Stochastic calculus, Monte-Carlo methods and Mathematical Finance

A Conference in honor of Professor Vlad Bally

Dates : October 6 (Tue) – 9 (Fri), 2015

Venue : Institut du Risque et de l'Assurance du Mans
(Le Mans, France)

Schedule

	Tuesday	**Wednesday**
09.00-9.40		P. Protter
9.40-10.20		J. Bertoin
10.20-10.45		*Coffee break*
10.45-12.25		R. Buckdahn
11.25-12.05	Registration (starting 11.45)	P. Imkeller
12.15-14.00	*Lunch*	*Lunch*
14.00-14.40	D. Nualart	A. Lejay
14.40-15.20	D. Lamberton	E. Nualart
15.20-15.45	*Coffee break*	*Coffee break*
15.45-16.25	D. Talay	B. Bouchard
16.25-17.05	N. Fournier	F. Delarue
17.05-17.45	N. Touzi	A. Millet
	Welcome cocktail (conference hall)	

	Thursday	**Friday**
09.00-9.40	E. Pardoux	M. Hairer
9.40-10.20	A. Kohatsu-Higa	L. Zambotti
10.20-10.45	*Coffee break*	*Coffee break*
10.45-12.25	G. Tessitore	D. Crisan
11.25-12.05	A. Sulem	G. Pagès
12.15-14.00	*Lunch*	*Closing remarks & Lunch*
14.00-14.40	E. Gobet	
14.40-15.20	A. Kebaier	
15.20-15.45	*Conference picture (15.20) & Coffee*	
15.45-16.25	C. Martini	
16.25-17.05	B. Jourdain	
	Visit of Le Mans & Conference dinner	

Invited talks

Tuesday 6 Oct 14.00 – 15.20

David Nualart (University of Kansas) - *Stochastic heat equation with rough multiplicative noise*
We present some results on the existence and uniqueness of solutions for the one-dimensional stochastic heat equation driven by a Gaussian noise which is white in time and it has the covariance of a fractional Brownian motion with Hurst parameter H¡1/2 in the space variable. In the linear case we will establish a Feynman-Kac formula for the moments and derive moment estimates and intermittency properties of the solution.

Damien Lamberton (Université Paris-Est Marne-La-Vallée) - *On the binomial approximation of the American put*
We prove that the order of convergence of the binomial approximation of the American put price (in the Black-Scholes model) is $O(\ln(n)^\alpha/n)$, where n is the number of steps and α depends on the difference between the interest rate and the dividend rate. Our estimates improve our earlier results (2002) and rely on quadratic estimates for the early exercise premium and on the asymptotics of the exercise boundary near maturity.

Tuesday 6 Oct 15.45 – 17.45

Denis Talay (INRIA Sophia-Antipolis) - *Convergence rate of fractional BM to BM first hitting time Laplace transforms*
The objective is to study the sensitivity of the first hitting time probability distribution of diffusion type models to the Hurst parameter of the noise around the pure BM value. We prove accurate estimates on the difference of the respective Laplace transforms. This work is notably motivated by stochastic modelling in neuroscience (Joint work with Alexandre Richard).

Nicolas Fournier (Université Pierre et Marie Curie) - *Statistics versus mean-field limit for Hawkes processes*
We consider a population of N individuals, of which we observe the number of actions as time evolves. For each couple of individuals (i, j), j may or not influence i, which we model by i.i.d. Bernoulli(p)-random variables, for some unknown parameter $p \in (0, 1]$. Each individual acts autonomously at some unknown rate $\mu > 0$ and acts by mimetism at some rate depending on the number of recent actions of the individuals which influence him, the age of these actions being taken into account through an unknown function φ (roughly, decreasing and with fast decay). The goal of this paper is to estimate p, which is the main charateristic of the graph of interactions, in the asymptotic $N \to \infty, t \to \infty$. The main issue is that the mean field limit (as $N \to \infty$) of this model is not identifiable, in that it only depends on the parameters μ and $p\varphi$.

Nizar Touzi (Ecole Polytechnique) - *Second Order BSDEs and Moral Hazard in the Principal-Agent Problem*
We consider a general formulation of the continuous-time Principal-Agent problem in finite horizon.

Our main result is a reduction of this problem to a standard stochastic control problem which may be analyzed by the standard tools of control theory. In particular, the performance index of the Agent appears naturally as a controlled state variable for the reduced Principal problem. Our argument relies on the backward SDE approach to non-Markov stochastic control, and more specifically the most recent extensions to the second order case.

