Advanced Control

January 25, 2013

École Centrale Paris, Châtenay-Malabry, France

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Course Overview

Date		Topic
Fri, 11.1.2013	DB	Introduction to Control, Examples of Advanced Control, Introduction to Fuzzy Logic
Fri, 18.1.2013	DB	Fuzzy Logic (cont'd), Introduction to Artificial Neural Networks
Fri, 25.1.2013	~~AA~~ DB	Bio-inspired Optimization, discrete search spaces
Fri, 1.2.2013	AA	The Traveling Salesperson Problem
Fri, 22.2.2013	AA	Continuous Optimization I
Fri, 1.3.2013	AA	Continuous Optimization II
Fr, 8.3.2013	DB	Controlling a Pole Cart
Do, 14.3.2013	DB	Advanced Optimization: multiobjective optimization, constraints, ...
Tue, 19.3.2013		written exam (paper and computer)

all classes + exam at **8h00-11h15** (incl. a 15min break around 9h30)

Remark to last lecture

All information also available at

`http://researchers.lille.inria.fr/~brockhof/advancedcontrol/`

(exercise sheets, lecture slides, additional information, links, ...)

Introduction to Bio-inspired Optimization

and Genetic Algorithms in particular

General Context Optimization

Given:

set of possible solutions — *Search space*

quality criterion — *Objective / Fitness function*

Objective:

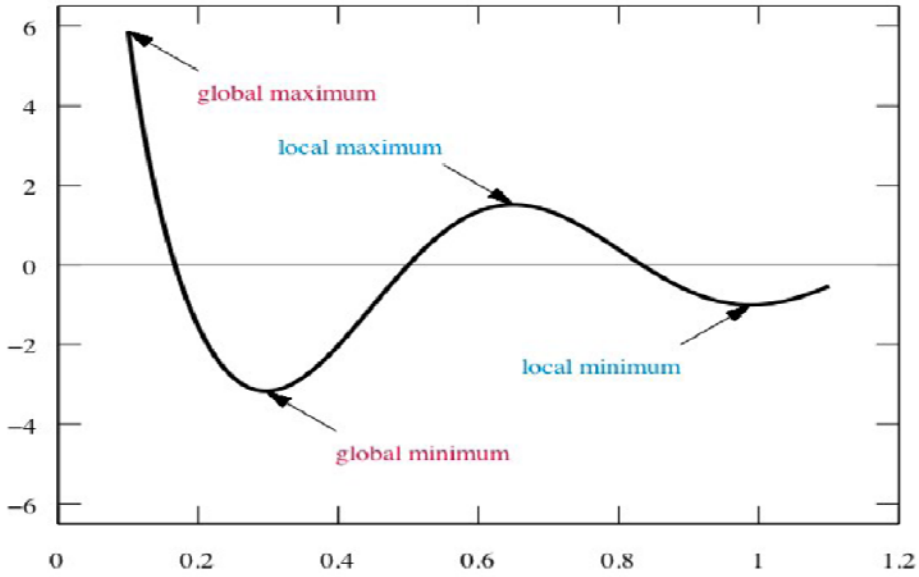
Find the best possible solution for the given criterion

Formally:

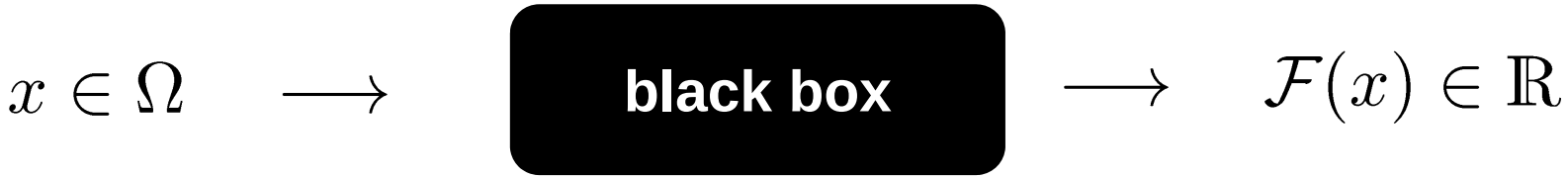
Maximize or minimize

$$\mathcal{F} : \Omega \mapsto \mathbb{R},$$

$$x \mapsto \mathcal{F}(x)$$



Black Box Scenario



Why are we interested in a black box scenario?

