# Mirrored Variants of the (1,2)-CMA-ES Compared on the Noiseless BBOB-2010 Testbed

[Black-Box Optimization Benchmarking Workshop]

Anne Auger, Dimo Brockhoff, and Nikolaus Hansen
Projet TAO, INRIA Saclay—Ile-de-France
LRI, Bât 490, Univ. Paris-Sud
91405 Orsay Cedex, France
firstname.lastname@inria.fr

## ABSTRACT

Derandomization by means of mirroring has been recently introduced to enhance the performances of $(1,\lambda)$-Evolution-Strategies (ESs) with the aim of designing fast robust local search stochastic algorithms. This paper compares on the BBOB-2010 noiseless benchmark testbed two variants of the (1,2)-CMA-ES where the mirroring method is implemented. Independent restarts are conducted till a total budget of $10^4 D$ function evaluations per trial is reached, where $D$ is the dimension of the search space. The results show that the improved variants increase the success probability on 5 (respectively 7) out of 24 test functions in 20D and at the same time are significantly faster on 9 (10) functions in 20D by a factor of about 2–3 (2–4) for a target value of $10^{-7}$ while in no case, the baseline (1,2)-CMA-ES is significantly faster on any tested target function value in 5D and 20D.

## Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

## General Terms

Algorithms

## 1. INTRODUCTION

Evolution Strategies (ESs) are robust stochastic search algorithms for numerical optimization where the function to be minimized, $f$, maps the continuous search space $\mathbb{R}^D$ into $\mathbb{R}$. Recently, a new derandomization technique replacing the independent sampling of new solutions by mirrored sampling has been introduced to enhance the performances of ESs [1]. While mirrored samples were introduced with the aim of designing fast robust local search algorithms, investigation of convergence speed was mainly carried out on the sphere function [1]. In this paper, we want to assess quantitatively the improvements that can be brought with the mirroring method on a wider range of problems. To do so, we compare on the BBOB-2010 noiseless testbed the (1,2)-CMA-ES with two variants implementing the mirrored samples: first the $(1,2_{\mathrm{m}})$-CMA-ES where every second mutation step is derandomized, and second the $(1,2_{\mathrm{m}}^{\mathrm{s}})$-CMA-ES that in addition to the mirroring idea implements sequential selection [1]. Both variants are described in Sec. 2.



## 2. THE ALGORITHMS TESTED

The three algorithms tested are variants of the well-known CMA-ES [8] where at each iteration $n$, $\lambda$ new solutions are generated by sampling *independently* $\lambda$ random vectors $(\mathcal{N}_i(\mathbf{0}, \boldsymbol{C}_n))_{1\leq i\leq\lambda}$ following a multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{C}_n$. The vectors are added to the current solution $\boldsymbol{X}_n$ to create the $\lambda$ new solutions or offspring $\boldsymbol{X}_n^i = \boldsymbol{X}_n + \sigma_n\mathcal{N}_i(\mathbf{0}, \boldsymbol{C}_n)$, where $\sigma_n$ is the strictly positive step-size. In the standard (1,2)-CMA-ES, the number of offspring $\lambda$ equals 2 and $\boldsymbol{X}_{n+1}$ is set to the best solution among $\boldsymbol{X}_n^1$ and $\boldsymbol{X}_n^2$, i.e., $\boldsymbol{X}_{n+1} = \mathrm{argmin}\{f(\boldsymbol{X}_n^1), f(\boldsymbol{X}_n^2)\}$.

In the mirrored variant, denoted $(1,2_{\mathrm{m}})$-CMA-ES, the second offspring is symmetric to the first offspring with respect to $\boldsymbol{X}_n$, namely $\boldsymbol{X}_n^2 = \boldsymbol{X}_n - \sigma_n\mathcal{N}_1(\mathbf{0}, \boldsymbol{C}_n)$, where $\sigma_n\mathcal{N}_1(\mathbf{0}, \boldsymbol{C}_n)$ is the random vector *added* to $\boldsymbol{X}_n$ to create $\boldsymbol{X}_n^1$. We see that the first and second added vector are negatively correlated (with correlation coefficient one). The update of $\boldsymbol{X}_{n+1}$ is then identical to the (1,2)-CMA-ES, namely $\boldsymbol{X}_{n+1} = \mathrm{argmin}\{f(\boldsymbol{X}_n^1), f(\boldsymbol{X}_n^2)\}$.

In the $(1,2_{\mathrm{m}}^{\mathrm{s}})$-CMA-ES, sequential selection is implemented. The offspring solutions are generated with mirrored sampling. Evaluations are carried out in a sequential manner. After evaluating $\boldsymbol{X}_n^1$, it is compared to $\boldsymbol{X}_n$ and if $f(\boldsymbol{X}_n^1) \leq f(\boldsymbol{X}_n)$, the sequence of evaluations is stopped and $\boldsymbol{X}_{n+1} = \boldsymbol{X}_n^1$. In case both offspring are worse than $\boldsymbol{X}_n$, $\boldsymbol{X}_{n+1} = \mathrm{argmin}\{f(\boldsymbol{X}_n^1), f(\boldsymbol{X}_n^2)\}$ according to the comma selection. The number of offspring evaluated is a random variable ranging from 1 to 2—reducing the number of offspring adaptively as long as improvements are easy to achieve [1].

Covariance matrix and step-size are updated using the selected steps [8, 1].

**Independent restarts:** Similar to [2], we independently restarted the (1,2)-CMA-ES, $(1,2_{\mathrm{m}})$-CMA-ES and $(1,2_{\mathrm{m}}^{\mathrm{s}})$-

CMA-ES as long as function evaluations were left, where $10^4 \cdot D$ was the maximal number of function evaluations.

### 2.1 Parameter setting

We used the default parameter and termination settings (cf. [1, 4, 7]) found in the source code on the WWW[1] with two exceptions. We rectified the learning rate of the rank-one update of the covariance matrix for small values of $\lambda$, setting $c_1 = \min(2, \lambda/3)/((D+1.3)^2 + \mu_{\text{eff}})$. The original value was not designed to work for $\lambda < 5$. We modified the damping parameter for the step-size to $d_\sigma = 0.3 + 2\mu_{\text{eff}}/\lambda + c_\sigma$. The setting was found by performing experiments on the sphere function, $f_1$: $d_\sigma$ was set as large as possible while still showing close to optimal performance, but, at least as large such that decreasing it by a factor of two did not lead to inacceptable performance. For $\mu_{\text{eff}}/\lambda = 0.35$ and $\mu_{\text{eff}} \leq D + 2$ the former setting of $d_\sigma$ is recovered. For a smaller ratio of $\mu_{\text{eff}}/\lambda$ or for $\mu_{\text{eff}} > D + 2$, the new setting allows larger (i.e. faster) changes of $\sigma$. Here, $\mu_{\text{eff}} = 1$. For $\lambda \geq 3$, the new setting might be harmful in a noisy or too rugged landscape. Finally, the step-size multiplier was clamped from above at $\exp(1)$, while we do not believe this had any effect in the presented experiments. Each initial solution $\boldsymbol{X}_0$ was uniformly sampled in $[-4, 4]^D$ and the step-size $\sigma_0$ was initialized to 2. The source code used for the experiments is available at[2].

As the same parameter setting has been used in all experiments for all test functions, the crafting effort CrE of all three algorithms is 0.

## 3. CPU TIMING EXPERIMENTS

For the timing experiment, all three algorithms were run on $f_8$ with a maximum of $10^4 D$ function evaluations and restarted until at least 30 seconds have passed (according to Figure 2 in [5]). The experiments have been conducted with an 8 core Intel Xeon E5520 machine with 2.27 GHz under Ubuntu 9.1 linux and Matlab R2008a. The time per function evaluation was 5.9; 5.9; 6.2; 6.1; 6.8; 9.1 times $10^{-4}$ seconds for (1,2)-CMA-ES, 6.6; 6.0; 5.9; 6.0; 6.8; 8.3 times $10^{-4}$ seconds for $(1,2_{\text{m}})$-CMA-ES, and 8.4; 8.6; 8.6; 9.2; 9.2; 11 times $10^{-4}$ seconds for $(1,2_{\text{m}}^{\text{s}})$-CMA-ES in dimensions 2; 3; 5; 10; 20; 40 respectively. Note that MATLAB distributes the computations over all 8 cores only for 20D and 40D.

