Advanced Control

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Course Overview

Date		Topic
Fri, 10.1.2014	DB	Introduction to Control, Examples of Advanced Control
Fri, 17.1.2014	DB	Introduction to Fuzzy Logic
Fri, 24.1.2014	DB	Introduction to Artificial Neural Networks, Bio-inspired Optimization, discrete search spaces
Fri, 31.1.2014	AA	Continuous Optimization I
Fri, 7.2.2014	AA	Continuous Optimization II
break		
Fri, 28.2.2014	AA	The Traveling Salesperson Problem
Fr, 7.3.2014	DB	Controlling a Pole Cart
Fr, 14.3.2014		written exam (paper and computer)

all classes at 8h00-11h15 (incl. a 15min break around 9h30)

next week: exam at **8h00-11h15**

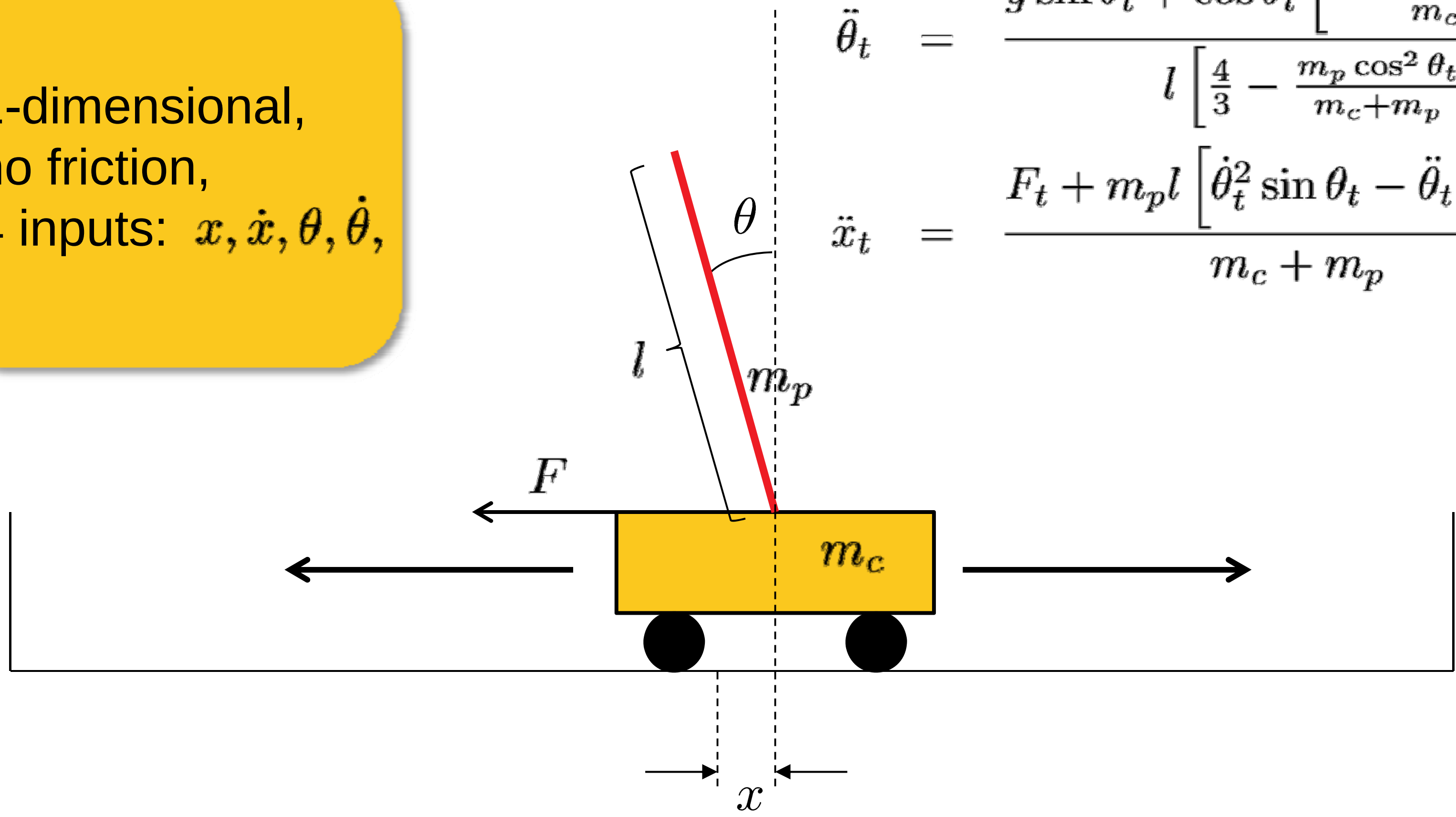
Exercise: Pole Balancing with ANNs and CMA-ES

