



GECCO 2026 Tutorial

More Than Tables: Visualizing Anytime Performance in Single- and Multiobjective Optimization

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The final slides are available at

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Overview

1. Presentation (~60 min)

- Motivation
- Quality indicators
- Runtime profiles
- Benchmarking tools
- cocoviz

1. Hands-on session (~30 min)

1. Q & A

Details Hands-On-Session

you are encouraged to follow along **with your own data...**

function evaluations	value of interest (a.k.a. quality indicator)
1	56.38765
2	43.06690
10	22.56282
1498	21.57811
...	...

Benchmarking Motivation

Benchmarking is ubiquitous in optimization:

- to understand algorithms
- to validate new designs
- to recommend good solvers

not specific to any domain (e.g. multiobjective)

but some details might be domain-specific

here in particular: blackbox optimization

What IS Benchmarking (in Our View)?

- comparing solvers experimentally
- running them on a set of function (instances)
- reporting the results
- analyzing/interpreting the results

What ISN'T Benchmarking (in Our View)?

- competition

Where Do We Start?

Running a solver on a single function instance

“time”, e.g. function evaluations	value of interest, e.g. f-value
1	56.38765
2	43.06690
3	22.56282
4	21.57811
...	...

Where Do We Start?

Running a solver on a single function instance

function evaluations	f
1	56.38765
2	43.06690
3	22.56282
4	21.57811
...	...

Where Do We Start?

Running a solver on a single function instance

function evaluations	f_1	f_2	f_3
1	56.38765	4.8724	5.982
2	43.06690	5.1222	3.228
3	22.56282	6.2645	5.269
4	21.57811	5.8465	7.364
...	...	...	...

Where Do We Start?

Running a solver on a single function instance

function evaluations	f_1	f_2	f_3	c_1	c_2
1	56.38765	4.8724	5.982	0.4627	11.856
2	43.06690	5.1222	3.228	2.4722	98.2837
3	22.56282	6.2645	5.269	-0.8472	2.2286
4	21.57811	5.8465	7.364	-0.2645	-736.11
...	...	...	...	...	...

Where Do We Start?

Running a solver on a single function instance

function evaluations	f_1	f_2	f_3	c_1	c_2	quality indicator	I
1	56.38765	4.8724	5.982	0.4627	11.856		1.20
2	43.06690	5.1222	3.228	2.4722	98.2837		1.10
3	22.56282	6.2645	5.269	-0.8472	2.2286		1.05
4	21.57811	5.8465	7.364	-0.2645	-736.11		0.98
...	...	...	...	...	...		...

Tables, Tables, Tables

Problem	MOCSA				NSGA2			
	$\langle d \rangle$	S	GD	ER	$\langle d \rangle$	S	GD	ER
ZDT1	0.0404	0.0055	0.0000	0	0.0270	0.0156	0.0011	0.04
ZDT2	0.0404	0.0082	0.0000	0	0.0292	0.0146	0.0212	0.02
ZDT3	0.0438	0.0148	0.0001	0	0.0329	0.0201	0.0020	0.02
ZDT4	0.0404	0.0097	0.0000	0	0.0328	0.0159	0.0006	0.02
ZDT6	0.0327	0.0150	0.0000	0	0.0216	0.0119	0.0000	0
DTLZ1	0.1114	0.0068	0.0000	0	0.0615	0.0319	0.0000	0
DTLZ2	0.2319	0.0646	0.0021	0.02	0.1361	0.0683	0.0020	0.04
DTLZ3	0.2770	0.0225	0.0000	0	0.1139	0.0739	0.0000	0
DTLZ4	0.2478	0.0424	0.0009	0	0.1630	0.0898	0.0019	0.02
DTLZ5	0.0487	0.0059	0.0000	0	0.0309	0.0176	0.0610	0.06
DTLZ6	0.0484	0.0156	0.0000	0	0.0306	0.0135	0.0000	0
DTLZ7	0.2897	0.0510	0.0011	0.04	0.1880	0.1322	0.0071	0.22

arXiv, 2012

Tables, Tables, Tables

Table 4: Influence of different κ values on the performance of the cone ϵ -MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for κ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		κ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
	SD	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	0.0001	0.0001
Δ	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴
H	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	< 10 ⁻⁴	< 10 ⁻⁴	0.2537	0.3457	0.4842	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	0.1826	0.1826	0.1826

Metric		κ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
Δ	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
H	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		κ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
Δ	M	0.0503	0.6066	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	0.2173	0.1982
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	0.0295	0.0239
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	0.7470	0.7471
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0003
H	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	95.00	94.00
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	2.2614	1.7100

Metric		κ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	0.0489	0.0534
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0389	0.0281	0.0284	0.0241	0.0304
Δ	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	0.3304	0.3319
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	0.0254	0.0259
H	M	14.00	79.50	90.00	92.00	95.00	96.00	95.50	97.00	98.00	95.50	97.00
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	4.6233	4.3423

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ -MOEA	cone ϵ -MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	γ	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	Δ	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	γ	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	Δ	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	γ	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	Δ	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	γ	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	Δ	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	0.04±2e-3	0.13±3e-3	0.01±6e-4	0.08±3e-3	0.07±3e-3
Zdt3	γ	0.16±2e-2	0.15±1e-2	0.17±10e-3	0.19±1e-2	0.35±3e-2	0.24±2e-2
	Δ	0.67±1e-2	0.85±1e-2	0.57±2e-2	0.49±1e-2	0.33±7e-3	0.50±2e-2
	HV	0.98±2e-3	0.97±3e-3	0.97±2e-3	0.97±2e-3	0.95±4e-3	0.97±3e-3
	CS	0.41±3e-2	0.36±3e-2	0.33±2e-2	0.29±2e-2	0.12±2e-2	0.24±3e-2
Zdt4	γ	0.27±2e-2	0.32±2e-2	0.35±2e-2	0.47±2e-2	0.69±2e-2	0.57±2e-2
	Δ	0.61±9e-3	0.59±1e-2	0.48±1e-2	0.58±1e-2	0.53±1e-2	0.68±1e-2
	HV	0.84±9e-3	0.80±1e-2	0.79±1e-2	0.73±1e-2	0.62±1e-2	0.66±2e-2
	CS	0.73±3e-2	0.58±4e-2	0.57±3e-2	0.41±4e-2	0.15±2e-2	0.21±4e-2
Zdt6	γ	0.06±3e-2	0.04±1e-2	0.04±2e-2	0.04±1e-2	0.01±3e-3	0.03±2e-2
	Δ	0.52±1e-2	0.24±1e-2	0.27±1e-2	0.37±9e-3	0.18±8e-3	0.37±2e-2
	HV	0.96±1e-2	0.98±7e-3	0.98±10e-3	0.97±7e-3	0.99±1e-3	0.98±10e-3
	CS	0.18±2e-2	0.11±2e-2	0.18±2e-2	0.10±1e-2	0.21±2e-2	0.21±2e-2

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ -MOEA	cone- ϵ -MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	γ	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	Δ	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	γ	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	Δ	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.90±7e-4	0.93±4e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	γ	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	Δ	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	γ	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	Δ	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.02±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	γ	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	Δ	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
Dtlz6	γ	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	Δ	0.78±1e-2	0.99±6e-4	0.45±6e-4	0.51±7e-3	0.32±5e-3	0.62±1e-2
	HV	0.99±1e-4	0.99±4e-5	0.99±2e-5	0.99±9e-5	0.99±3e-5	0.99±1e-4
	CS	0.02±6e-4	0.03±8e-4	0.04±8e-4	0.03±8e-4	0.03±8e-4	0.01±7e-4
Dtlz7	γ	0.74±9e-3	0.47±2e-3	0.74±10e-3	0.86±9e-3	0.67±8e-3	0.74±10e-3
	Δ	0.78±1e-2	0.55±9e-3	0.71±9e-3	0.56±1e-2	0.52±8e-3	0.78±8e-3
	HV	0.92±1e-3	0.91±1e-3	0.93±7e-4	0.91±1e-3	0.94±6e-4	0.92±9e-4
	CS	0.06±2e-3	0.08±2e-3	0.06±2e-3	0.02±1e-3	0.09±3e-3	0.07±2e-3
Dtlz8	γ	0.45±2e-2	0.03±10e-4	0.24±5e-3	0.52±1e-2	0.83±9e-3	0.58±1e-2
	Δ	0.82±9e-3	0.69±9e-3	0.70±9e-3	0.49±8e-3	0.36±8e-3	0.80±1e-2
	CS	0.11±4e-3	0.20±3e-3	0.24±4e-3	0.10±3e-3	0.10±3e-3	0.09±3e-3
Dtlz9	γ	0.90±5e-3	0.16±7e-3	0.57±5e-3	0.64±7e-3	0.77±7e-3	0.90±5e-3
	Δ	0.62±5e-3	0.71±1e-2	0.52±6e-3	0.49±6e-3	0.45±4e-3	0.61±6e-3
	CS	0.11±8e-3	0.51±1e-2	0.34±7e-3	0.23±8e-3	0.24±8e-3	0.13±7e-3

Tables, Tables, Tables

Table 4: Influence of different κ values on the performance of the cone ϵ -MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for κ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		κ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
	SD	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴
Δ	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴
H	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	< 10 ⁻⁴	< 10 ⁻⁴	0.2537	0.3457	0.4842	< 10 ⁻⁴	< 10 ⁻⁴	< 10 ⁻⁴	0.1826	0.1826	0.1826

Metric		κ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
Δ	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
H	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		κ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
Δ	M	0.0503	0.6068	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	0.2177	0.2177
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	0.0306	0.0306
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	0.7470	0.7470
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005
H	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	95.00	95.00
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	2.0197	2.0197

Metric		κ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	0.0590	0.0590
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0389	0.0281	0.0284	0.0284	0.0284
Δ	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	0.3253	0.3253
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	0.0210	0.0210
H	M	14.00	79.50	90.00	92.00	95.00	96.00	95.00	97.00	98.00	98.00	98.00
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	4.5945	4.5945

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ -MOEA	cone ϵ -MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	γ	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	Δ	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	γ	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	Δ	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	γ	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	Δ	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	γ	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	Δ	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	0.04±2e-3	0.13±3e-3	0.01±6e-4	0.08±3e-3	0.07±3e-3
Zdt3	γ	0.16±2e-2	0.15±1e-2	0.17±10e-3	0.19±1e-2	0.35±3e-2	0.24±2e-2
	Δ	0.67±1e-2	0.85±1e-2	0.57±2e-2	0.49±1e-2	0.33±7e-3	0.50±2e-2
	HV	0.98±2e-3	0.97±3e-3	0.97±2e-3	0.97±2e-3	0.95±4e-3	0.97±3e-3
	CS	0.41±3e-2	0.36±3e-2	0.33±2e-2	0.29±2e-2	0.12±2e-2	0.24±3e-2
Zdt4	γ	0.27±2e-2	0.32±2e-2	0.35±2e-2	0.47±2e-2	0.69±2e-2	0.57±2e-2
	Δ	0.61±9e-3	0.59±1e-2	0.48±1e-2	0.58±1e-2	0.53±1e-2	0.68±1e-2
	HV	0.84±9e-3	0.80±1e-2	0.79±1e-2	0.73±1e-2	0.62±1e-2	0.66±2e-2
	CS	0.73±3e-2	0.58±4e-2	0.57±3e-2	0.41±4e-2	0.15±2e-2	0.21±4e-2
Zdt6	γ	0.06±3e-2	0.04±1e-2	0.04±2e-2	0.04±1e-2	0.01±3e-3	0.03±2e-2
	Δ	0.52±1e-2	0.24±1e-2	0.27±1e-2	0.37±9e-3	0.18±8e-3	0.37±2e-2
	HV	0.96±1e-2	0.98±7e-3	0.98±10e-3	0.97±7e-3	0.99±1e-3	0.98±10e-3
	CS	0.18±2e-2	0.11±2e-2	0.18±2e-2	0.10±1e-2	0.21±2e-2	0.21±2e-2

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ -MOEA	cone- ϵ -MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	γ	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	Δ	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	γ	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	Δ	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.90±7e-4	0.93±4e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	γ	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	Δ	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	γ	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	Δ	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.02±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	γ	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	Δ	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
Dtlz6	γ	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	Δ	0.78±1e-2	0.99±6e-4	0.45±6e-4	0.51±7e-3	0.32±5e-3	0.62±1e-2
	HV	0.99±1e-4	0.99±4e-5	0.99±2e-5	0.99±9e-5	0.99±3e-5	0.99±1e-4
	CS	0.02±6e-4	0.03±8e-4	0.04±8e-4	0.03±8e-4	0.03±8e-4	0.01±7e-4
Dtlz7	γ	0.74±9e-3	0.47±2e-3	0.74±10e-3	0.86±9e-3	0.67±8e-3	0.74±10e-3
	Δ	0.78±1e-2	0.55±9e-3	0.71±9e-3	0.56±1e-2	0.52±8e-3	0.78±8e-3
	HV	0.92±1e-3	0.91±1e-3	0.93±7e-4	0.91±1e-3	0.94±6e-4	0.92±9e-4
	CS	0.06±2e-3	0.08±2e-3	0.06±2e-3	0.02±1e-3	0.09±3e-3	0.07±2e-3
Dtlz8	γ	0.45±2e-2	0.03±10e-4	0.24±5e-3	0.52±1e-2	0.83±9e-3	0.58±1e-2
	Δ	0.82±9e-3	0.69±9e-3	0.70±9e-3	0.49±8e-3	0.36±8e-3	0.80±1e-2
	HV	0.98±9e-5	0.99±9e-5	0.99±9e-5	0.99±9e-5	0.99±9e-5	0.99±9e-5
	CS	0.03±1e-3	0.10±7e-3	0.24±4e-3	0.10±3e-3	0.10±3e-3	0.09±3e-3
Dtlz9	γ	0.33±9e-3	0.16±7e-3	0.57±5e-3	0.64±7e-3	0.77±7e-3	0.90±5e-3
	Δ	0.78±1e-2	0.71±1e-2	0.52±6e-3	0.49±6e-3	0.45±4e-3	0.61±6e-3
	HV	0.98±9e-5	0.99±9e-5	0.99±9e-5	0.99±9e-5	0.99±9e-5	0.99±9e-5
	CS	0.03±1e-3	0.51±1e-2	0.34±7e-3	0.23±8e-3	0.24±8e-3	0.13±7e-3