objective function F often noisy, non-differentiable, or sometimes not even understood or available

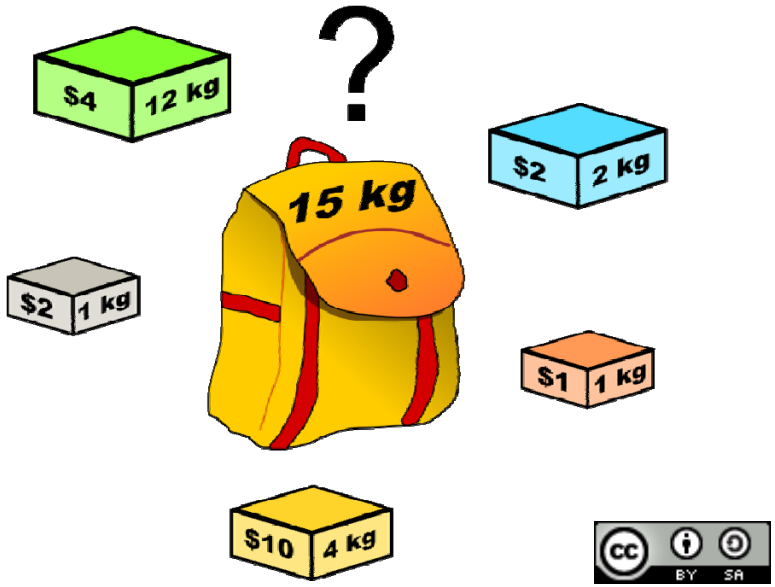
Objective: find x with small F(x) with as few function evaluations as possible

assumption: internal calculations of algo irrelevant

Example 1: Combinatorial Optimization

Knapsack Problem

- Given a set of objects with a given weight and value (profit)
- Find a subset of objects whose overall mass is below a certain limit and maximizing the total value of the objects



Dake

[Problem of ressource allocation with financial constraints]

$$\max. \sum_{j=1}^{n} p_j x_j \text{ with } x_j \in \{0,1\}$$

$$\text{s.t.} \sum_{j=1}^{n} w_j x_j \leq W$$

$$\Omega = \{0,1\}^n$$

Example 2: Combinatorial Optimization

Travelling Salesperson Problem (TSP)

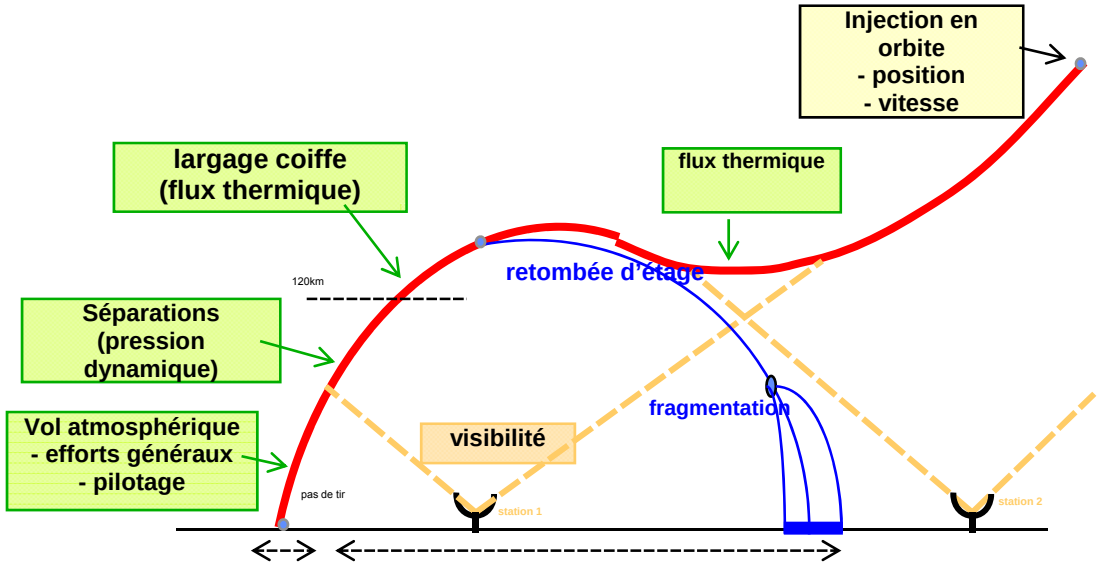
- Given a set of cities and their distances
- Find the shortest path going through all cities

