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Comparing Mirrored Mutations and Active Covariance Matrix Adaptation in the IPOP-CMA-ES on the Noiseless BBOB Testbed

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ABSTRACT

This paper investigates two variants of the well-known Covariance Matrix Adaptation Evolution Strategy (CMA-ES). *Active covariance matrix adaptation* allows for negative weights in the covariance matrix update rule such that "bad" steps are (actively) taken into account when updating the covariance matrix of the sample distribution. On the other hand, *mirrored mutations* via *selective mirroring* also take the "bad" steps into account. In this case, they are first evaluated when taken in the opposite direction (mirrored) and then considered for regular selection. In this study, we investigate the difference between the performance of the two variants empirically on the noiseless BBOB testbed. The CMA-ES with selectively mirrored mutations only outperforms the active CMA-ES on the sphere function while the active variant statistically significantly outperforms mirrored mutations on 10 of 24 functions in several dimensions.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

The covariance matrix adaptation evolution strategy (CMA-ES) is considered as a standard method for stochastic optimization in continuous domain. More recently, mirrored mutations for evolution strategies have been introduced and theoretically investigated in a number of papers [4, 1, 2]. In evolution strategies with weighted recombination and only positive recombination weights, mirrored mutations can improve the possible progress rate on the sphere function by about 56% [2]. Carefully implemented, mirrored mutations retain unbiasedness. In this paper, we use these mirrored mutations with CMA-ES and compare the performance with active covariance matrix adaptation [9]. The latter is also based on the idea to use bad examples, but in the context of covariance matrix adaptation. Active CMA-ES has shown to consistently outperform the standard CMA-ES variant on the BBOB testbed [8]. In this paper, both algorithms are compared using restarts with increasing population size (IPOP-CMA-ES, [3]).

2. THE CONSIDERED ALGORITHM VARIANTS

Mirrored mutations together with selective mirroring has been implemented according to [2] into the CMA-ES. In particular, selective mirroring with $\lambda_\mathrm{m} = \lfloor 0.5 + 0.159\lambda_\mathrm{iid} \rfloor$ is used together with the standard recombination weights [2]. We denote the corresponding algorithm by $\mathrm{CMA_m}$.

Active covariance matrix adaptation [9] has been implemented as in [8]. This algorithm will be referred to as $\mathrm{CMA_a}$. As a *baseline algorithm*, we also show results for the IPOP-CMA-ES that does neither use the active covariance matrix adaptation nor mirrored mutations. All three algorithms use the same parameter settings that are slightly different from those in [8]. They were restarted up to 9 times with the population size doubling each time and up to the maximal number of overall function evaluations of $2 \cdot 10^5 \cdot D$ with D the problem dimension. For the experiments, we used version 3.54.beta.mirrors of the MATLAB implementation which can be downloaded from `http://canadafrance.gforge.inria.fr/mirroring/`.

3. TIMING EXPERIMENTS

In order to see the dependency of the algorithms on the problem dimension, the requested BBOB'2012 timing experiment has been performed for the original IPOP-CMA-ES and the variants $\mathrm{CMA_m}$ with mirrored mutations and $\mathrm{CMA_a}$ with active covariance matrix adaptation on an Intel Core2 Duo T9600 laptop with 2.80GHz, 4.0GB of RAM, and

MATLAB R2008b on Windows Vista SP2. The algorithms have been restarted for up to $2 \cdot 10^5 N$ function evaluations until 30 seconds have passed. The per-function-evaluation-runtimes were 18, 14, 9.8, 5.5, 4.2, 4.3, 6.6 times 10^{-4} seconds for the IPOP-CMA-ES, 23, 16, 9.3, 5.3, 4.4, 4.9, 6.2 times 10^{-4} seconds for the CMA_m, and 25, 18, 13, 7.9, 5.5, 5.5, and 7.4 times 10^{-4} seconds for the CMA_a in 2, 3, 5, 10, 20, 40, and 80 dimensions respectively.

4. RESULTS

Results from experiments according to [6] on the benchmark functions given in [5, 7] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_\text{t} = f_\text{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [6, 10]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} as in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

A significant improvement due to mirrored mutations can be observed on the sphere function only. Mirrored mutations speed up CMA-ES by about 35% in this case. Otherwise, no statistically significant effect of mirrored mutations is observed within the given experimental setup. In particular, mirrored mutations also do not lead to a failure where the original algorithm succeeds. As observed already before, active CMA-ES improves the performance on many ill-conditioned unimodal problems, usually also by less than a factor of two.