Tables vs. Graphs

Tables

- show precise numerical data
- transfer raw data

Tables vs. Graphs

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- show precise numerical data
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Often better: Graphs

- simplify large datasets
- easier interpretation
- show how values change dynamically
- visualize trends over time
- compare relationships between variables

Tables vs. Graphs

Tables

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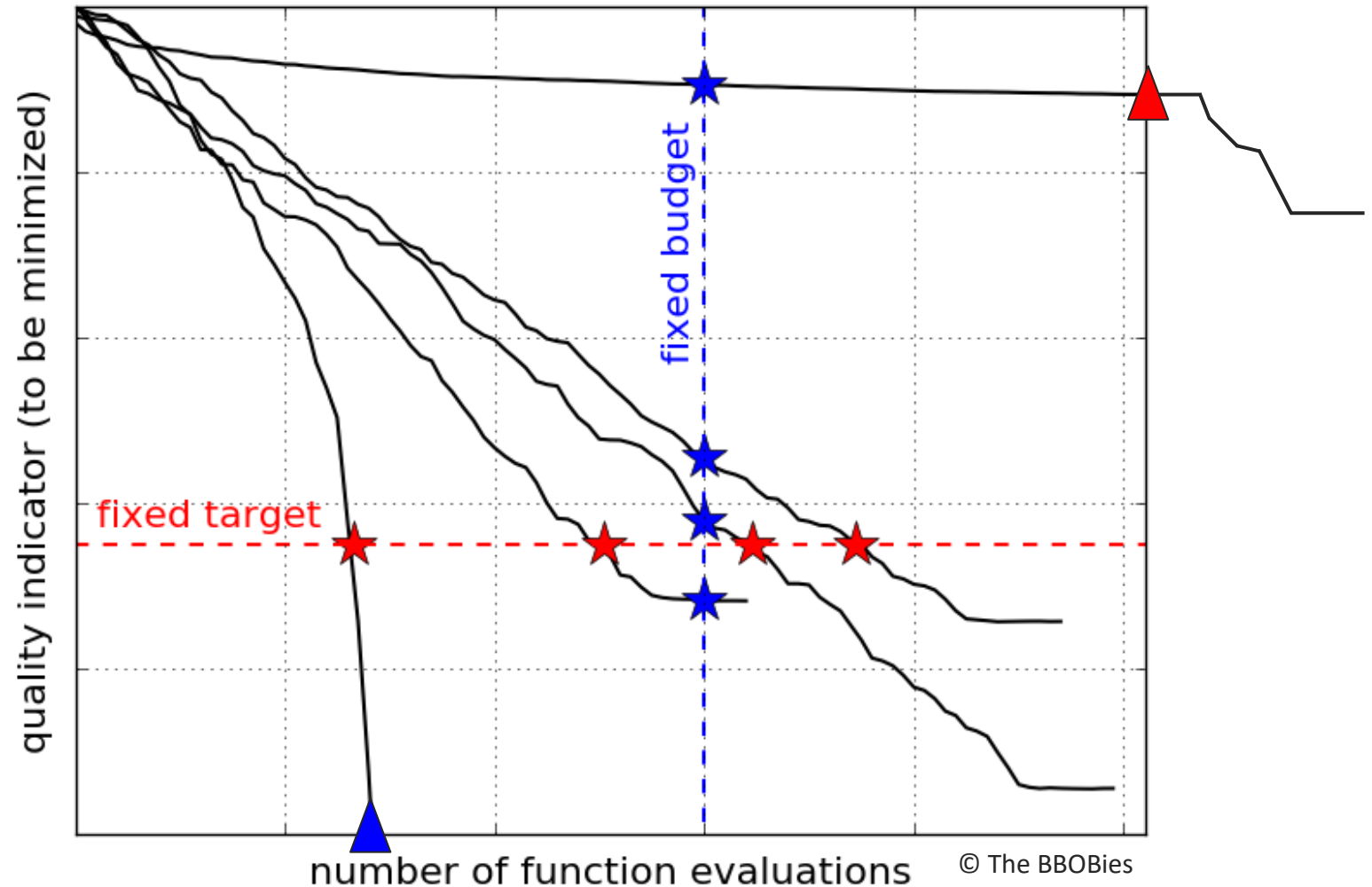
Often better: Graphs

- simplify large datasets
- easier interpretation
- show how values change dynamically
- visualize trends over time
- compare relationships between variables

Visual information is simply faster processed by our brains...
if done right :-)

Displaying Performance Visually

Convergence graphs
is all we have to start
with...



Indicators

simplest case of unconstrained, single-objective optimization:
quality = f-value (to be minimized)

Indicators

simplest case of unconstrained, single-objective optimization:

quality = f-value (to be minimized)

but already less clear in the case of constrained optimization:

quality could be, for example, $\max(f_{\text{opt}}, f(x)) + \sum_{k=0}^m \max(0, g_k(x))$

Multiobjective: Which Indicator?

Performance indicators in multiobjective optimization

Charles Audet^a, Jean Bignon^b, Dominique Cartier^c, Sébastien Le Digabel^a,
Ludovic Salomon^{a,1}

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C.P. 6079, Succ. Centre-ville, Montréal, Québec, H3C 3A7, Canada.

^bUniv. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France.

^cCollège de Maisonneuve, 3800 Rue Sherbrooke E, Montréal, Québec, H1X 2A2, Canada.

[Audet et al. 2021]

63 indicators

Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

Miqing Li, and Xin Yao¹

¹CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.

*Email: limitsing@gmail.com, x.yao@cs.bham.ac.uk

[Li and Yao 2019]

100 indicators

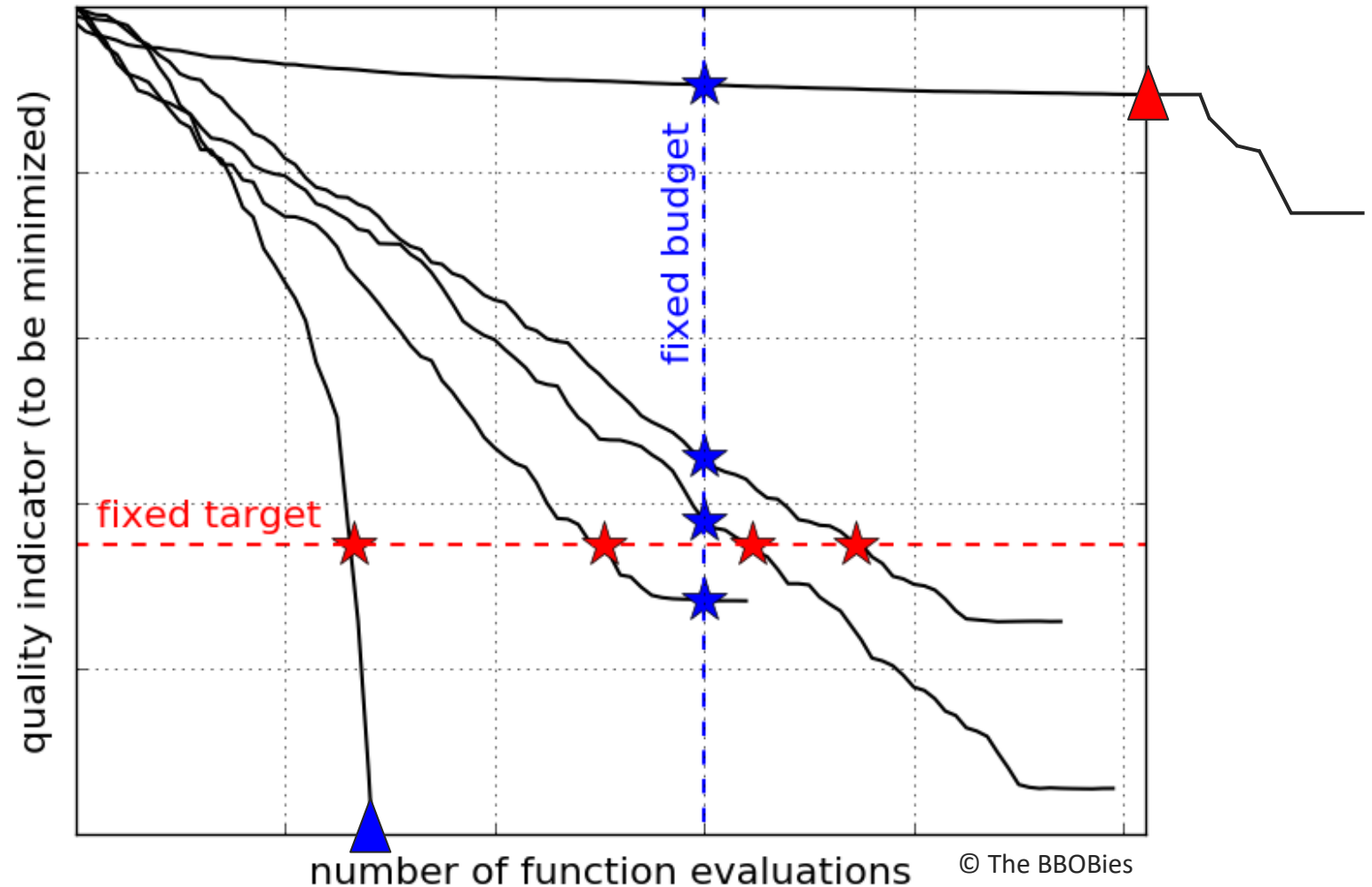
Multiobjective: Which Indicator?

Focus on indicators which are (strictly) monotone

- All hypervolume-based indicators [Zitzler et al. 2007]
- Unary epsilon indicator [Zitzler et al. 2003]
- R2 [Hansen and Jaszewicz 1998]
- IGD+ [Ishibuchi et al. 2015]

Measuring Performance Empirically

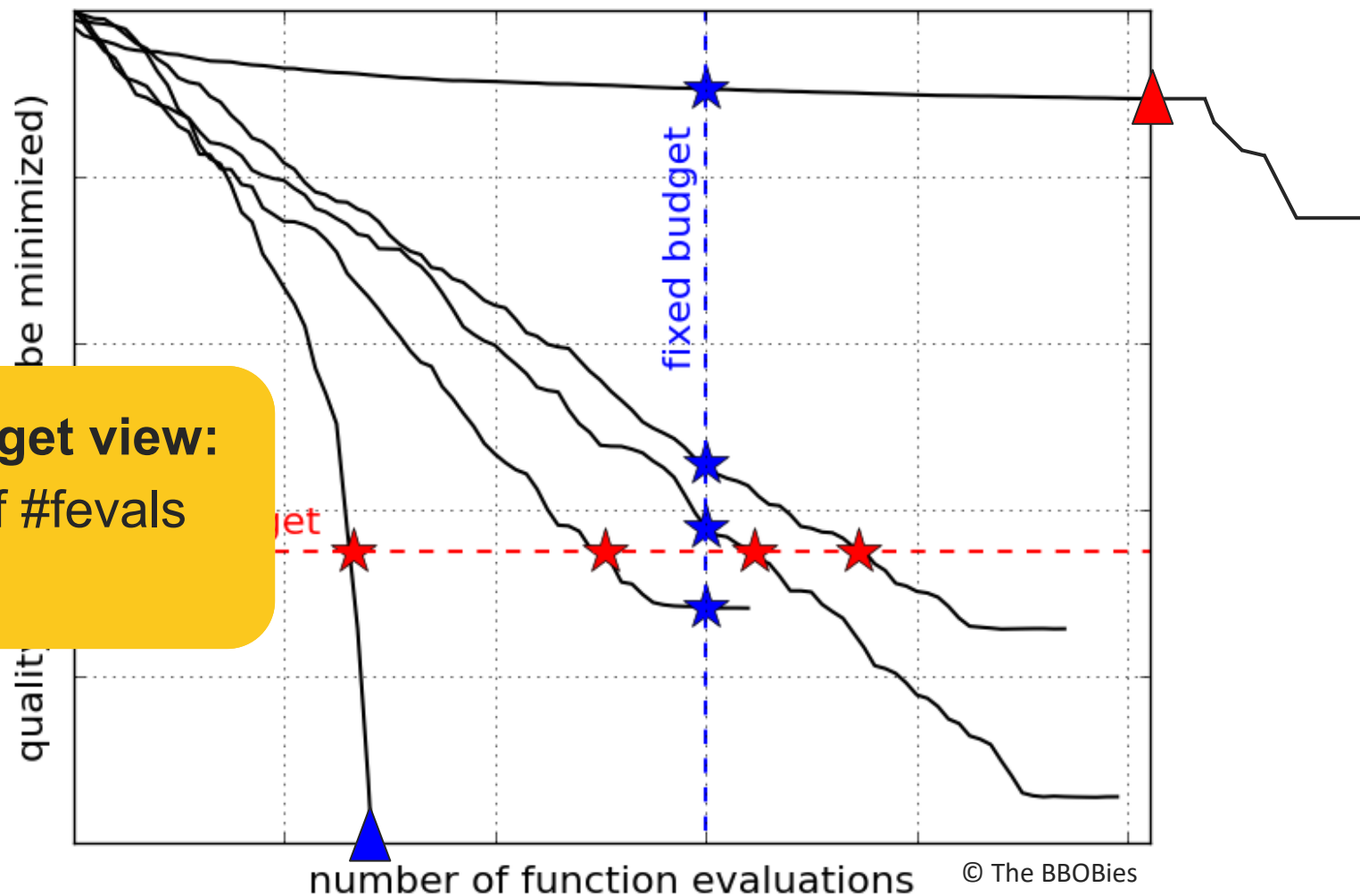
Convergence graphs
is all we have to start
with...



