Runtime Analysis of Simple Interactive Evolutionary Biobjective Optimization Algorithms

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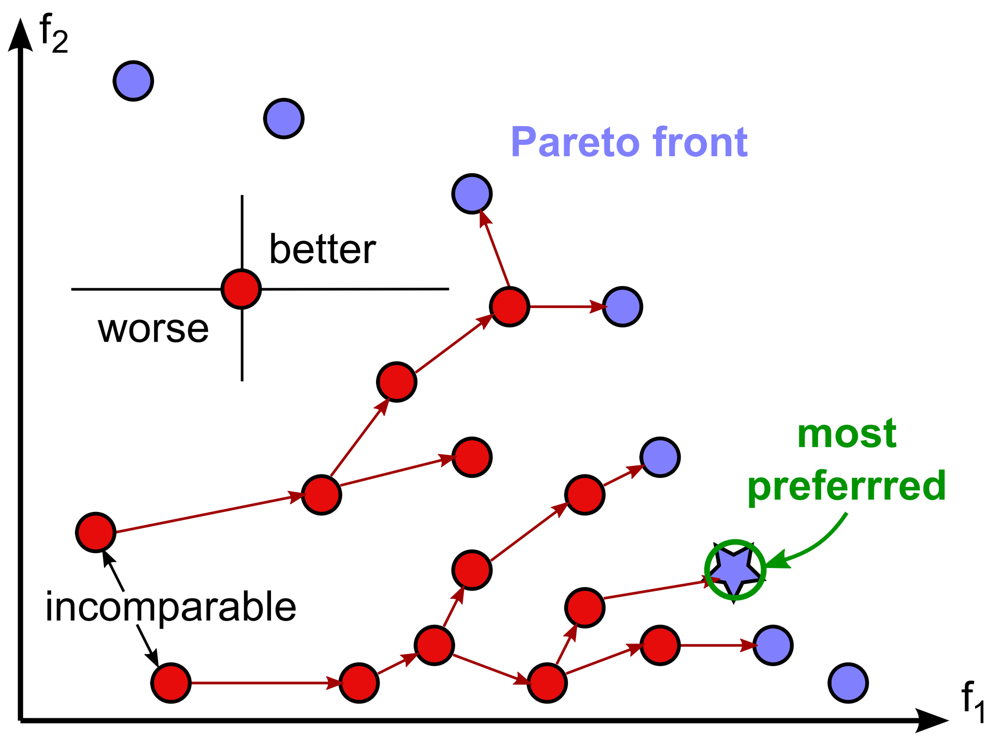
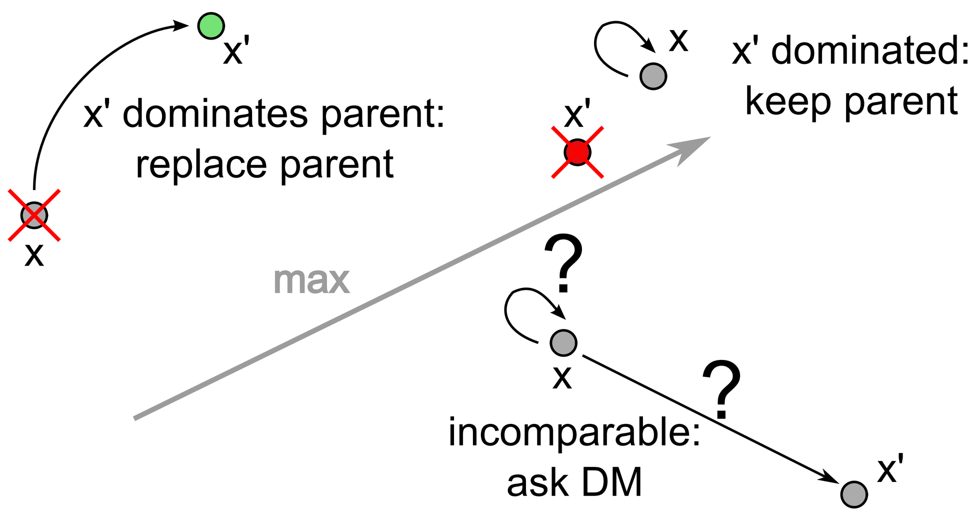
MOO