Reminder: The Pole Balancing Benchmark

Typical benchmark example of a system with “advanced control”:

The Pole Balancing Problem

1-dimensional,
no friction,
4 inputs: $x, \dot{x}, \theta, \dot{\theta}$,



$$\ddot{\theta}_t = \frac{g\sin\theta_t + \cos\theta_t\left[\frac{-F_t - m_p l \dot{\theta}_t^2 \sin\theta_t}{m_c + m_p}\right]}{l\left[\frac{4}{3} - \frac{m_p \cos^2\theta_t}{m_c + m_p}\right]}$$

$$\ddot{x}_t = \frac{F_t + m_p l\left[\dot{\theta}_t^2 \sin\theta_t - \ddot{\theta}_t \cos\theta_t\right]}{m_c + m_p}$$

`http://researchers.lille.inria.fr/~brockhof/advancedcontrol/`

Reminder: Simulated Pole Balancing

Given all the parameters of the system, what do we do with it?

Answer: simulate!

- starting point: certain (random) position and angle; velocities and accelerations are zero
- choose discretization time step (e.g. $\tau = 0.02s$)
- at each time step, do:
  - compute $\ddot{\theta}_t$ with values $\dot{\theta}_t$ and θ_t
  - compute $\ddot{x}_t$ with $\dot{\theta}_t, \theta_t$ and the new $\ddot{\theta}_t$
  - $$\begin{aligned} x_{t+1} &= x_t + \tau \dot{x}_t \\ \dot{x}_{t+1} &= \dot{x}_t + \tau \ddot{x}_t \\ \theta_{t+1} &= \theta_t + \tau \dot{\theta}_t \\ \dot{\theta}_{t+1} &= \dot{\theta}_t + \tau \ddot{\theta}_t \end{aligned}$$

Reminder: Linear Control Law

Remark:

if the values and velocities of both position and angle are measured, there exists a linear (bang-bang) controller of the form:

$$F_t = F_m \, \mathrm{sgn}(k_1 x_t + k_2 \dot{x}_t + k_3 \theta_t + k_4 \dot{\theta}_t)$$

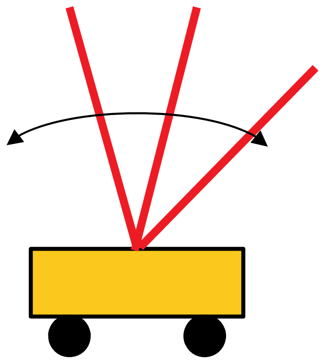
What we have seen:

random choice of k_1, k_2, k_3, k_4 enough to find a good controller most of the time

But

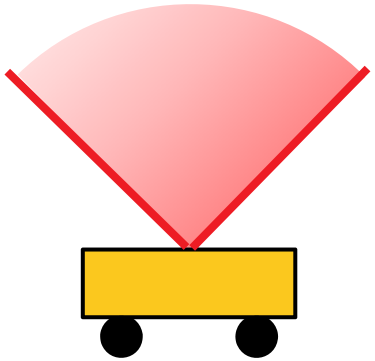
- this holds only for one specific initial condition of x_0 and θ_0
- parameters different for different initial conditions or random sampling of k_1, k_2, k_3, k_4 not enough anymore

Excursion: Robustness and Noise



A controller is robust if it works for different initial conditions - not only for one