Measuring Performance Empirically

Convergence graphs
is all we have to start
with...

Advantage of the fixed target view:
Ratio scale (interpretation of #fevals
easier than for f-values)



ECDF

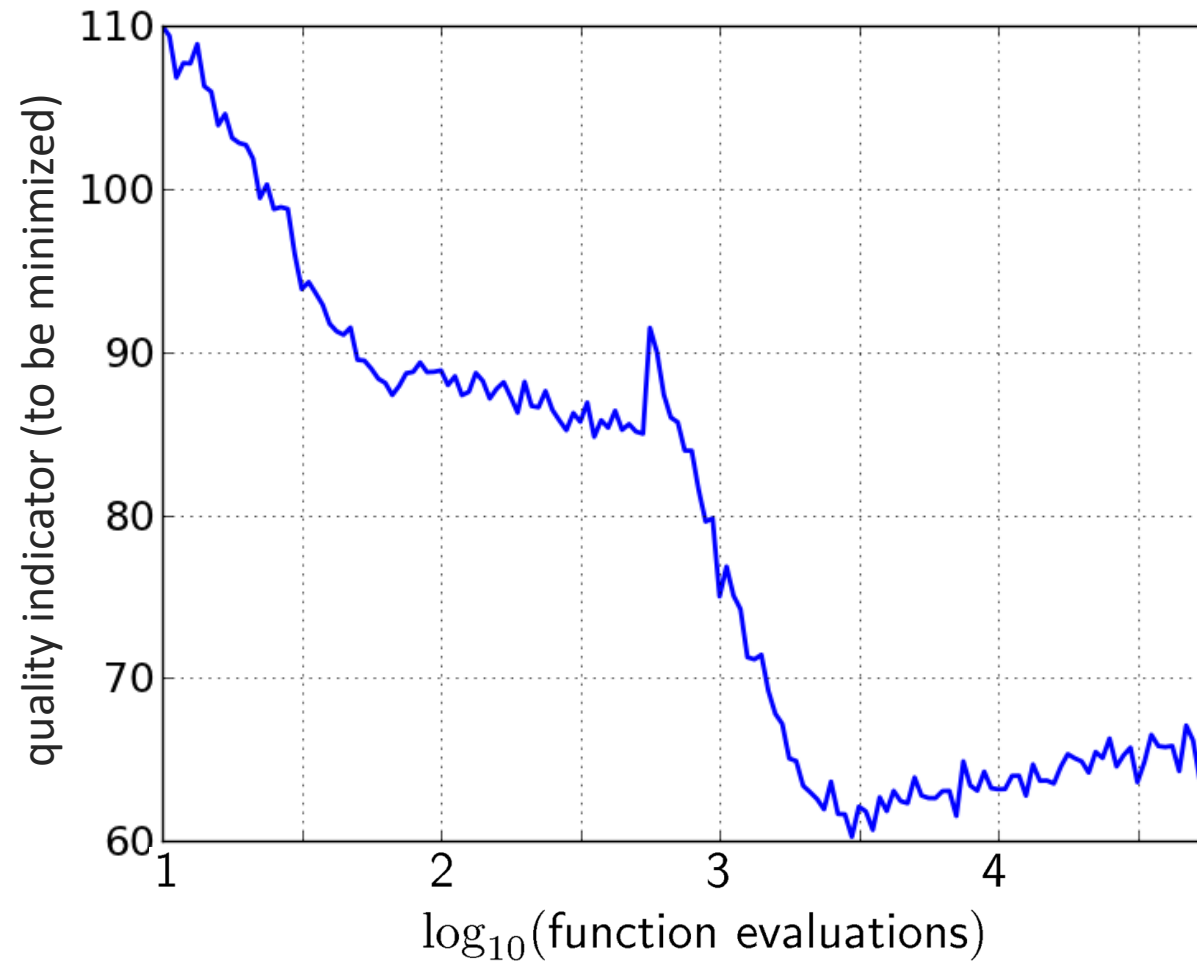
Empirical Cumulative Distribution Function of the Runtime

ECDF

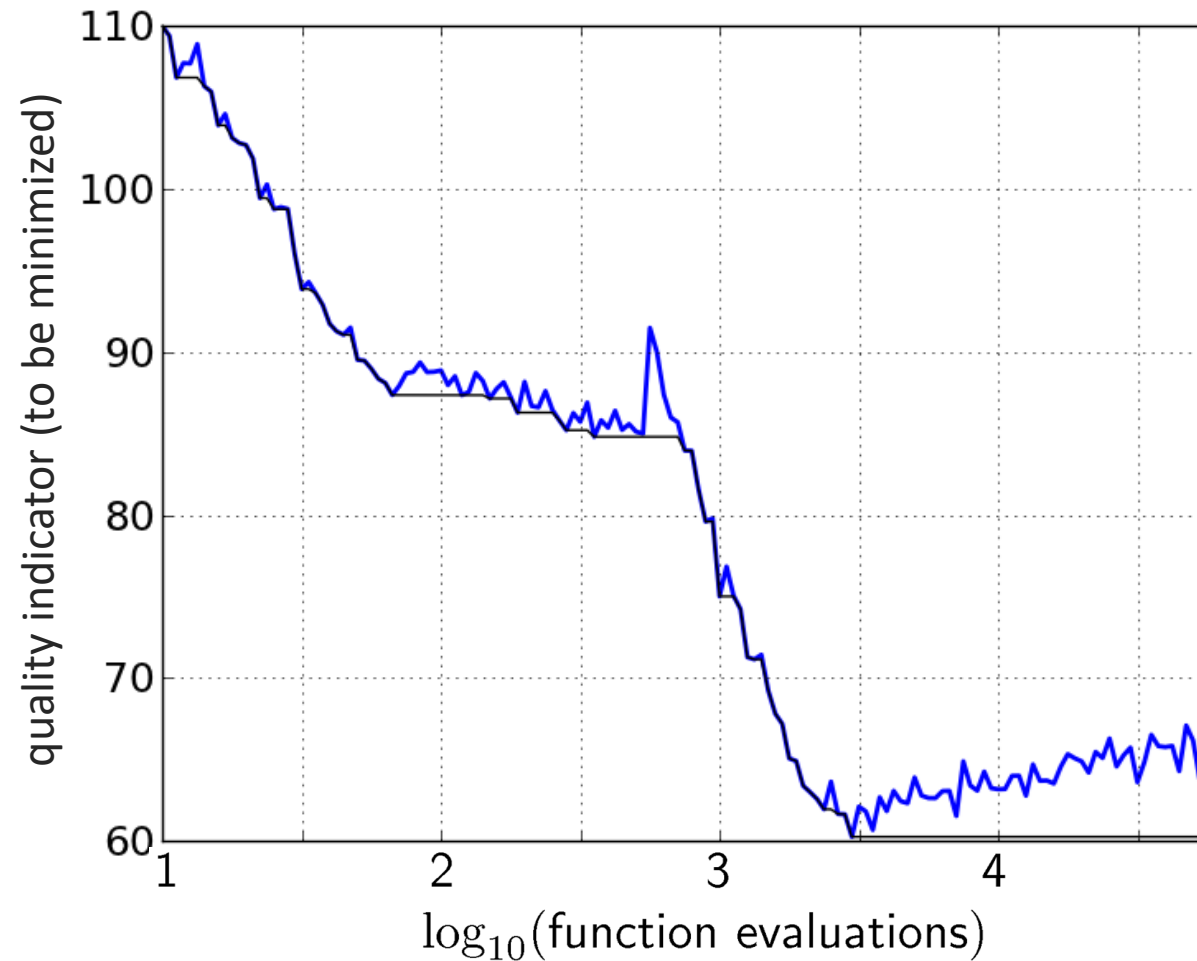
Empirical Cumulative Distribution Function of the Runtime
a.k.a. **runtime profile** a.k.a. runtime performance profile plot
a.k.a. data profile

[Hansen et al. 2021] [Moré and Wild 2009]