$$\Omega = S_n \text{ (set of all permutations)}$$

Example 3: Continuous Optimization

Design of a Launcher

Poppy



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- Scenario: multi-stage launcher brings a satellite into orbit
- Minimize the overall cost of a launch
- Parameters: propellant mass of each stage / diameter of each stage / flux of each engine / parameters of the command law

23 continuous parameters to optimize

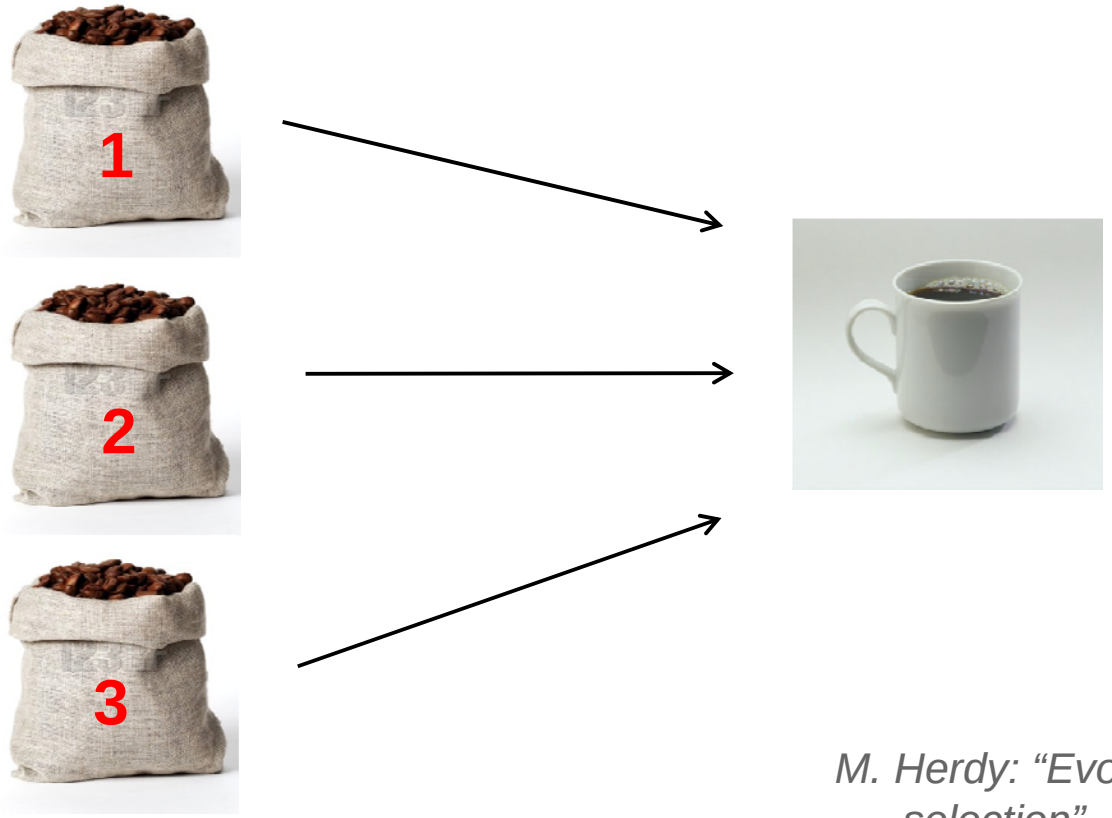
+ constraints

$$\Omega = \mathbb{R}^{23}$$

Example 4: Interactive Optimization

Coffee Tasting Problem

- Find a mixture of coffee in order to keep the coffee taste from one year to another
- Objective function = opinion of one expert



M. Herdy: "Evolution Strategies with subjective selection", 1996

What makes an optimization problem difficult?

