Investigating the Impact of Sequential Selection in the (1,4)-CMA-ES on the Noisy BBOB-2010 Testbed

[Black-Box Optimization Benchmarking Workshop]

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ABSTRACT

Sequential selection, introduced for Evolution Strategies (ESs) with the aim of accelerating their convergence, consists in performing the evaluations of the different offspring sequentially, stopping the sequence of evaluations as soon as an offspring is better than its parent and updating the new parent to this offspring solution. This paper investigates the impact of the application of sequential selection to the (1,4)-CMA-ES on the BBOB-2010 noisy benchmark testbed. The performance of the (1,4^s)-CMA-ES, where sequential selection is implemented, is compared to the baseline algorithm (1,4)-CMA-ES. Independent restarts for the two algorithms are conducted till a maximum of 10^4D function evaluations per trial was reached, where D is the dimension of the search space.

The results show that the sequential selection within the (1,4^s)-CMA-ES clearly outperforms the baseline algorithm (1,4)-CMA-ES by at least 12% on 7 functions in 20D whereas no statistically significant worsening can be observed. Moreover, the (1,4^s)-CMA-ES shows shorter expected running times on 6 functions of up to 32% compared to the function-wise best algorithm of the BBOB-2009 benchmarking (in 20D and for a target value of 10^{-7}).

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

Evolution Strategies (ESs) are robust stochastic search algorithms for numerical optimization where the objective function to be minimized, f, maps the continuous search space $\mathbb{R}^D$ into $\mathbb{R}$. In ESs, a population of λ candidate solutions is sampled at each iteration by adding to a current solution λ random vectors following a multivariate normal distribution. In the local search $(1, \lambda)$-ES we are interested in, the best of the λ solutions, i.e., the solution having the smallest objective function value, is selected to become the new current solution.

Sequential selection has been recently introduced for Evolution Strategies with the aim of accelerating their convergence [2]. When sequential selection is applied in a $(1, \lambda)$-ES, the evaluations are carried out sequentially and the sequence of evaluations is stopped as soon as an offspring turns out to be better than its parent. The parent for the next iteration is then set to this offspring. In this paper, we evaluate the impact of sequential selection on the (1,4)-Covariance-Matrix-Adaptation Evolution-Strategy (CMA-ES) using the BBOB-2010 noisy testbed. The performance of the (1,4^s)-CMA-ES implementing sequential selection is compared to the performance of the (1,4)-CMA-ES. The algorithms as well as the CPU timing experiments are described in a complementing paper in the same proceedings [1].

2. COMPARING THE (1,4) AND THE (1,4^s)-CMA-ES

Results from experiments comparing (1,4)-CMA-ES and (1,4^s)-CMA-ES according to [4] on the benchmark functions given in [3, 5] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_t = f_{opt} + \Delta f_t$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [4, 6]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

First of all, it is to mention that already the simple (1,4)-CMA-ES outperforms the function-wise best algorithm of the BBOB-2009 benchmarking in 20D on the Gallagher function with Cauchy noise (f_{130}) by about 40% (although only 11 of the 15 runs are successful) and that it shows the same expected running time than the BBOB-2009 function-wise best algorithm on the sphere function with moderate Cauchy noise (f_{103}).

Moreover, the sequential selection in the (1,4^s)-CMA-ES further improves over the (1,4)-CMA-ES on seven functions statistically significant in 20D and for a target value of 10^{-7}: on $f_{101-103}$, the improvement is between 12% and 20%, on f_{106} and f_{118}, the improvement is 40% and on f_{121} and f_{112}, the running time of the (1,4^s)-CMA-ES is smaller than the one of the (1,4)-CMA-ES by a factor of about 2 and 3 respectively (all results statistically significant). No statistically significant worsening on any function in 5D and 20D can be observed although the expected running times on f_{130} are approximately 50% higher for the (1,4^s)-CMA-ES than for the (1,4)-CMA-ES and also the success probability of the (1,4^s)-CMA-ES is smaller on this function (8 versus 11 instances solved).

Despite this result on f_{130}, the (1,4^s)-CMA-ES shows, in comparison to the function-wise best algorithm of the BBOB-2009 benchmarking, better results on all functions that are solved except for the comparably easy functions f_{101} and f_{102} as well as on f_{118} (in 20D and for a target value of 10^{-7}): on f_{106}, f_{121}, and f_{130}, the improvements are rather small ($\leq$ 10%) but on f_{103}, the (1,4^s)-CMA-ES is 18% faster, on f_{109} 32% faster, and on f_{112} 26% faster than the function-wise best algorithm of BBOB-2009, which was in all those cases the IPOP-SEP-CMA-ES of [7]—showing that incorporating the sequential selection idea into the separable CMA-ES of [7] might even further improve the results.