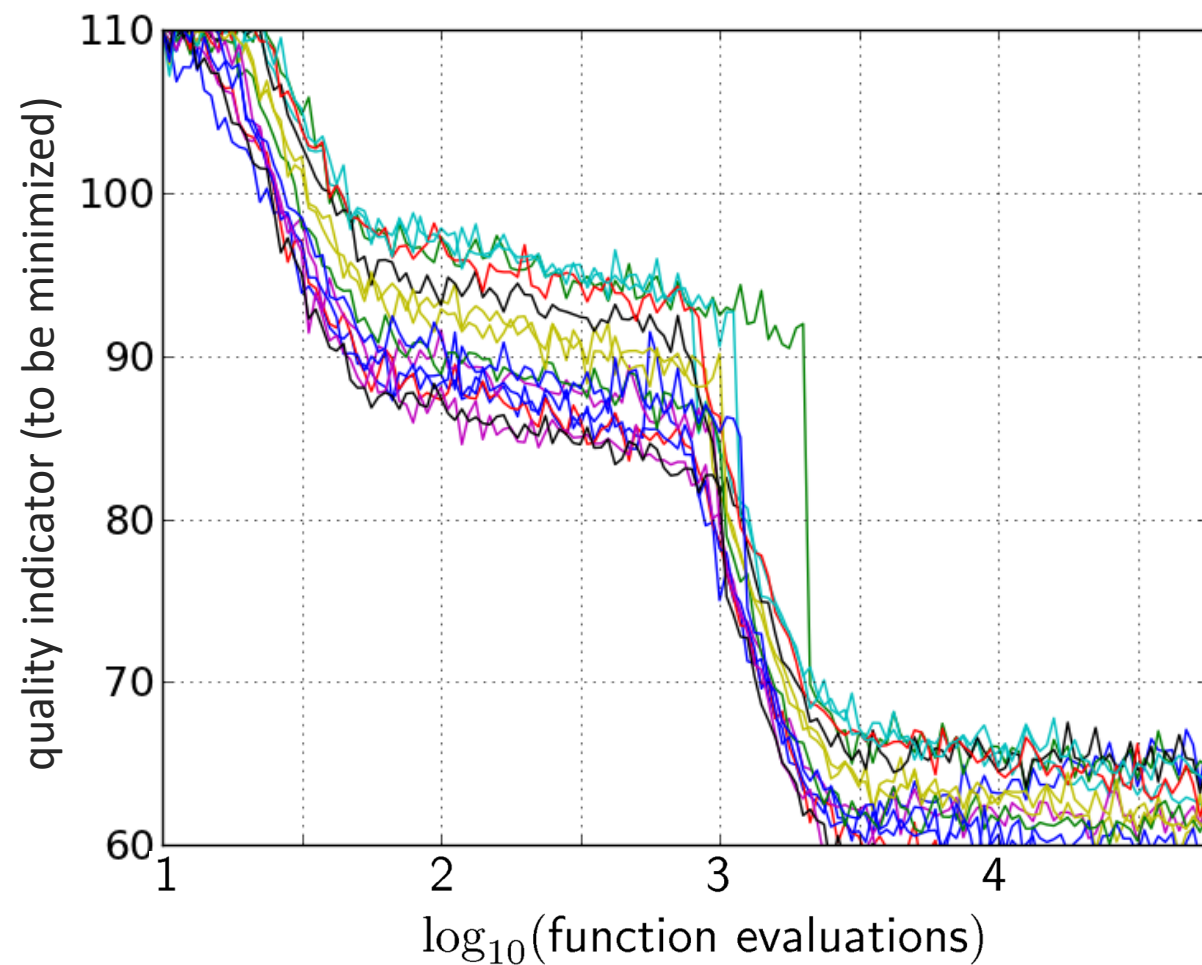
A Convergence Graph



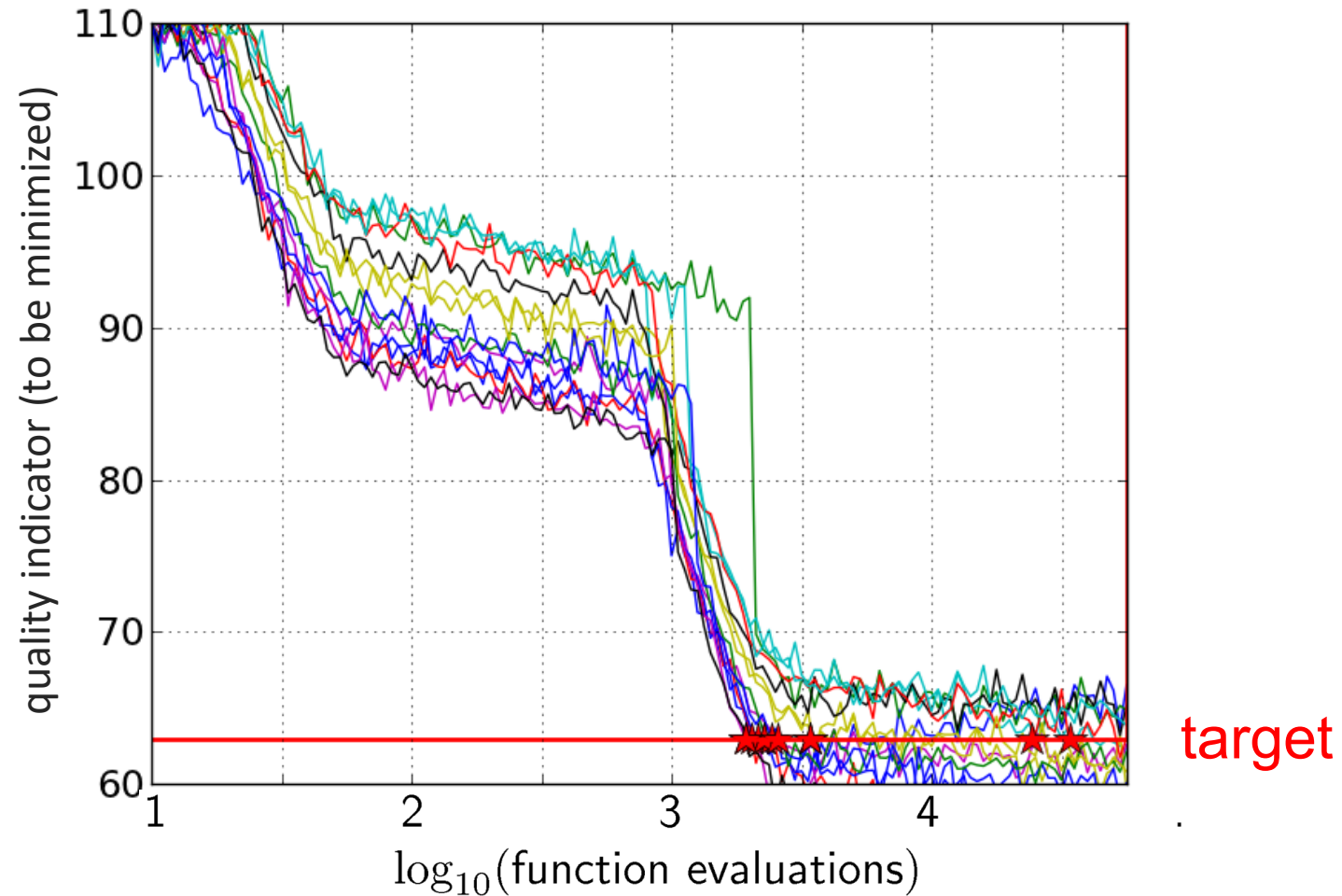
First Hitting Time is Monotonous



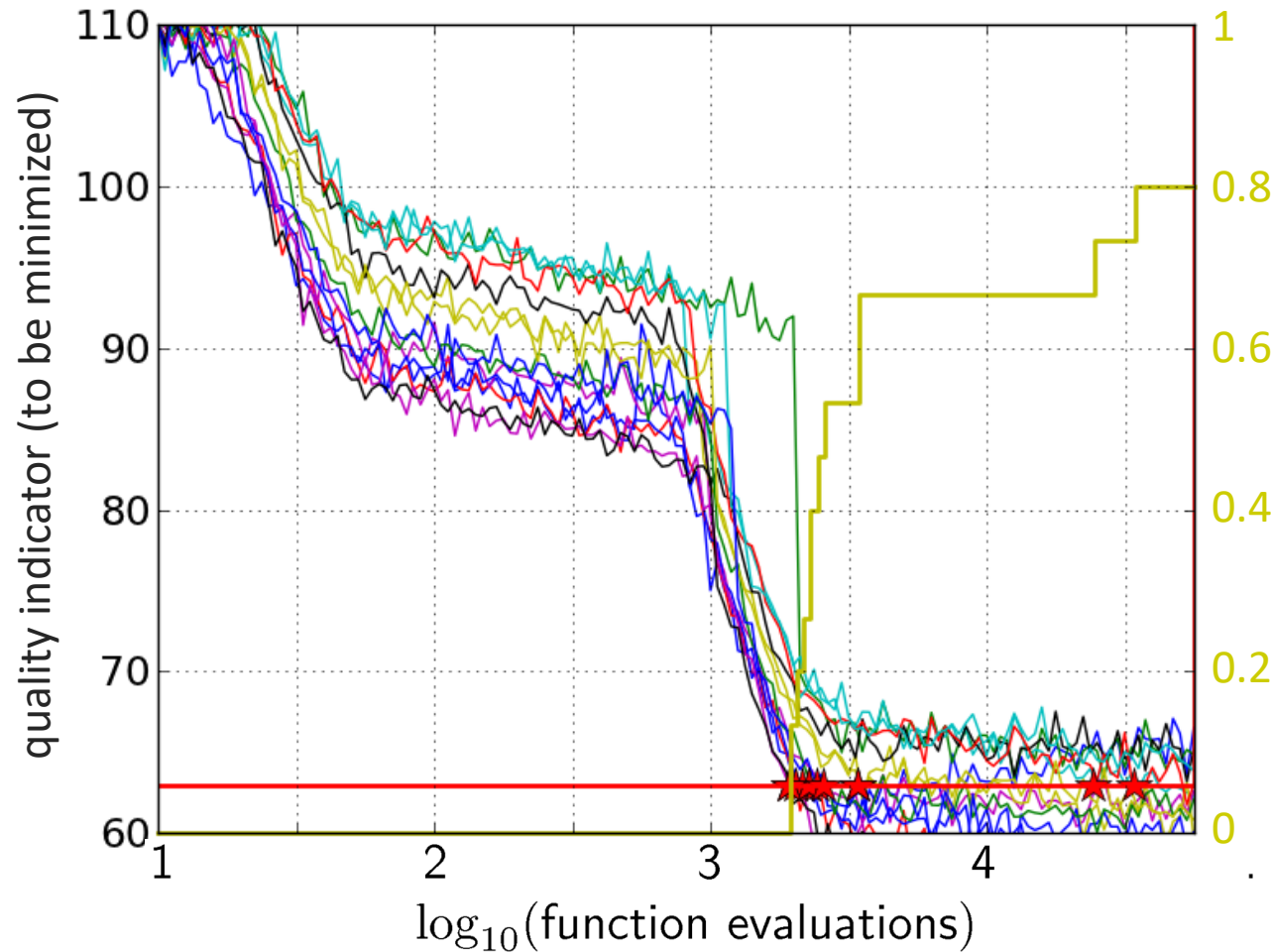
15 Runs



15 Runs $\leq$ 15 Runtime Data Points



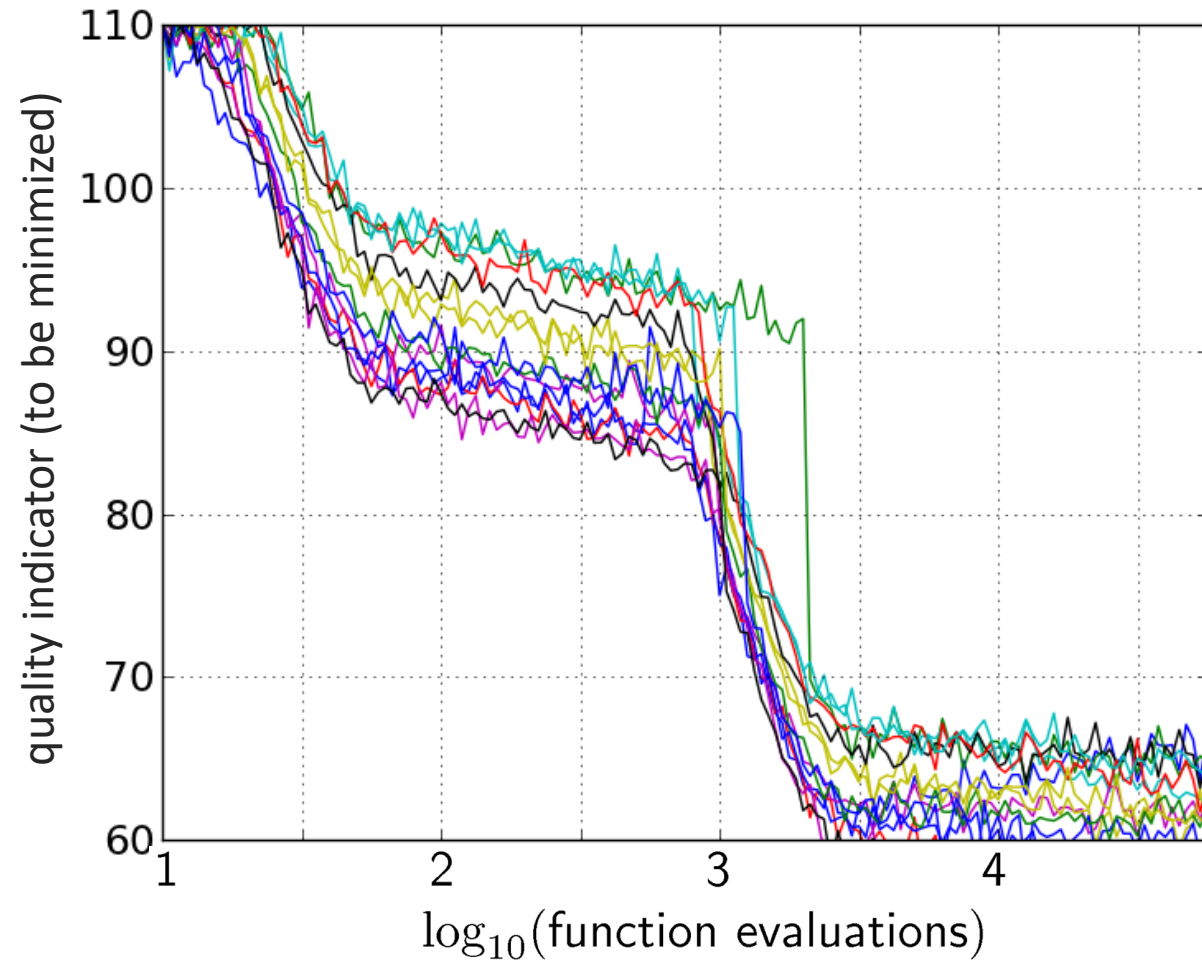
Runtime Profile



The **ECDF** of run lengths to reach the target

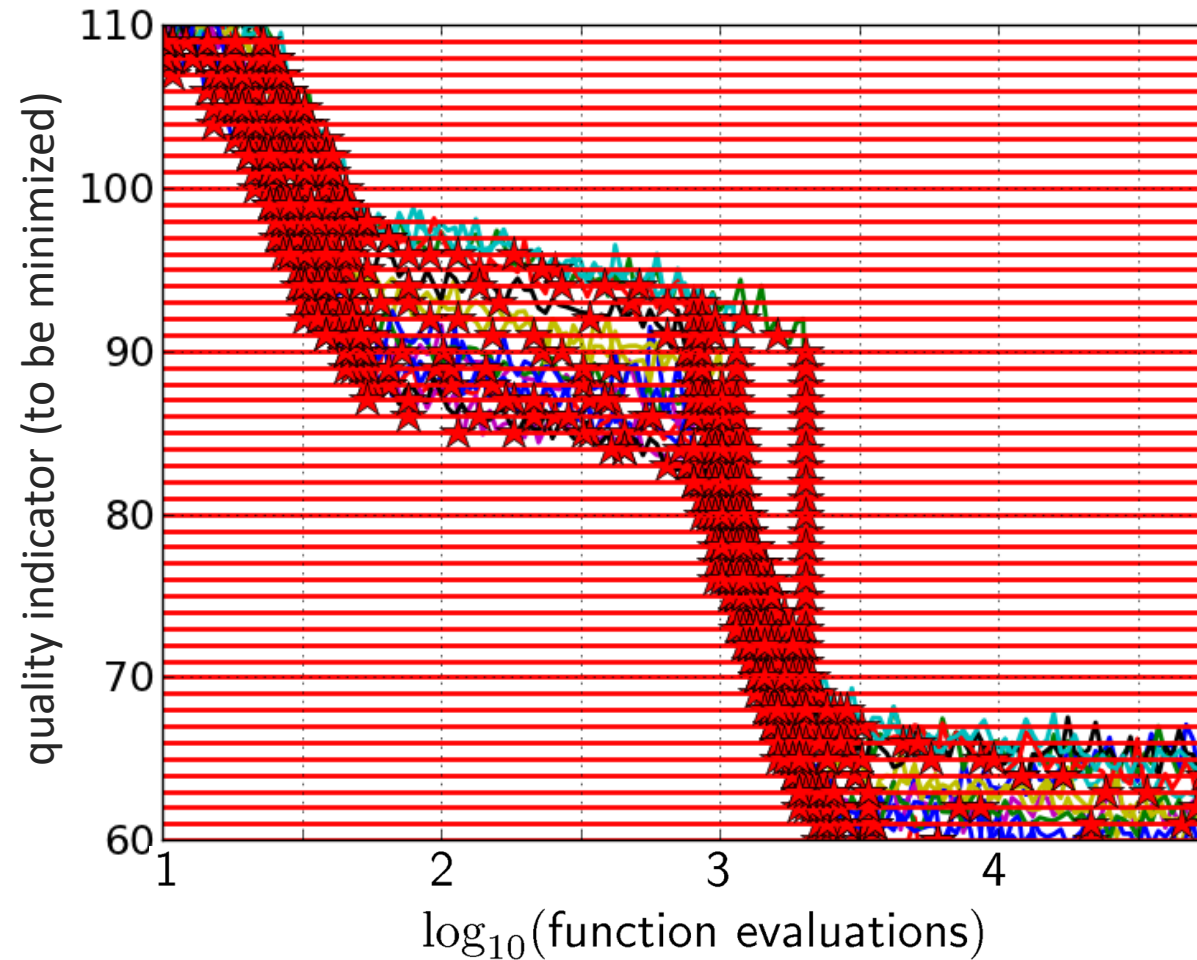
- Has for each data point a **vertical step of constant size**
- Displays for each x-value (budget) the count of observations to the left (first hitting times)