multiobjective optimization = search + decision making

Here: interactive DM



Algorithm details

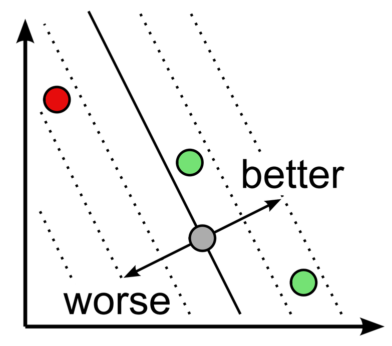
Selection $(1+1)$ + decision maker (DM)
answers are stored

Variation
iRLS: single bit flip
iEA: standard mutation, $p = \frac{1}{n}$

Goal: runtime analysis

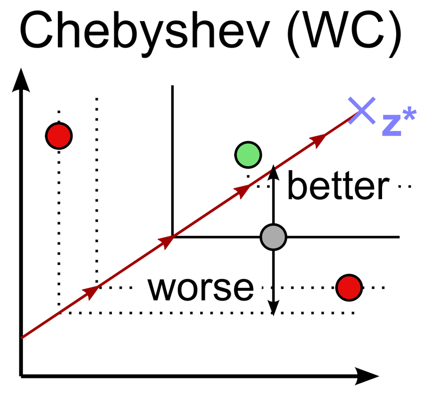
How many function evaluations (FE)?
How many DM queries (Qs)?

Models of DM

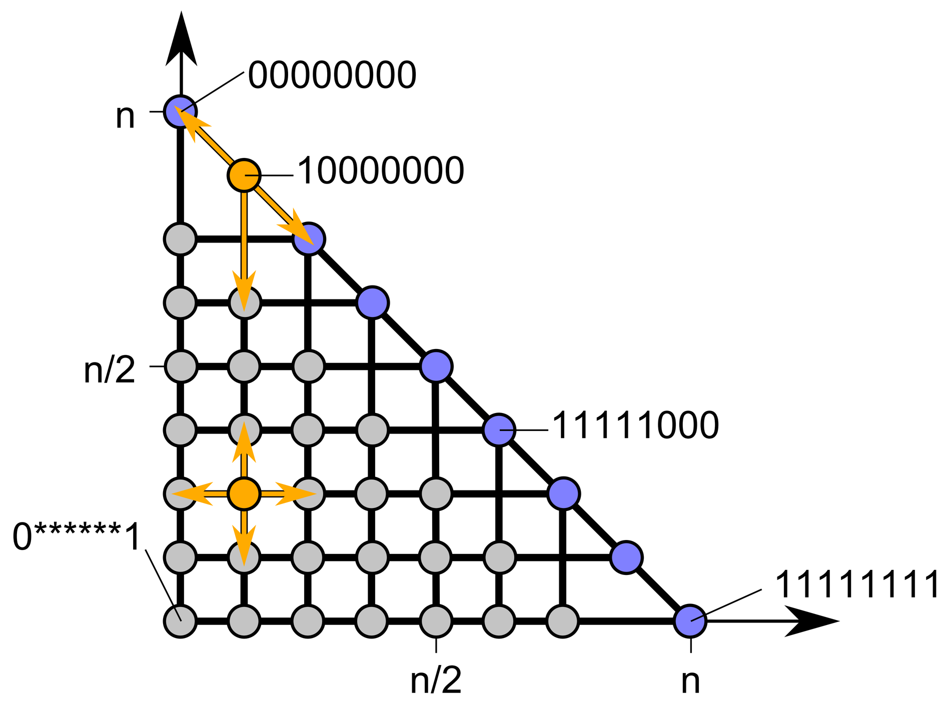


weighted sum (WS)

weighted Chebyshev (WC)