Why using (bio-inspired) search heuristics?

- Search space too large

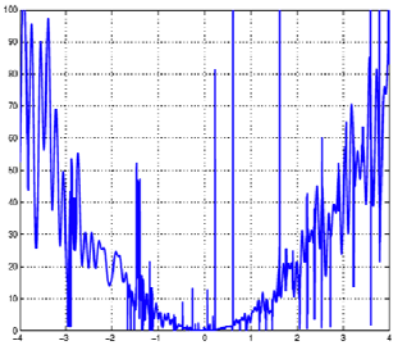
  exhaustive search impossible

- Non conventional objective function or search space

  mixed space, function that cannot be computed

- Complex objective function

  non-smooth, non differentiable, Noisy, ...

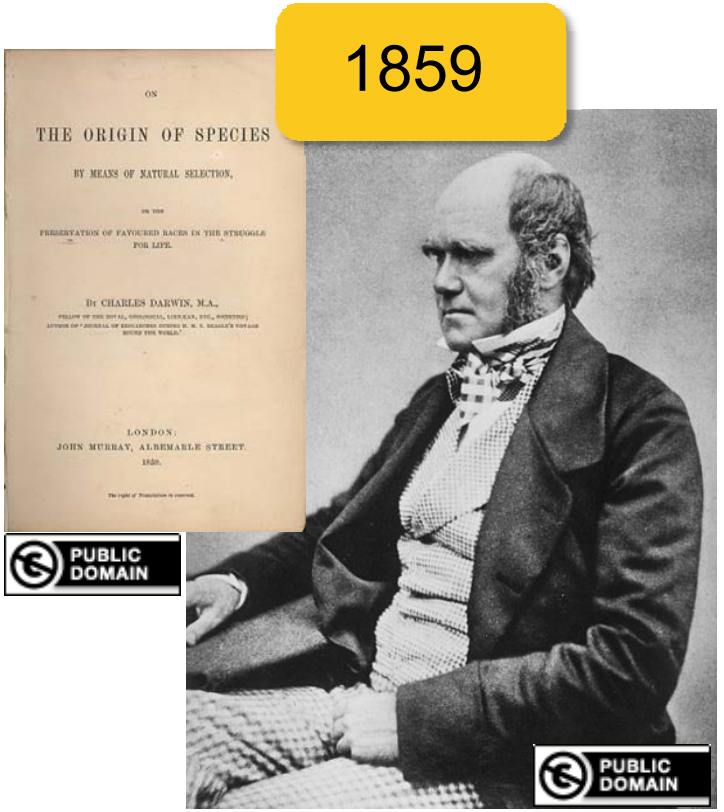


Basic Algorithms

Bio-inspired Stochastic Optimization Algorithms

One class of bio-inspired stochastic optimization algorithms: Evolutionary Algorithms (EAs)

