}(\omega)\hat{s}^j(\omega, \mathbf{y}),$$

Here the unperturbed noise source distribution is uniform over the circle $\mathcal{C}$ and has power spectral density $\hat{F}(\omega)$, and $\hat{s}^j(\omega, \mathbf{y})$ is the daily perturbation of the power spectral density at location $\mathbf{y}$. We have two different representations for $\hat{s}^j$:

1. The daily perturbation is uniform with respect to the locations of the sources: $\hat{s}^j(\omega, \mathbf{y}) = \hat{f}^j(\omega)l(\mathbf{y})$,
2. The daily perturbation is not uniform and we cannot write it in a separable form.

Simulated results

We simulate 360 days so we construct the Reference CC-function by averaging all 360 daily CC-functions and the Current CC-function for day j by averaging 7 days around day j, $j = 1, \ldots, 360$.
We use a velocity that remains constant and a velocity with a small change for a 30 day period. The results are shown in Figure 4.

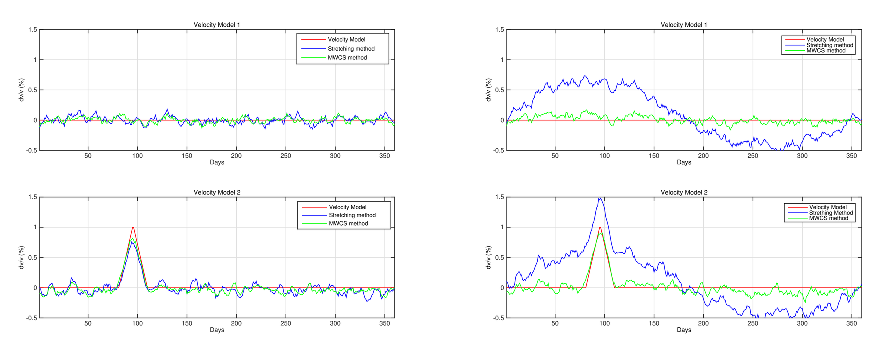

Figure 4: *The estimation using SM and MWCS methods without seasonal variations (Left) and when we have seasonal variations (Right) that are uniform with respect to the position of the noise sources.*

Spectral Whitening
We clearly observe the effect of seasonal variations on the estimation provided by the stretching method. A simple way to eliminate the effect of seasonal variations is to perform spectral whitening on the daily CC-functions (normalise the amplitude spectra). The results are shown in Figure 5.

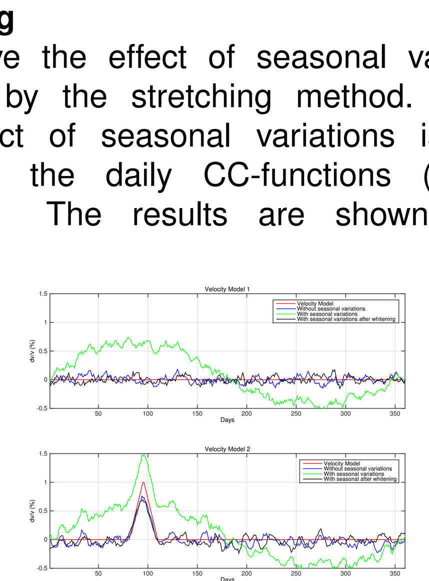

Figure 5: *The effect of Spectral Whitening in the estimations using the Stretching Method.*

Spectral whitening can remove the effect of seasonal variations that are uniform with respect to the position of the sources. In Figure 6 we show an example with non-uniform seasonal variations in which case spectral whitening is not successful.

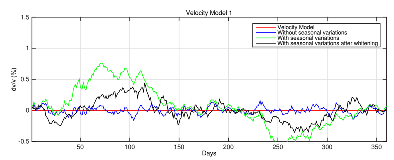

Figure 6: *The effect of Spectral Whitening in the case of non-uniform seasonal variations.*

SNR increase

We will illustrate now the performance of this simple signal processing on real data. In Figure 7 we apply the stretching method on data recorded on Milos, an island in Aegean sea, for two stations 6 Km apart.