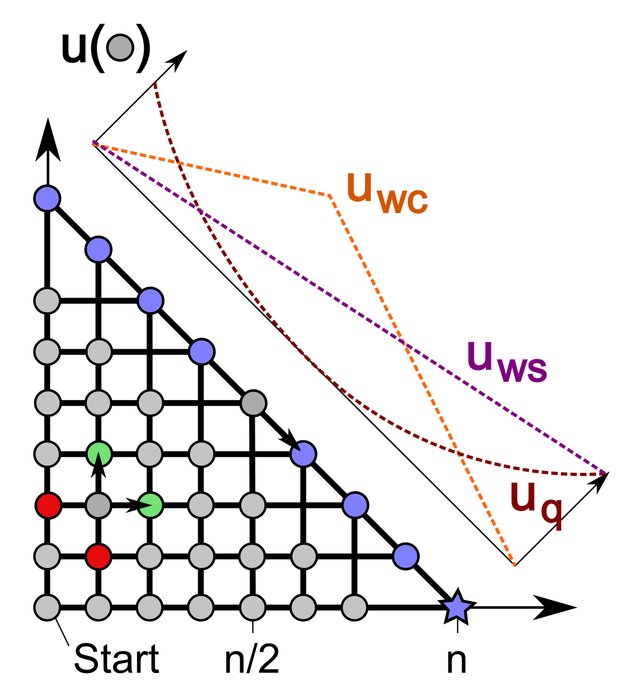
LOTZ



LOTZ	WS	
iRLS	$\Theta(n^2)$	FE
(also WC)	$O(n)$	Qs
iEA	$O(n^2)$	FE
	$O(n)$	Qs

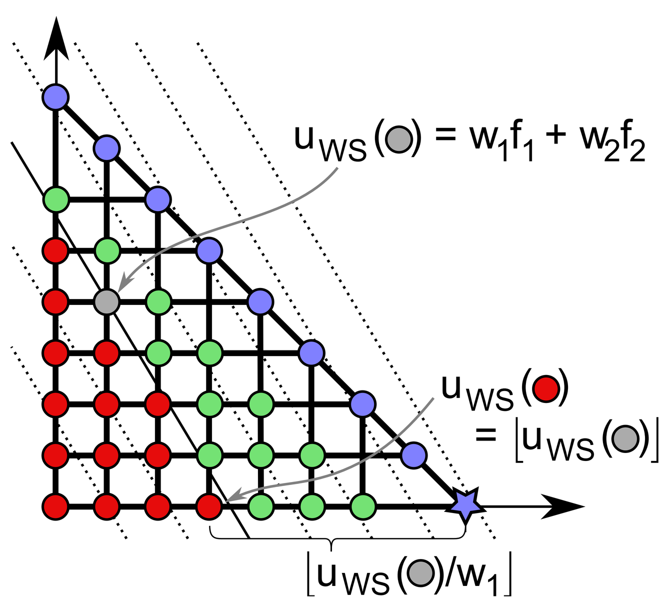
Quadratic utilities:
#FE can be ∞

Analysis iRLS



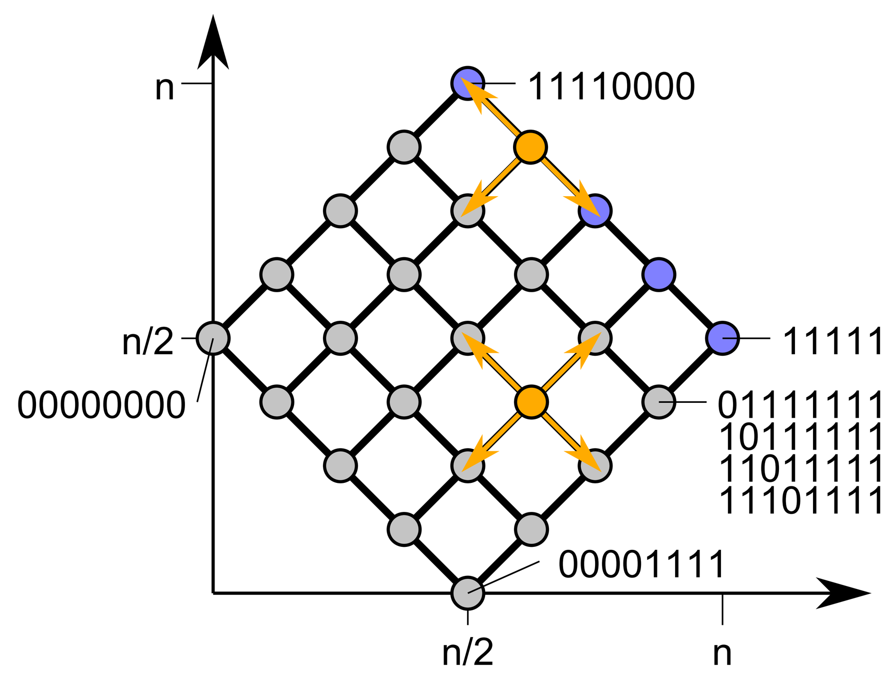
- Unless PF reached:
  up or right accepted,
  down or left rejected,
  no queries
- High prob. to start near $0 * 1$
  $\Rightarrow O(n)$ moves to PF $\Rightarrow O(n^2)$ FE
- As soon as PF reached:
  can only move on PF
  $\Rightarrow$ queries needed!
  flip last $1 \to 0$ or $0 \to 1$,
  $\Rightarrow$ need at most n improvements
  $\Rightarrow O(n^2)$ FE,
  at most $O(2n) = O(n)$ queries