3. CONCLUSIONS

The idea behind the sequential selection scheme introduced in [2] is to finish the iteration as soon as an offspring is evaluated which is better than the current solution and thereby save some of the λ function evaluations per iteration in a $(1 \overset{+}{,} \lambda)$-ES. Here, the concept of sequential selection has been integrated into a comma-strategy, the so-called (1,4^s)-CMA-ES, and compared with the corresponding baseline (1,4)-CMA-ES on the noisy BBOB-2010 testbed.

The results show that the (1,4)-CMA-ES and its improved version (1,4^s)-CMA-ES with sequential selection solve 9 of the 30 functions overall. No statistically significantly worse results can be observed for the (1,4^s)-CMA-ES although the expected running times on f_{130} are approximately 50% higher for the (1,4^s)-CMA-ES than for the (1,4)-CMA-ES. Instead, the sequential selection in the (1,4^s)-CMA-ES improves over the (1,4)-CMA-ES on seven functions statistically significantly in 20D with improvements of at least 12%.

Moreover, the (1,4^s)-CMA-ES even shows an improved performance over the overall best algorithm from the BBOB-2009 benchmarking on 6 functions (in 20D and for a target value of 10^{-7}). Interestingly, all those 6 functions belong to the class of functions with additional Cauchy noise. The largest improvements are obtained on f_{103} (18% faster than the best algorithm of the BBOB-2009 benchmarking on that function), on f_{109} (32% faster), and on f_{112} (26% faster).

4. REFERENCES

[1] A. Auger, D. Brockhoff, and N. Hansen. Investigating the impact of sequential selection in the (1,4)-CMA-ES on the noiseless BBOB-2010 testbed. In *GECCO (Companion)*, 2010.

[2] A. Auger, D. Brockhoff, and N. Hansen. Mirrored sampling and sequential selection for evolution strategies. Rapport de Recherche RR-7249, INRIA Saclay—Île-de-France, April 2010.

[3] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2010: Presentation of the noisy functions. Technical Report 2009/21, Research Center PPE, 2010.

[4] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2010: Experimental setup. Technical Report RR-7215, INRIA, 2010.

[5] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noisy functions definitions. Technical Report RR-6869, INRIA, 2009. Updated February 2010.

[6] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.

[7] R. Ros. Benchmarking sep-CMA-ES on the BBOB-2009 noisy testbed. In F. Rothlauf, editor, *GECCO (Companion)*, pages 2441–2446. ACM, 2009.

