Do Additional Objectives Make a Problem Harder?

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Starting Point

Most optimization problems are multiobjective in nature…

→ Include as many objective as possible would be desirable

~1995: formulated as single-objective problem

~2005: formulated as two- or three-objective problem

Today: many-objective problems *Wagner et al. (2007)*

Are the algorithms bad or are the problems more difficult or both?

We haven't understood yet how additional objectives affects the problem complexity!

Adding Objectives: What Is Known Today…

Statements are contradictory: some studies say that…

problems may become harder

- *Fonseca and Fleming (1995), Deb (2001), Coello et al. (2002), and others:*
  - conflicts between objectives
  - Pareto front size ↑
  - # incomparable solutions ↑
- *Winkler (1985):*
  - *theoretical work for random objectives*

problems may become easier

- *Knowles et al. (2001):*
  - multiobjectivization
- *Jensen (2004):*
  - helper-objectives
- *Scharnow et al. (2002), Neumann and Wegener (2006):*
  - theoretical investigations
  - 2D faster than 1D
  - decomposition

Question Addressed In This Study

Given:

- single-objective problem
- algorithm for single- and multiobjective optimization: SEMO - (1+1)EA

Now:

add an objective, but keep the search space the same

Question:

What is the runtime complexity to generate all Pareto optima in comparison to the original problem?

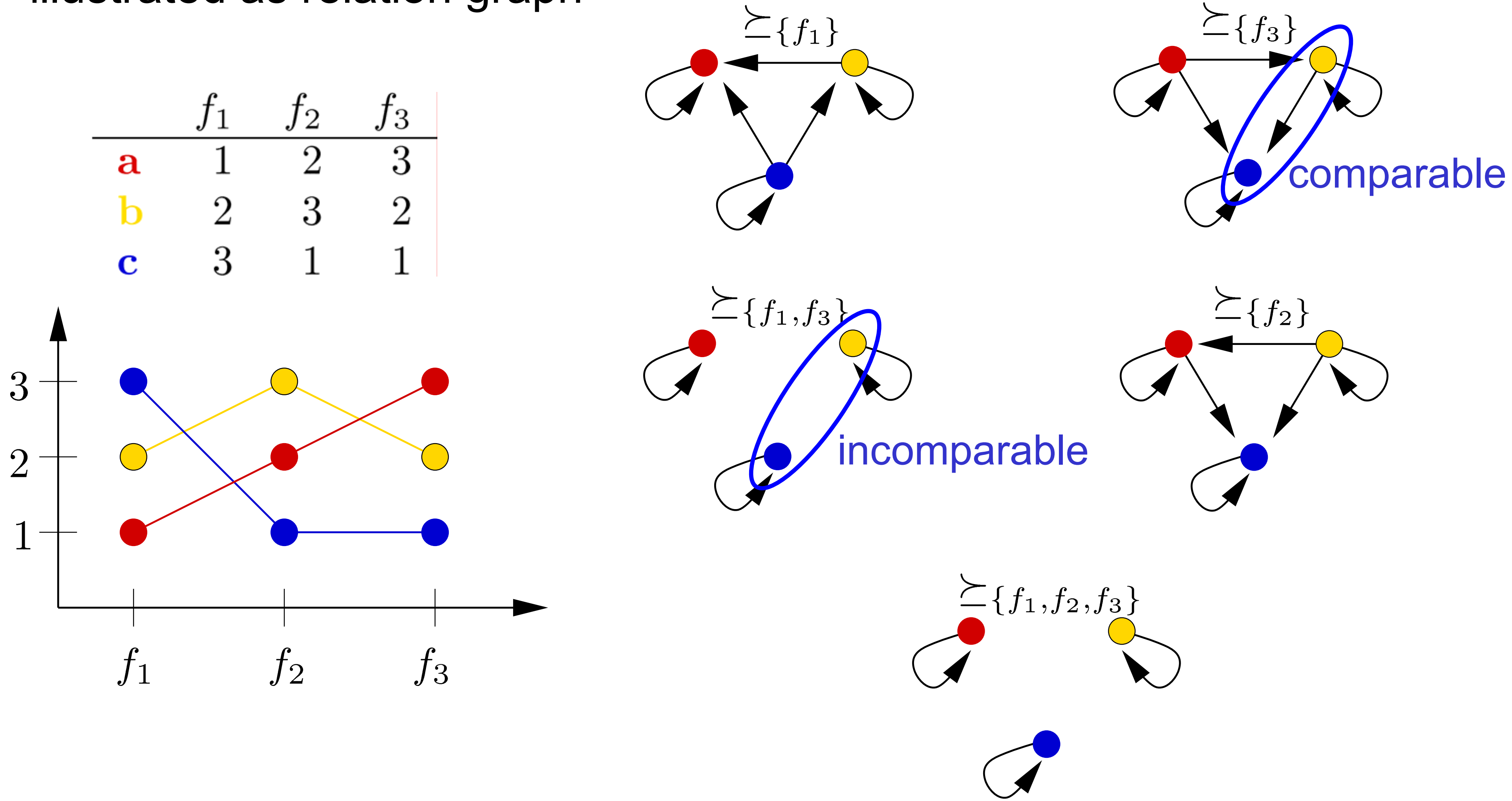
Note:

single-objective optimum is part of the Pareto-optimal set

General Considerations On Adding Objectives

Weak Pareto-dominance relation $\succeq_{\mathcal{F}} := \{(x,y) \in X \mid \forall f_i \in \mathcal{F} : f_i(x) \geq f_i(y)\}$
illustrated as relation graph

	f_1	f_2	f_3
a	1	2	3
b	2	3	2
c	3	1	1



A Simple Evolutionary Multiobjective Optimizer

with 1 objective: SEMO ≅ (1+1)EA

Choose $x \in \{0,1\}^n$ uniformly at random

Determine $f(x)$

$P \leftarrow \{x\}$

Repeat

  Choose $x \in P$ uniformly at random

  Create x' by flipping each bit of x with probability $1/n$

  Determine $f(x')$

  If x' is not dominated by any other search point in P,

    include x' into P and delete all solutions dominated

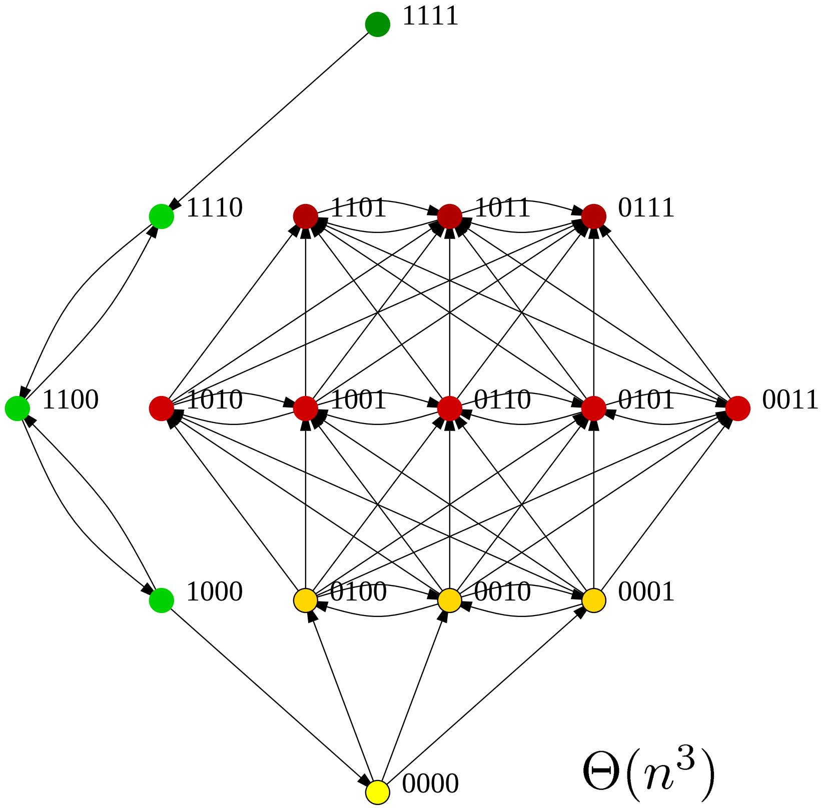
    by x' or with fitness vector $f(x')$ from P

initially 1 solution

standard bit mutation

all solutions in P incomparable

$$\textsc{Plateau}_1(x) := \begin{cases} |x|_0 & : \quad x \notin \{1^i 0^{n-i}, 1 \le i \le n\} \\ n+1 & : \quad x \in \{1^i 0^{n-i}, 1 \le i < n\} \\ n+2 & : \quad x = 1^n. \end{cases}$$