## 4. RESULTS

### 4.1 Comparing (1,2)- and $(1,2_{\text{m}})$-CMA-ES

Results from experiments comparing (1,2)-CMA-ES and $(1,2_{\text{m}})$-CMA-ES according to [5] on the benchmark functions given in [3, 6] are presented in Figures 1 and 2 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_{\text{t}} = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach $f_{\text{t}}$, summed over all trials and divided by the number of trials that actually reached $f_{\text{t}}$ [5, 9]. **Statistical significance** is tested with the rank-sum test for a given target $\Delta f_{\text{t}}$ using, for each trial, either the number of needed function evaluations to reach $\Delta f_{\text{t}}$ (inverted and multiplied by $-1$), or, if the target was not reached, the best $\Delta f$-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

The experiments show a big improvement of the $(1,2_{\text{m}})$-CMA-ES over the (1,2)-CMA-ES, in particular on the sphere ($f_1$, in 20D 44% faster), ellipsoid ($f_2$ and $f_{10}$, both about 2 times faster), Rosenbrock ($f_8$ and $f_9$, 2–3 times faster), as well as on $f_{11}$ (42% better), $f_{12}$ (about 50% better), and $f_{14}$ (factor of 2.4 better). Figure 2 suggests that the speedup is larger in 20D than in 5D where the largest improvement can be seen for the moderate and separable functions.

The attractive sector function $f_6$, that is not solved by the (1,2)-CMA-ES, is reliably solved (for 13 of the 15 instances) by the mirrored version up to the target value $10^{-8}$ (Table 1). The success probability for the $(1,2_{\text{m}})$-CMA-ES is also slightly higher on the Gallagher functions $f_{21}$ and $f_{22}$ but the differences in expected running time are not statistically significant. Overall, the $(1,2_{\text{m}})$-CMA-ES outperforms the (1,2)-CMA-ES statistically significantly on 9 (respectively 10) out of the 24 functions in 20D (5D) and is never worse. However, there are still 12 problems that the $(1,2_{\text{m}})$-CMA-ES cannot solve in dimension 20 within $10^4 D$ function evaluations, compared to 13 problems for the (1,2)-CMA-ES; in 5D, the (1,2)-CMA-ES has no successful run for 11 test problems and the $(1,2_{\text{m}})$-CMA-ES for 9.

### 4.2 Comparing $(1,2_{\text{m}})$- and $(1,2_{\text{m}}^{\text{s}})$-CMA-ES

The results from experiments comparing the (1,2)-CMA-ES and the $(1,2_{\text{m}}^{\text{s}})$-CMA-ES according to [5] on the benchmark functions given in [3, 6] are presented in Figures 3 and 4 and in Table 2. The statistical tests and the definition of the ERT is the same than above.

The results indicate that the $(1,2_{\text{m}}^{\text{s}})$-CMA-ES is even faster than the $(1,2_{\text{m}})$-CMA-ES for 11 functions in 20D of which only 6 functions show a significant outperformance ($p \leq 0.05$). The largest speedups are on the sphere function, on $f_2$ (on both functions approx. 10% faster), on $f_6$ (about a factor of 2 faster, but not significant), and on $f_8$ (factor of 1.6 faster). On the other hand, the $(1,2_{\text{m}})$-CMA-ES is never significantly faster (only on $f_{13}$ and $f_{21}$ it is slightly faster than the $(1,2_{\text{m}}^{\text{s}})$-CMA-ES which is not statistically significant). Results on 5D are similar, although the difference between the algorithms is larger in higher dimensions.

### 4.3 Comparing (1,2)- and $(1,2_{\text{m}}^{\text{s}})$-CMA-ES

Regarding this comparison, we cannot show the results in the same form as above due to space limitations but can state that, in principle, the results are comparable to the first comparison above—showing an even larger improvement for the $(1,2_{\text{m}}^{\text{s}})$-CMA-ES here.

## 5. CONCLUSIONS

The idea behind derandomization by means of mirroring introduced in [1] is to use only one random sample from a multivariate normal distribution to create two (negatively correlated or *mirrored*) offspring. Thereby, the first offspring is generated by adding a random sample to the parent solution and the second offspring then equals the solution which is symmetric to the first offspring with respect to the parent (by adding the negative sample to the parent). Here, this concept of mirroring has been integrated within two variants of a simple (1,2)-CMA-ES (of which one uses se-

[1] `cmaes.m`, version `3.41.beta`, from `http://www.lri.fr/~hansen/cmaes_inmatlab.html`