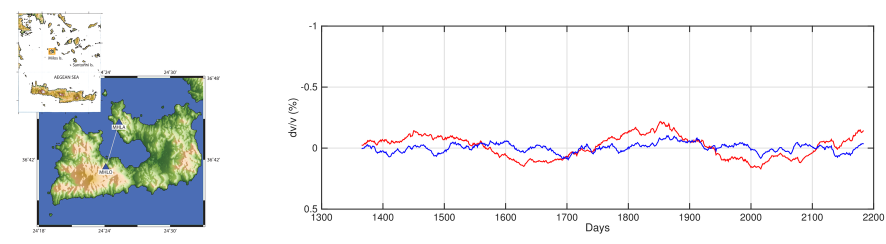

Figure 7: *Red: dv/v estimation without using Spectral Whitening. Blue: dv/v estimation using Spectral Whitening*

Santorini 2011-2012 unrest

The Santorini 2011-2012 seismic unrest begun on January 2011 and ended on February 2012. During the unrest a total uplift of 10 cm was measured with GPS on the caldera of Santorini.

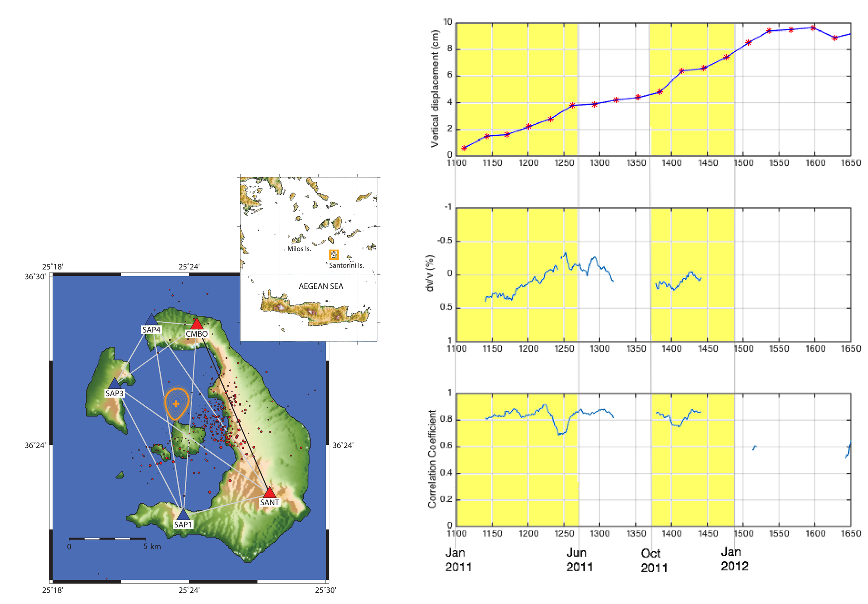

Figure 8: *The map of Santorini with the seismic stations(Left) and the results using SM(Right). On top we have the GPS measurement, on the middle we have the dv/v measurement and on the bottom we have the Correlation Coefficient.*

Conclusion

- *The dv/v estimation of the Stretching Method is affected from seasonal variations of the ambient noise sources.*
- *Under the reasonable assumption (for closely located receivers) of uniform variations, this problem can be avoided using spectral whitening on the CC functions. This improves the SNR of the method significantly.*
- *In the example of Santorini it was necessary to use spectral whitening in order to distinguish the real dv/v from the effect of seasonal variations.*

Acknowledgements

The work of G. Papanicolaou was partially supported by AFOSR grant FA9550-11-1-0266. The work of J. Garnier was partially supported by ERC Advanced Grant Project MULTIMOD- 26718. The work of E. Daskalakis and C. Tsogka was partially supported by the ERC Starting Grant Project ADAPTIVES-239959.

References

[1] Garnier, J. and Papanicolaou, G.(2009), *Passive sensor imaging using cross correlations of noisy signals in a scattering medium. SIAM J. Imaging Sciences* , 2:396–437.

[2] Konstantinou, K., Evangelidis, C., Liang, W.-T., Melis, N., and Kalogeras, I.(2013), *Seismicity, vp/vs and shear wave anisotropy variations during the 2011 unrest at Santorini caldera, southern Aegean.*, Journal of Volcanology and Geothermal Research, 267(0):57 – 67.

[3] Meier, U., Shapiro, N. M., and Brenguier, F.(2010), *Detecting seasonal variations in seismic ve- locities within Los Angeles basin from correlations of ambient seismic noise.*, Geophysical Journal International, 181(2):985–996.

[4] Saltogianni, V., Stiros, S. C., Newman, A. V., Flanagan, K., and Moschas, F. (2014), "Time-space modeling of the dynamics of Santorini volcano (Greece) during the 2011–2012 unrest.", J. Geophys. Res. Solid Earth, 119:8517–8537.

[5] Zhan, Z., Tsai, V., and Clayton, R. (2013), *Spurious velocity changes caused by temporal variations in ambient noise frequency content.*, Geophysical Journal International, 194:1574–1581.