Analysis iEA



- High prob. to start near $0 * 1$
- Unless PF reached:
  incomparable points possible
  $\Rightarrow$ queries needed!
  utility larger $\Leftrightarrow$ accept!
  can decrease LOTZ level!
  drift function:
  $g(x) = n - \lfloor f_1(x) + \frac{1-w_1}{w_1} f_2(x) \rfloor$
  at most $g(x) \leq n$ 1-flips apart from PF
  need less than $n \cdot O(n) = O(n^2)$ FE
- As soon as PF reached:
  either stay on PF or LOTZ level
  decreased (see above)
- For #queries: less than constant
  #queries per improvement
  (details: see paper)

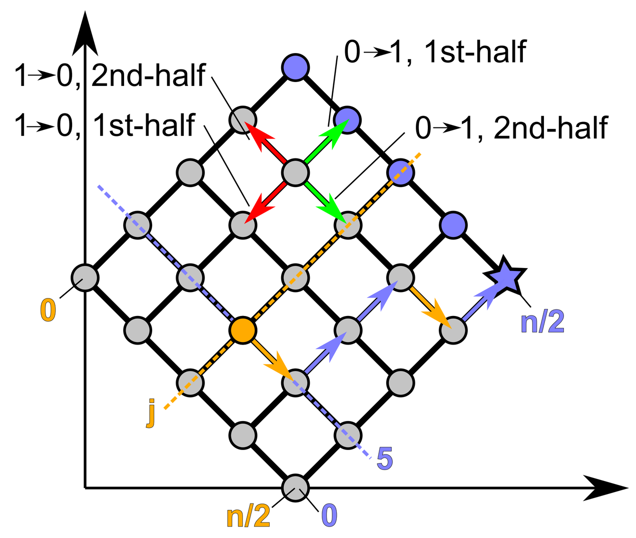
COCZ



COCZ	WS	
iRLS	$\Theta(n \log n)$	FE
	$O(n)$	Qs
iEA	not here	

Even valid for linear objective functions

Analysis iRLS



- $f_1 =$ #1
  $f_2 =$ #1 in 1st half + #0 in 2nd half
- Movements towards/along front independent
- In both “ONEMAX” scenario
- Prob(improve towards front) $= \frac{n/2-i}{n}$
  $\Rightarrow \Theta(n \log n)$ FE towards front
- Prob(improve along front) $= \frac{n/2-j}{n}$
  $\Rightarrow O(n \log n)$ FE along front
- In total: $\Theta(n \log n)$ FE
- At most $\frac{n}{2}$ queries along front

Conclusions

- Many DM queries necessary even for simple problems
  $\Rightarrow$ incorporation of DM model required to be practical
- Number of FE strongly depending on DM model
  $\Rightarrow$ DM model must be chosen carefully
- DM model can be
  – provided completely/partially by DM
  – learned online/offline

Acknowledgments

- Dagstuhl for hosting the “*Learning in Multi-Objective Optimization*” seminar
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Erratum

First equation in the proof of Theorem 2 should be

$$g(x) = n - \left\lfloor f_1(x) + \frac{1-w_1}{w_1} f_2(x) \right\rfloor$$

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