[2] `http://coco.gforge.inria.fr/doku.php?id=bbob-2010-results`

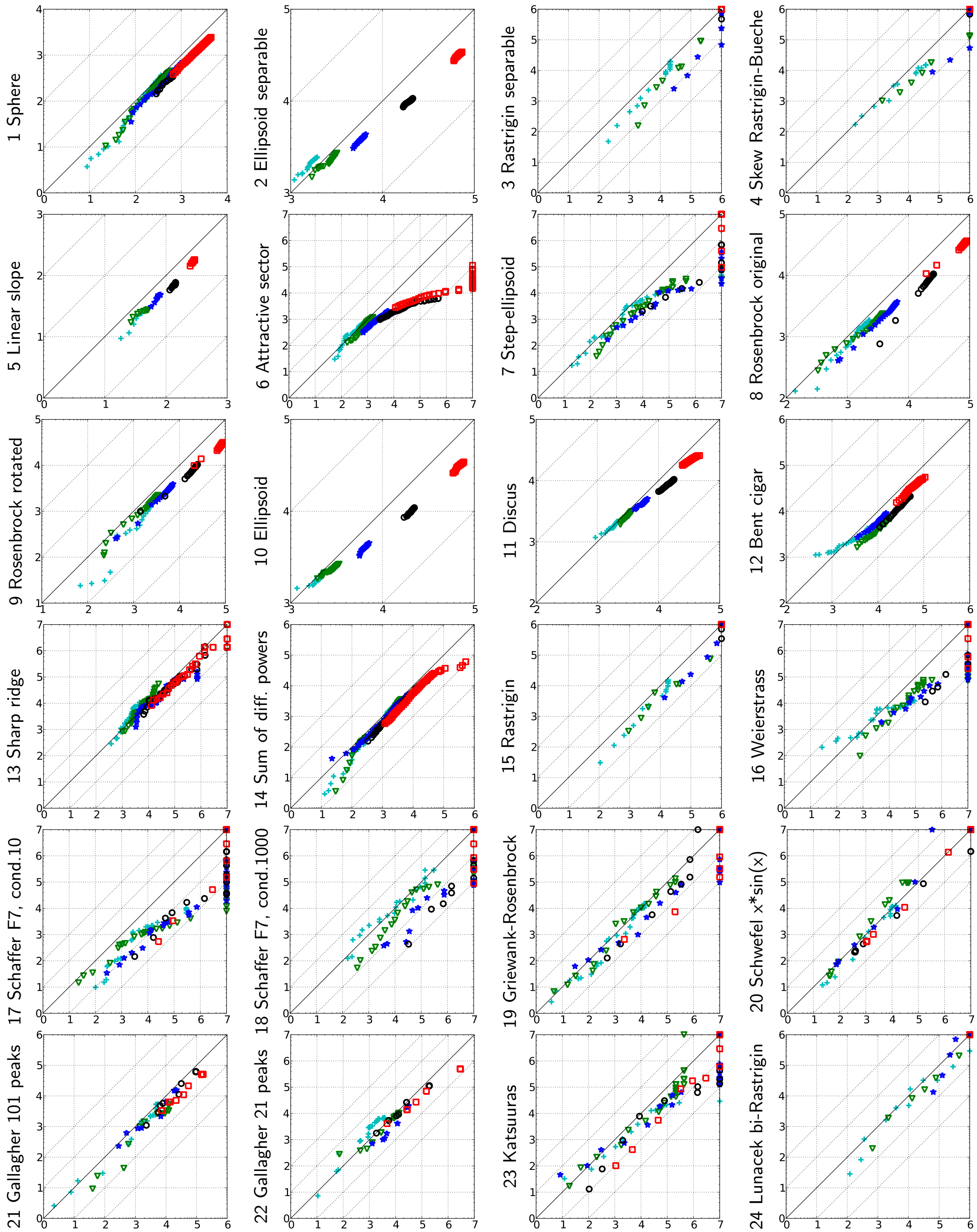


**Figure 1: Expected running time (ERT in log10 of number of function evaluations) of (1,2$_\mathrm{m}$)-CMA-ES versus (1,2)-CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions $f_1$–$f_{24}$. Markers on the upper or right edge indicate that the target value was never reached by (1,2$_\mathrm{m}$)-CMA-ES or (1,2)-CMA-ES respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□.**

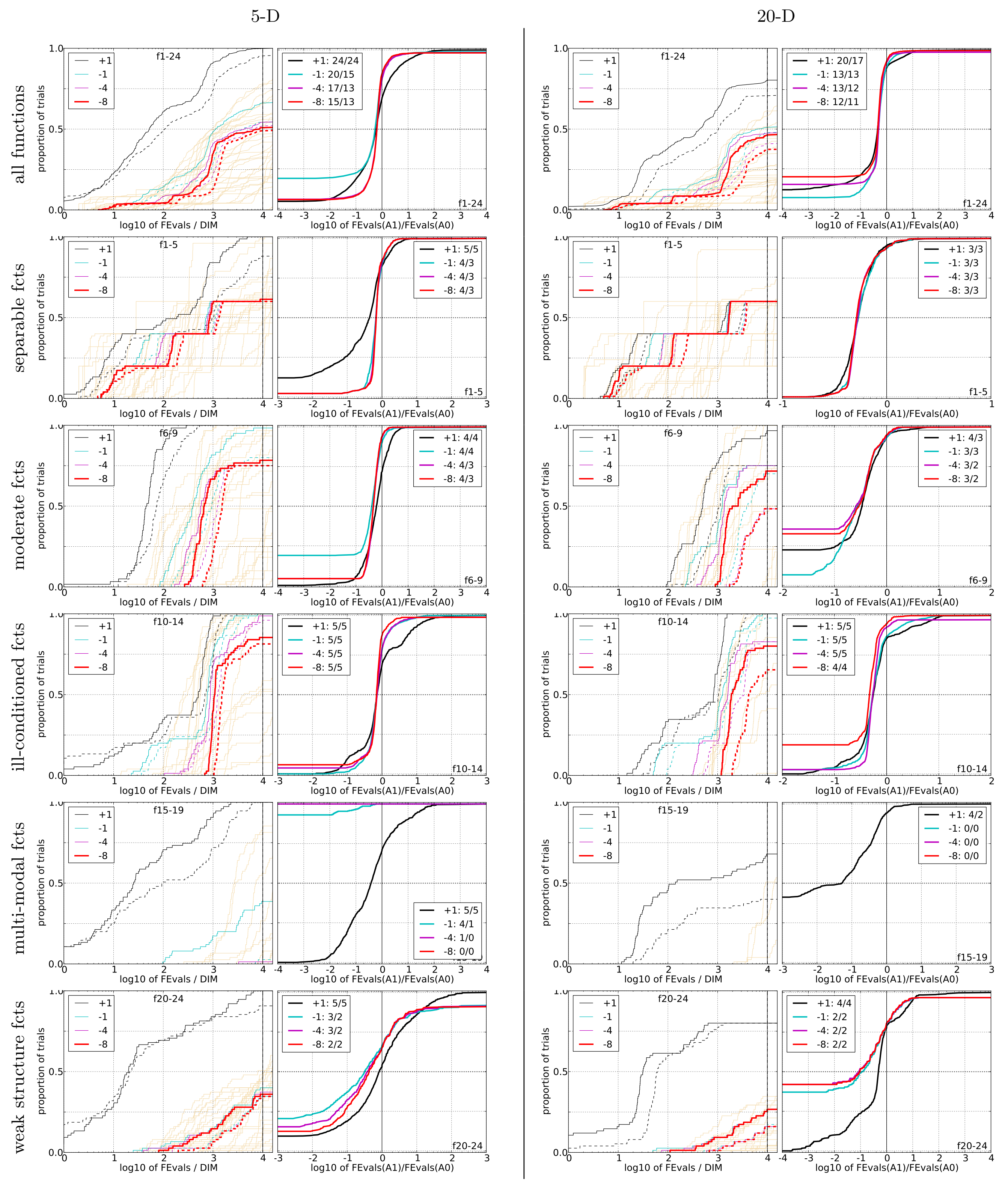


**Figure 2: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension $D$ (FEvals/D) to reach a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for (1,2$_{\mathrm{m}}$)-CMA-ES (solid) and (1,2)-CMA-ES (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of (1,2$_{\mathrm{m}}$)-CMA-ES divided by (1,2)-CMA-ES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being $> 0$ or $< 1$. The legends indicate the number of functions that were solved in at least one trial ((1,2$_{\mathrm{m}}$)-CMA-ES first).**