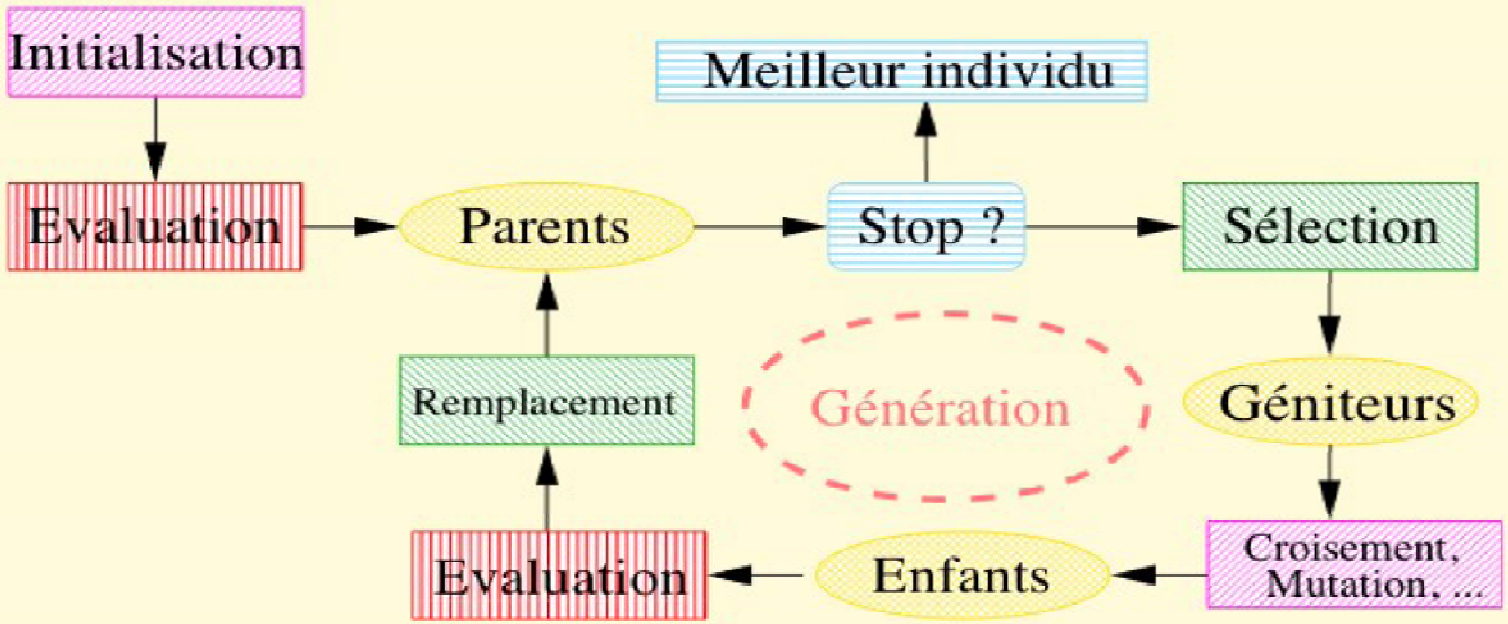
- Class of optimization algorithms inspired by the idea of biological evolution
- selection, mutation, recombination



Metaphors

Classical Optimization	Evolutionary Computation
candidate solution vector of decision variables / design variables / object variables	individual, offspring, parent
set of candidate solutions	population
objective function loss function cost function error function	fitness function
iteration	generation

Generic Framework of an EA



Opérateurs stochastiques: Dépendent de la représentation

"Darwinisme" (stochastique ou déterministe)

Coût calcul

Critère d'arrêt, statistiques, ...

Important:
representation (search space)

The Historic Roots of EAs

Genetic Algorithms (GA)

J. Holland 1975 and D. Goldberg (USA)

$$\Omega = \{0,1\}^n$$

Evolution Strategies (ES)

I. Rechenberg and H.P. Schwefel, 1965 (Berlin)

$$\Omega = \mathbb{R}^n$$

Evolutionary Programming (EP)

L.J. Fogel 1966 (USA)

Genetic Programming (GP)

J. Koza 1990 (USA)

$$\Omega = \text{space of all programs}$$

nowadays one umbrella term: evolutionary algorithms

Examples for some EA parts

Selection

Selection is the major determinant for specifying the trade-off between exploitation and exploration