→ simulate for different initial conditions



- however, amount of “testable” initial conditions is typically limited
- but one would like to find a controller that works for all initial conditions

→ simulate for different *random* conditions

random initialization introduces noisy measurements in terms of number of stable simulation steps

→ interested in *robust* solutions

More General Issue: Uncertainty

Uncertainty is always an important aspect in practice:

- the objective function is only a model of what we want

  measuring/simulation/modeling errors

- the problem formulation is static while reality is dynamic

  temperature, atmospheric pressure, ... changes

  material wears down

- even if we can detect the optimum, we might not be able to produce it

based on H.G Beyer and B. Sendhoff: “Robust Optimization – A Comprehensive Survey”. In Computer Methods in Applied Mechanics and Engineering, 196(33-34):3190-3218, 2007

Exercise Part I:
Is the linear controller robust?

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http://researchers.lille.inria.fr/~brockhof/advancedcontrol/
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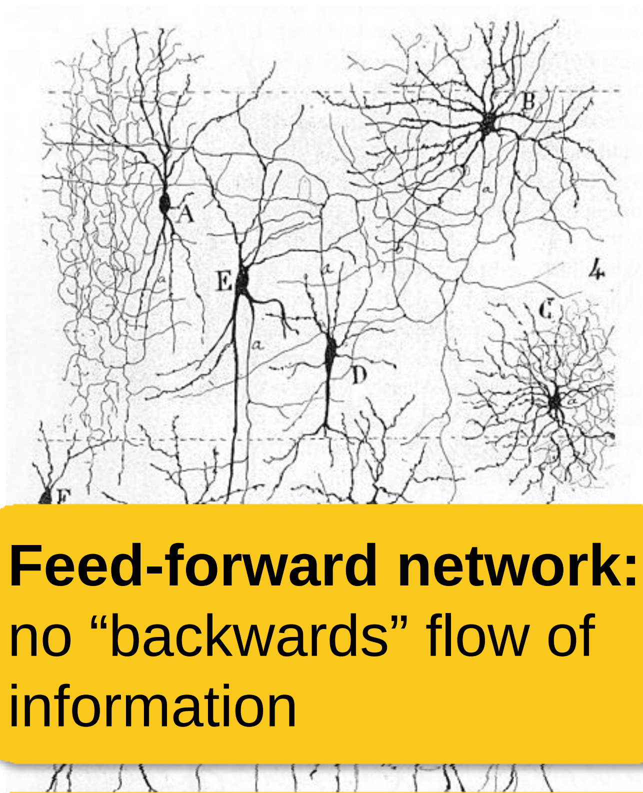
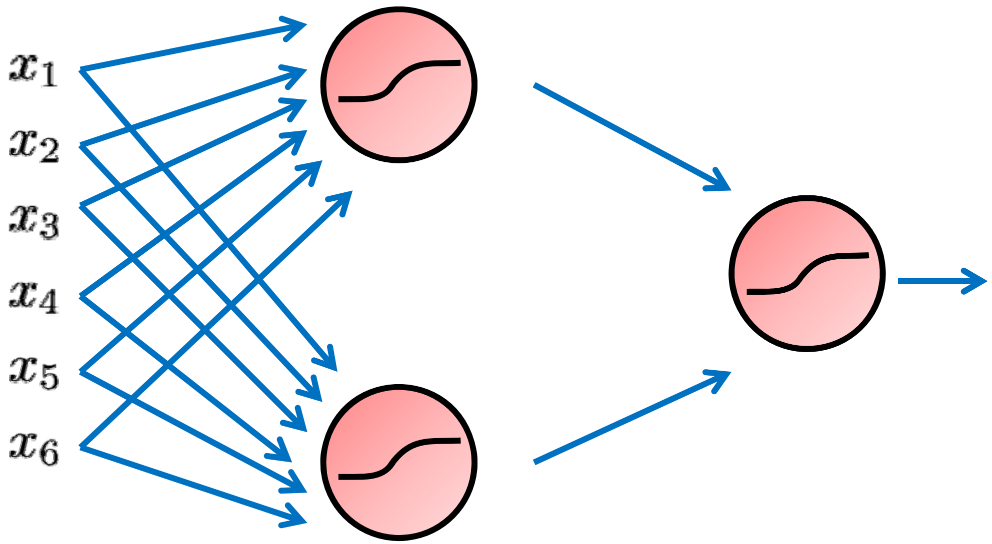
Combining Artificial Neurons