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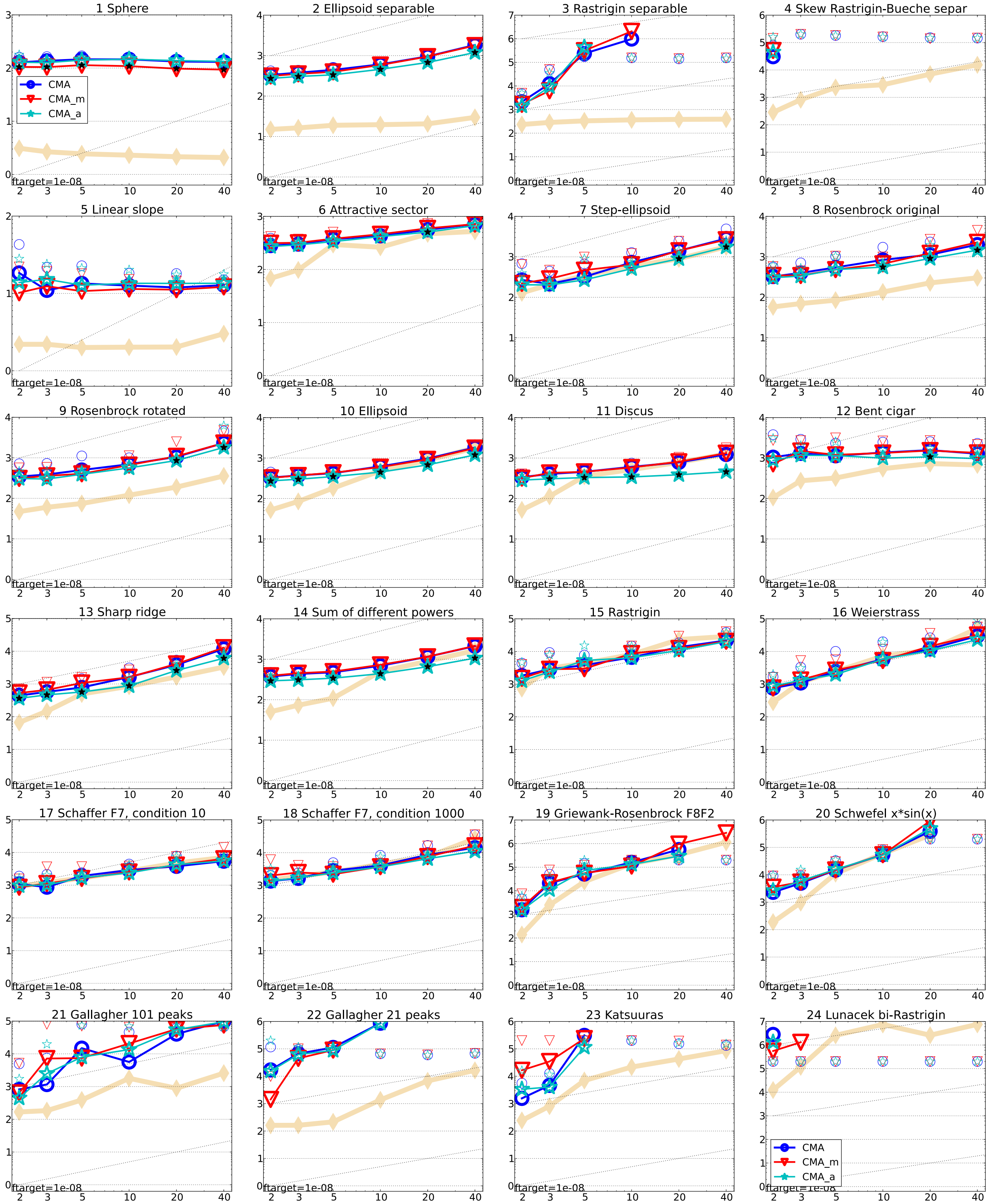

Figure 1: Expected running time (ERT in number of f-evaluations) divided by dimension for target function value 10^{-8} as $\log_{10}$ values versus dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: $\circ$:CMA, $\triangledown$:CMA_m, $\star$:CMA_a.