Wednesday 7 Oct 9.00 – 10.20

Philip Protter (Columbia University) - *Financial Bubbles : New results*
Using the mathematical model of a financial bubble, couple with a statistically based test and a methodology to determine the birth and death of bubbles, we are able to incorporate these with massive data analysis to determine the empirical distribution of the lifetimes of bubbles.

Jean Bertoin (University of Zurich) - *Probabilistic aspects of critical growth-fragmentation equations*
Based on a joint work with Alex Waston. Growth-fragmentation equation describes the evolution of a medium in which particles grow and divide as time proceeds, with the growth and splitting of each particle depending only upon its size. The critical case of the equation, in which the growth and division rates balance one another, was considered by Doumic and Escobedo in the homogeneous setting where the rates do not depend on the particle size. Here, we study the general self-similar setting, using a probabilistic approach based on Lvy processes and positive self-similar Markov processes which also permits us to analyse quite general splitting rates. Whereas existence and uniqueness of the solution are rather easy to establish in the homogeneous setting, the equation in the non-homogeneous setting has some surprising features. In particular, using the fact that certain self-similar Markov processes can enter $(0, \infty)$ continuously from either 0 or $+\infty$, we exhibit unexpected spontaneous generation of mass in the solutions.

Wednesday 7 Oct 10.45 – 12.05

Rainer Buckdahn (Université de Bretagne Occidentale) - *Representation formulas for long run averaging optimal control problem*
We investigate an optimal control problem with averaging cost. The asymptotic behavior of the values is a classical problem in ergodic control. To study the long run averaging we consider both Cesro and Abel means. A main result of the paper saysthat there is at most one possible accumulation point - in the uniform convergence topology -of the values, when the time horizon of the Cesàro means converges to infinity or the discount factor of the Abel means converges to zero. This unique accumulation point is explicitly described by representationformulas involving probability measures on the state and control spaces. As a byproduct we obtain the existence of a limit value whenever the Cesàro or Abel values are equicontinuous. Our approach allows to generalise several results in ergodic control, and in particular it allows to cope with cases where the limit value is not constant with respect to the initial condition. The talk is based on a joint work with M.Quincampoix (UBO, Brest) and Jérome Renault (TSE, Toulouse 1).

Peter Imkeller (Humboldt University) - *A Fourier analysis based approach of integration*
In 1961, Ciesielski established a remarkable isomorphism of spaces of Holder continuous functions and Banach spaces of real valued sequences. The isomorphism can be established along Fourier type expansions of (rough) Holder continuous functions by means of the Haar–Schauder wavelet. We will use Schauder representations for a pathwise approach of the integral of one rough function with respect to another one, via Ciesielski's isomorphism. In a more general and analytical setting, this approach of rough path analysis can be understood in terms of Gubinelli's concept of controlled paths, Paley-Littlewood decompositions of distributions, and Bony paraproducts in Besov spaces. It is well suited for Hairer's approach of SPDE. In a stochastic analysis context, the resulting integral is closely related to Stratonovich's or Ogawa's concepts. To recover Ito's integral for instance in Föollmer's pathwise approach requires some additional knowledge of the quadratic variation. This talk is based on work in progress with M. Gubinelli (U Paris-Dauphine) and N. Perkowski (HU Berlin).

Wednesday 7 Oct 14.00 – 15.20

Antoine Lejay (INRIA - Nancy) - *Simulation of SDE with discontinuous drift*
We propose a method to estimate the weak rate of convergence of a SDE with a discontinuous drift coefficient. This method relies on a stochastic formulation of the perturbation semi-group, and could be applied to deal with several classes of diffusion coefficients and terminal condition. From a joint with with A. Kohatsu-Higa and K. Yasuda

Eulalia Nualart (Universitat Pompeu Fabra) - *LAN property for stochastic differential equations with additive fractional noise and continuous time observation*
We consider a stochastic differential equation with additive fractional noise of Hurst parameter $H > 1/2$, and a non-linear drift depending on an unknown parameter. We show the Local Asymptotic Normality property (LAN) of this parametric model with rate $\sqrt{\tau}$ as $\tau \to \infty$, when the solution is observed continuously on the time interval $[0, \tau]$. The proof uses ergodic properties of the equation and a Poincaré type inequality.

Wednesday 7 Oct 15.45 – 17.45

Bruno Bouchard (Université Paris Dauphine) - *A Doob-Meyer decomposition for general g-supermatingale systems, without right-continuity*
We provide a general Doob-Meyer decomposition for g-supermartingale systems, which does not require any right-continuity on the system, nor the quasi-left continuity of the filtration. In particular, it generalizes the Doob-Meyer decomposition of Mertens for classical supermartingales, as well as Peng's version for right-continuous g-supermartingales. As examples of application, we prove an optional decomposition theorem for g-supermartingale systems, and also obtain a general version of the well-known dual formation for BSDEs with constraint on the gains-process, using very simple arguments.