Aggregation



15 runs

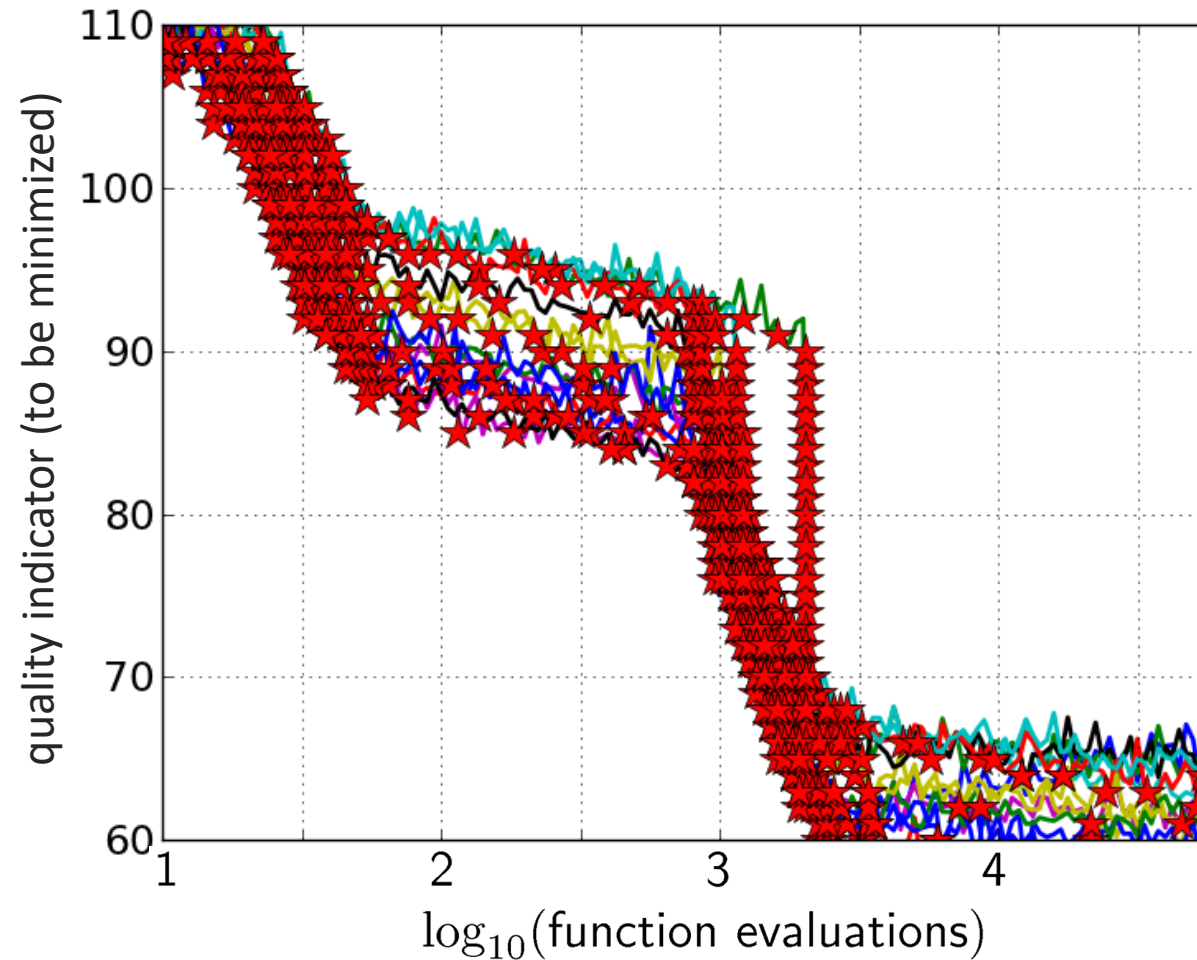
Aggregation



15 runs

50 targets

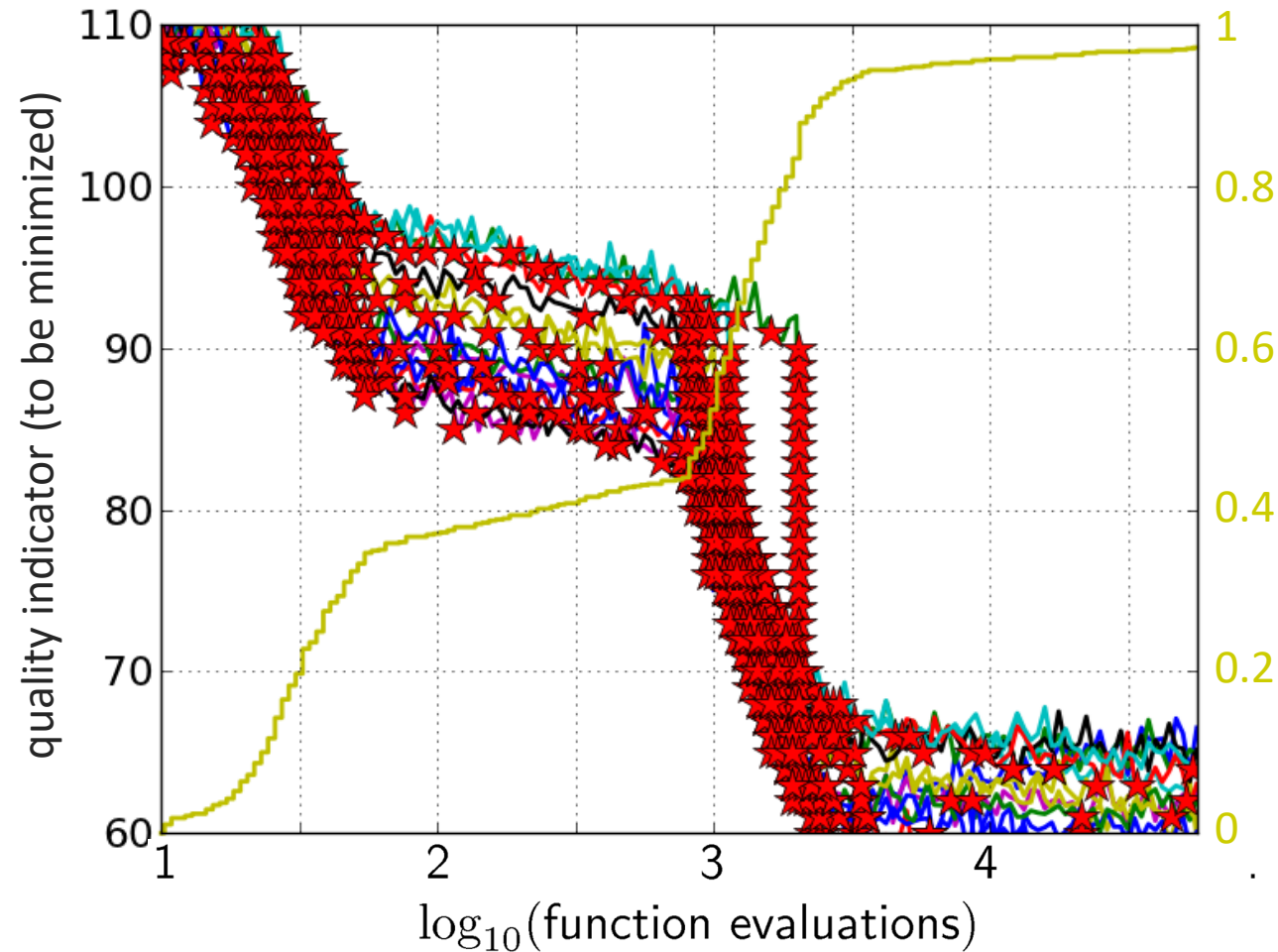
Aggregation



15 runs

50 targets

Aggregation

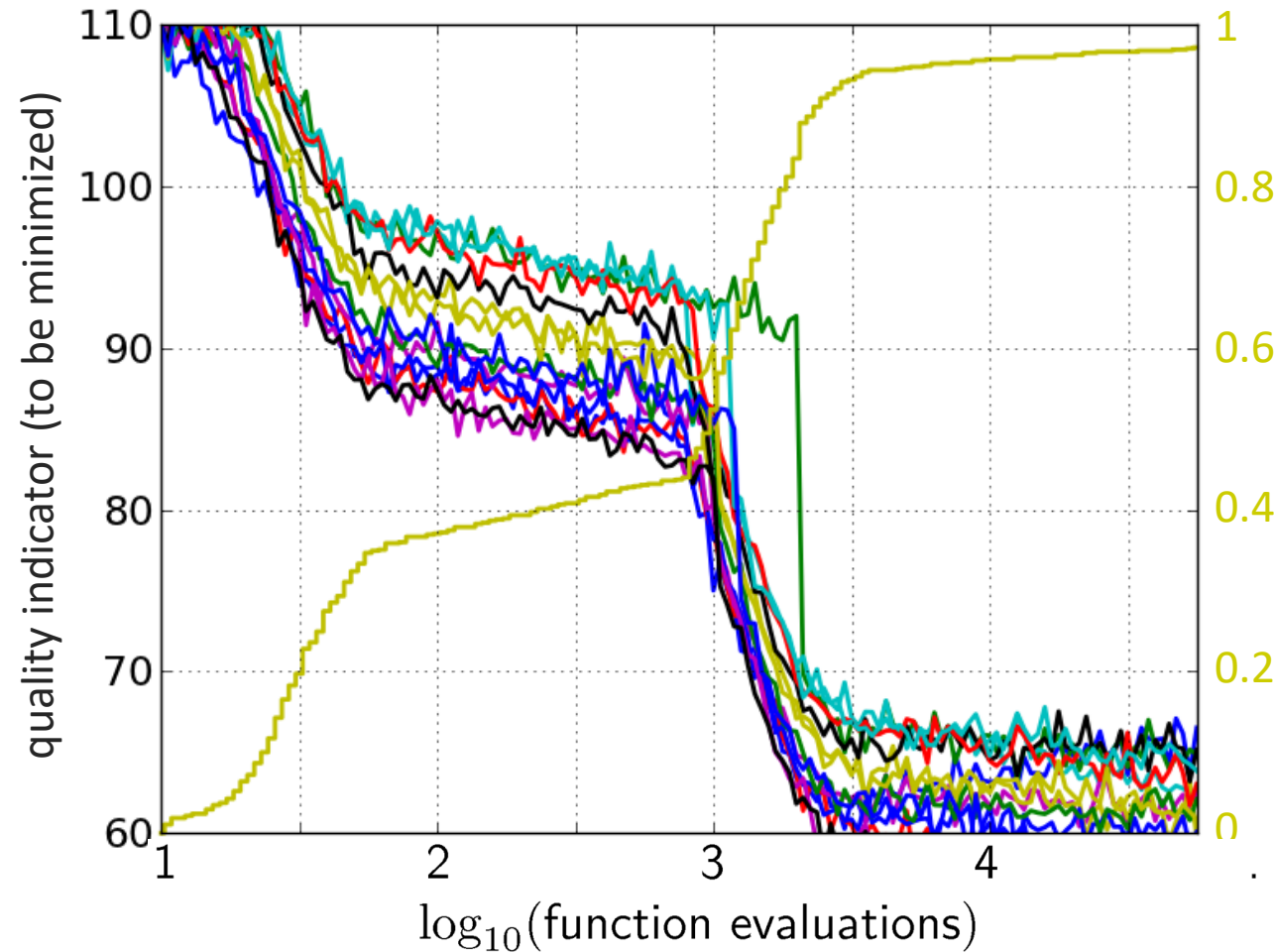


15 runs

50 targets

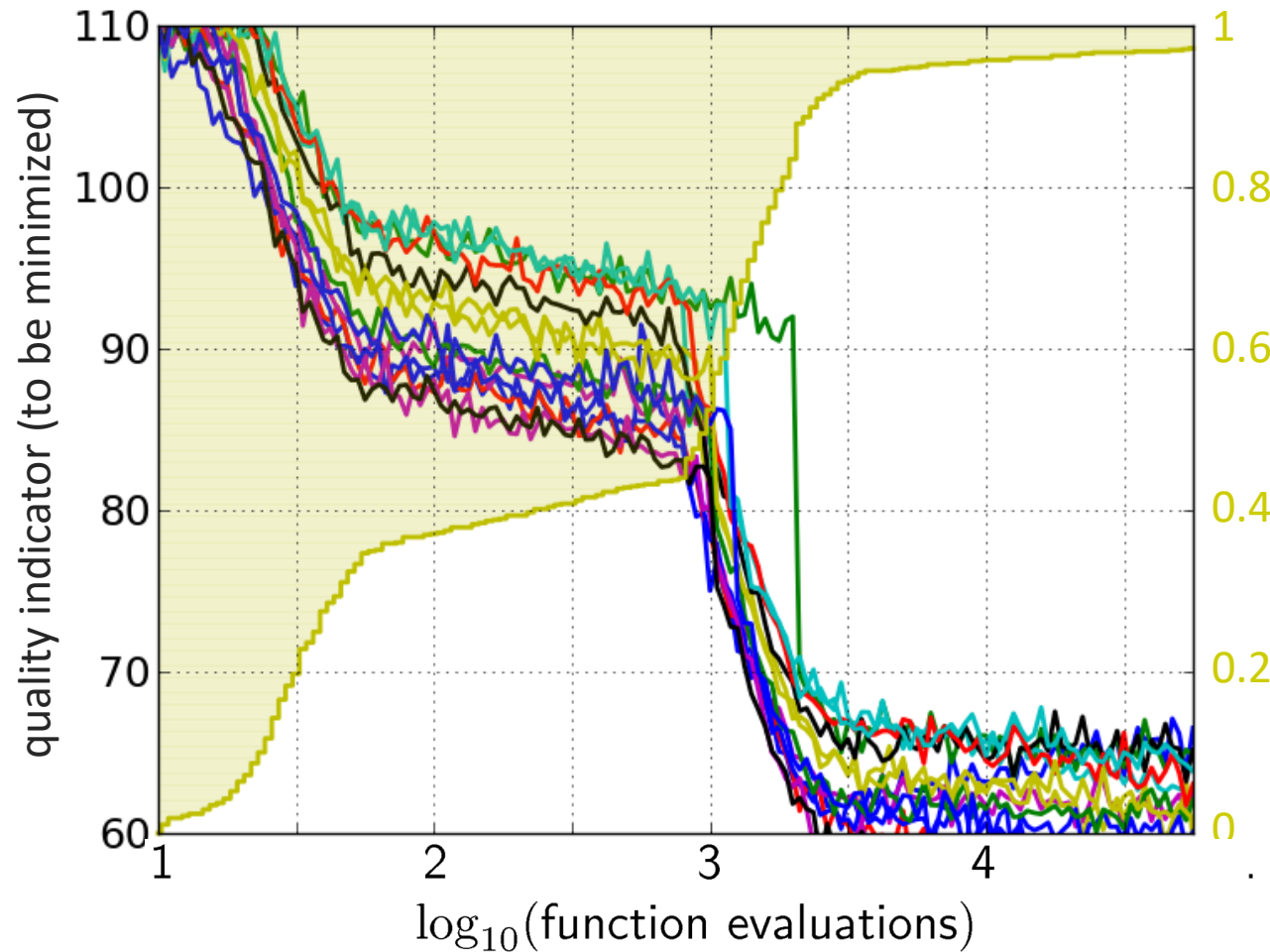
ECDF with 750 steps

Aggregation



50 targets from 15 runs
integrated in a single
graph

Interpretation



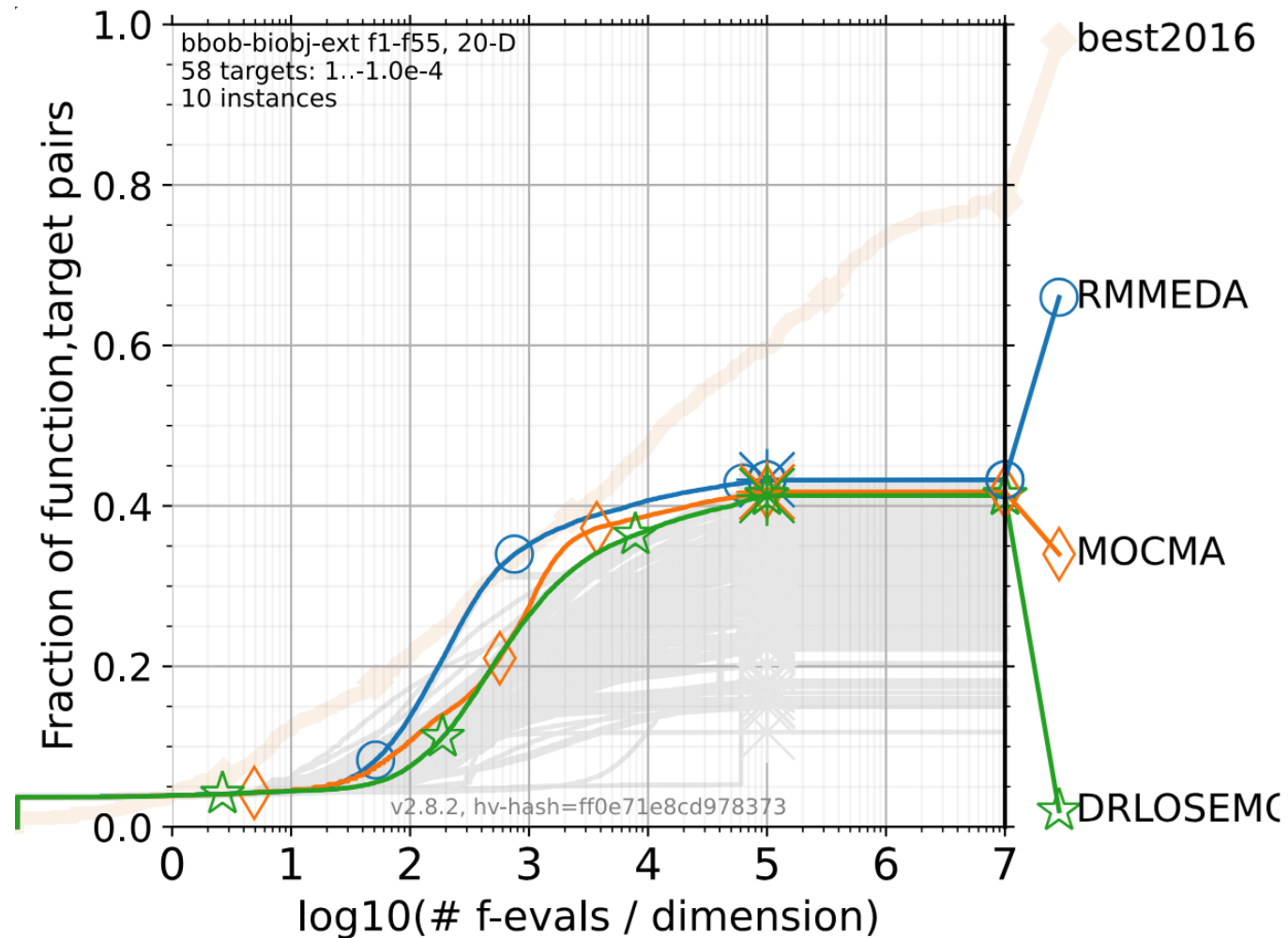
50 targets from 15 runs
integrated in a single
graph

area over the ECDF curve
=

average log runtime
(or geometric avg. runtime)
over all targets (difficult and
easy) and all runs

Example

<https://steigner.github.io/Benchmarking-PlatEMO-MultiObj-Opti-Algos-on-COCO-Problems/>



unfortunately, there is more to benchmarking...

unfortunately, there is more to benchmarking...
but because it is repetitive & time-consuming, there are tools

Components of benchmarking



Algorithms

Algorithms