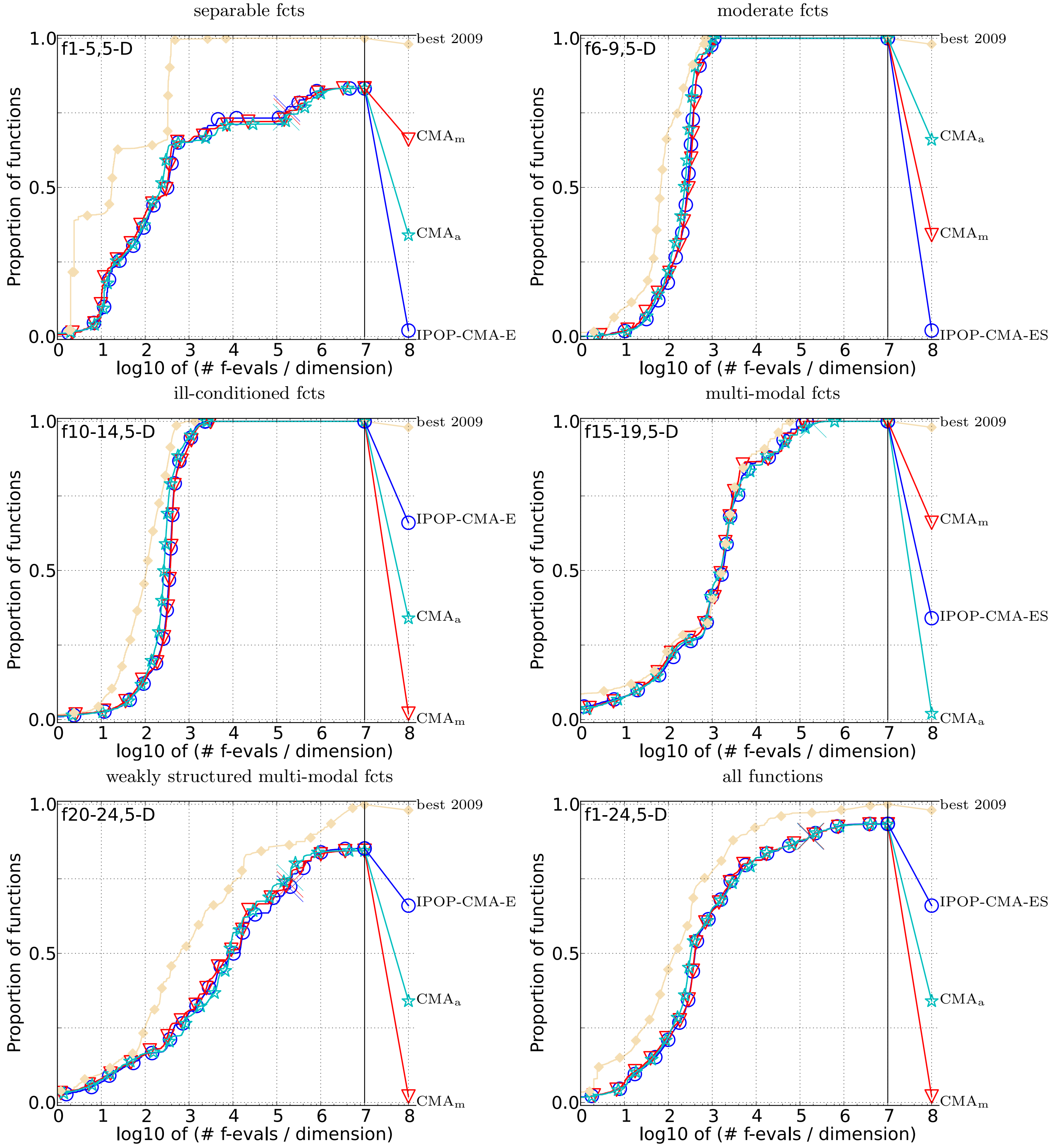

Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

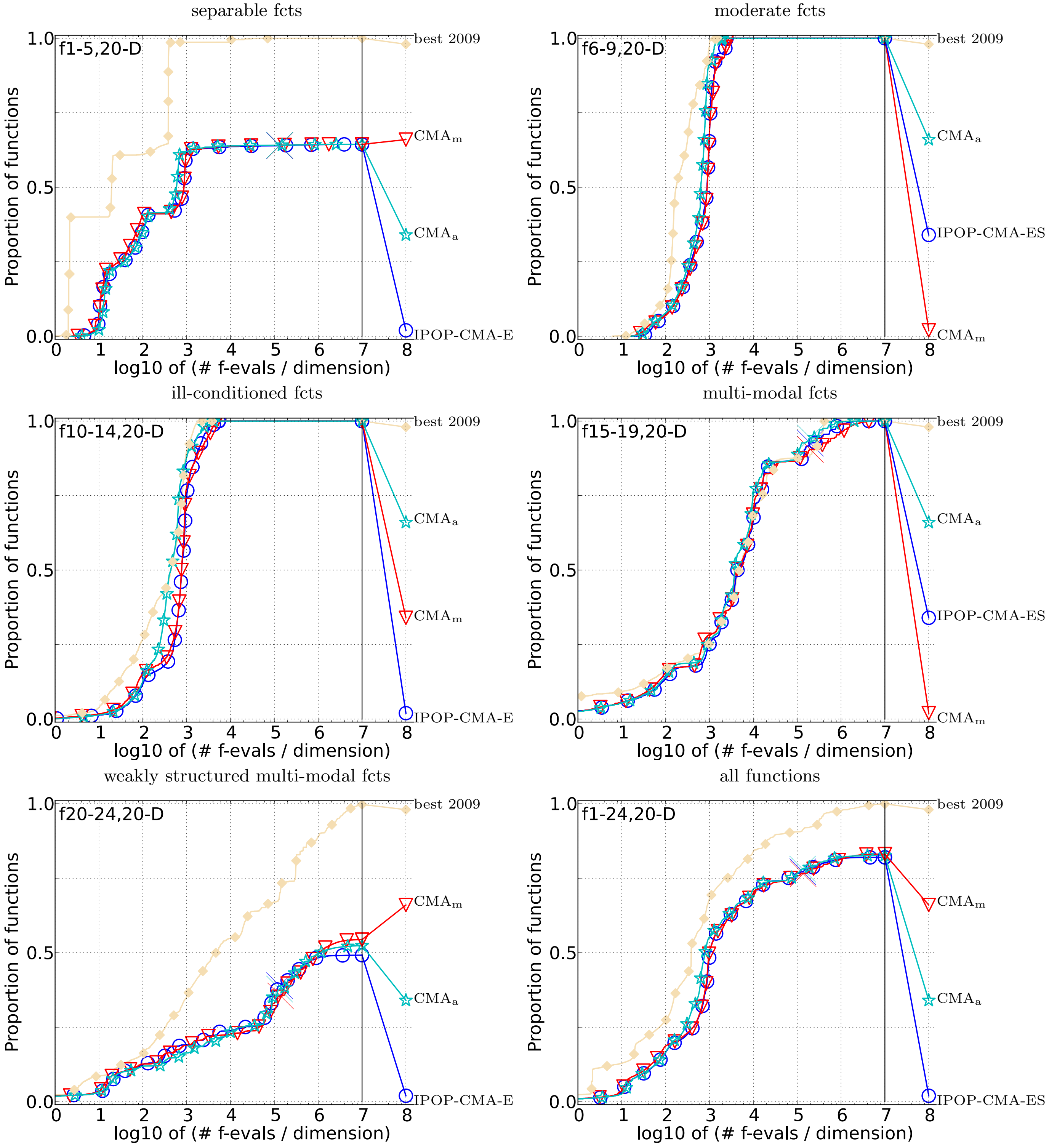

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	15/15
CMA	2.6(3)	9.3(4)	15(4)	28(4)	40(5)	54(6)	15/15
mir	2.8(2)	**7.6**(2)	**12**(3)	**22**(4)	**30**(5)$^{\star 3}$	**41**(5)$^{\star 2}$	15/15
act	**2.5**(2)	8.1(4)	15(4)	25(5)	38(4)	51(8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	83	87	88	90	92	94	15/15
CMA	14(4)	16(4)	17(4)	20(3)	22(3)	23(2)	15/15
mir	13(5)	16(4)	17(4)	19(2)	20(1)	21(1.0)	15/15
act	**10**(3)	**12**(2)	**13**(2)	**15**(1)$^{\star 3}$	**16**(2)$^{\star 4}$	**17**(1)$^{\star 4}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1646	1650	1654	15/15
CMA	1.4(2)	7.9(6)	**718**(939)	**715**(936)	**713**(932)	**712**(945)	6/15
mir	1.1(1)	**7.0**(10)	959(1347)	955(1367)	953(1162)	951(1218)	5/15
act	**0.91**(0.6)	30(7)	1333(1489)	1326(1644)	1323(1663)	1321(1644)	4/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	809	1633	1688	1817	1886	1903	15/15
CMA	2.7(3)	∞	∞	∞	∞	∞ *9e5*	0/15
mir	2.9(3)	∞	∞	∞	∞	∞ *9e5*	0/15
act	**1.7**(2)	∞	∞	∞	∞	∞ *9e5*	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	10	10	10	10	10	10	15/15
CMA	4.4(2)	6.5(2)	6.7(2)	6.7(2)	6.7(2)	6.7(2)	15/15
mir	**3.9**(2)	**5.1**(2)	**5.3**(1)	**5.4**(1)	**5.4**(1)	**5.4**(1)	15/15
act	4.2(2)	6.0(2)	6.3(2)	6.4(2)	6.4(2)	6.4(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	114	214	281	580	1038	1332	15/15
CMA	2.5(0.9)	2.2(0.4)	2.2(0.3)	1.7(0.2)	1.3(0.1)	1.2(0.1)	15/15
mir	2.2(1)	2.0(0.8)	2.1(0.6)	1.7(0.5)	1.3(0.2)	1.3(0.2)	15/15
act	**2.0**(0.6)	**1.9**(0.4)	**2.0**(0.3)	**1.5**(0.2)	**1.2**(0.1)	**1.1**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	24	324	1171	1572	1572	1597	15/15
CMA	4.7(3)	1.5(1)	**0.88**(0.4)	0.92(0.7)	0.92(0.7)	0.94(0.7)	15/15
mir	**3.9**(2)	**1.1**(1.0)	1.3(1)	1.3(1)	1.3(1)	1.4(1)	15/15
act	7.3(3)	1.1(1)	0.88(0.6)	**0.77**(0.5)	**0.77**(0.5)	**0.79**(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	73	273	336	391	410	422	15/15
CMA	3.4(1.0)	5.1(5)	5.7(4)	5.8(4)	6.0(4)	6.3(4)	15/15
mir	**2.6**(0.8)	**4.2**(5)	**4.9**(4)	5.2(3)	5.4(3)	5.6(3)	15/15
act	2.7(1.0)	4.5(5)	4.9(5)	**5.1**(4)	**5.3**(4)	**5.5**(4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	35	127	214	300	335	369	15/15