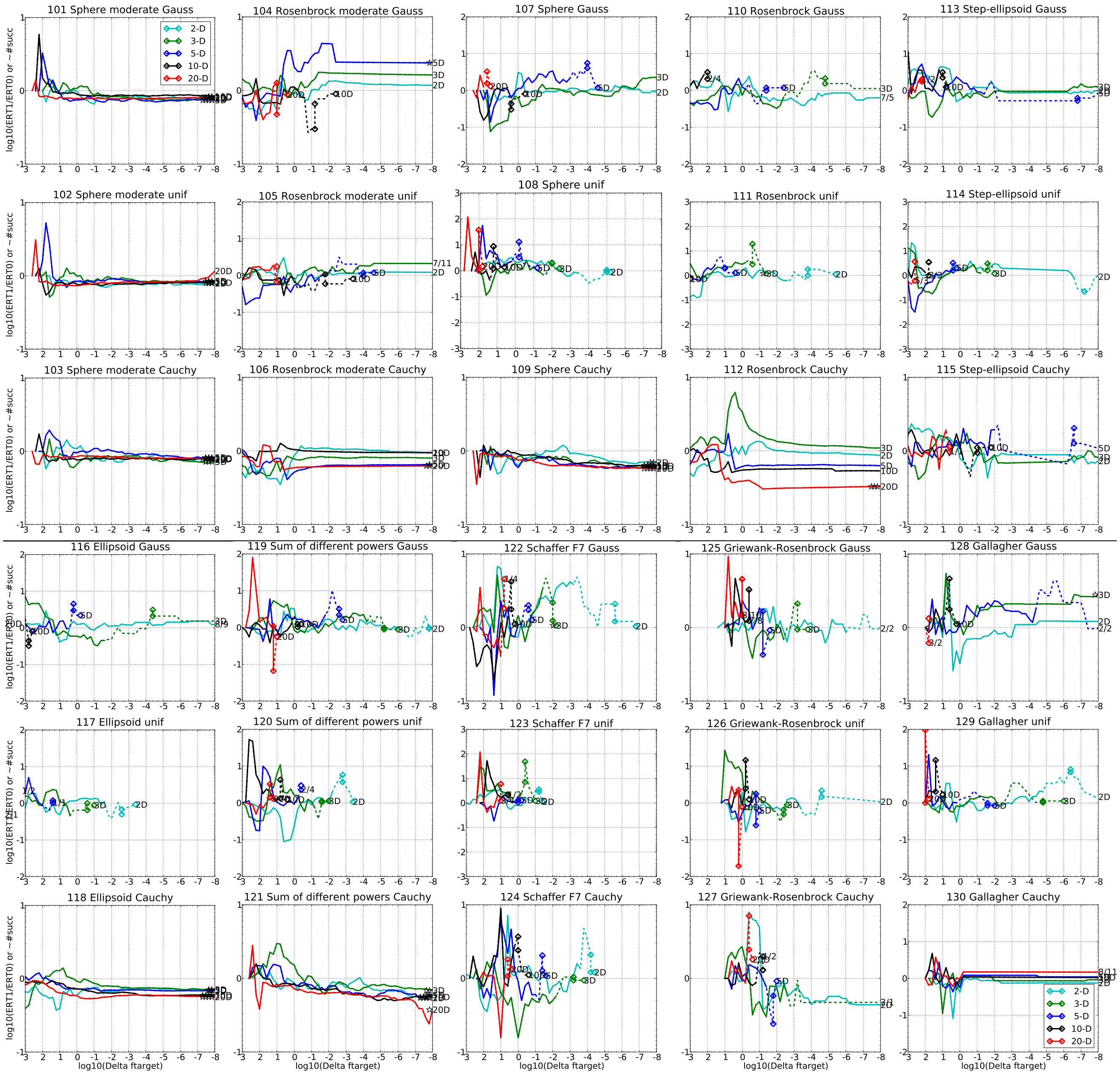

Figure 1: Ratio of the expected running times (ERT) of (1,4^s)-CMA-ES divided by (1,4)-CMA-ES versus $\log_{10}(\Delta f)$ for f_{101}–f_{130} in 2, 3, 5, 10, 20. Ratios $< 10^0$ indicate an advantage of (1,4^s)-CMA-ES, smaller values are always better. The line gets dashed when for any algorithm the ERT exceeds thrice the median of the trial-wise overall number of f-evaluations for the same algorithm on this function. Symbols indicate the best achieved Δf-value of one algorithm (ERT gets undefined to the right). The dashed line continues as the fraction of successful trials of the other algorithm, where 0 means 0% and the y-axis limits mean 100%, values below zero for (1,4^s)-CMA-ES. The line ends when no algorithm reaches Δf anymore. The number of successful trials is given, only if it was in $\{1 \ldots 9\}$ for (1,4^s)-CMA-ES (1st number) and non-zero for (1,4)-CMA-ES (2nd number). Results are statistically significant with $p = 0.05$ for one star and $p = 10^{-\#\star}$ otherwise, with Bonferroni correction within each figure.