$\Theta(n^3)$

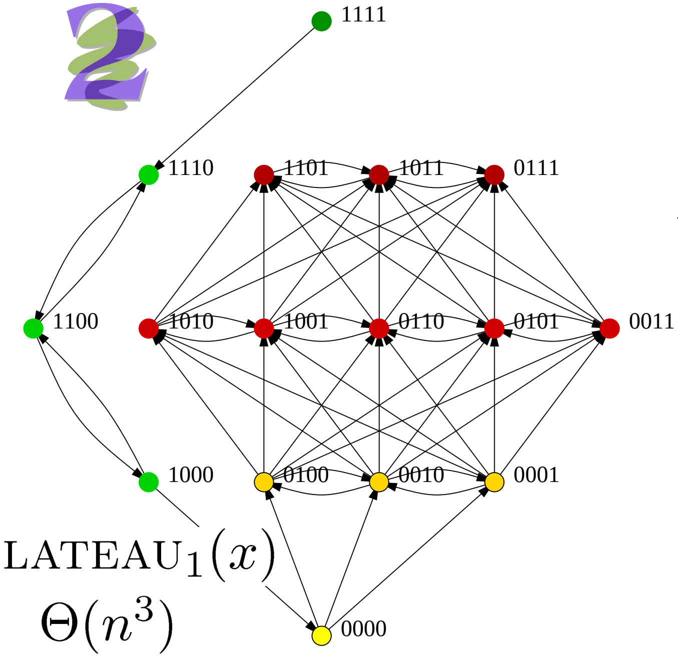
Add $\textsc{OneMax}(x)$

Faster: $O(n^2 \log n)$

Add $\textsc{ZeroMax}(x)$

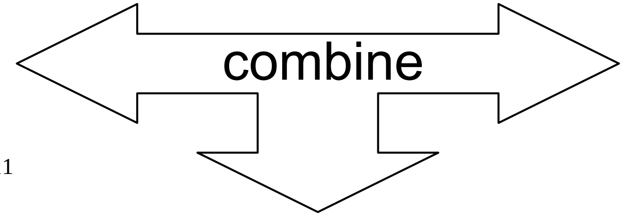
Slower: exponential w.h.p.