CMA	7.0(3)	9.0(11)	7.7(7)	6.7(5)	6.7(5)	6.5(4)	15/15
mir	**5.4**(4)	**6.5**(2)	6.2(1)	5.6(1.0)	5.5(0.8)	5.3(0.8)	15/15
act	6.1(2)	6.5(2)	**5.9**(1)	**5.2**(1)	**5.2**(1.0)	**5.2**(0.9)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	349	500	574	626	829	880	15/15
CMA	2.8(1)	2.7(0.8)	2.6(0.6)	2.8(0.4)	2.3(0.3)	2.3(0.3)	15/15
mir	3.9(0.9)	3.2(0.7)	3.0(0.3)	3.0(0.3)	2.4(0.2)	2.4(0.2)	15/15
act	**2.6**(0.8)	**2.2**(0.4)	**2.1**(0.2)	**2.2**(0.2)$^{\star 3}$	**1.8**(0.2)$^{\star 3}$	**1.9**(0.2)$^{\star 3}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	143	202	763	1177	1467	1673	15/15
CMA	8.7(2)	7.6(1)	2.2(0.4)	1.6(0.2)	1.4(0.2)	1.3(0.2)	15/15
mir	8.4(3)	7.8(1)	2.3(0.3)	1.7(0.2)	1.4(0.2)	1.3(0.1)	15/15
act	**5.2**(1.0)$^{\star 2}$	**4.6**(0.7)$^{\star 4}$	**1.4**(0.2)$^{\star 4}$	**1.1**(0.1)$^{\star 4}$	**0.95**(0.1)$^{\star 4}$	**0.93**(0.1)$^{\star 4}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	108	268	371	461	1303	1494	15/15
CMA	10(8)	8.3(5)	8.4(5)	8.6(5)	3.7(2)	**3.7**(3)	15/15
mir	**7.4**(8)	7.5(6)	8.2(6)	8.7(6)	3.9(3)	3.9(3)	15/15
act	8.7(6)	**7.2**(6)	**7.9**(6)	**8.5**(6)	**3.7**(2)	3.7(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	132	195	250	1310	1752	2255	15/15
CMA	3.3(2)	5.3(2)	5.5(2)	1.4(0.3)	1.6(0.3)	1.5(0.3)	15/15
mir	4.0(3)	5.0(2)	4.7(2)	1.7(0.7)	1.8(0.8)	2.0(0.8)	15/15
act	**2.9**(0.7)	**4.1**(2)	**4.5**(1)	**1.2**(0.2)	**1.2**(0.1)*	**1.2**(0.1)$^{\star 3}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	10	41	58	139	251	476	15/15
CMA	2.3(3)	2.8(0.9)	3.5(1)	4.2(1)	5.4(0.5)	4.4(0.6)	15/15
mir	**1.6**(2)	2.8(1)	**3.2**(2)	4.1(1.0)	5.4(1)	4.4(0.6)	15/15
act	2.5(3)	**2.7**(1)	3.5(1)	**4.0**(0.7)	**3.9**(0.4)$^{\star 2}$	**3.1**(0.4)$^{\star 4}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	511	9310	19369	20073	20769	21359	14/15
CMA	1.6(2)	**0.74**(0.5)	0.86(0.6)	0.86(0.6)	0.86(0.6)	0.87(0.6)	15/15
mir	1.8(2)	0.74(0.6)	**0.66**(0.4)	**0.67**(0.4)	**0.67**(0.3)	**0.68**(0.3)	15/15
act	**1.5**(2)	1.1(0.7)	1.2(0.6)	1.2(0.7)	1.2(0.6)	1.2(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	120	612	2662	10449	11644	12095	15/15
CMA	2.3(2)	3.1(3)	**1.9**(2)	1.1(1)	1.00(1)	1.00(1)	15/15
mir	2.9(4)	5.0(6)	3.0(2)	1.0(0.6)	1.1(0.7)	1.1(0.7)	15/15
act	**1.7**(1)	**2.8**(3)	2.2(2)	**0.84**(0.6)	**0.80**(0.5)	**0.80**(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	5.2	215	899	3669	6351	7934	15/15