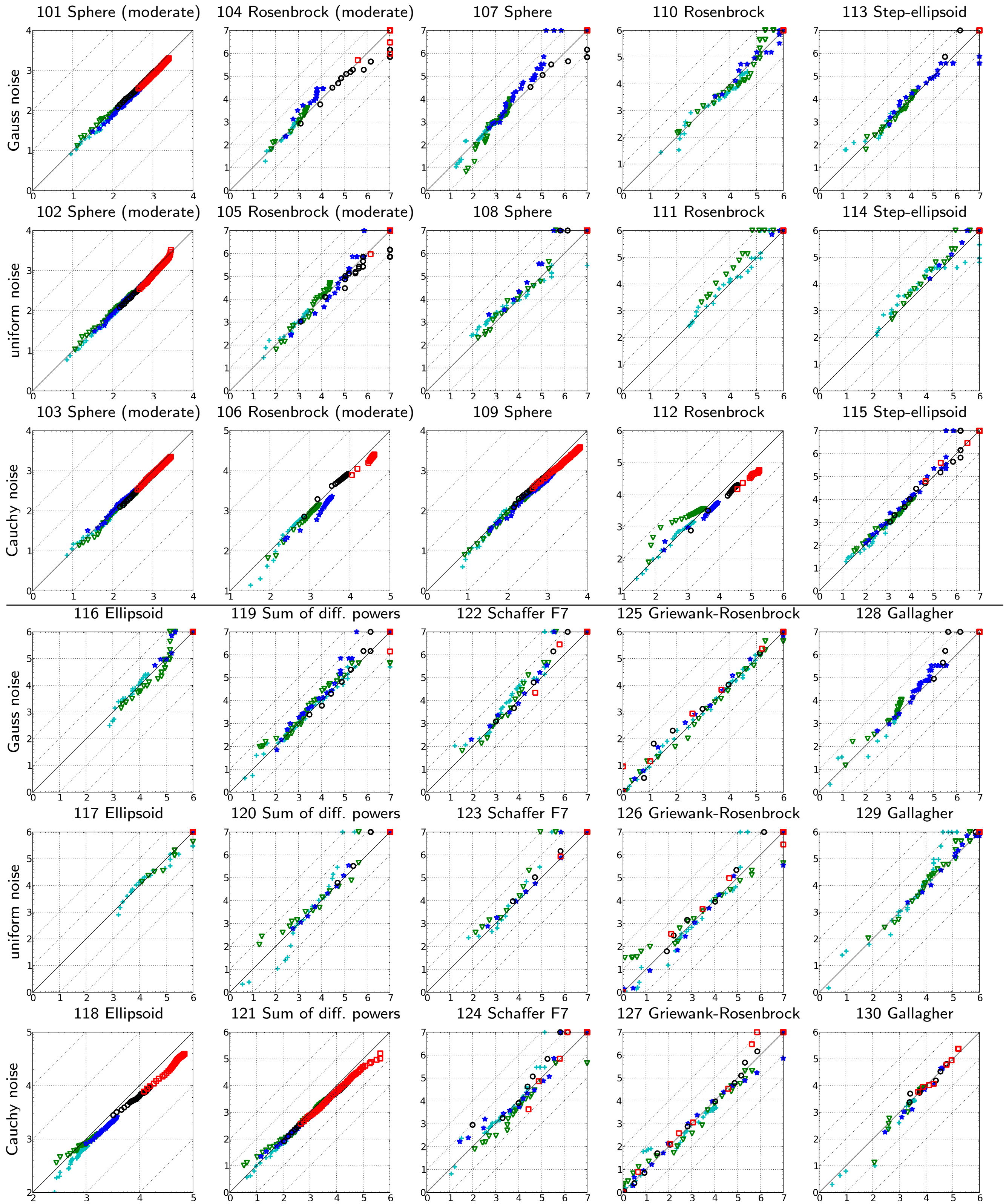

Figure 2: Expected running time (ERT in log10 of number of function evaluations) of (1,4^{s})-CMA-ES versus (1,4)-CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_{101}–f_{130}. Markers on the upper or right edge indicate that the target value was never reached by (1,4^{s})-CMA-ES or (1,4)-CMA-ES respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□.