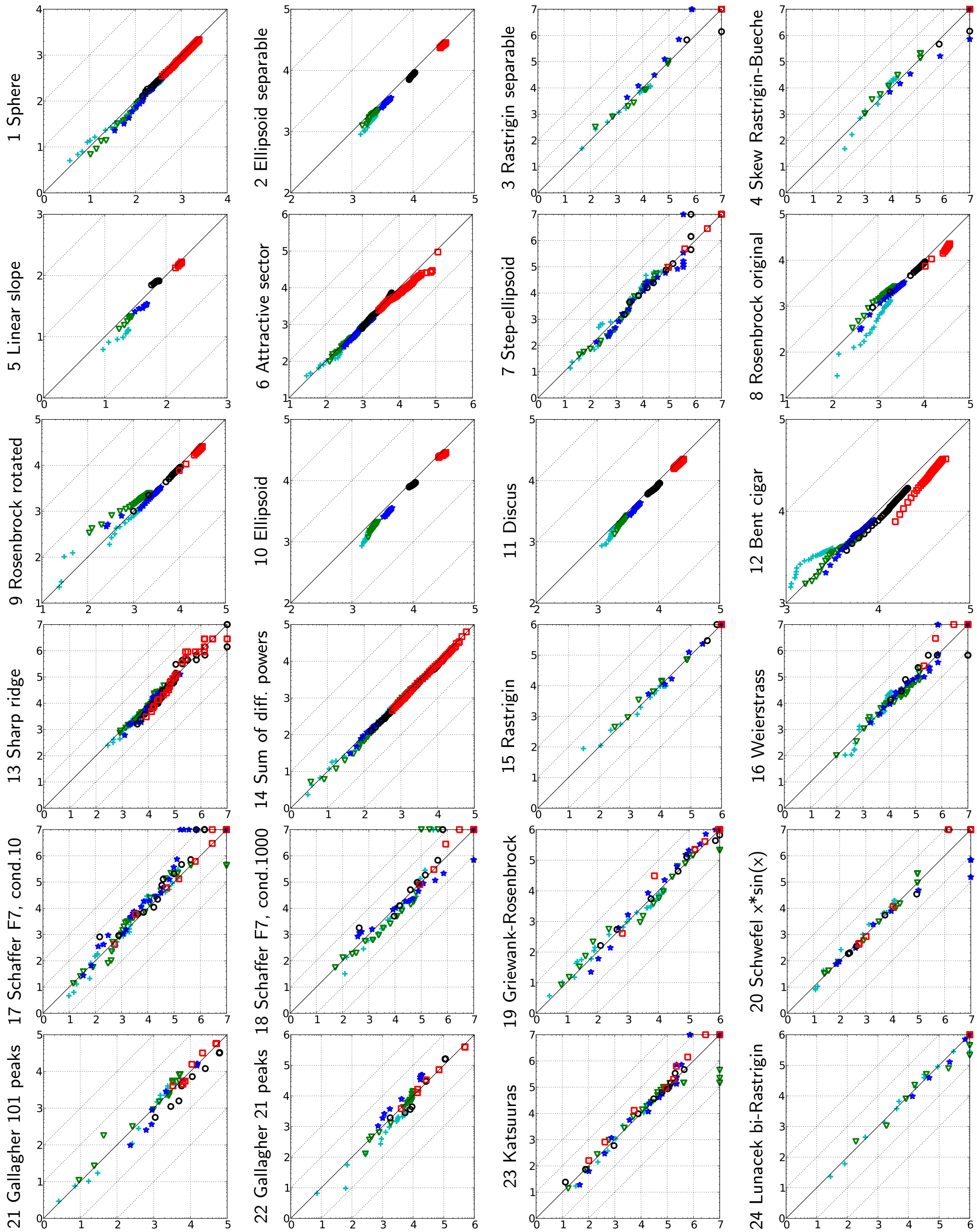


**Figure 3: Expected running time (ERT in log10 of number of function evaluations) of (1,2$^{\mathrm{s}}_{\mathrm{m}}$)-CMA-ES versus (1,2$_{\mathrm{m}}$)-CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions $f_1$–$f_{24}$. Markers on the upper or right edge indicate that the target value was never reached by (1,2$^{\mathrm{s}}_{\mathrm{m}}$)-CMA-ES or (1,2$_{\mathrm{m}}$)-CMA-ES respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□.**

5-D

| $\Delta f$ | 1e+1 | 1e+0 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|---|
| **f1** | 11 | 12 | 12 | 12 | 12 | 12 | 15/15 |
| (1,2)-CMA-ES | 7.4 | 15 | 27 | 47 | 63 | 80 | 15/15 |
| (1,2m)-CMA-ES | 3.2 | 10 | **15**$^{\star 2}$ | **28**$^{\star 3}$ | **40**$^{\star 3}$ | **53**$^{\star 3}$ | 15/15 |
| **f2** | 83 | 87 | 88 | 90 | 92 | 94 | 15/15 |
| (1,2)-CMA-ES | 57 | 61 | 64 | 66 | 68 | 69 | 15/15 |
| (1,2m)-CMA-ES | **37**$^{\star}$ | **40**$^{\star 2}$ | **42**$^{\star 3}$ | **44**$^{\star 3}$ | **45**$^{\star 3}$ | **46**$^{\star 3}$ | 15/15 |
| **f3** | 720 | 1600 | 1600 | 1600 | 1700 | 1700 | 15/15 |
| (1,2)-CMA-ES | 39 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | **3.5**$^{\star 3}$ | **150**$^{\star 2}$ | **450**$^{\star 2}$ | **450**$^{\star 2}$ | **450**$^{\star 2}$ | **450**$^{\star 2}$ | 1/15 |
| **f4** | 810 | 1600 | 1700 | 1800 | 1900 | 1900 | 15/15 |
| (1,2)-CMA-ES | 75 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | **11**$^{\star}$ | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **f5** | 10 | 10 | 10 | 10 | 10 | 10 | 15/15 |
| (1,2)-CMA-ES | 5.7 | 7.5 | 8.1 | 8.1 | 8.1 | 8.1 | 15/15 |
| (1,2m)-CMA-ES | 3.1 | 4.8 | 4.8 | 4.9 | 4.9 | 4.9 | 15/15 |
| **f6** | 110 | 210 | 280 | 580 | 1000 | 1300 | 15/15 |
| (1,2)-CMA-ES | 5.9 | 5.1 | 5 | 4.3 | 3.9 | 4 | 15/15 |
| (1,2m)-CMA-ES | 2.7 | 2.4 | **2.5**$^{\star 2}$ | **1.8**$^{\star 3}$ | **1.4**$^{\star 3}$ | **1.4**$^{\star 3}$ | 15/15 |
| **f7** | 24 | 320 | 1200 | 1600 | 1600 | 1600 | 15/15 |
| (1,2)-CMA-ES | 19 | 28 | 190 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 7.2 | **5.3**$^{\star 2}$ | **11**$^{\star 2}$ | **220**$^{\star 2}$ | **220**$^{\star 2}$ | **220**$^{\star 2}$ | 2/15 |
| **f8** | 73 | 270 | 340 | 390 | 410 | 420 | 15/15 |
| (1,2)-CMA-ES | 9.7 | 10 | 14 | 15 | 15 | 15 | 15/15 |
| (1,2m)-CMA-ES | 5.6 | 5.7 | **7.2**$^{\star}$ | **8.1**$^{\star 2}$ | **8.4**$^{\star 2}$ | **8.7**$^{\star 2}$ | 15/15 |
| **f9** | 35 | 130 | 210 | 300 | 340 | 370 | 15/15 |
| (1,2)-CMA-ES | 12 | 27 | 24 | 21 | 20 | 19 | 15/15 |
| (1,2m)-CMA-ES | 7.4 | 14 | **12**$^{\star}$ | **11**$^{\star}$ | **11**$^{\star}$ | **10**$^{\star}$ | 15/15 |
| **f10** | 350 | 500 | 570 | 630 | 830 | 880 | 15/15 |
| (1,2)-CMA-ES | 16 | 12 | 11 | 10 | 8.2 | 8 | 15/15 |
| (1,2m)-CMA-ES | **9.4**$^{\star 3}$ | **7.6**$^{\star 3}$ | **6.8**$^{\star 3}$ | **6.5**$^{\star 3}$ | **5.1**$^{\star 3}$ | **5.1**$^{\star 3}$ | 15/15 |
| **f11** | 140 | 200 | 760 | 1200 | 1500 | 1700 | 15/15 |
| (1,2)-CMA-ES | 28 | 25 | 7.3 | 5.1 | 4.3 | 3.9 | 15/15 |
| (1,2m)-CMA-ES | 24 | 20 | **5.6**$^{\star}$ | **3.9**$^{\star 2}$ | **3.3**$^{\star 2}$ | **3**$^{\star 2}$ | 15/15 |
| **f12** | 110 | 270 | 370 | 460 | 1300 | 1500 | 15/15 |
| (1,2)-CMA-ES | 33 | 26 | 24 | 24 | 10 | 9.8 | 15/15 |
| (1,2m)-CMA-ES | 25 | 16 | 14 | 14 | 6 | 5.9 | 15/15 |
| **f13** | 130 | 190 | 250 | 1300 | 1800 | 2300 | 15/15 |
| (1,2)-CMA-ES | 25 | 27 | 37 | 22 | 28 | 98 | 1/15 |
| (1,2m)-CMA-ES | 9.4 | 30 | 34 | 14 | 27 | 37 | 4/15 |
| **f14** | 9.8 | 41 | 58 | 140 | 250 | 480 | 15/15 |
| (1,2)-CMA-ES | 2.3 | 5 | 5.9 | 8.3 | 13 | 13 | 15/15 |
| (1,2m)-CMA-ES | 4.3 | 3.5 | 3.9 | 5.8 | 11 | **9.3**$^{\star 2}$ | 15/15 |
| **f15** | 510 | 9300 | 1.9e4 | 2.0e4 | 2.1e4 | 2.1e4 | 14/15 |
| (1,2)-CMA-ES | 27 | 76 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 8.1 | 26 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **f16** | 120 | 610 | 2700 | 1.0e4 | 1.2e4 | 1.2e4 | 15/15 |
| (1,2)-CMA-ES | 42 | 260 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 16 | **29**$^{\star}$ | **45**$^{\star 2}$ | **69**$^{\star 2}$ | **62**$^{\star 2}$ | ∞ *5.0e4* | 0/15 |
| **f17** | 5.2 | 210 | 900 | 3700 | 6400 | 7900 | 15/15 |
| (1,2)-CMA-ES | 52 | 54 | 400 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 6.7 | **5.3**$^{\star}$ | **6.2**$^{\star 3}$ | **64**$^{\star 3}$ | ∞ | ∞ *5.0e4* | 0/15 |
| **f18** | 100 | 380 | 4000 | 9300 | 1.1e4 | 1.2e4 | 15/15 |
| (1,2)-CMA-ES | 36 | 200 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 3.7 | **28**$^{\star}$ | **23**$^{\star 2}$ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **f19** | 1 | 1 | 240 | 1.2e5 | 1.2e5 | 1.2e5 | 15/15 |
| (1,2)-CMA-ES | 31 | 1.6e4 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 62 | 4.6e3 | 940 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **f20** | 16 | 850 | 3.8e4 | 5.4e4 | 5.5e4 | 5.5e4 | 14/15 |
| (1,2)-CMA-ES | 4.7 | 19 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 4.5 | 11 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **f21** | 41 | 1200 | 1700 | 1700 | 1700 | 1800 | 14/15 |
| (1,2)-CMA-ES | 6.9 | 8.6 | 13 | 13 | 13 | 12 | 14/15 |
| (1,2m)-CMA-ES | 5.7 | 5.8 | 9 | 8.9 | 8.9 | 8.8 | 13/15 |
| **f22** | 71 | 390 | 940 | 1000 | 1000 | 1100 | 14/15 |
| (1,2)-CMA-ES | 18 | 31 | 31 | 30 | 29 | 29 | 12/15 |
| (1,2m)-CMA-ES | 10 | 11 | 20 | 19 | 18 | 18 | 14/15 |
| **f23** | 3 | 520 | 1.4e4 | 3.2e4 | 3.3e4 | 3.4e4 | 15/15 |
| (1,2)-CMA-ES | 2.8 | 100 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 15 | 37 | 16 | 23 | 22 | 21 | 0/15 |
| **f24** | 1600 | 2.2e5 | 6.4e6 | 9.6e6 | 1.3e7 | 1.3e7 | 3/15 |
| (1,2)-CMA-ES | 40 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,2m)-CMA-ES | 8.3 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |

20-D

| $\Delta f$ | 1e+1 | 1e+0 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|---|
| **f1** | 43 | 43 | 43 | 43 | 43 | 43 | 15/15 |
| (1,2)-CMA-ES | 15 | 25 | 34 | 53 | 72 | 93 | 15/15 |
| (1,2m)-CMA-ES | **8.9**$^{\star 3}$ | **15**$^{\star 3}$ | **20**$^{\star 3}$ | **30**$^{\star 3}$ | **41**$^{\star 3}$ | **52**$^{\star 3}$ | 15/15 |
| **f2** | 380 | 390 | 390 | 390 | 390 | 390 | 15/15 |
| (1,2)-CMA-ES | 150 | 170 | 170 | 180 | 180 | 180 | 15/15 |
| (1,2m)-CMA-ES | **71**$^{\star 3}$ | **80**$^{\star 3}$ | **82**$^{\star 3}$ | **84**$^{\star 3}$ | **85**$^{\star 3}$ | **86**$^{\star 3}$ | 15/15 |
| **f3** | 5100 | 7600 | 7600 | 7600 | 7600 | 7700 | 15/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f4** | 4700 | 7600 | 7700 | 7700 | 7800 | 1.4e5 | 9/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f5** | 41 | 41 | 41 | 41 | 41 | 41 | 15/15 |
| (1,2)-CMA-ES | 6.1 | 7 | 7.2 | 7.2 | 7.2 | 7.2 | 15/15 |
| (1,2m)-CMA-ES | **3.5**$^{\star}$ | 4.4 | 4.4 | 4.4 | 4.4 | 4.4 | 15/15 |
| **f6** | 1300 | 2300 | 3400 | 5200 | 6700 | 8400 | 15/15 |
| (1,2)-CMA-ES | 8.9 | 12 | 26 | 570 | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **2.2**$^{\star 3}$ | **1.8**$^{\star 3}$ | **1.8**$^{\star 3}$ | **2.5**$^{\star 3}$ | **3.5**$^{\star 3}$ | **6.7**$^{\star 3}$ | 13/15 |
| **f7** | 1400 | 4300 | 9500 | 1.7e4 | 1.7e4 | 1.7e4 | 15/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **71**$^{\star 3}$ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f8** | 2000 | 3900 | 4000 | 4200 | 4400 | 4500 | 15/15 |
| (1,2)-CMA-ES | 9.5 | 19 | 20 | 20 | 20 | 20 | 14/15 |
| (1,2m)-CMA-ES | **5.3**$^{\star}$ | 7.8 | **8.2**$^{\star}$ | **8.3**$^{\star}$ | **8.2**$^{\star}$ | **8.1**$^{\star}$ | 15/15 |
| **f9** | 1700 | 3100 | 3300 | 3500 | 3600 | 3700 | 15/15 |
| (1,2)-CMA-ES | 12 | 24 | 25 | 25 | 24 | 24 | 15/15 |
| (1,2m)-CMA-ES | **5.8**$^{\star}$ | **8.1**$^{\star 2}$ | **8.5**$^{\star 2}$ | **8.6**$^{\star 2}$ | **8.5**$^{\star 3}$ | **8.4**$^{\star 3}$ | 15/15 |
| **f10** | 7400 | 8700 | 1.1e4 | 1.5e4 | 1.7e4 | 1.7e4 | 15/15 |
| (1,2)-CMA-ES | 7.9 | 7.4 | 6.4 | 4.9 | 4.4 | 4.3 | 15/15 |
| (1,2m)-CMA-ES | **3.5**$^{\star 3}$ | **3.5**$^{\star 3}$ | **2.9**$^{\star 3}$ | **2.2**$^{\star 3}$ | **2**$^{\star 3}$ | **2**$^{\star 3}$ | 15/15 |
| **f11** | 1000 | 2200 | 6300 | 9800 | 1.2e4 | 1.5e4 | 15/15 |
| (1,2)-CMA-ES | 24 | 12 | 4.8 | 3.6 | 3.2 | 2.9 | 15/15 |
| (1,2m)-CMA-ES | **18**$^{\star 3}$ | **8.6**$^{\star 3}$ | **3.3**$^{\star 3}$ | **2.3**$^{\star 3}$ | **2**$^{\star 3}$ | **1.7**$^{\star 3}$ | 15/15 |
| **f12** | 1000 | 1900 | 2700 | 4100 | 1.2e4 | 1.4e4 | 15/15 |
| (1,2)-CMA-ES | 24 | 21 | 19 | 16 | 6.4 | 6.7 | 14/15 |
| (1,2m)-CMA-ES | 15 | 13 | 12 | 9.3 | 3.6 | **3.6**$^{\star}$ | 15/15 |
| **f13** | 650 | 2000 | 2800 | 1.9e4 | 2.4e4 | 3.0e4 | 15/15 |
| (1,2)-CMA-ES | 20 | 25 | 41 | 47 | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | 13 | 12 | 24 | 33 | 55 | 93 | 0/15 |
| **f14** | 75 | 240 | 300 | 930 | 1600 | 1.6e4 | 15/15 |
| (1,2)-CMA-ES | 16 | 8.2 | 8.2 | 7.8 | 14 | 4.8 | 5/15 |
| (1,2m)-CMA-ES | **7.7**$^{\star 2}$ | **3.8**$^{\star 3}$ | **4.1**$^{\star 3}$ | **4.2**$^{\star 3}$ | **8.1**$^{\star 3}$ | **2**$^{\star 3}$ | 15/15 |
| **f15** | 3.0e4 | 1.5e5 | 3.1e5 | 3.2e5 | 4.5e5 | 4.6e5 | 15/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f16** | 1400 | 2.7e4 | 7.7e4 | 1.9e5 | 2.0e5 | 2.2e5 | 15/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **150**$^{\star 3}$ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f17** | 63 | 1000 | 4000 | 3.1e4 | 5.6e4 | 8.0e4 | 15/15 |
| (1,2)-CMA-ES | 400 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **8.6**$^{\star}$ | **2.8e3**$^{\star 3}$ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f18** | 620 | 4000 | 2.0e4 | 6.8e4 | 1.3e5 | 1.5e5 | 15/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **140**$^{\star 3}$ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f19** | 1 | 1 | 3.4e5 | 6.2e6 | 6.7e6 | 6.7e6 | 15/15 |
| (1,2)-CMA-ES | 2.3e3 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **660**$^{\star 2}$ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f20** | 82 | 4.6e4 | 3.1e6 | 5.5e6 | 5.6e6 | 5.6e6 | 14/15 |
| (1,2)-CMA-ES | 13 | 30 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | **6.5**$^{\star 3}$ | 30 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f21** | 560 | 6500 | 1.4e4 | 1.5e4 | 1.6e4 | 1.8e4 | 15/15 |
| (1,2)-CMA-ES | 13 | 8.3 | 10 | 11 | 10 | 9.1 | 11/15 |
| (1,2m)-CMA-ES | 5.9 | 3.3 | 3.6 | 3.4 | 3.3 | 2.9 | 15/15 |
| **f22** | 470 | 5600 | 2.3e4 | 2.5e4 | 2.7e4 | 1.3e5 | 12/15 |
| (1,2)-CMA-ES | 10 | 27 | 120 | 120 | 110 | 22 | 1/15 |
| (1,2m)-CMA-ES | 8.8 | 13 | 21 | 20 | 18 | 3.7 | 5/15 |
| **f23** | 3.2 | 1600 | 6.7e4 | 4.9e5 | 8.1e5 | 8.4e5 | 15/15 |
| (1,2)-CMA-ES | 340 | 1.8e3 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | 32 | **140**$^{\star}$ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **f24** | 1.3e6 | 7.5e6 | 5.2e7 | 5.2e7 | 5.2e7 | 5.2e7 | 3/15 |
| (1,2)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,2m)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |

**Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for the algorithms (1,2)-CMA-ES and is (1,2$_{\rm m}$)-CMA-ES for different $\Delta f$ values for functions $f_1$–$f_{24}$. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\rm opt} + 10^{-8}$. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.**

quential selection [1] in addition) and the three algorithms are compared on the BBOB-2010 testbed.

The results show that using mirroring gives an improvement by a factor of about 2 on seven functions, and by a factor of 3 on the rotated Rosenbrock function. Using mirroring is also never worse than the standard (1,2)-CMA-ES. The improvement is even larger when using the sequential selection of [1] in addition.

## Acknowledgments

This work received support by the French national research agency (ANR) within the SYSCOMM project ANR-08-SYSC-017 and within the COSINUS project ANR-08-COSI-007-12.

## 6. REFERENCES

[1] A. Auger, D. Brockhoff, and N. Hansen. Mirrored sampling and sequential selection for evolution

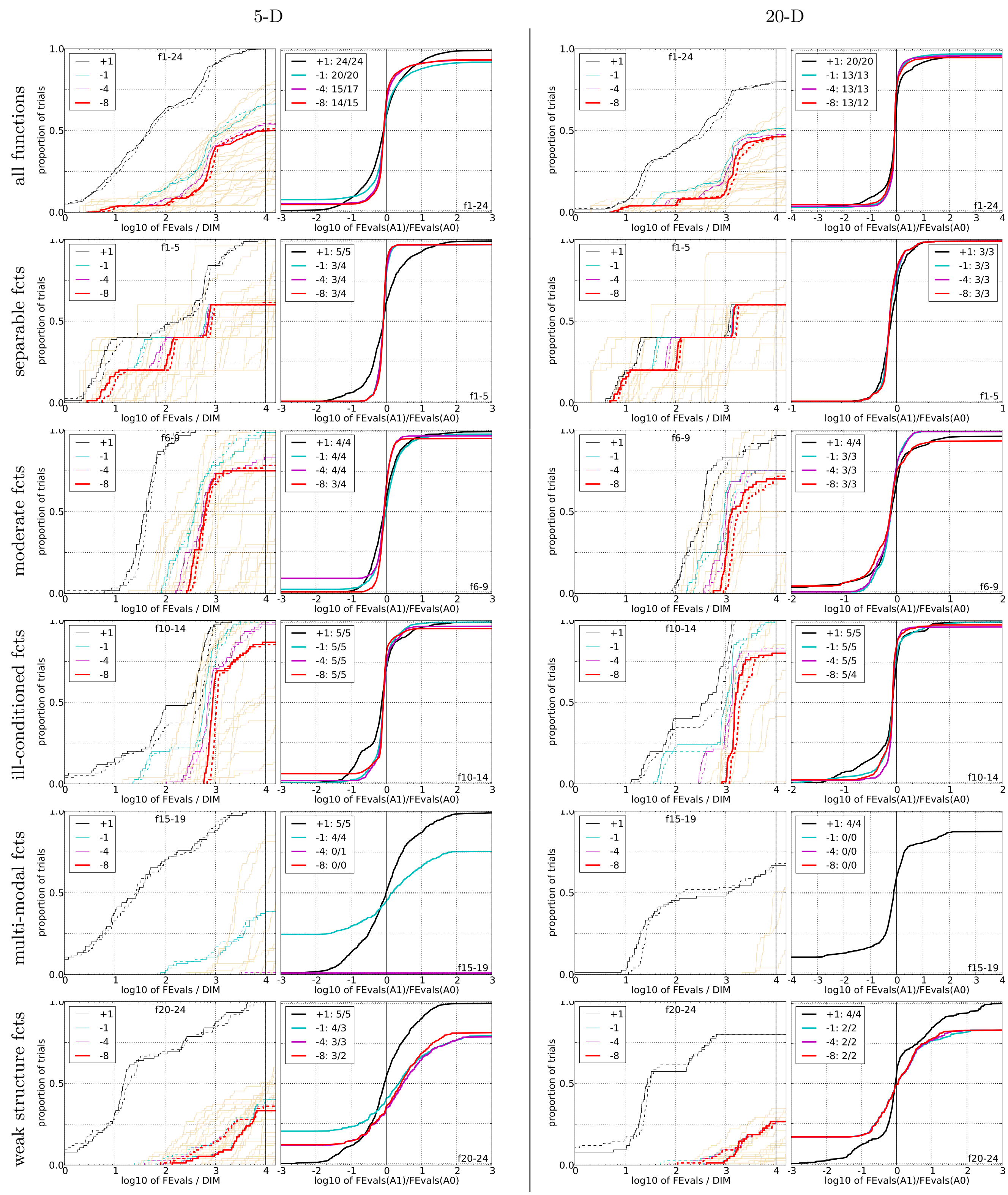


**Figure 4: Empirical cumulative distributions of run lengths for (1,$2^{s}_{m}$)-CMA-ES (solid) and (1,$2_{m}$)-CMA-ES (dashed) and speed-up ratios of the former divided by the latter in 5-D (left) and 20-D (right) as in Fig. 2.**

strategies. Rapport de Recherche RR-7249, INRIA Saclay—Île-de-France, April 2010.