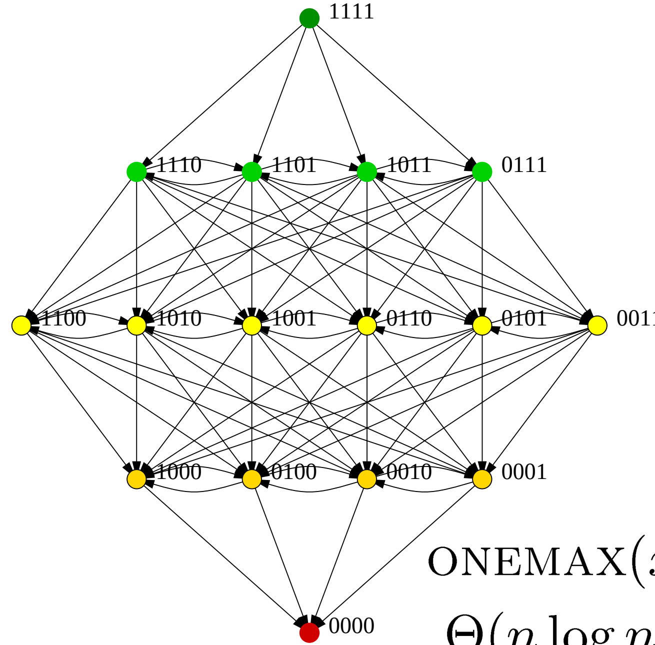
Adding ONEMAX



$\mathrm{PLATEAU}_1(x)$

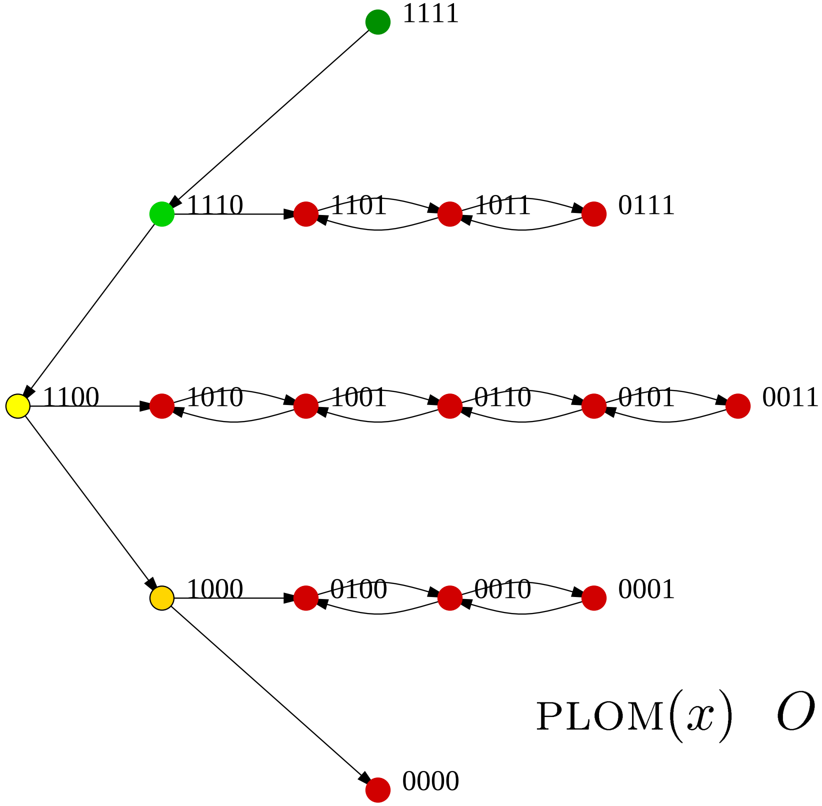
$\Theta(n^3)$





$\mathrm{ONEMAX}(x)$

$\Theta(n \log n)$

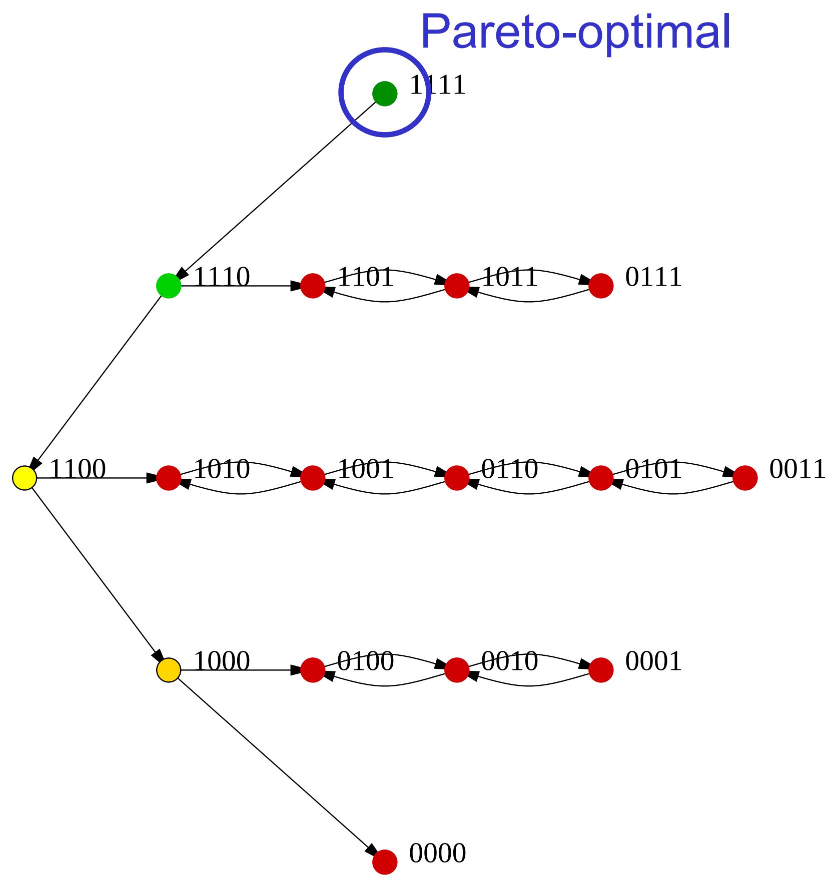


$\mathrm{PLOM}(x)$ $O(n^2 \log n)$

Adding ONEMAX (Proof)