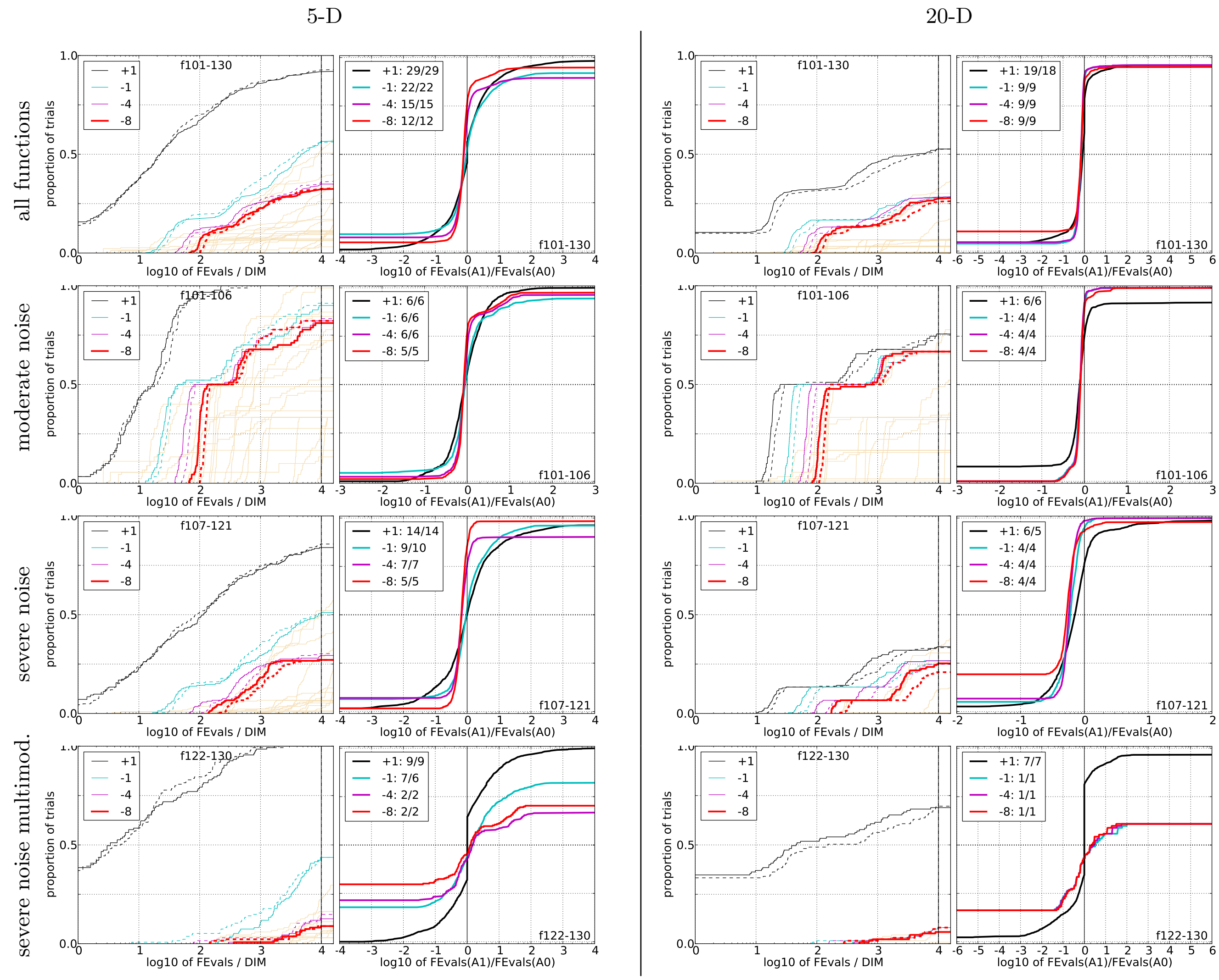

Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of necessary function evaluations divided by dimension D (FEvals/D) to reached a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for (1,4$^{\mathrm{s}}$)-CMA-ES (solid) and (1,4)-CMA-ES (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of (1,4$^{\mathrm{s}}$)-CMA-ES divided by (1,4)-CMA-ES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial ((1,4$^{\mathrm{s}}$)-CMA-ES first).