François Delarue (Université de Nice) - *Restoration of uniqueness in mean-field games*

Mean-field games theory was introduced by Lasry and Lions in 2006. The purpose is to describe the asymptotic behavior of Nash equilibria over a large population of controlled players interacting with one another in a mean-field way. In this framework, very few criteria are known to ensureuniqueness of the asymptotic equilibria. Inspired by the theory of ordinary and stochastic differential equations,we here address the question of restoration of uniqueness by randomization of the equilibria.

Annie Millet (Université Paris 1) - *On the Richardson acceleration of finite elements schemes for parabolic SPDEs*
We consider some finite elements approximations u^h of the solution u to a semi-linear parabolic SPDE whose coefficients satisfy the classical stochastic parabolicity condition, and $h > 0$ is a scaling factor. We obtain an approximation of $u^h - u$ which is polynomial in h. This yields the corresponding Richardson acceleration method for such finite element approximations of u by u^h given any prescribed speed. This joint work with I. Gyöngy.

Thursday 8 Oct 9.00 – 10.20

Etienne Pardoux(Université Aix-Marseille) - *Homogenization of semilinear heat equation with a highly oscillating random potential*
We prove a law of large numbers and a central limit theorem for the homogenization of a random semilinear heat equation in one space dimension, with a rapidly oscillating (both in time and space) random potential. This is joint work with M. Hairer and A. Piatnitski.

Arturo Kohatsu-Higa (Ritsumeikan University) - *The parametrix method applied to skew diffusions*
The parametrix method is a method in partial differential equations that allows the expansion of solutions for uniformly elliptic parabolic equations with Holder coefficients. In this talk, I will present a methodology to analyse the behaviour of the density of the so-called skew diffusions. This class of one dimensional diffusions have discontinuous coefficients at one point. We show how to apply the parametrix method using skew brownian motion. We also give some conclusions about the regularity of the density and a probabilistic representation.

Thursday 8 Oct 10.45 – 12.05

Gianmario Tessitore (Milano-Bicocca University) - *Singular perturbation control problems : a BSDE approach*
We consider a control problem for a two scale stochastic system in infinite dimensions. We represent the value function as the solution of a BSDE and show that it converges (as the speed of the fast equation increases) towards a limit BSDE. The non linearity of the limit equation is expressed in terms of the solution to a parametrized ergodic BSDE (Joint work with Francois Delarue and Giuseppina Guatteri).

Agnes Sulem (INRIA-Rocquencourt) - *Weak Dynamic Programming Principle for Combined Optimal Stopping / Stochastic Control with f-Expectation*

We study combined optimal control/stopping problems with f-expectations in the Markovian framework on a finite horizon of time T. We establish a weak dynamic programming principle. To this purpose, we prove some measurability properties and a "splitting" result stating that, given an intermediary time $t \leq T$, the problem can be decomposed into two independent parts, one corresponding to the past (before t) and one to the future (after t). Using this weak dynamic programming principle and properties of reflected backward stochastic differential equations, we prove that the value function of our combined control problem is a weak viscosity solution of a nonlinear Hamilton-Jacobi-Bellman variational inequality. Some illustrating examples in mathematical finance are provided (Joint work with Roxana Dumitrescu and M.-C. Quenez).

Thursday 8 Oct 14.00 – 15.20

Emmanuel Gobet (Ecole Polytechnique) - *Stratified Regression Monte-Carlo Scheme for BSDEs with Large-Scale Parallelization on GPUs*
In this paper, we design a novel algorithm based on least-squares Monte Carlo (LSMC) in order toapproximate the solution of discrete-time backward stochastic differential equations (BSDEs). Ouralgorithm allows massive parallelization of the computations on multicore devices such as graphicsprocessing units (GPUs). Our approach consists of a novel method of stratification which appearsto be crucial for large scale parallelization (Joint work with J. G. Lopez-Salas, P. Turkedjiev, and C. Vazquez).

Ahmed Kebaier (Université Paris 13) - *Multilevel Sample Average Approximation*
In the present work, we study an original adaptation of the Multilevel Monte Carlo method. More precisely, we propose an adaptive Multilevel Monte Carlo method based on a deterministic optimization of the optimal parameter in the associated importance sampling procedure. This innovative estimator leads to a robust and efficient procedure that reduces at the same time the variance and the computational effort. In the setting of discretized diffusions, we establish for this poposed estimator a new strong law of large numbers and a new central limit theorem as well. Finally, we illustrate the effectiveness of our method through several numerical tests dedicated to quantitative finance (joint work with Jérôme Lelong, Université Grenoble Alpes).