CMA	**1.8**(2)	**0.82**(0.3)	0.93(2)	0.89(0.6)	1.1(0.7)	1.2(0.4)	15/15
mir	3.4(3)	0.85(0.5)	**0.58**(0.1)	**0.73**(0.4)	**0.77**(0.5)	**0.91**(0.3)	15/15
act	2.6(2)	1.3(0.4)	0.77(1.0)	0.89(0.5)	0.81(0.3)	1.0(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	103	378	3968	9280	10905	12469	15/15
CMA	1.2(0.9)	1.6(0.8)	1.6(2)	0.97(0.8)	1.0(0.7)	1.1(0.5)	15/15
mir	0.94(0.7)	**0.77**(0.3)	0.53(0.6)	0.79(0.4)	0.82(0.3)	**0.85**(0.3)	15/15
act	**0.82**(0.3)	1.7(0.3)	**0.44**(0.5)	**0.66**(0.3)$_{\downarrow}$	**0.76**(0.3)	0.94(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	242	1.2e5	1.2e5	1.2e5	15/15
CMA	**15**(14)	1796(1570)	572(573)	**2.1**(2)	**2.1**(2)	**2.1**(2)	15/15
mir	20(16)	**1379**(1430)	551(660)	2.3(2)	2.3(2)	2.3(2)	15/15
act	24(10)	6888(1525)	**462**(416)	3.0(3)	3.0(3)	3.0(3)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	16	851	38111	54470	54861	55313	14/15
CMA	3.7(2)	**8.3**(6)	1.7(0.8)	**1.3**(0.6)	**1.3**(0.6)	**1.3**(0.6)	15/15
mir	3.0(3)	9.0(4)	**1.7**(0.8)	1.3(0.6)	1.3(0.6)	1.3(0.6)	15/15
act	**2.5**(2)	9.1(3)	1.7(1)	1.4(1)	1.4(1)	1.4(1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	41	1157	1674	1705	1729	1757	14/15
CMA	6.6(16)	**7.3**(14)	43(107)	43(104)	43(102)	42(103)	13/15
mir	**1.4**(1)	26(8)	**21**(20)	**21**(21)	**21**(21)	**21**(21)	14/15
act	1.9(1)	28(14)	23(20)	23(21)	23(21)	22(22)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	71	386	938	1008	1040	1068	14/15
CMA	**10**(11)	87(40)	292(395)	554(738)	537(709)	524(698)	6/15
mir	12(23)	**17**(26)	**144**(233)	444(562)	431(557)	421(532)	7/15
act	15(24)	87(30)	379(466)	**433**(559)	**421**(552)	**411**(522)	7/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.0	518	14249	31654	33030	34256	15/15
CMA	2.7(3)	20(18)	107(141)	49(64)	47(61)	45(59)	6/15
mir	**2.2**(2)	**16**(17)	63(105)	37(49)	36(47)	35(45)	7/15
act	2.4(3)	29(17)	**39**(71)	**18**(32)	**17**(30)	**17**(16)	10/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1622	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
CMA	1.9(1)	**9.4**(12)	∞	∞	∞	∞ *1e6*	0/15
mir	2.2(2)	19(23)	∞	∞	∞	∞ *1e6*	0/15
act	**1.5**(2)	13(16)	∞	∞	∞	∞ *1e6*	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	15/15
CMA	7.3(1)	13(1)	19(1)	32(2)	43(2)	56(2)	15/15