5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_{101}}$	11	37	44	62	69	75	15/15
(1,4)-CMA-ES	2.8	2.8	3.6	4.6	6.1	7.1	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	2.7	2.1	2.7	$\mathbf{3.5}^{\star}$	$\mathbf{4.4}^{\star 3}$	$\mathbf{5.3}^{\star 3}$	15/15
$\mathbf{f_{102}}$	11	35	50	72	86	99	15/15
(1,4)-CMA-ES	3.2	2.5	2.9	3.7	4.6	5.3	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	2.8	2.1	2.4	$\mathbf{3.2}^{\star}$	$\mathbf{3.6}^{\star 3}$	$\mathbf{4.4}^{\star 2}$	15/15
$\mathbf{f_{103}}$	11	28	30	31	35	120	15/15
(1,4)-CMA-ES	2.1	3	4.4	8.5	12	5	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	2.9	2.9	4.8	8	10	4.2	15/15
$\mathbf{f_{104}}$	170	770	1300	1800	2000	2300	15/15
(1,4)-CMA-ES	1.6	4.2	4.9	6.6	5.8	5.3	14/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.3	8.9	14	16	14	13	13/15
$\mathbf{f_{105}}$	170	1400	5200	1.0e4	1.1e4	1.1e4	15/15
(1,4)-CMA-ES	2.8	12	14	34	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.6	6.6	16	70	∞	∞ *5.0e4*	0/15
$\mathbf{f_{106}}$	86	530	1100	2700	2900	3100	15/15
(1,4)-CMA-ES	2.8	3.6	2.4	1.2	1.1	1.1	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	2.3	1.8	1.5	$0.74^{\downarrow}$	$0.74^{\downarrow}$	$0.73^{\downarrow}$	15/15
$\mathbf{f_{107}}$	40	230	450	940	1400	1900	15/15
(1,4)-CMA-ES	13	8.9	13	57	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	14	10	26	120	∞	∞ *5.0e4*	0/15
$\mathbf{f_{108}}$	87	5100	1.4e4	3.1e4	5.9e4	8.1e4	15/15
(1,4)-CMA-ES	5.9	15	51	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	25	69	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{109}}$	11	57	220	570	870	950	15/15
(1,4)-CMA-ES	3.4	2.2	1	1	1.1	1.4	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	3	1.3	0.79	$0.68^{\downarrow}$	$\mathbf{0.66}^{\star 3 \downarrow 2}$	$\mathbf{0.88}^{\star 3}$	15/15
$\mathbf{f_{110}}$	950	3.4e4	1.2e5	5.9e5	6.0e5	6.1e5	15/15
(1,4)-CMA-ES	3	1.3	6	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	3.6	1.6	2.8	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{111}}$	6900	6.1e5	8.8e6	2.3e7	3.1e7	3.1e7	3/15
(1,4)-CMA-ES	54	∞	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	100	∞	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{112}}$	110	1700	3400	4500	5100	5600	15/15
(1,4)-CMA-ES	1.8	2.4	1.8	1.7	1.6	1.6	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.8	1.5	1.1	1.1	1.1	1	15/15
$\mathbf{f_{113}}$	130	1900	8100	2.4e4	2.4e4	2.4e4	15/15
(1,4)-CMA-ES	8.8	3.3	11	29	29	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	6	6.4	10	15	15	15	1/15
$\mathbf{f_{114}}$	770	1.5e4	5.6e4	8.3e4	8.3e4	8.5e4	15/15
(1,4)-CMA-ES	18	∞	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	21	∞	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{115}}$	64	490	1800	2600	2600	3000	15/15
(1,4)-CMA-ES	1.7	1.7	5.5	130	130	120	1/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.9	1.8	6.9	89	89	∞ *5.0e4*	0/15
$\mathbf{f_{116}}$	5700	1.4e4	2.2e4	2.7e4	3.0e4	3.2e4	15/15
(1,4)-CMA-ES	6.6	15	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	9.8	∞	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{117}}$	2.7e4	7.6e4	1.1e5	1.4e5	1.7e5	1.9e5	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{118}}$	430	1200	1600	2000	2400	2900	15/15
(1,4)-CMA-ES	2.3	1.4	1.3	1.3	1.3	1.2	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.9	1	0.96	$\mathbf{0.95}^{\star}$	$\mathbf{0.88}^{\star}$	$\mathbf{0.84}^{\star}$	15/15
$\mathbf{f_{119}}$	12	660	1100	1.0e4	3.5e4	5.0e4	15/15
(1,4)-CMA-ES	9.6	2.2	11	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	6	4	12	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{120}}$	16	2900	1.9e4	7.2e4	3.3e5	5.5e5	15/15
(1,4)-CMA-ES	36	16	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	38	15	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{121}}$	8.6	110	270	1600	3900	6200	15/15