Including

- any variants
- parameter settings

Components of benchmarking

Algorithms

Test problems

Test problems

Including known optimum or Pareto set/front approximation

Components of benchmarking

Algorithms

Test problems

Experimental
protocol

Experimental protocol

- budget/budget-free
- number of repetitions
- random seed settings
- ...

Components of benchmarking

Algorithms

Performance
measures

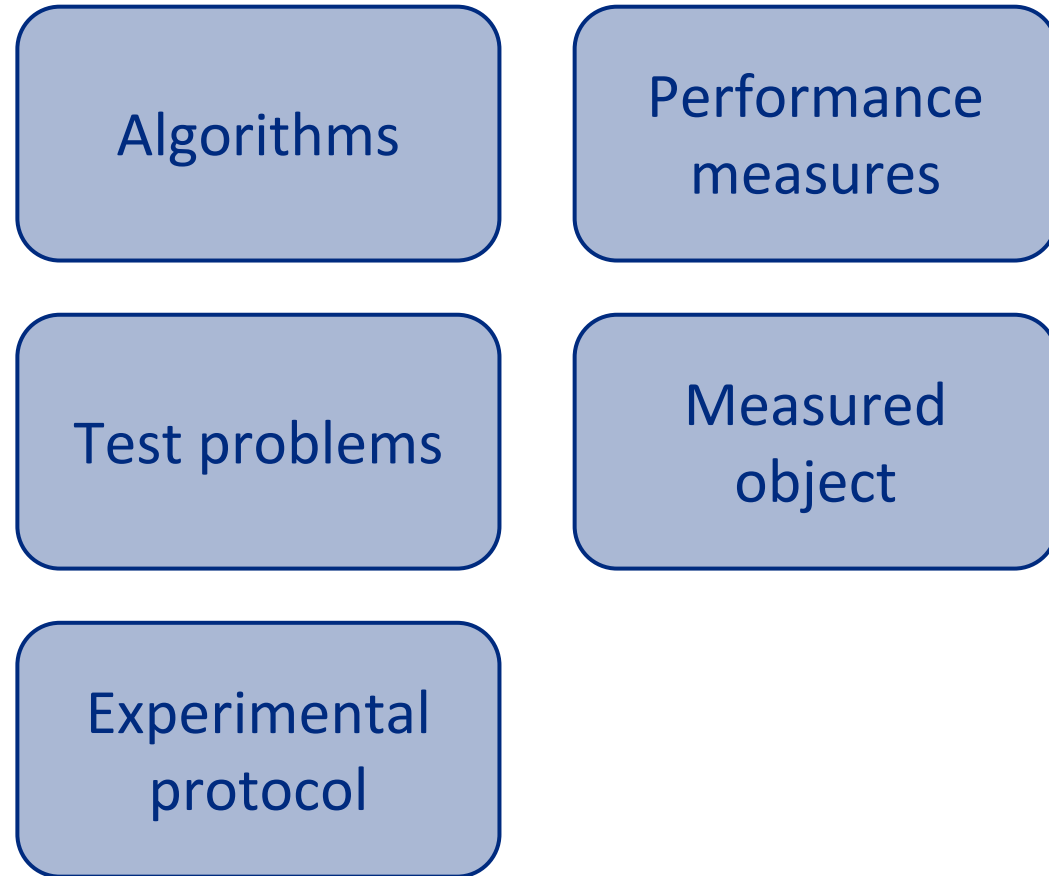
Test problems

Experimental
protocol

Performance measures

- function value
- distance to optimal value
- quality indicator
- ...

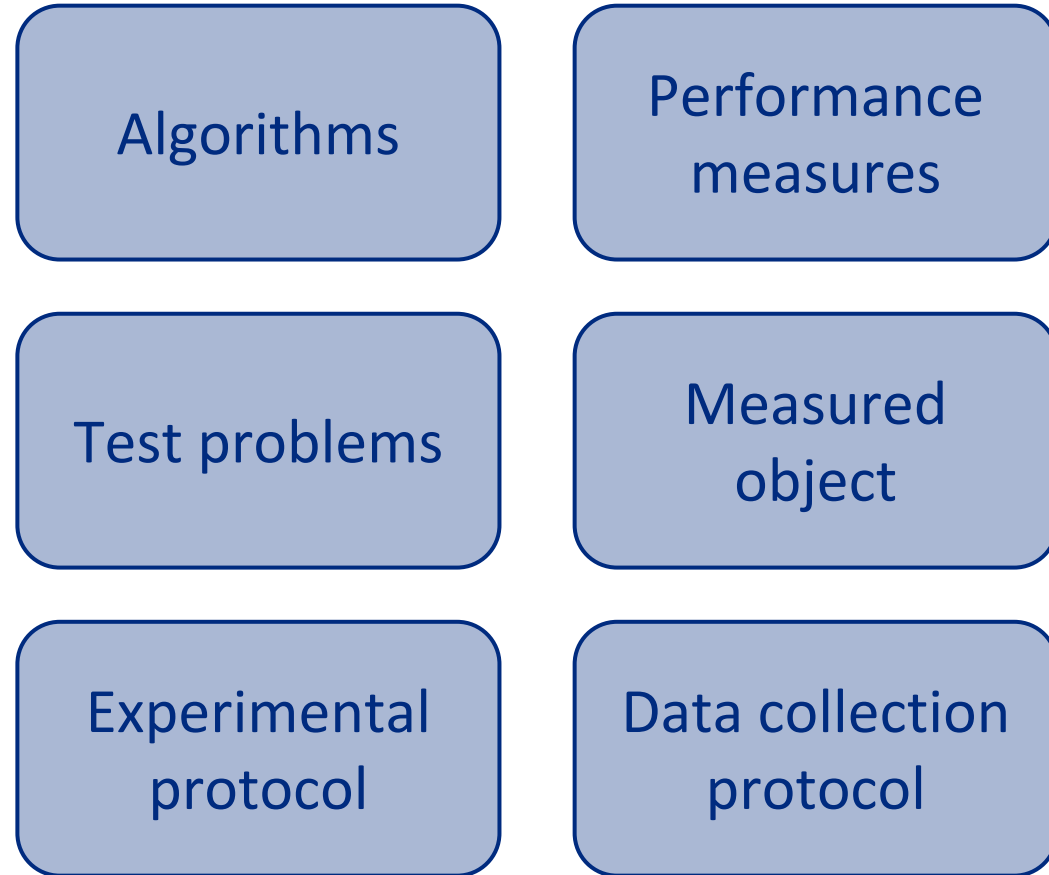
Components of benchmarking



Measured object

- current solution
- current population
- current archive
- feasible subset
- ...

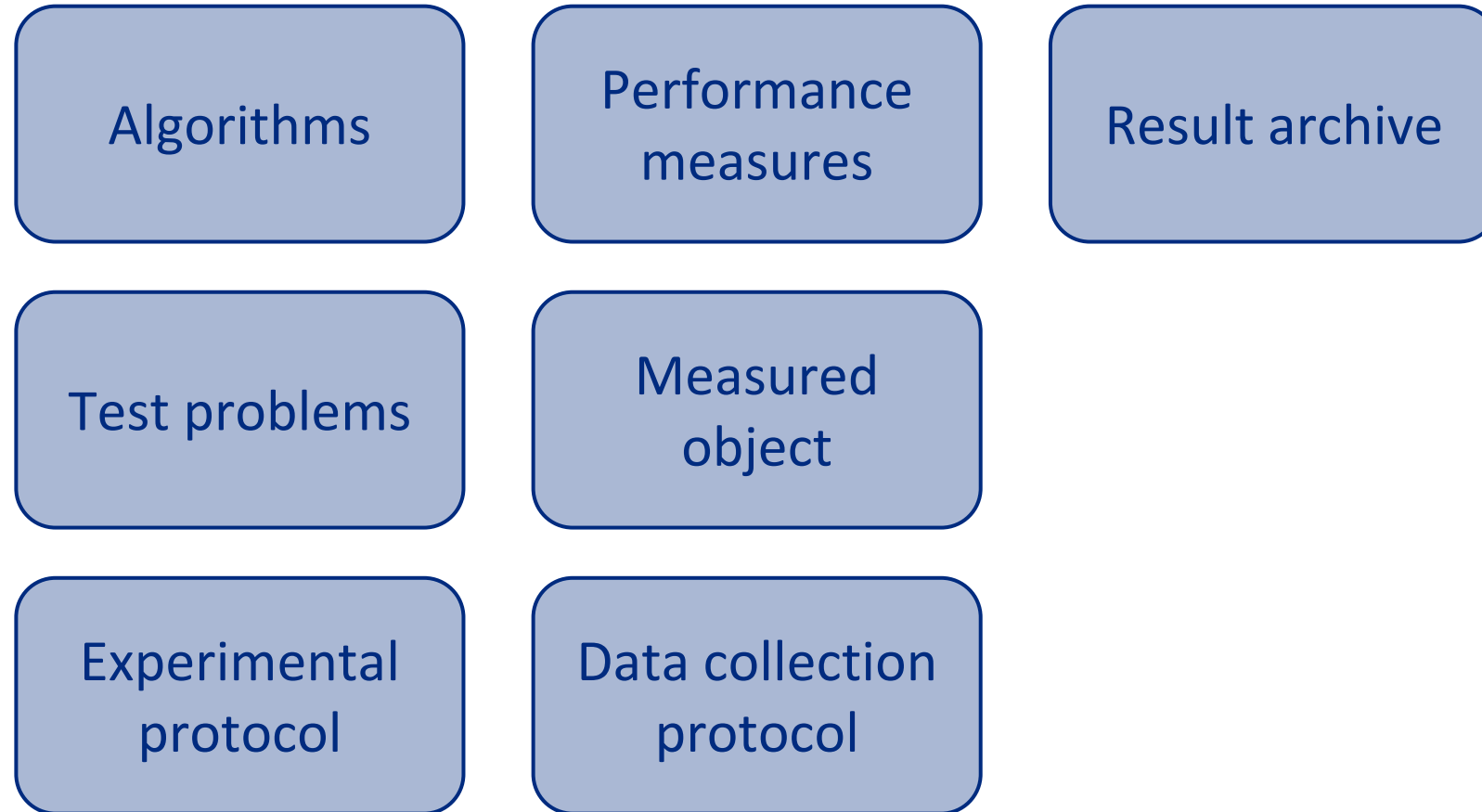
Components of benchmarking



Data collection protocol

- every evaluation
- every iteration (generation)
- every improvement
- every target hit
- ...

