An Inverse Problem in Hydrodynamics
Un problème inverse en hydrodynamique

Equipe DeFI, INRIA Saclay/CMAP-Ecole Polytechnique

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▷ **Scientific Context :** The Stokes equations for the velocity u and the pressure p,

$$\Delta u + \nabla p = 0, \qquad \nabla \cdot u = 0,$$

describe incompressible Newtonian fluids at very low Reynolds number. This model is obtained through the Navier-Stokes equations by neglecting, amongst others, the convection term. In this project we study these equations in a bounded domain $\Omega \subset \mathbb{R}^3$, where we immerse a bounded impenetrable and fixed "defect" $D \subset \Omega$. Since we suppose that there is no flow into the defect, the velocity u vanishes on ∂D. Hence, if the excitation of the system is given by a flow f on the boundary of the domain, we are given two boundary conditions for $u : \Omega \setminus \overline{D} \to \mathbb{R}^3$, first $u|_{\partial\Omega} = f$, and second $u|_{\partial D} = 0$. Since u is divergence free, we need to impose that $\int_{\partial\Omega} f \cdot \nu \, ds = 0$ where ν is the outward unit normal field to Ω. To obtain uniqueness of the pressure, we impose that $\int_{\Omega\setminus\overline{D}} p \, dx = 0$.

The interest of this project is to study the inverse Stokes problem: Given measurements of the traction

$$t(u) := ((\nabla u + \nabla u^\top) - pI) \cdot \nu$$

on (a part of) the boundary of Ω, reconstruct the defect D! A possible application of this mathematical problem is the industrial process of mould filling, during which gas bubbles (corresponding to the defect D) might get trapped inside the material while solidifying. Then, the problem arises to decide from boundary measurements whether or not there are gas bubbles inside the material, and, if so, to determine their location. This is a non-destructive testing problem.

▷ **Project :**

To solve the non-destructive testing problem, we will use so-called sampling methods for inverse problems [3, 2]. These techniques have recently gained a lot of interest for inverse problems in acoustics, electromagnetics, and elasticity. For the inverse Stokes problem, only one related question has been investigated in [1], however, in this PhD thesis the author considers direct and inverse Stokes problem *in all of* $\mathbb{R}^2$, that is, the solution pair (u, p) lives in an unbounded domain. For the non-destructive testing problem we have in mind, the setting of a bounded domain Ω seems to be better suited. The aim of this project is to analyze a sampling method for the inverse Stokes problem in this setting and to implement this sampling method numerically.

▷ **Requirements :** This internship is suitable for Master students. It requires basic knowledge on (linear, elliptic, variational) PDE theory. Since a part of the project is devoted to numerical implementation, knowledge of MATLAB or C/C++ is helpful. No preliminary knowledge on inverse problems is necessary. The internship might lead into a PhD thesis project.

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References

[1] Viktor Tsiporin, *Charakterisierung eines Gebiets durch Spektraldaten eines Dirichletproblems zur Stokesgleichung*, Mathematisch-naturwissenschaftliche Fakultäten, Georg-August-Universität Göttingen, Germany, 2004

[2] Andreas Kirsch and Natalia Grinberg, *The factorization method for inverse problems*, Oxford Lecture Series in Mathematics and Its Applications, 2008

[3] Fioralba Cakoni and David Colton, *Qualitative Methods in Inverse Scattering Theory. An Introduction.*, Springer, 2006.