(1,4)-CMA-ES	1.7	1.2	0.91	0.96	1.2	1.3	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	2.6	0.98	0.75	$0.62^{\downarrow}$	0.77	$\mathbf{0.8}^{\star}$	15/15
$\mathbf{f_{122}}$	10	1700	9200	3.0e4	5.4e4	1.1e5	15/15
(1,4)-CMA-ES	8.7	11	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	20	12	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{123}}$	11	1.6e4	8.2e4	3.4e5	6.7e5	2.2e6	15/15
(1,4)-CMA-ES	40	44	∞	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	69	46	∞	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{124}}$	9.7	200	1000	2.0e4	4.5e4	9.5e4	15/15
(1,4)-CMA-ES	2.8	21	120	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	17	19	72	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{125}}$	1	1	1	2.4e5	2.4e5	2.5e5	15/15
(1,4)-CMA-ES	1	21	1.3e4	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1	48	2.3e4	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{126}}$	1	1	1	∞	∞	∞	0
(1,4)-CMA-ES	1	530	∞	∞	∞	∞	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.2	280	3.6e5	∞	∞	∞	0/15
$\mathbf{f_{127}}$	1	1	1	3.4e5	3.9e5	4.0e5	15/15
(1,4)-CMA-ES	1	18	7.9e3	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1	17	9.7e3	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{128}}$	110	4200	7800	1.2e4	1.7e4	2.1e4	15/15
(1,4)-CMA-ES	3.1	2.8	2.9	3.6	4.6	7.4	2/15
(1,4$^{\mathrm{s}}$)-CMA-ES	4.3	3.1	4.1	6	13	16	2/15
$\mathbf{f_{129}}$	64	1.1e4	5.9e4	2.8e5	5.1e5	5.8e5	15/15
(1,4)-CMA-ES	32	12	12	∞	∞	∞ *5.0e4*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	37	15	12	∞	∞	∞ *5.0e4*	0/15
$\mathbf{f_{130}}$	55	810	3000	3.3e4	3.4e4	3.5e4	10/15
(1,4)-CMA-ES	5.4	25	12	1.1	1.2	1.2	10/15
(1,4$^{\mathrm{s}}$)-CMA-ES	3.3	14	15	1.4	1.3	1.3	10/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_{101}}$	59	360	510	700	740	780	15/15
(1,4)-CMA-ES	7.4	1.8	1.7	1.9	2.5	2.9	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	6	$\mathbf{1.4}^{\star}$	$\mathbf{1.3}^{\star 2}$	$\mathbf{1.5}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2.3}^{\star 3}$	15/15
$\mathbf{f_{102}}$	230	400	580	920	1200	1400	15/15
(1,4)-CMA-ES	2	1.8	1.6	1.6	1.7	1.8	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	$\mathbf{1.5}^{\star}$	$\mathbf{1.3}^{\star 3}$	$\mathbf{1.3}^{\star 2}$	$\mathbf{1.3}^{\star 2}$	$\mathbf{1.4}^{\star 2}$	$\mathbf{1.6}^{\star}$	15/15
$\mathbf{f_{103}}$	65	420	630	1300	1900	2500	14/15
(1,4)-CMA-ES	6.3	1.5	1.4	1.1	1	1	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	5.2	1.3	1.2	$\mathbf{0.86}^{\star 2 \downarrow}$	$\mathbf{0.83}^{\star 3 \downarrow 3}$	$\mathbf{0.82}^{\star 2 \downarrow 3}$	15/15
$\mathbf{f_{104}}$	2.4e4	8.6e4	1.7e5	1.8e5	1.9e5	2.0e5	15/15
(1,4)-CMA-ES	17	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	21	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{105}}$	1.9e5	6.1e5	6.3e5	6.5e5	6.6e5	6.7e5	15/15
(1,4)-CMA-ES	7.4	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	4.9	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{106}}$	1.1e4	2.2e4	2.4e4	2.5e4	2.6e4	2.7e4	15/15
(1,4)-CMA-ES	0.97	1.5	1.5	1.5	1.5	1.5	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	0.68	0.89	0.91	0.91	0.91	0.91	15/15
$\mathbf{f_{107}}$	8600	1.4e4	1.6e4	2.7e4	5.2e4	6.5e4	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{108}}$	5.8e4	9.7e4	2.0e5	4.5e5	6.3e5	9.0e5	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{109}}$	330	630	1100	2300	3600	5000	15/15
(1,4)-CMA-ES	1.3	1.4	1.3	1.1	1.2	1.2	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.1	$\mathbf{1}^{\star 2}$	$\mathbf{0.89}^{\star 2}$	$\mathbf{0.73}^{\star 3 \downarrow 3}$	$\mathbf{0.7}^{\star 3 \downarrow 3}$	$\mathbf{0.68}^{\star 3 \downarrow 4}$	15/15
$\mathbf{f_{110}}$	∞	∞	∞	∞	∞	∞	0