mir	**6.1**(1)*	**10**(1)$^{\star 3}$	**14**(2)$^{\star 4}$	**23**(1)$^{\star 4}$	**32**(1)$^{\star 4}$	**41**(2)$^{\star 4}$	15/15
act	7.8(1)	14(2)	20(2)	32(2)	45(3)	58(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	385	386	387	390	391	393	15/15
CMA	34(5)	40(6)	43(3)	45(3)	47(1)	48(1)	15/15
mir	34(6)	39(6)	42(5)	45(2)	47(2)	48(2)	15/15
act	**23**(3)$^{\star 3}$	**27**(3)$^{\star 3}$	**29**(3)$^{\star 4}$	**31**(2)$^{\star 4}$	**32**(2)$^{\star 4}$	**34**(2)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	5066	7626	7635	7643	7646	7651	15/15
CMA	13(9)	∞	∞	∞	∞	∞ *3e6*	0/15
mir	**8.5**(6)	∞	∞	∞	∞	∞ *3e6*	0/15
act	8.7(7)	∞	∞	∞	∞	∞ *3e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	4722	7628	7666	7700	7758	1.4e5	9/15
CMA	∞	∞	∞	∞	∞	∞ *3e6*	0/15
mir	∞	∞	∞	∞	∞	∞ *3e6*	0/15
act	∞	∞	∞	∞	∞	∞ *3e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	41	41	41	41	41	41	15/15
CMA	4.9(1)	5.7(0.9)	5.9(1)	5.9(1)	5.9(1)	5.9(1)	15/15
mir	**4.4**(1)	**5.4**(1)	**5.5**(1)	**5.5**(1)	**5.5**(1)	**5.5**(1)	15/15
act	5.5(1)	6.5(2)	6.6(2)	6.6(2)	6.6(2)	6.6(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	1296	2343	3413	5220	6728	8409	15/15
CMA	1.7(0.2)	1.3(0.2)	1.2(0.1)	1.2(0.1)	1.2(0.1)	1.2(0.1)	15/15
mir	1.7(0.3)	1.3(0.2)	1.2(0.1)	1.2(0.2)	1.2(0.2)	1.3(0.1)	15/15
act	**1.6**(0.3)	**1.3**(0.2)	**1.1**(0.1)	**1.1**(0.1)	**1.1**(0.1)	**1.1**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	1351	4274	9503	16524	16524	16969	15/15
CMA	1.7(1)	3.9(1)	2.7(2)	1.7(1.0)	1.7(1.0)	1.6(0.9)	15/15
mir	1.7(1)	4.2(2)	2.7(1.0)	1.7(0.6)	1.7(0.6)	1.6(0.6)	15/15
act	**1.0**(1.0)	**2.3**(1.0)	**1.7**(0.7)*	**1.1**(0.4)*	**1.1**(0.4)*	**1.0**(0.4)*	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	2039	3871	4040	4219	4371	4484	15/15
CMA	3.7(0.6)	4.4(0.3)	4.7(0.3)	4.9(0.3)	4.9(0.3)	5.0(0.3)	15/15
mir	3.9(0.7)	5.0(4)	5.3(4)	5.4(3)	5.4(3)	5.4(3)	15/15
act	**3.6**(0.7)	**3.5**(0.6)$^{\star 2}$	**3.8**(0.6)$^{\star 2}$	**4.0**(0.6)$^{\star 2}$	**4.0**(0.6)$^{\star 2}$	**4.0**(0.6)$^{\star 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	1716	3102	3277	3455	3594	3727	15/15
CMA	4.7(0.9)	5.1(0.6)	5.4(0.6)	5.6(0.5)	5.6(0.5)	5.6(0.5)	15/15
mir	4.1(1)	5.4(0.7)	5.7(0.6)	5.8(0.6)	5.8(0.6)	5.8(0.6)	15/15