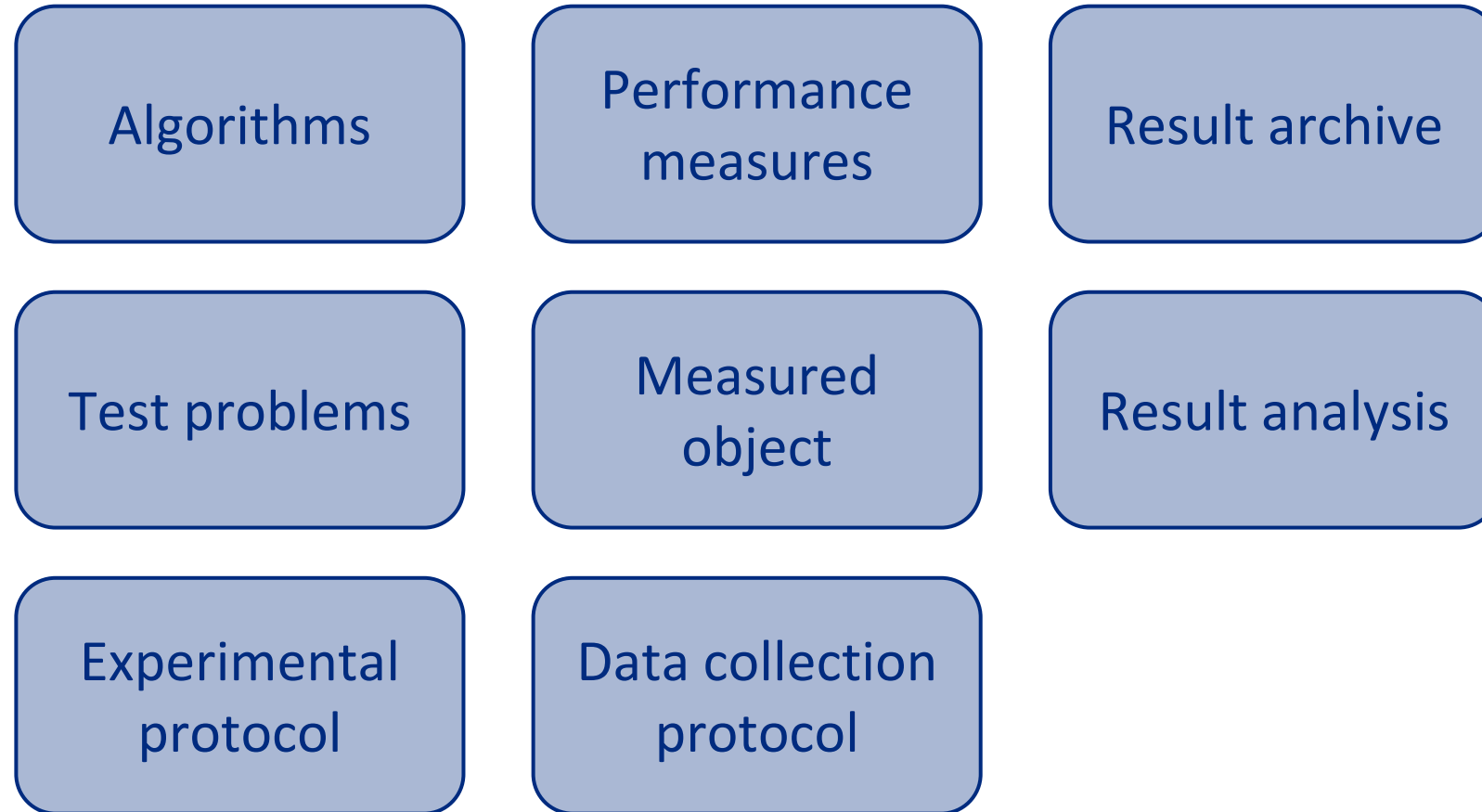
Components of benchmarking



Result archive

Long-term storage of results, needs to include metadata

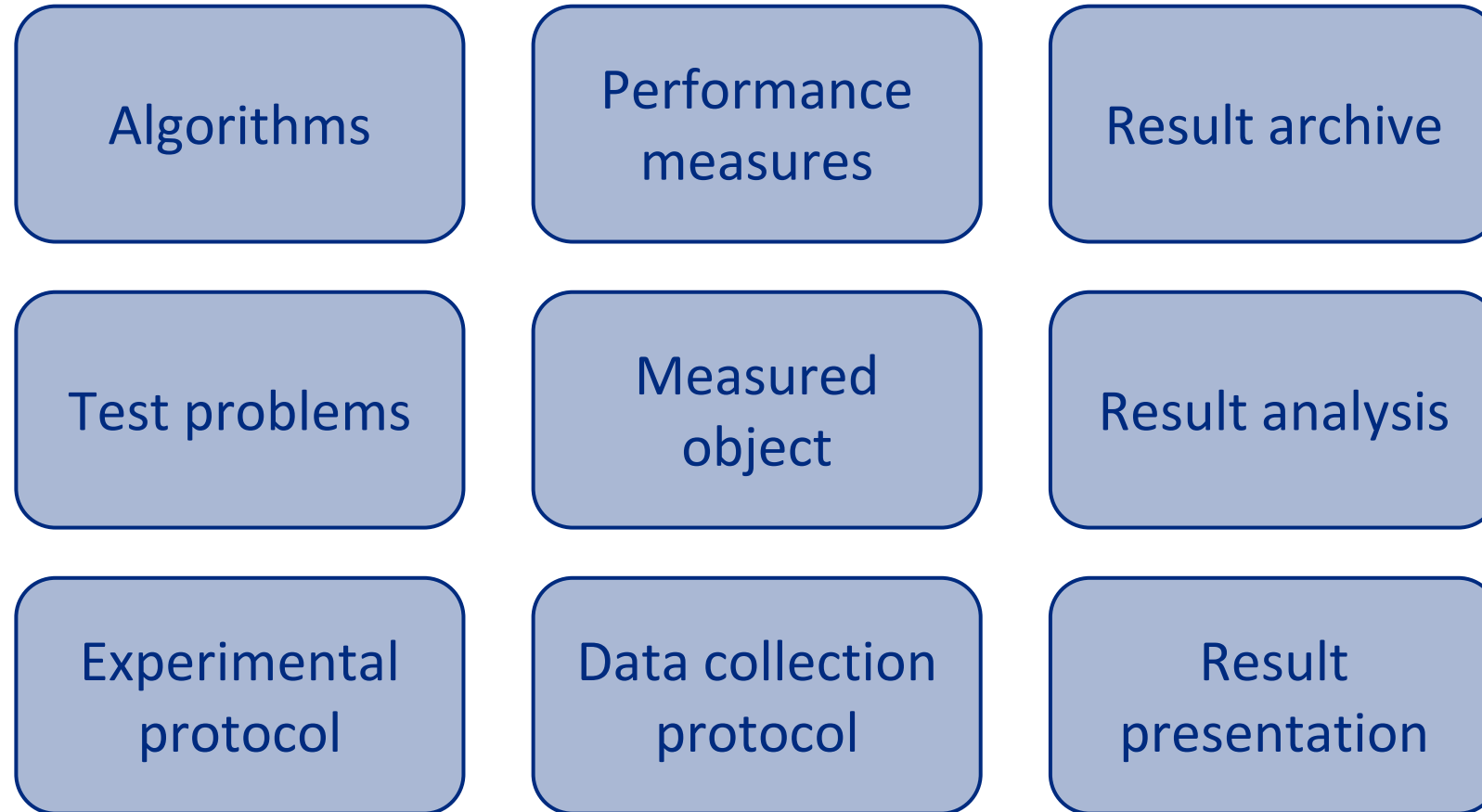
Components of benchmarking



Result analysis

- statistical comparisons
- result aggregation
- runtime profiles
- ...

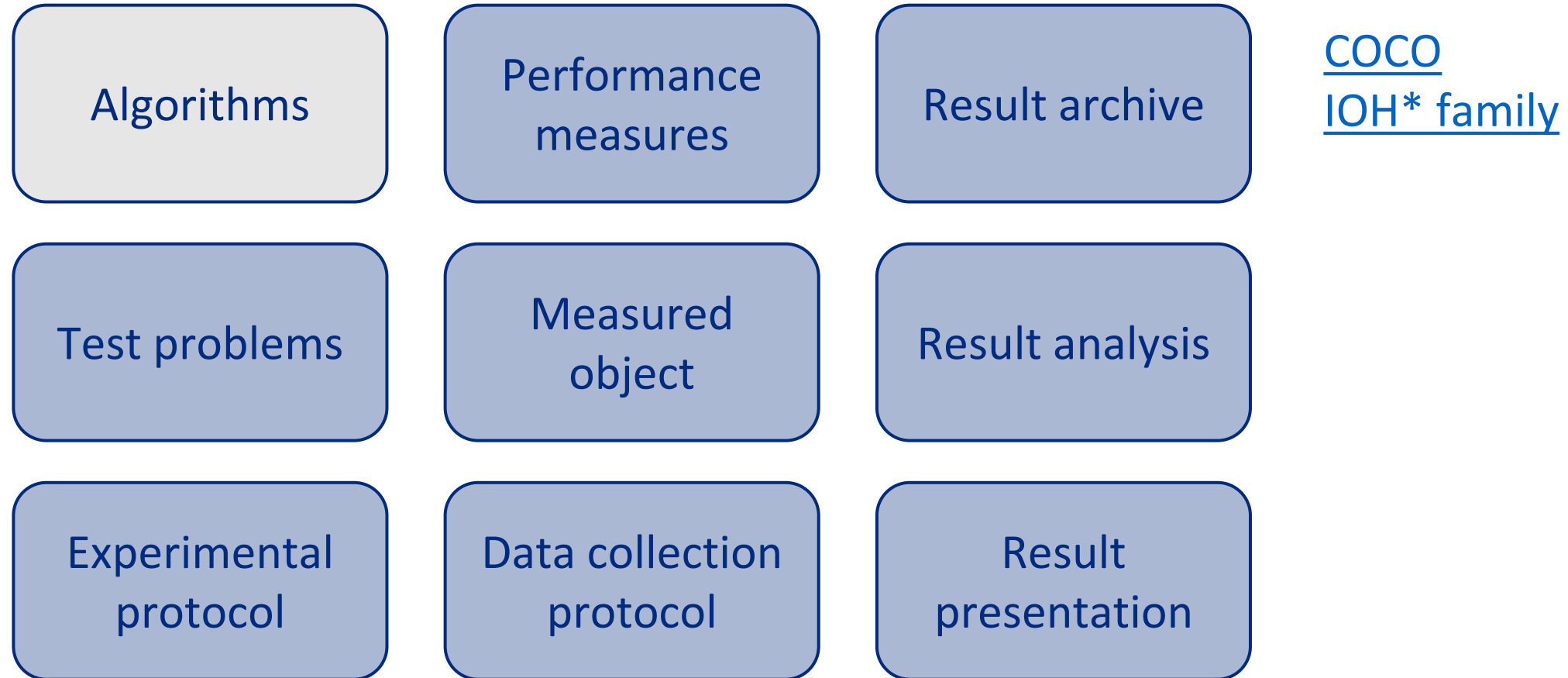
Components of benchmarking



Result presentation

- tables
- plots
- interactive displays
- ...