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
$\mathbf{f_{111}}$	∞	∞	∞	∞	∞	∞	0
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
$\mathbf{f_{112}}$	2.6e4	6.4e4	7.0e4	7.4e4	7.6e4	7.8e4	15/15
(1,4)-CMA-ES	1.4	1.6	2.1	2.3	2.3	2.3	11/15
(1,4$^{\mathrm{s}}$)-CMA-ES	$\mathbf{0.59}^{\star 3 \downarrow 3}$	$\mathbf{0.63}^{\star \downarrow 2}$	$\mathbf{0.69}^{\star 2 \downarrow 2}$	$\mathbf{0.72}^{\star 2 \downarrow 2}$	$\mathbf{0.73}^{\star 2 \downarrow 2}$	$\mathbf{0.74}^{\star 2 \downarrow}$	15/15
$\mathbf{f_{113}}$	5.0e4	3.6e5	5.6e5	5.9e5	5.9e5	5.9e5	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{114}}$	2.1e5	1.1e6	1.4e6	1.6e6	1.6e6	1.6e6	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{115}}$	2400	3.0e4	9.2e4	1.3e5	1.3e5	1.3e5	15/15
(1,4)-CMA-ES	18	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	26	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{116}}$	5.0e5	6.9e5	8.9e5	1.0e6	1.1e6	1.1e6	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{117}}$	1.8e6	2.5e6	2.6e6	2.9e6	3.2e6	3.6e6	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{118}}$	6900	1.2e4	1.8e4	2.6e4	3.0e4	3.3e4	15/15
(1,4)-CMA-ES	1.8	1.9	2.1	1.9	2	2	15/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.1	$\mathbf{1.1}^{\star}$	$\mathbf{1.1}^{\star 2}$	$\mathbf{1.1}^{\star 3}$	$\mathbf{1.2}^{\star 3}$	$\mathbf{1.2}^{\star 3}$	15/15
$\mathbf{f_{119}}$	2800	2.9e4	3.6e4	4.1e5	1.4e6	1.9e6	15/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	500	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{120}}$	3.6e4	1.8e5	2.8e5	1.6e6	6.7e6	1.4e7	13/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{121}}$	250	770	1400	9300	3.4e4	5.7e4	15/15
(1,4)-CMA-ES	1.8	1.5	1.4	$0.87^{\downarrow 4}$	1	2.1	6/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.5	$\mathbf{1.1}^{\star}$	$\mathbf{0.9}^{\star 3}$	$\mathbf{0.55}^{\star 3 \downarrow 4}$	$\mathbf{0.58}^{\star 3 \downarrow 3}$	$\mathbf{0.98}^{\star 2}$	12/15
$\mathbf{f_{122}}$	690	5.2e4	1.4e5	7.9e5	2.0e6	5.8e6	15/15
(1,4)-CMA-ES	78	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	32	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{123}}$	1100	5.3e5	1.5e6	5.3e6	2.7e7	1.6e8	0
(1,4)-CMA-ES	650	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	800	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{124}}$	190	2000	4.1e4	1.3e5	3.9e5	8.0e5	15/15
(1,4)-CMA-ES	140	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	22	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{125}}$	1	1	1	2.5e7	8.0e7	8.1e7	4/15
(1,4)-CMA-ES	1	1.6e5	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1	2.4e5	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{126}}$	1	1	1	∞	∞	∞	0
(1,4)-CMA-ES	1	∞	∞	∞	∞	∞	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1	2.9e6	∞	∞	∞	∞	0/15
$\mathbf{f_{127}}$	1	1	1	4.4e6	7.3e6	7.4e6	15/15
(1,4)-CMA-ES	1	1.1e3	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1	1.1e3	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{128}}$	1.4e5	1.3e7	1.7e7	1.7e7	1.7e7	1.7e7	9/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{129}}$	7.8e6	4.1e7	4.2e7	4.2e7	4.2e7	4.2e7	5/15
(1,4)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
(1,4$^{\mathrm{s}}$)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$\mathbf{f_{130}}$	4900	9.3e4	2.5e5	2.5e5	2.6e5	2.6e5	7/15
(1,4)-CMA-ES	1	0.97	0.63	0.63	0.63	0.63	11/15
(1,4$^{\mathrm{s}}$)-CMA-ES	1.2	0.95	0.95	0.94	0.94	0.94	8/15

Table 1: ERT in number of function evaluations divided by the best ERT measured during BBOB-2009 (given in the respective first row) for the algorithms (1,4)-CMA-ES and (1,4$^{\mathrm{s}}$)-CMA-ES for different Δf values for functions f_{101}–f_{130}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 60.