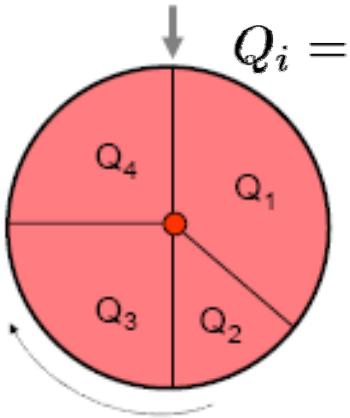
Selection is either

stochastic or deterministic

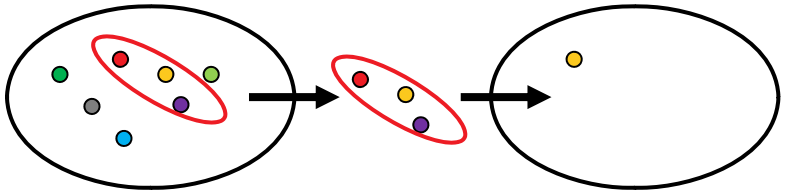
e.g. fitness proportional

$$Q_i = \frac{f(x_i)}{\sum_{j=1}^{\mu} f(x_j)}$$

Disadvantage: depends on scaling of f



e.g. via a tournament



e.g. (μ+λ), (μ,λ)

Mating selection (selection for variation): usually stochastic

Environmental selection (selection for survival): often deterministic

Variation Operators

Variation aims at generating new individuals on the basis of those individuals selected for mating

Variation = Mutation and Recombination/Crossover

mutation: *mut:* $\Omega \to \Omega$

recombination: *recomb:* $\Omega^r \to \Omega^s$ where $r \geq 2$ and $s \geq 1$

- choice always depends on the problem and the chosen representation
- however, there are some operators that are applicable to a wide range of problems and tailored to standard representations such as vectors, permutations, trees, etc.

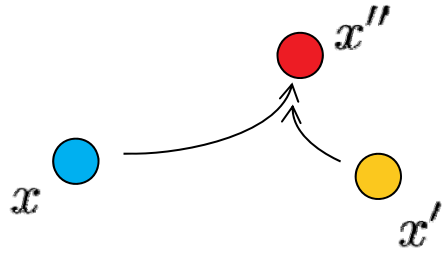
Variation Operators: Guidelines

Two desirable properties for mutation operators:

- every solution can be generation from every other with a probability greater than 0 (“exhaustiveness”)
- $d(x, x') < d(x, x'') => Prob(\mathrm{mut}(x) = x') > Prob(\mathrm{mut}(x) = x'')$ (“locality”)

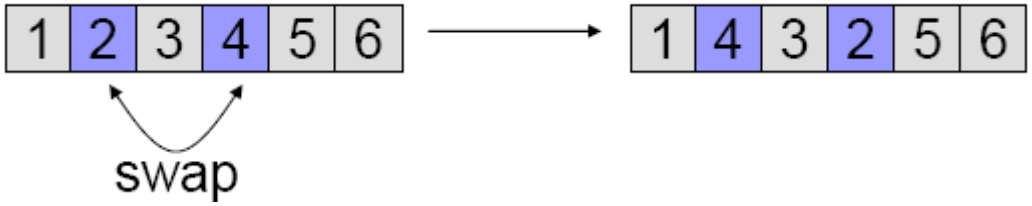
Desirable property of recombination operators (“in-between-ness”):

$$x'' = \mathrm{recomb}(x, x') \Rightarrow d(x'', x) \leq d(x, x) \wedge d(x'', x) \leq d(x, x')$$

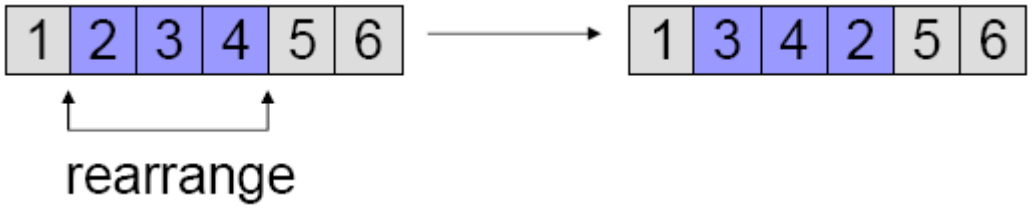


Examples of Mutation Operators on Permutations

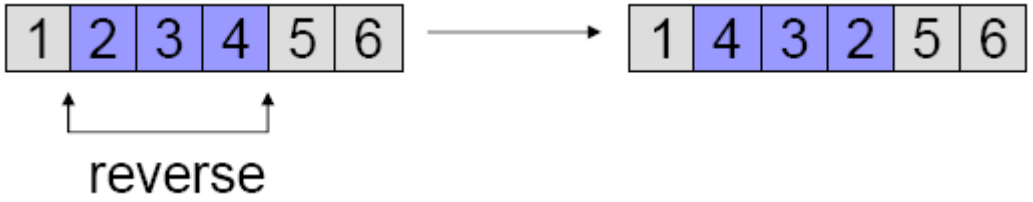
Swap:



Scramble:



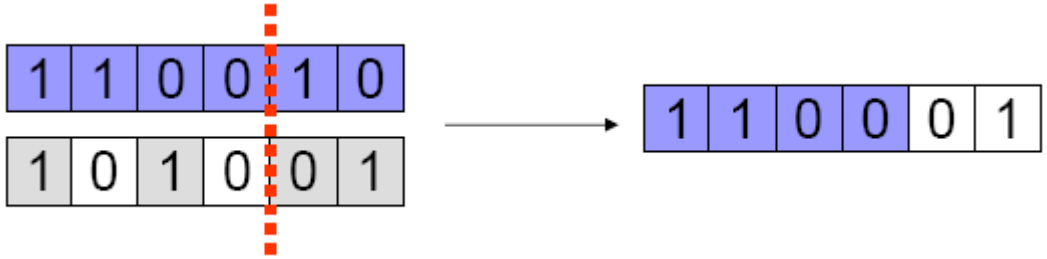
Invert:



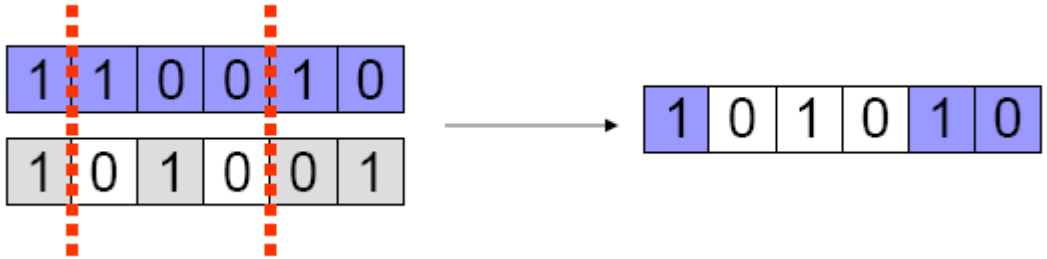
Insert:

Examples of Recombination Operators: $\{0,1\}^n$

1-point crossover



n-point crossover



uniform crossover

choose each bit independently from one parent or another

A Canonical Genetic Algorithm

- binary search space, maximization
- uniform initialization
- generational cycle: of the population
  - evaluation of solutions
  - mating selection (e.g. roulette wheel)
  - crossover (e.g. 1-point)
  - environmental selection (e.g. plus-selection)

First Conclusions of Introductory Part

- EAs are generic algorithms (randomized search heuristics, meta-heuristics, ...) for black box optimization

  no or almost no assumptions on the objective function

- They are typically less efficient than problem-specific (exact) algorithms (in terms of #funevals)

  not the case in the continuous case (we will see later)

- Allow for an easy and rapid implementation and therefore to find good solutions fast

  easy to incorporate (and recommended!) to incorporate problem-specific knowledge to improve the algorithm