act	**3.9**(0.7)	**4.1**(0.4)$^{\star 2}$	**4.4**(0.4)$^{\star 2}$	**4.5**(0.4)$^{\star 2}$	**4.5**(0.4)$^{\star 2}$	**4.5**(0.4)$^{\star 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	7413	8661	10735	14920	17073	17476	15/15
CMA	1.8(0.3)	1.8(0.2)	1.6(0.1)	1.2(0.1)	1.1(0.0)	1.1(0.0)	15/15
mir	1.8(0.2)	1.8(0.2)	1.6(0.1)	1.2(0.0)	1.1(0.0)	1.1(0.0)	15/15
act	**1.2**(0.2)$^{\star 3}$	**1.2**(0.2)$^{\star 4}$	**1.0**(0.1)$^{\star 4}$	**0.82**(0.0)$^{\star 4}_{\downarrow 4}$	**0.75**(0.0)$^{\star 4}_{\downarrow 4}$	**0.76**(0.0)$^{\star 4}_{\downarrow 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	1002	2228	6278	9762	12285	14831	15/15
CMA	10(1.0)	5.1(0.3)	1.9(0.1)	1.4(0.0)	1.2(0.0)	1.0(0.0)	15/15
mir	11(0.7)	5.4(0.4)	2.0(0.1)	1.4(0.1)	1.2(0.1)	1.1(0.0)	15/15
act	**4.5**(0.2)$^{\star 4}$	**2.2**(0.1)$^{\star 4}$	**0.86**(0.0)$^{\star 4}$	**0.63**(0.0)$^{\star 4}_{\downarrow 4}$	**0.55**(0.0)$^{\star 4}_{\downarrow 4}$	**0.50**(0.0)$^{\star 4}_{\downarrow 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	1042	1938	2740	4140	12407	13827	15/15
CMA	3.4(5)	5.4(4)	5.6(3)	5.1(2)	2.1(0.9)	2.2(0.9)	15/15
mir	3.2(4)	4.1(5)	4.8(5)	4.6(3)	2.0(1)	2.1(1.0)	15/15
act	**2.4**(0.2)	**3.4**(2)	**3.4**(2)	**3.4**(1)	**1.4**(0.5)	**1.5**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	18749	24455	30201	15/15
CMA	2.5(0.4)	5.1(4)	7.5(6)	1.7(1)	1.9(0.9)	2.0(1)	15/15
mir	3.1(4)	**3.2**(4)	6.2(4)	1.7(1)	2.4(0.9)	2.4(0.7)	15/15
act	**2.4**(0.3)	3.5(3)	**4.5**(3)	**1.1**(0.8)	**1.2**(0.7)	**1.5**(1.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	75	239	304	932	1648	15661	15/15
CMA	4.5(2)	2.9(0.6)	3.7(0.5)	4.1(0.4)	6.1(0.5)	1.2(0.1)	15/15
mir	**2.9**(1)	**2.3**(0.4)	**2.8**(0.3)$^{\star 2}$	3.7(0.4)	6.3(0.6)	1.2(0.1)	15/15
act	3.8(1)	2.7(0.3)	3.5(0.5)	**3.1**(0.2)$^{\star 3}$	**3.9**(0.2)$^{\star 4}$	**0.69**(0.0)$^{\star 4}_{\downarrow 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	30378	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
CMA	0.98(0.7)	**0.98**(0.4)	0.76(0.2)	0.77(0.2)	0.57(0.2)$_{\downarrow}$	0.58(0.2)$_{\downarrow}$	15/15
mir	**0.81**(0.6)	1.1(0.3)	0.69(0.3)	0.70(0.3)	0.52(0.3)$_{\downarrow 2}$	0.53(0.3)$_{\downarrow 2}$	15/15
act	0.90(0.5)	1.0(0.3)	**0.60**(0.3)	**0.61**(0.3)	**0.45**(0.2)$_{\downarrow 3}$	**0.46**(0.3)$_{\downarrow 3}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	1384	27265	77015	1.9e5	2.0e5	2.2e5	15/15