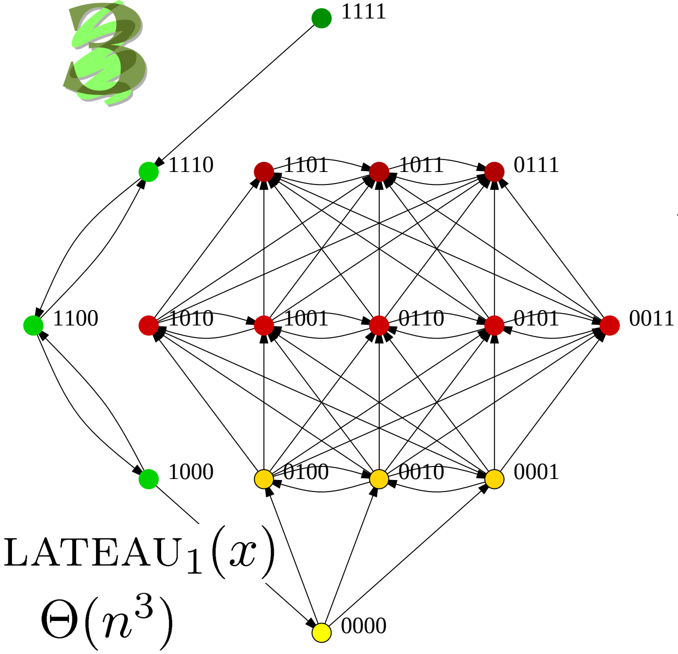
Theorem 2 *The expected optimization time of Global SEMO on* PLOM *is* $O(n^2 \log n)$.

- |Population| $= O(n)$
- Initial point $\in SP$:
  - $O(n^2 \log n)$ to reach 1^n
- Initial point $\notin SP$:
  - SEMO produces trade-off solutions
  - $O(n \cdot n/(n-k))$ expected steps necessary to increase maximal number k of ones by one
  - $$O(\sum_{k=1}^{n} n^2/(n-k)) = O(n^2 \sum_{k=1}^{n} 1/k) = O(n^2 \log n)$$ $\square$