Exercise:
Pure Random Search
and the (1+1)EA

`http://researchers.lille.inria.fr/~brockhof/advancedcontrol/`

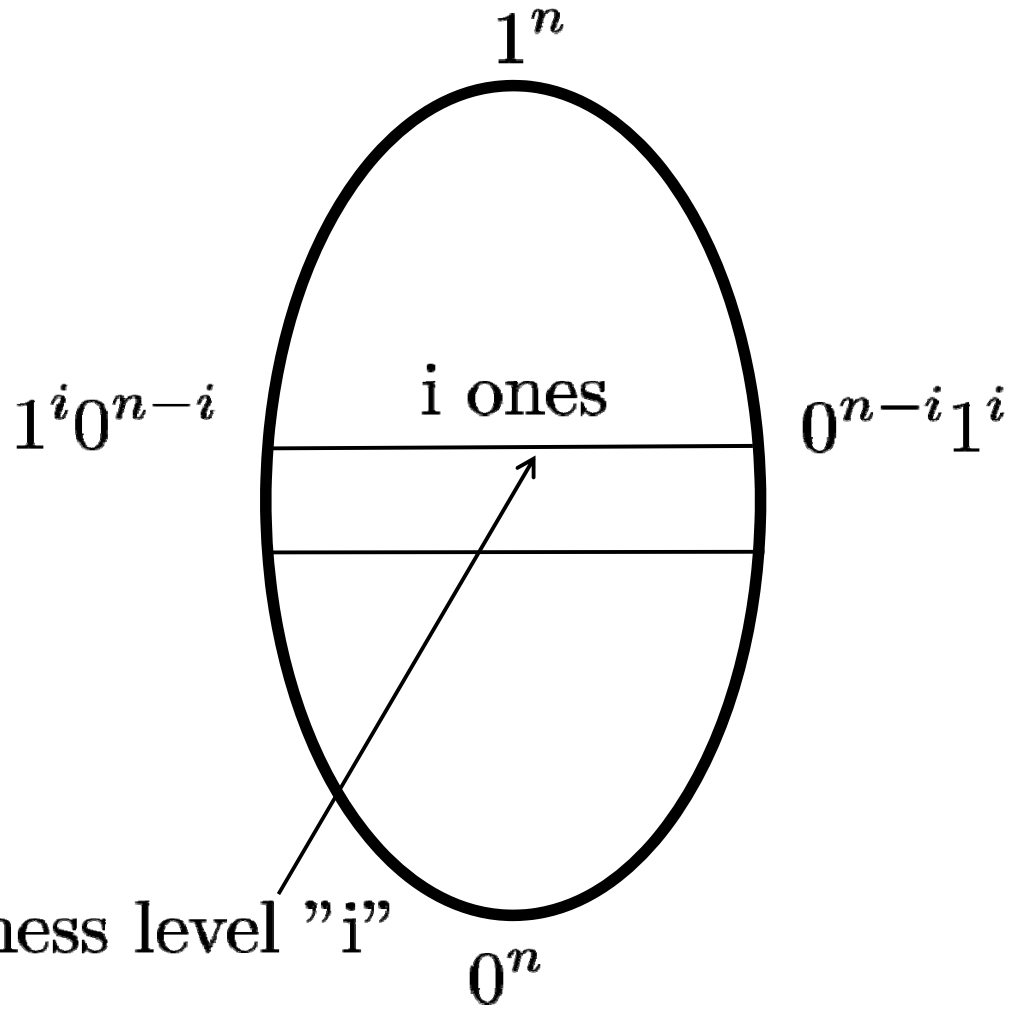
Proof Technique Fitness-based Partitions

$$\Omega = \{0,1\}^n$$

$$f(x) = \sum_{i=1}^{n} x_i$$

$$P_i = \{x | f(x) = i\}$$

$$\Omega = \bigcup_{i=0}^{n} P_i$$



Upper Runtime Bound for (1+1)EA on ONEMAX

$$T = \inf\{t \in \mathbb{N}, X_t = (1, \ldots, 1)\}$$

X_t estimate of solution at iteration t
T_i time to leave fitness level "i"

$$E(T) \leq \sum_{i=0}^{n-1} E(T_i)$$

$\text{Prob}(\text{leave} P_i) \geq \frac{1}{n}(1-\frac{1}{n})^{n-1} \times (n-i)$ (proba to flip one and only one of the $(n-i)$ remaining 0)

$$(1-\frac{1}{n})^{n-1} \geq \frac{1}{e}$$

$$\text{Prob}(\text{leave} P_i) \geq \frac{n-i}{en}$$

Upper Runtime Bound for (1+1)EA on ONEMAX

$$E(T_i) \leq \frac{en}{n-i}$$

$$E(T) \leq \sum_{i=0}^{n-1} \frac{en}{n-i} \leq e.n(\log n + 1)$$