[2] A. Auger and N. Hansen. Performance evaluation of an advanced local search evolutionary algorithm. In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2005)*, pages 1777–1784, 2005.

5-D

| $\Delta f$ | 1e+1 | 1e+0 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|---|
| **$f_1$** | 11 | 12 | 12 | 12 | 12 | 12 | 15/15 |
| (1,$2_m$)-CMA-ES | 3.2 | 10 | 15 | 28 | 40 | 53 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 2.1 | 7.2 | 13 | 24 | 35 | 45 | 15/15 |
| **$f_2$** | 83 | 87 | 88 | 90 | 92 | 94 | 15/15 |
| (1,$2_m$)-CMA-ES | 37 | 40 | 42 | 44 | 45 | 46 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 30 | **34**$^{\star}$ | **35**$^{\star 2}$ | **36**$^{\star 2}$ | **37**$^{\star 3}$ | **37**$^{\star 3}$ | 15/15 |
| **$f_3$** | 720 | 1600 | 1600 | 1600 | 1700 | 1700 | 15/15 |
| (1,$2_m$)-CMA-ES | 3.5 | 150 | 450 | 450 | 450 | 450 | 1/15 |
| (1,$2_m^s$)-CMA-ES | 6.2 | 440 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_4$** | 810 | 1600 | 1700 | 1800 | 1900 | 1900 | 15/15 |
| (1,$2_m$)-CMA-ES | 11 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 8.8 | 450 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_5$** | 10 | 10 | 10 | 10 | 10 | 10 | 15/15 |
| (1,$2_m$)-CMA-ES | 3.1 | 4.8 | 4.8 | 4.9 | 4.9 | 4.9 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 2.6 | 3.3 | 3.5 | 3.5 | 3.5 | 3.5 | 15/15 |
| **$f_6$** | 110 | 210 | 280 | 580 | 1000 | 1300 | 15/15 |
| (1,$2_m$)-CMA-ES | 2.7 | 2.4 | 2.5 | 1.8 | 1.4 | 1.4 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 2.2 | 2 | 2.1 | 1.6 | **1.1**$^{\star}$ | **1.1**$^{\star}$ | 15/15 |
| **$f_7$** | 24 | 320 | 1200 | 1600 | 1600 | 1600 | 15/15 |
| (1,$2_m$)-CMA-ES | 7.2 | 5.3 | 11 | 220 | 220 | 220 | 2/15 |
| (1,$2_m^s$)-CMA-ES | 5.8 | 5 | 17 | 81 | 81 | 100 | 0/15 |
| **$f_8$** | 73 | 270 | 340 | 390 | 410 | 420 | 15/15 |
| (1,$2_m$)-CMA-ES | 5.6 | 5.7 | 7.2 | 8.1 | 8.4 | 8.7 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 4.2 | 5.8 | 6.8 | 7.5 | 7.6 | 7.8 | 15/15 |
| **$f_9$** | 35 | 130 | 210 | 300 | 340 | 370 | 15/15 |
| (1,$2_m$)-CMA-ES | 7.4 | 14 | 12 | 11 | 11 | 10 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 14 | 12 | 11 | 9.6 | 9.2 | 8.8 | 15/15 |
| **$f_{10}$** | 350 | 500 | 570 | 630 | 830 | 880 | 15/15 |
| (1,$2_m$)-CMA-ES | 9.4 | 7.6 | 6.8 | 6.5 | 5.1 | 5.1 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 7.3 | **5.8**$^{\star 2}$ | **5.4**$^{\star 2}$ | **5.2**$^{\star 2}$ | **4.1**$^{\star 3}$ | **4**$^{\star 3}$ | 15/15 |
| **$f_{11}$** | 140 | 200 | 760 | 1200 | 1500 | 1700 | 15/15 |
| (1,$2_m$)-CMA-ES | 24 | 20 | 5.6 | 3.9 | 3.3 | 3 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 19 | 16 | 4.8 | 3.3 | **2.8**$^{\star}$ | **2.6**$^{\star}$ | 15/15 |
| **$f_{12}$** | 110 | 270 | 370 | 460 | 1300 | 1500 | 15/15 |
| (1,$2_m$)-CMA-ES | 25 | 16 | 14 | 14 | 6 | 5.9 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 20 | 15 | 14 | 13 | 5.4 | 5.2 | 15/15 |
| **$f_{13}$** | 130 | 190 | 250 | 1300 | 1800 | 2300 | 15/15 |
| (1,$2_m$)-CMA-ES | 9.4 | 30 | 34 | 14 | 27 | 37 | 4/15 |
| (1,$2_m^s$)-CMA-ES | 4.6 | 17 | 21 | 14 | 18 | 36 | 5/15 |
| **$f_{14}$** | 9.8 | 41 | 58 | 140 | 250 | 480 | 15/15 |
| (1,$2_m$)-CMA-ES | 4.3 | 3.5 | 3.9 | 5.8 | 11 | 9.3 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 3.1 | 3.2 | 3.3 | 4.8 | 9.2 | 8.1 | 15/15 |
| **$f_{15}$** | 510 | 9300 | 1.9e4 | 2.0e4 | 2.1e4 | 2.1e4 | 14/15 |
| (1,$2_m$)-CMA-ES | 8.1 | 26 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 11 | 25 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_{16}$** | 120 | 610 | 2700 | 1.0e4 | 1.2e4 | 1.2e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 16 | 29 | 45 | 69 | 62 | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 15 | 43 | 38 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_{17}$** | 5.2 | 210 | 900 | 3700 | 6400 | 7900 | 15/15 |
| (1,$2_m$)-CMA-ES | 6.7 | 5.3 | 6.2 | 64 | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 5.3 | 5 | 13 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_{18}$** | 100 | 380 | 4000 | 9300 | 1.1e4 | 1.2e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 3.7 | 28 | 23 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 8.2 | 27 | 21 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_{19}$** | 1 | 1 | 240 | 1.2e5 | 1.2e5 | 1.2e5 | 15/15 |
| (1,$2_m$)-CMA-ES | 62 | 4.6e3 | 940 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 22 | 8.5e3 | 1.4e3 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_{20}$** | 16 | 850 | 3.8e4 | 5.4e4 | 5.5e4 | 5.5e4 | 14/15 |
| (1,$2_m$)-CMA-ES | 4.5 | 11 | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 4.7 | 9.3 | 18 | 13 | 13 | 13 | 1/15 |
| **$f_{21}$** | 41 | 1200 | 1700 | 1700 | 1700 | 1800 | 14/15 |
| (1,$2_m$)-CMA-ES | 5.7 | 5.8 | 9 | 8.9 | 8.9 | 8.8 | 13/15 |
| (1,$2_m^s$)-CMA-ES | 2.4 | 5 | 9.5 | 9.4 | 9.3 | 9.2 | 15/15 |
| **$f_{22}$** | 71 | 390 | 940 | 1000 | 1000 | 1100 | 14/15 |
| (1,$2_m$)-CMA-ES | 10 | 11 | 20 | 19 | 18 | 18 | 14/15 |
| (1,$2_m^s$)-CMA-ES | 15 | 21 | 51 | 48 | 47 | 46 | 9/15 |
| **$f_{23}$** | 3 | 520 | 1.4e4 | 3.2e4 | 3.3e4 | 3.4e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 15 | 37 | 16 | 23 | 22 | 21 | 0/15 |
| (1,$2_m^s$)-CMA-ES | 6.4 | 23 | 9.5 | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| **$f_{24}$** | 1600 | 2.2e5 | 6.4e6 | 9.6e6 | 1.3e7 | 1.3e7 | 3/15 |
| (1,$2_m$)-CMA-ES | 8.3 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 6 | ∞ | ∞ | ∞ | ∞ | ∞ *5.0e4* | 0/15 |