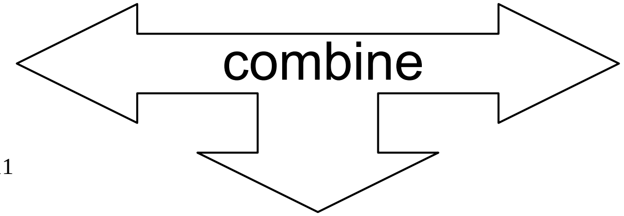
$\text{PLOM}(x)$

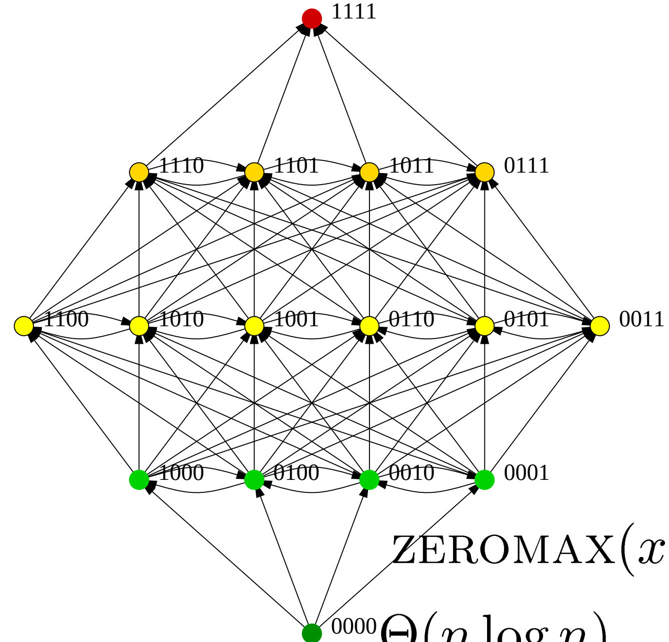
Adding ZEROMAX



$\text{PLATEAU}_1(x)$

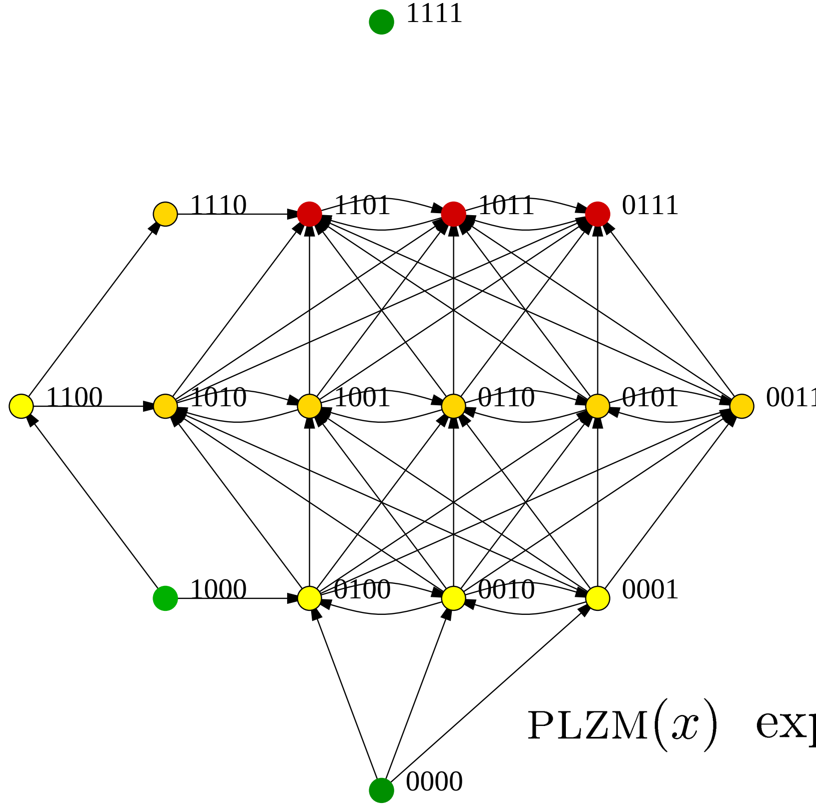
$\Theta(n^3)$





$\text{ZEROMAX}(x)$

$\Theta(n \log n)$



$\text{PLZM}(x)$ exponential w.h.p.

Wait a minute…

Results:

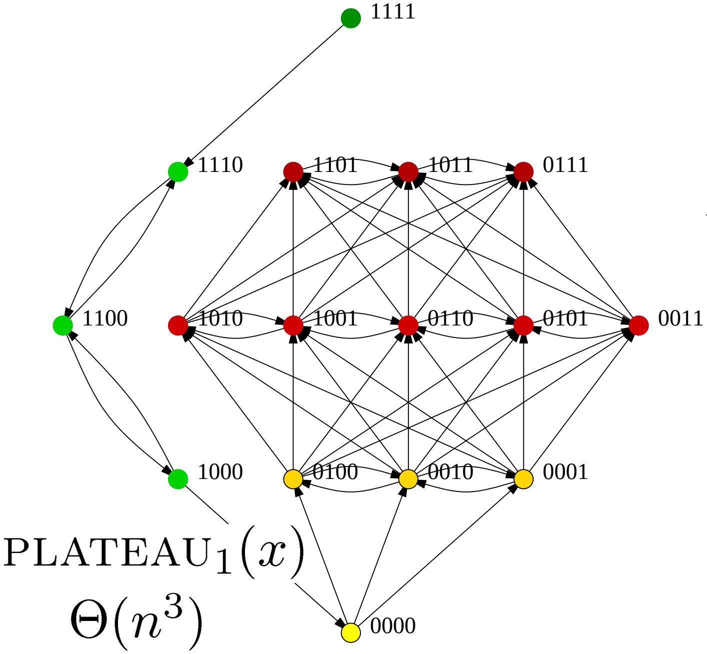
- Added simple problem to a more difficult problem
  $\rightarrow$ combination less complex (PLATEAU_1 + $\text{ONEMAX} \rightarrow \text{PLOM}$)
- Other perspective: added difficult problem to an easy one
  $\rightarrow$ more complex (ONEMAX + $\text{PLATEAU}_1 \rightarrow \text{PLOM}$)

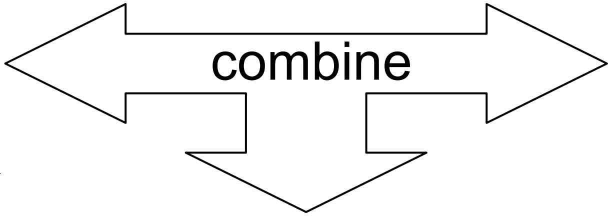
Question:

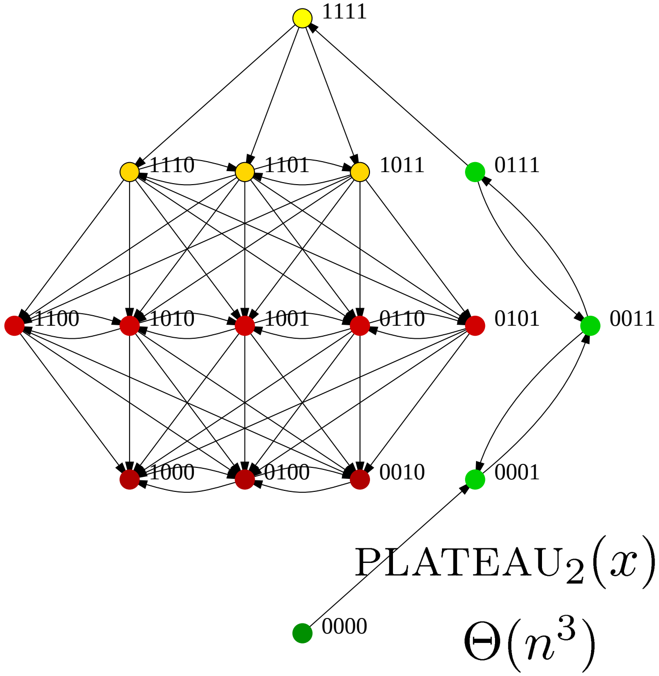
What happens if we combine two equally complex problems?

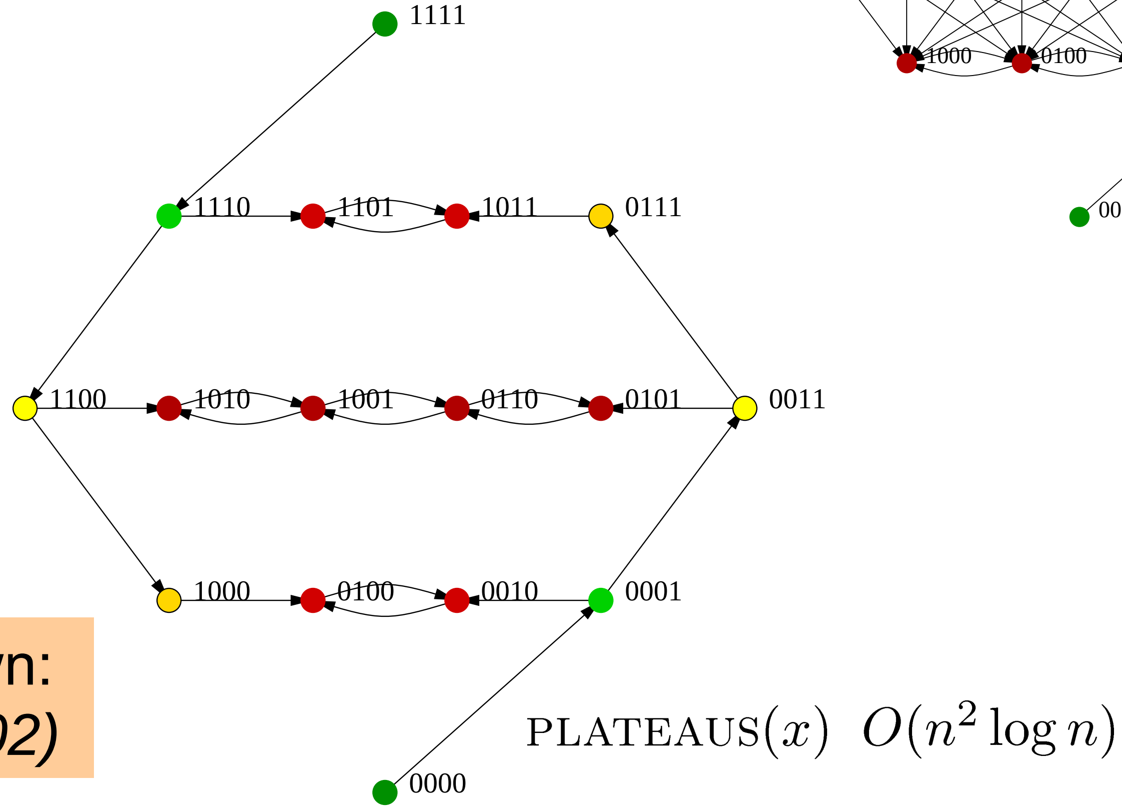
In other words:

Combination more complex or even less complex?









