Home Exercise 7: Turing Machines

Algorithms and Complexity lecture
at CentraleSupelec/ESSEC

Dimo Brockhoff
firstname.lastname@inria.fr

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Abstract

Please send your solutions by email to Dimo Brockhoff (preferably in PDF format) with a clear indication of your full name until the submission deadline on November 15, 2019 (a Friday). Groups of up to 4 students are explicitly allowed and even encouraged. In the case of group submissions, please make sure that you submit maximally four times with the same partner!

1 What is it doing? (10 points)

Consider the following Turing machine $\mathcal{M}$. The states are $Q = \{q_0, \ldots, q_4\}$ without an explicit accepting state ($F = \emptyset$) and where q_0 is the initial state. The input alphabet is $\Sigma = \{0, 1\}$ and the band alphabet is $\Gamma = \{0, 1, B\}$ with B the blank character. The transition function δ is given by the following table.

	0	1	B
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	(q_1, B, L)
q_1	(q_2, B, R)	(q_3, B, R)	STOP
q_2	$(q_4, 0, L)$	$(q_4, 0, L)$	$(q_4, 0, L)$
q_3	$(q_4, 1, L)$	$(q_4, 1, L)$	$(q_4, 1, L)$
q_4	$(q_4, 1, R)$	$(q_4, 0, R)$	(q_1, B, L)

The stop sign () indicates that the Turing machine stops.

Assume that the band of $\mathcal{M}$ initially contains a bitstring $w \in \Sigma^n$ of length n starting at position 1, the initial position of the reading/writing head. How did the content of the band change after $\mathcal{M}$ stopped?

2 Writing Ones Game (10 points)

Let us consider Turing machines that always start on an empty band and that are allowed to only write 1s, i.e., the band alphabet is $\Gamma = \{1, B\}$. We play the following "game": design a Turing machine with k states $q_0, \ldots, q_{k-1}$ that, when run on the empty band, halts after a finite number of steps and, during its operation, writes the largest possible block of contiguous 1s to the band.

Give the transition function δ of a Turing machine with $k = 2$ states for the above "writing ones game" with the maximal number of 1s written (hint: the best possible Turing machine with $k = 2$ states writes a block of four 1s. To this end, fill out the following transition table:

	1	B
q_0		
q_1		