Artificial Neural Networks (ANNs) = a network of artificial neurons





Feed-forward network:
no “backwards” flow of information

Transfer functions:
output of each neuron based on inputs

$$y = \varphi\left(\sum_{i=1}^{n} w_i x_i\right)$$

Exercise Part II: Implementing an Artificial Neural Network

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The Algorithm CMA-ES

Input: $\boldsymbol{m} \in \mathbb{R}^n$, $\sigma \in \mathbb{R}_+$, λ
Initialize: $\mathbf{C} = \mathbf{I}$, and $\boldsymbol{p}_\mathbf{c} = \mathbf{0}$, $\boldsymbol{p}_\sigma = \mathbf{0}$,
Set: $c_\mathbf{c} \approx 4/n$, $c_\sigma \approx 4/n$, $c_1 \approx 2/n^2$, $c_\mu \approx \mu_w/n^2$, $c_1 + c_\mu \le 1$, $d_\sigma \approx 1 + \sqrt{\frac{\mu_w}{n}}$,
and $w_{i=1\ldots\lambda}$ such that $\mu_w = \frac{1}{\sum_{i=1}^{\mu} w_i{}^2} \approx 0.3\,\lambda$
While not terminate

$$\boldsymbol{x}_i = \boldsymbol{m} + \sigma \boldsymbol{y}_i, \quad \boldsymbol{y}_i \sim \mathcal{N}_i(\mathbf{0}, \mathbf{C}), \quad \text{for } i = 1, \ldots, \lambda \qquad \text{sampling}$$

$$\boldsymbol{m} \leftarrow \sum_{i=1}^{\mu} w_i \boldsymbol{x}_{i:\lambda} = \boldsymbol{m} + \sigma \boldsymbol{y}_w \quad \text{where } \boldsymbol{y}_w = \sum_{i=1}^{\mu} w_i \boldsymbol{y}_{i:\lambda} \qquad \text{update mean}$$

$$\boldsymbol{p}_\mathbf{c} \leftarrow (1 - c_\mathbf{c})\boldsymbol{p}_\mathbf{c} + \mathbb{1}_{\{\|\boldsymbol{p}_\sigma\| < 1.5\sqrt{n}\}} \sqrt{1 - (1 - c_\mathbf{c})^2} \sqrt{\mu_w}\, \boldsymbol{y}_w \qquad \text{cumulation for } \mathbf{C}$$