Harder already shown:
Laumanns et al. (2002)

Conclusions

General result:

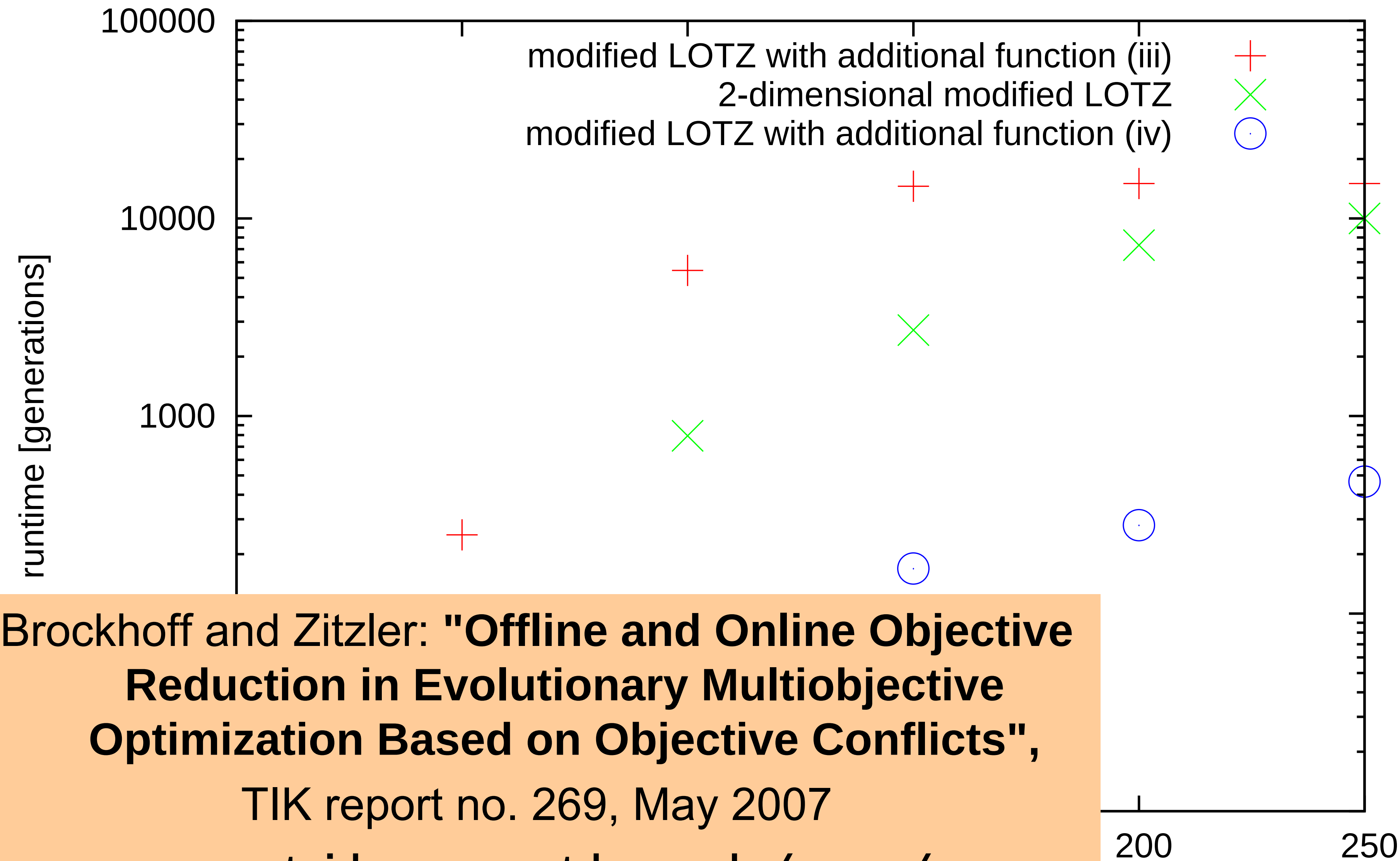
gained general understanding of how problem structure changes with additional objectives (introducing plateaus, deceptive/helping)

- Introducing incomparable solutions is not always a problem
- Performance of EA depends on the objectives chosen

Specific Results:

- Running time analyses for simple EAs
  - One and the same plateau function can become harder and easier with an additional objective
  - Example, where the combination of two problems as a bi-objective problem is easier than solving the single problems individually
- All proofs also for an asymmetric mutation operator

More Objectives…



Brockhoff and Zitzler: **"Offline and Online Objective Reduction in Evolutionary Multiobjective Optimization Based on Objective Conflicts",** TIK report no. 269, May 2007

`www.tik.ee.ethz.ch/sop/`