Potential Exam Questions

Introduction to Optimization lecture
Master AIC at Université Paris-Saclay

Dimo Brockhoff
firstname.lastname@inria.fr

October 16, 2017

Abstract

Below, you find a few questions that shall give you an idea of expected difficulties in the upcoming exam for the lecture "Introduction to Optimization" in the AIC Master of Paris-Saclay. Please note that there is no guarantee that all final questions are related to the ones below or that they will be all of similar difficulty than the questions provided here.

The exam will be a multiple choice test with 3-4 answers for each question. Each question will have at least one correct answer (maybe 2, maybe 3, maybe 4). Last year's exam had 28 questions in total, ranging from simpler (2 points) to harder questions (10 points). Both the discrete and continuous part were equally weighted (i.e. 50%) but this might change slightly as this year, we did cover a larger portion of continuous optimization (4 lectures) than discrete optimization (2 lectures). Note that wrong answers will be graded with negative points such that you rather leave a question unanswered if you are not fully sure.

1 Discrete Optimization

1. (2 points) Which of the following statements is correct?
   A. $n \log n = O(n \log n)$
   B. $2n^2 \cdot \log^2(n) = O(n \log n)$
   C. $\frac{n}{\log n} = O(n \log n)$
   D. $\frac{\log n}{n} = O(n \log n)$

2. (2 points) Which of the following statements about graphs is correct?
   A. Each forest is a walk.
   B. Each cycle is a trail.
   C. Each Hamiltonian cycle is a closed walk.
   D. Each closed walk is a Hamiltonian cycle.

3. (10 points) For which of the following coin sets does the greedy algorithm compute the optimum for the money change problem of the lecture with a change of W=37?
   A. 1 cent, 2 cents, 3 cents
   B. 1 cent, 5 cents, 15 cents
   C. 1 cent, 17 cents, 22 cents
   D. 1 cent, 16 cents, 33 cents

4. (10 points) The following trees are supposed to showcase the working principles of a branch and bound algorithm. Unfortunately, several errors have been introduced when creating the example. What is the first erroneous time step, for which the example cannot be produced by a branch and bound algorithm anymore?

5. (10 points) The shortest path from node u to node v in the following graph is of which length?

2 Continuous Optimization

6. (6 points) We consider the function $f(x) = \frac{1}{2}\left(x_1^2 + \sum_{i=2}^{n}(10^3)^{\frac{i-1}{n-1}} x_i^2\right)$. Which of the following statements are correct?

   A. The minimum of f equals $\frac{1}{2}$.
   B. The function f is convex.
   C. The gradient of f in $x = 0$ is the zero vector.
   D. The gradient of f in $x = (1, 1, \ldots, 1)$ equals $(1, 1, \ldots, 1)$.

7. (6 points) We consider the 2-dimensional *constrained* optimization problem consisting in minimizing $f(x_1, x_2) = x_1^2 + x_2^2$ while satisfying the constraint $g(x_1, x_2) = x_1 \leq 0$.

   Let $a \in \mathbb{R}^2$ be an optimum of the problem. Which of the following statements are correct?

   A. The problem's constraint is active in a.
   B. There exists $\lambda \in \mathbb{R}$ such that $\nabla f(a) + \lambda \nabla g(a) = 0$.
   C. $g(a) = 0$.
   D. $f(a) = 0$.

8. (2 points) We consider the following function $f(x) =$ ANY GIVEN FORMULA. Which of the following plot(s) correspond(s) to the level sets of the function?

9. (10 points) Given a (small) set of objective functions and a (small) set of algorithms such as CMA-ES and BFGS that have been run on the given functions, match given concrete convergence graphs without labels with the algorithm names.

10. (10 points) Given a (small) set of objective functions and the output of a CMA-ES run on them, match the given concrete convergence graphs without labels with the given function names.