Thursday 8 Oct 15.45 – 17.05

Claude Martini (Zeliade Systems) - *On the support of extremal martingale measures with given marginals : the countable case*
The extremal points in the set of all measures with pre-specified marginals, *without the martingale constraint*, have been extensively studied by many authors in the past (e.g. Denny, Douglas, Letac, Klopotowski to cite only a few). In the discrete (countable) 2 marginals *martingale* problem studied by Beiglböck-Juillet, with special cases provided by Henry-Labordère and Touzi, Hobson and Klimmeck, Hobson and Neuberger, and Laachir, we will give examples of extremal and non-extremal points, and provide a candidate characterization of the support of extremal points (joint work with L. Campi, LSE).

Benjamin Jourdain (Ecole des Ponts ParisTech CERMICS) - *Strong convergence properties of the Ninomiya Victoir scheme and applications to multilevel Monte Carlo*
We prove that the strong convergence rate of the Ninomiya-Victoir scheme is 1/2. The normalized error term converges stably to the solution of an affine SDE with a source term involving the commutators between the Brownian vector fields. When the Brownian vector fields commute, this limit vanishes and we show that the strong convergence rate improves to 1. We last show that averaging the order of integration of the Brownian fields leads to a scheme which couples with strong order 1 to the scheme recently proposed by Giles and Szpruch in order to achieve the optimal complexity in the multilevel Monte Carlo method.

Friday 9 Oct 9.00 – 10.20

Martin Hairer (Warwick University)- *Motion of a random string*
A rubber band constrained to remain on a manifold evolves by trying to shorten its length, eventually settling on some minimal closed geodesic, or collapsing entirely. It is natural to try to consider a noisy version of such a model where each segment of the band gets pulled in random directions. Trying to build such a model turns out to be surprisingly difficult and generates a number of nice geometric insights, as well as some beautiful algebraic and analytical objects.

Lorenzo Zambotti (Université Pierre et Marie Curie) - *Renormalisation in regularity structures*
We describe a general approach to the renormalisation step in the theory of regularity structures based on Hopf algebras of labelled trees (joint work with Yvain Bruned and Martin Hairer).

Friday 9 Oct 10.45 – 12.05

Dan Crisan (Imperial College London) - *Gradient bounds for the solution of the filtering equation*
In this talk I will discuss sharp gradient bounds for perturbed diffusion semigroups. In contrast with existing results, the perturbation is here random and the bounds obtained are pathwise. Our approach builds on the classical work of Kusuoka and Stroock and extends their program developed for the heat semi-group to solutions of stochastic partial differential equations. The work is motivated by and applied to nonlinear filtering. The analysis allows us to derive pathwise gradient bounds for the un-normalised conditional distribution of a partially observed signal. The estimates we derive have sharp small time asymptotics. This is joint work with Terry Lyons (Oxford University) and Christian Litterer (University of York) and is based on the paper

Crisan, Dan ; Litterer, Christian ; Lyons, Terry. Kusuoka-Stroock gradient bounds for the solution of the filtering equation. J. Funct. Anal. 268 (2015), No. 7, 1928-1971.

Gilles Pages (Université Pierre et Marie Curie) - *Improved error bounds for quantization based numerical schemes*
We present improved error bounds satisfied by quantization-based numerical schemes for *BSDE*s and non-linear filtering theory. These bounds also hold in under lighter assumptions in both cases than the original ones ([Bally-P., 2003] and [P. -Pham, 2005] respectively). These improvements take advantage of recent new results for optimal vector quantization theory : a quantized Pythagoras like

theorem for the quadratic approximation of conditional expectations and the distortion mismatch property which rules the rate of decay of the L^q-mean quantization error induced by a sequence of L^p-optimal quantizers of an R^d-valued random vector when q lies in $(p, p+d)$.

Evening activities

Tuesday

17.45 **Welcome cocktail**
In the hall of Institut du Risque et de l'Assurance du Mans, just out of the conference room.

Thursday

18.00 – 19.30 **Guided tour of historic city center & Le Mans cathedral**
Meeting point : place du jet d'eau, next to the cathedral.

20.00 **Conference dinner**
Restaurant *La Villa*, 26 place de la République, 72000 Le Mans
The guided tour ends close to place de la République (within walking distance)

Organizers Stefano De Marco (Ecole Polytechnique)
Laurent Denis (Université du Maine)
Anis Matoussi (Université du Maine)

Organizing committee Nicolas Fournier, Said Hamadène, Damien Lamberton, Gilles Pagès, Nizar Touzi

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