CMA	1.8(1)	1.1(0.4)	**0.82**(0.7)	1.1(0.9)	1.2(0.9)	1.1(0.8)	15/15
mir	**1.3**(0.6)	0.85(0.5)	1.3(1)	1.4(1)	1.4(1)	1.3(1)	15/15
act	1.9(0.6)	**0.76**(0.3)	0.83(0.7)	**0.81**(0.5)	**1.00**(0.9)	**0.95**(0.8)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	63	1030	4005	30677	56288	80472	15/15
CMA	**2.2**(1)	1.00(0.3)	1.5(2)	0.81(0.3)	0.93(0.4)	**0.91**(0.3)	15/15
mir	2.2(0.5)	**0.82**(0.3)	1.4(1)	**0.59**(0.3)$_{\downarrow 2}$	0.82(0.4)	0.92(0.1)	15/15
act	2.3(1)	0.87(0.2)	**0.52**(0.2)	0.70(0.3)	**0.80**(0.4)	0.92(0.2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	621	3972	19561	67569	1.3e5	1.5e5	15/15
CMA	0.96(0.2)	**0.70**(0.4)	0.89(0.7)	0.98(0.3)	1.1(0.8)	1.1(0.8)	15/15
mir	**0.81**(0.3)	1.0(1)	**0.61**(0.7)	0.89(0.3)	0.97(0.3)	0.95(0.3)	15/15
act	0.96(0.3)	0.96(2)	0.96(0.9)	**0.79**(0.3)	**0.85**(0.4)	**0.87**(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
CMA	170(56)	3.1e4(3e4)	2.0(3)	0.94(0.7)	1.7(2)	1.7(2)	5/15
mir	**134**(58)	**1.8e4**(1e4)	**1.1**(0.6)	2.3(2)	2.9(3)	2.8(3)	3/15
act	156(72)	7.7e4(1e4)	2.5(4)	**0.73**(0.6)	**0.88**(0.9)	**0.88**(0.8)	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	82	46150	3.1e6	5.5e6	5.6e6	5.6e6	14/15
CMA	4.8(1)	5.4(2)	**0.79**(0.4)	1.2(1)	1.2(1)	**1.2**(1)	6/15
mir	**3.4**(0.7)$^{\star 2}$	5.4(3)	1.1(0.7)	1.4(1)	2.0(2)	3.3(4)	3/15
act	4.8(1)	**3.2**(1)	0.90(0.4)	**1.1**(0.9)	**1.1**(1)	1.7(2)	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	561	6541	14103	14643	15567	17589	15/15
CMA	5.0(5)	122(180)	**57**(84)	**55**(80)	**52**(76)	**46**(68)	9/15
mir	3.5(4)	109(177)	80(110)	77(113)	73(111)	65(96)	8/15
act	**3.2**(4)	**95**(175)	77(105)	74(87)	70(85)	62(97)	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	467	5580	23491	24948	26847	1.3e5	12/15
CMA	12(14)	433(551)	∞	∞	∞	∞ *1e6*	0/15
mir	**7.0**(12)	**188**(225)	∞	∞	∞	∞ *1e6*	0/15
act	10(13)	232(306)	∞	∞	∞	∞ *1e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.2	1614	67457	4.9e5	8.1e5	8.4e5	15/15
CMA	**3.4**(4)	∞	∞	∞	∞	∞ *3e6*	0/15
mir	3.9(4)	1.1e4(1e4)	577(664)	∞	∞	∞ *3e6*	0/15
act	6.5(5)	**1.1e4**(1e4)	**556**(593)	∞	∞	∞ *3e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
CMA	∞	∞	∞	∞	∞	∞ *4e6*	0/15
mir	**12**(15)	**3.6**(4)	∞	∞	∞	∞ *4e6*	0/15
act	42(48)	∞	∞	∞	∞	∞ *4e6*	0/15

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.