Home Exercise 1: O-Notation, Data Structures

Algorithms and Complexity lecture
at CentraleSupélec/ESSEC

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Abstract

Please send your solutions by email to Dimo Brockhoff in PDF format (with a clear indication of your full name(s) in the email and the PDF file name, see below) until the submission deadline on Monday, October 4, 2021 at 11:59pm, Paris time. Groups of 5 students are explicitly allowed and encouraged. In the case of group submissions, please make sure that you submit only once with the same student!
Important: Please name your PDF file according to your last names (sorted in alphabetical order and separated by an underscore), for example like `Monet_Renoir_Toulouse-Lautrec.pdf`.

1 Tennis Event (5 points)

Propose a game plan with as little games as possible for a tennis tournament in which the first and second place are given fairly to the actual best and second best players. We assume that in each game, the better player always wins. How many games are needed when there are n players in the tournament? What changes for the numbers of matches needed (and why) if we are only interested in finding out the second-best player?

2 O notation (5 points)

Which of the following "equations" hold? Please give either a proof or a counter example. Assume that $n \in \mathbb{N}_{>0}$ is always a positive integer. For the two last "equations", $O(f_1) \circ O(f_2)$ denotes the set $\{g_1 \circ g_2 \mid g_1 \in O(f_1), g_2 \in O(f_2)\}$ for a given operator $\circ \in \{+, -\}$ and functions $f_1, f_2 : \mathbb{N}_{>0} \to \mathbb{R}$.

1. $100n + 1000 = O(n)$
2. $\log(n) = O(1)$
3. $n^2 + n^3 = \Omega(n^3)$
4. $O(f_1) + O(f_2) = O(f_1 + f_2)$
5. $O(f_1) - O(f_2) = O(f_1 - f_2)$

3 Binary Search Tree (5 points)

Insert the following integer numbers in the given order into an empty binary search tree: 6, 9, 2, 10, 1, 3, 7, 5, 4. Afterwards, the numbers 10, 3, and 6 should be deleted (in this order). Draw for each of the 12 steps the resulting binary search tree.

4 Inserting Into A Hash Table (5 points)

We consider a hash table with space for 17 data sets and the corresponding hash function $h(x) = x \mod 17$. Insert the following (key, value) pairs: (388, "first"), (68, "second"), (296, "third"), (63, "fourth"), (160, "fifth"), (264, "sixth"), (8, "seventh"). In case of a collision, consider the next empty table cell (modulo 17).

Explain what happens and draw the content of the hash table after each insert.