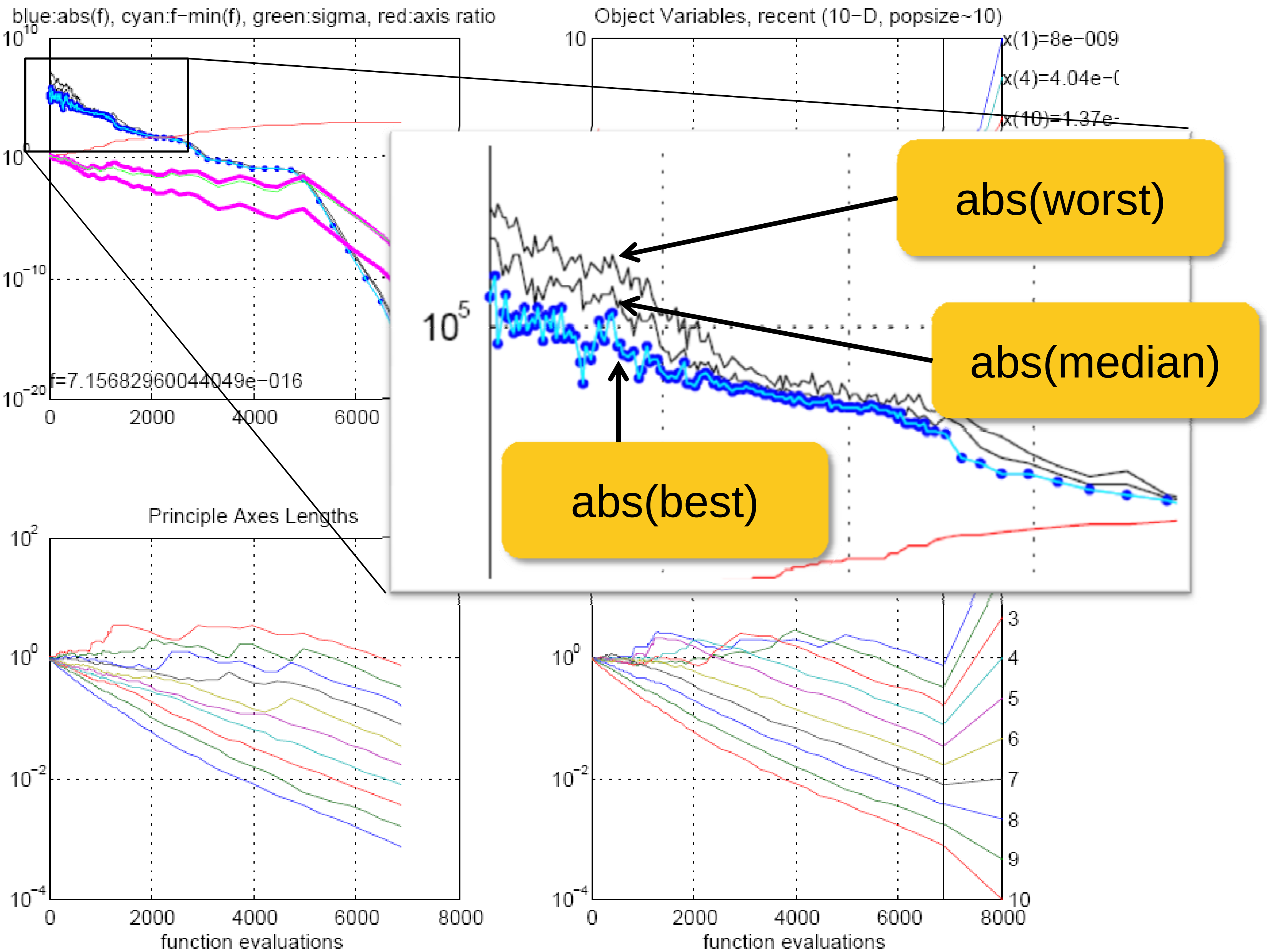
$$\boldsymbol{p}_\sigma \leftarrow (1 - c_\sigma)\boldsymbol{p}_\sigma + \sqrt{1 - (1 - c_\sigma)^2} \sqrt{\mu_w}\, \mathbf{C}^{-\frac{1}{2}} \boldsymbol{y}_w \qquad \text{cumulation for } \sigma$$

$$\mathbf{C} \leftarrow (1 - c_1 - c_\mu)\,\mathbf{C} + c_1 \boldsymbol{p}_\mathbf{c} \boldsymbol{p}_\mathbf{c}^\mathrm{T} + c_\mu \sum_{i=1}^{\mu} w_i \boldsymbol{y}_{i:\lambda} \boldsymbol{y}_{i:\lambda}^\mathrm{T} \qquad \text{update } \mathbf{C}$$

$$\sigma \leftarrow \sigma \times \exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\|\boldsymbol{p}_\sigma\|}{\mathsf{E}\|\mathcal{N}(\mathbf{0},\mathbf{I})\|} - 1\right)\right) \qquad \text{update of } \sigma$$

Not covered on this slide: termination, restarts, useful output, boundaries and encoding

The output of CMA-ES



Issues on the Representation

Observation

- The weights of ANNs are typically normalized and lie within $[0, 1]$
- But CMA-ES does not restrict the variables in the standard setting

Hence, we have to set the bound constraints correctly:

```
opts.LBounds = 0;
opts.UBounds = 1;
```

Exercise Part III:
Using CMA-ES to Optimize the Weights of our ANN controller

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