20-D

| $\Delta f$ | 1e+1 | 1e+0 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|---|
| **$f_1$** | 43 | 43 | 43 | 43 | 43 | 43 | 15/15 |
| (1,$2_m$)-CMA-ES | 8.9 | 15 | 20 | 30 | 41 | 52 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 8.1 | **13**$^{\star}$ | 18 | 27 | **36**$^{\star}$ | **47**$^{\star}$ | 15/15 |
| **$f_2$** | 380 | 390 | 390 | 390 | 390 | 390 | 15/15 |
| (1,$2_m$)-CMA-ES | 71 | 80 | 82 | 84 | 85 | 86 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 61 | **67**$^{\star 2}$ | **69**$^{\star 2}$ | **70**$^{\star 3}$ | **71**$^{\star 3}$ | **72**$^{\star 3}$ | 15/15 |
| **$f_3$** | 5100 | 7600 | 7600 | 7600 | 7600 | 7700 | 15/15 |
| (1,$2_m$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_4$** | 4700 | 7600 | 7700 | 7700 | 7800 | 1.4e5 | 9/15 |
| (1,$2_m$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_5$** | 41 | 41 | 41 | 41 | 41 | 41 | 15/15 |
| (1,$2_m$)-CMA-ES | 3.5 | 4.4 | 4.4 | 4.4 | 4.4 | 4.4 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 3.3 | 3.9 | 4.1 | 4.1 | 4.1 | 4.1 | 15/15 |
| **$f_6$** | 1300 | 2300 | 3400 | 5200 | 6700 | 8400 | 15/15 |
| (1,$2_m$)-CMA-ES | 2.2 | 1.8 | 1.8 | 2.5 | 3.5 | 6.7 | 13/15 |
| (1,$2_m^s$)-CMA-ES | 1.9 | 1.6 | 1.5 | 1.7 | 2.1 | 3.1 | 12/15 |
| **$f_7$** | 1400 | 4300 | 9500 | 1.7e4 | 1.7e4 | 1.7e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 71 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 71 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_8$** | 2000 | 3900 | 4000 | 4200 | 4400 | 4500 | 15/15 |
| (1,$2_m$)-CMA-ES | 5.3 | 7.8 | 8.2 | 8.3 | 8.2 | 8.1 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 3.6 | **4.5**$^{\star}$ | **4.8**$^{\star 2}$ | **5**$^{\star 2}$ | **5**$^{\star 2}$ | **5**$^{\star 2}$ | 15/15 |
| **$f_9$** | 1700 | 3100 | 3300 | 3500 | 3600 | 3700 | 15/15 |
| (1,$2_m$)-CMA-ES | 5.8 | 8.1 | 8.5 | 8.6 | 8.5 | 8.4 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 4.5 | 6.5 | 6.8 | 6.9 | 6.9 | 6.8 | 15/15 |
| **$f_{10}$** | 7400 | 8700 | 1.1e4 | 1.5e4 | 1.7e4 | 1.7e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 3.5 | 3.5 | 2.9 | 2.2 | 2 | 2 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 3.2 | **3**$^{\star 2}$ | **2.5**$^{\star 2}$ | **1.8**$^{\star 3}$ | **1.6**$^{\star 3}$ | **1.6**$^{\star 3}$ | 15/15 |
| **$f_{11}$** | 1000 | 2200 | 6300 | 9800 | 1.2e4 | 1.5e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 18 | 8.6 | 3.3 | 2.3 | 2 | 1.7 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 16 | 7.7 | 2.9 | 2 | 1.7 | **1.5**$^{\star}$ | 15/15 |
| **$f_{12}$** | 1000 | 1900 | 2700 | 4100 | 1.2e4 | 1.4e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 15 | 13 | 12 | 9.3 | 3.6 | 3.6 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 7.4 | 8 | 7.6 | 6.4 | 2.6 | 2.6 | 15/15 |
| **$f_{13}$** | 650 | 2000 | 2800 | 1.9e4 | 2.4e4 | 3.0e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 13 | 12 | 24 | 33 | 55 | 93 | 0/15 |
| (1,$2_m^s$)-CMA-ES | 4.6 | 6.9 | 23 | 49 | 120 | 94 | 1/15 |
| **$f_{14}$** | 75 | 240 | 300 | 930 | 1600 | 1.6e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 7.7 | 3.8 | 4.1 | 4.2 | 8.1 | 2 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 6 | 3.1 | **3.3**$^{\star}$ | **3.6**$^{\star}$ | **6.6**$^{\star 2}$ | **1.6**$^{\star}$ | 14/15 |
| **$f_{15}$** | 3.0e4 | 1.5e5 | 3.1e5 | 3.2e5 | 4.5e5 | 4.6e5 | 15/15 |
| (1,$2_m$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{16}$** | 1400 | 2.7e4 | 7.7e4 | 1.9e5 | 2.0e5 | 2.2e5 | 15/15 |
| (1,$2_m$)-CMA-ES | 150 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 190 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{17}$** | 63 | 1000 | 4000 | 3.1e4 | 5.6e4 | 8.0e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 8.6 | 2.8e3 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 6.7 | 2.9e3 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{18}$** | 620 | 4000 | 2.0e4 | 6.8e4 | 1.3e5 | 1.5e5 | 15/15 |
| (1,$2_m$)-CMA-ES | 140 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 130 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{19}$** | 1 | 1 | 3.4e5 | 6.2e6 | 6.7e6 | 6.7e6 | 15/15 |
| (1,$2_m$)-CMA-ES | 660 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 410 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{20}$** | 82 | 4.6e4 | 3.1e6 | 5.5e6 | 5.6e6 | 5.6e6 | 14/15 |
| (1,$2_m$)-CMA-ES | 6.5 | 30 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 5.3 | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{21}$** | 560 | 6500 | 1.4e4 | 1.5e4 | 1.6e4 | 1.8e4 | 15/15 |
| (1,$2_m$)-CMA-ES | 5.9 | 3.3 | 3.6 | 3.4 | 3.3 | 2.9 | 15/15 |
| (1,$2_m^s$)-CMA-ES | 7.2 | 4.9 | 4 | 3.9 | 3.7 | 3.3 | 14/15 |
| **$f_{22}$** | 470 | 5600 | 2.3e4 | 2.5e4 | 2.7e4 | 1.3e5 | 12/15 |
| (1,$2_m$)-CMA-ES | 8.8 | 13 | 21 | 20 | 18 | 3.7 | 5/15 |
| (1,$2_m^s$)-CMA-ES | 8.2 | 13 | 17 | 16 | 15 | 3 | 6/15 |
| **$f_{23}$** | 3.2 | 1600 | 6.7e4 | 4.9e5 | 8.1e5 | 8.4e5 | 15/15 |
| (1,$2_m$)-CMA-ES | 32 | 140 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | 50 | 390 | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| **$f_{24}$** | 1.3e6 | 7.5e6 | 5.2e7 | 5.2e7 | 5.2e7 | 5.2e7 | 3/15 |
| (1,$2_m$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |
| (1,$2_m^s$)-CMA-ES | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ *2.0e5* | 0/15 |

**Table 2: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 as in Table 1 but now comparing (1,$2_m$)-CMA-ES and (1,$2_m^s$)-CMA-ES.**


[3] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009. Updated February 2010.

[4] N. Hansen. Benchmarking a BI-population CMA-ES on the BBOB-2009 function testbed. In F. Rothlauf, editor, *GECCO (Companion)*, pages 2389–2396. ACM, 2009.

[5] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2010: Experimental setup. Technical Report RR-7215, INRIA, 2010.

[6] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.

[7] N. Hansen and S. Kern. Evaluating the CMA evolution strategy on multimodal test functions. In X. Yao et al., editors, *Parallel Problem Solving from Nature PPSN VIII*, volume 3242 of *LNCS*, pages 282–291. Springer, 2004.

[8] N. Hansen, S. D. Müller, and P. Koumoutsakos. Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation. *Evolutionary Computation*, 11(1):1–18, 2003.

[9] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.