Dedicated benchmarking platforms



Optimization frameworks with benchmarking support

Algorithms

Performance
measures

Result archive

Test problems

Measured
object

Result analysis

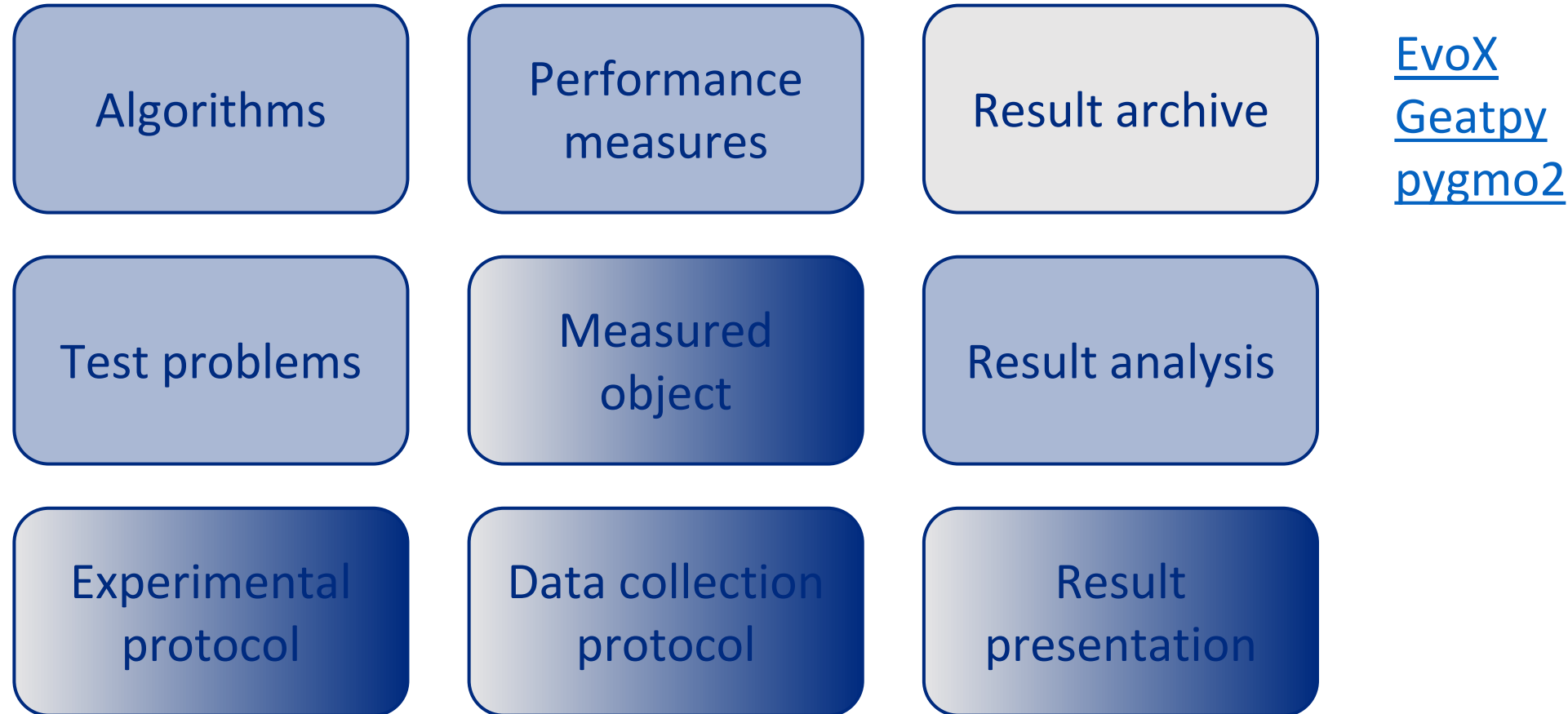
Experimental
protocol

Data collection
protocol

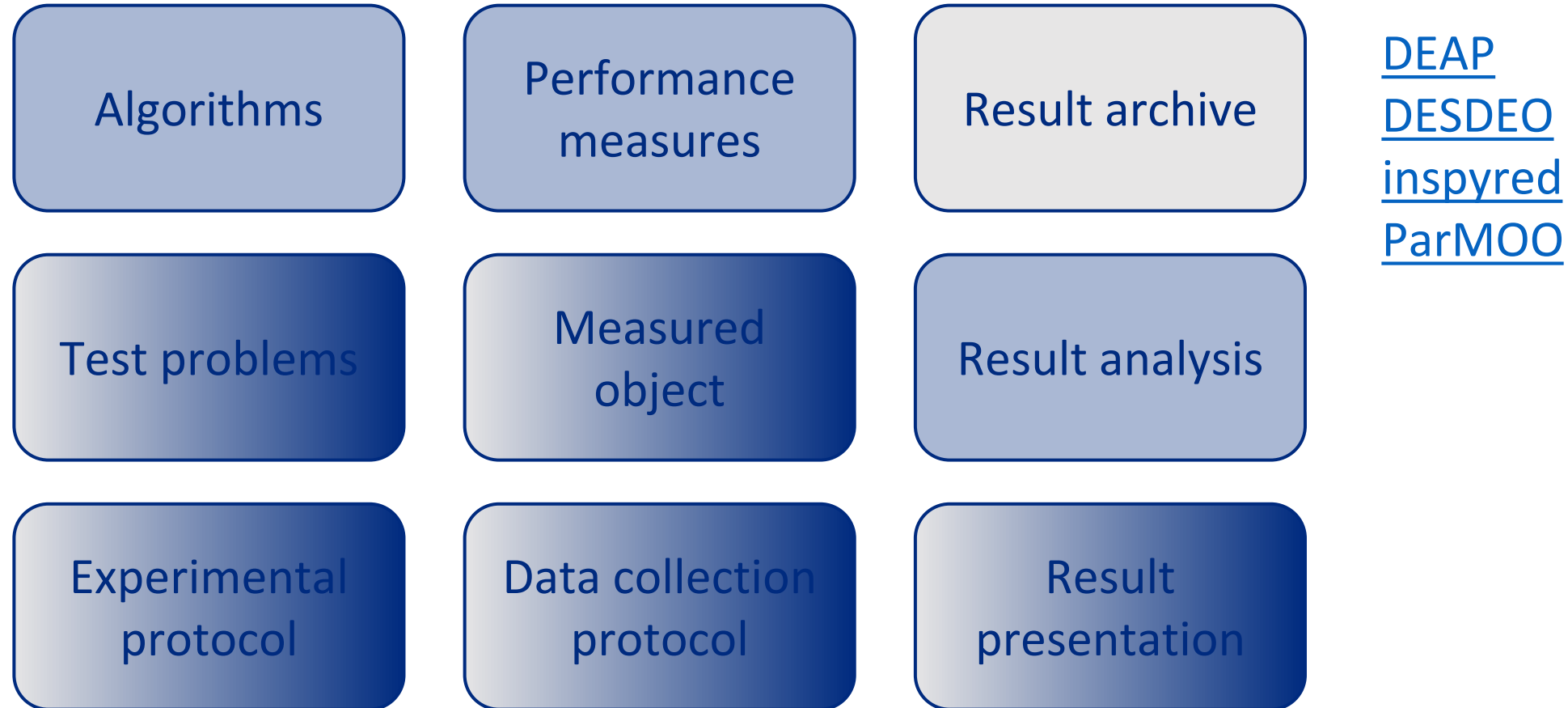
Result
presentation

[jMetal](#)
[jMetalPy](#)
[MOEA Framework](#)
[Nevergrad](#)
[PlatEMO](#)
[pymoo](#)

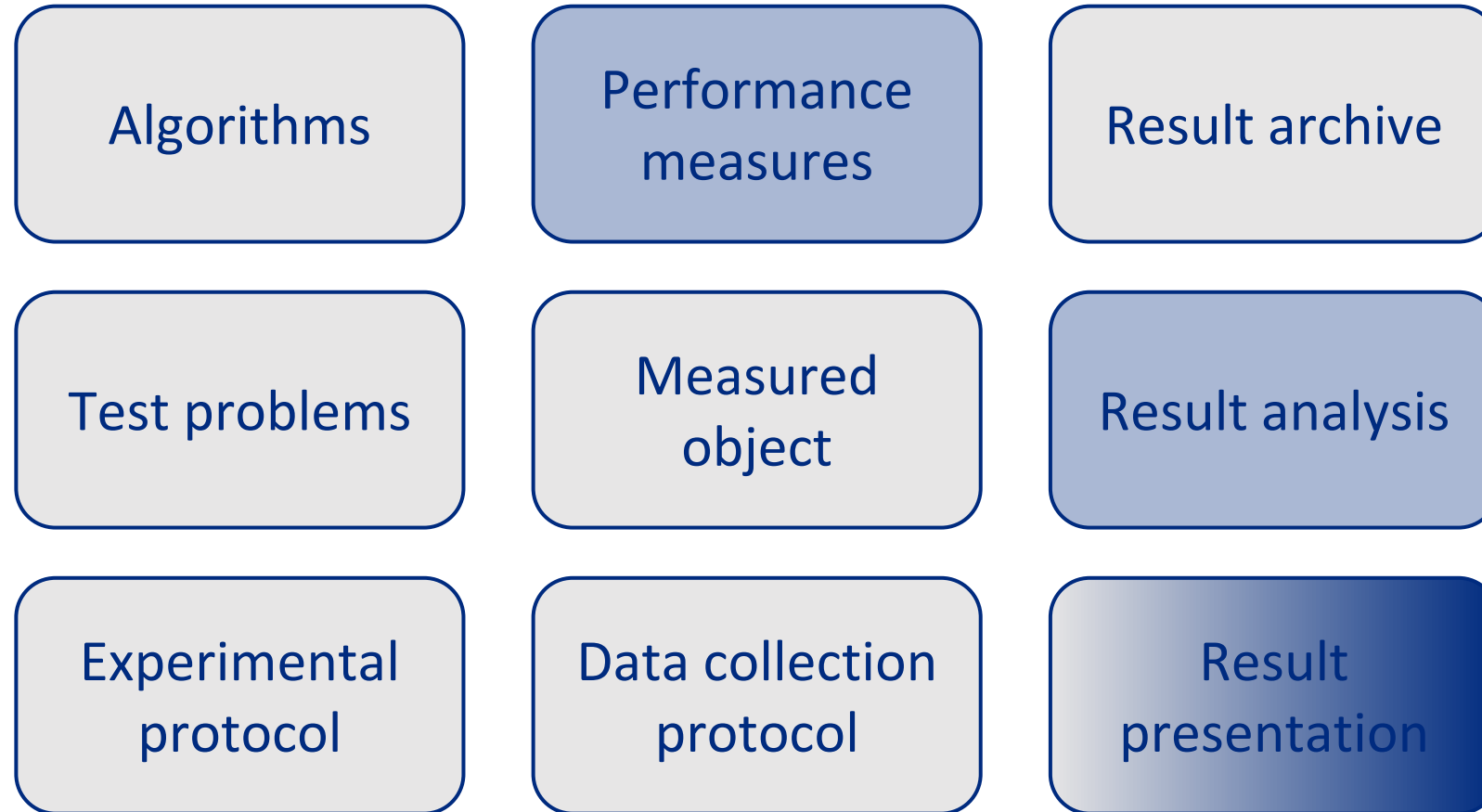
Optimization frameworks with some benchmarking support



Prototyping frameworks



Tools supporting result analysis



[moarchiving](#)
[moocore](#)
[mooplot](#)

Light-weight benchmarking libraries

Algorithms

Performance
measures

Result archive

[cocoviz](#)
[IOHinspector](#)

Test problems

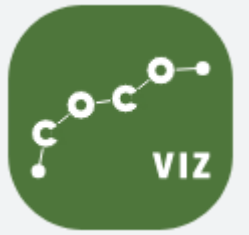
Measured
object

Result analysis

Experimental
protocol

Data collection
protocol

Result
presentation



A minimalist Python package to visualize benchmark experiments

- Do one thing: runtime profile (ECDF) plots
- A toolkit, not a framework
 - No harness to run algorithms
 - No fixed data format
 - No need to define targets
 - ... but lots of little helpers to support your research
- Good defaults for most situations

Core Objects of cocoviz: Problem

Since we are possibly aggregating results over different problems, cocoviz needs to know some properties of your problems.

For that, we have a `ProblemDescription` object with four fields:

- `name`
- `instance`
- `number_of_variables`
- `number_of_objectives`

```
from cocoviz import ProblemDescription

problem = ProblemDescription(
    name="my_problem",
    instance=1,
    number_of_variables=10,
    number_of_objectives=2
)
```

Core Objects of cocoviz: Result

cocoviz assumes that you collected the raw results of a single run of an algorithm on a problem in a DataFrame like structure (pandas or polars version work equally well).

You also need to provide a name for the algorithm so that cocoviz can aggregate results from the same algorithm and a `ProblemDescription` object to describe the problem. Finally, cocoviz needs to know which column of `raw_results` contains the time (measured in number of function evaluations) at which the indicator values were recorded.

```
from cocoviz import Result

res = Result(
    algorithm="my_alg",
    problem=problem,
    data=raw_results,
    fevals_column="nevals"
)
```

Core Objects of cocoviz: ResultSet

To aggregate over different results, you need to collect them in a ResultSet object. It acts just like a Python list with some extra methods.

```
from cocoviz import ResultSet

results = ResultSet()
for problem in problems:
    for alg in algorithms:
        res = Result(
            alg.name,
            problem.description,
            run(alg, problem),
            "nevals"
        )
        results.append(res)
```

Subsetting ResultSets

Once you've collected all results in a `ResultSet`, you can slice and dice the results into suitable subsets for analysis using the following methods:

- `by_number_of_variables`: One subset for each number of variables
- `by_number_of_objectives`: One subset for each number of objectives
- `by_algorithm`: One subset for each algorithm
- `by_problem_name`: One subset for each unique problem name
- `by_problem`: One subset for each problem instance

```
results = ... # A ResultSet

for n_bar, var in results.by_number_of_variables():
    for name, var_name in var.by_problem_name():
        for alg, var_name_alg in var_name.by_algorithm():
            # Do some analysis on subset
```

cocoviz hands-on session



<https://t1p.de/mtt2026>



International Conference on Evolutionary Multi-Criterion Optimization

5-8 April, 2027 | Exeter, UK

Keynote Speakers

- **Benjamin Doerr**, École Polytechnique, France
- **Sanaz Mostaghim**, Otto-von-Guericke-Universität Magdeburg, Germany
- **Margaret Wiecek**, Clemson University, USA

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Author Notification	30 November 2026
Tutorial Proposal Deadline	2 December 2026



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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dima Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. After postdoctoral research positions at Inria Saclay Ile-de-France in Orsay and at Ecole Polytechnique in Palaiseau, both in France, Dima has been a permanent researcher at Inria: from 2011 till 2016 with the Inria Lille - Nord Europe research center and since October 2016 with the Saclay - Ile-de-France research center, co-located with CMAP, Ecole Polytechnique, IP Paris. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

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