An Introduction to Evolutionary Multiobjective Optimization

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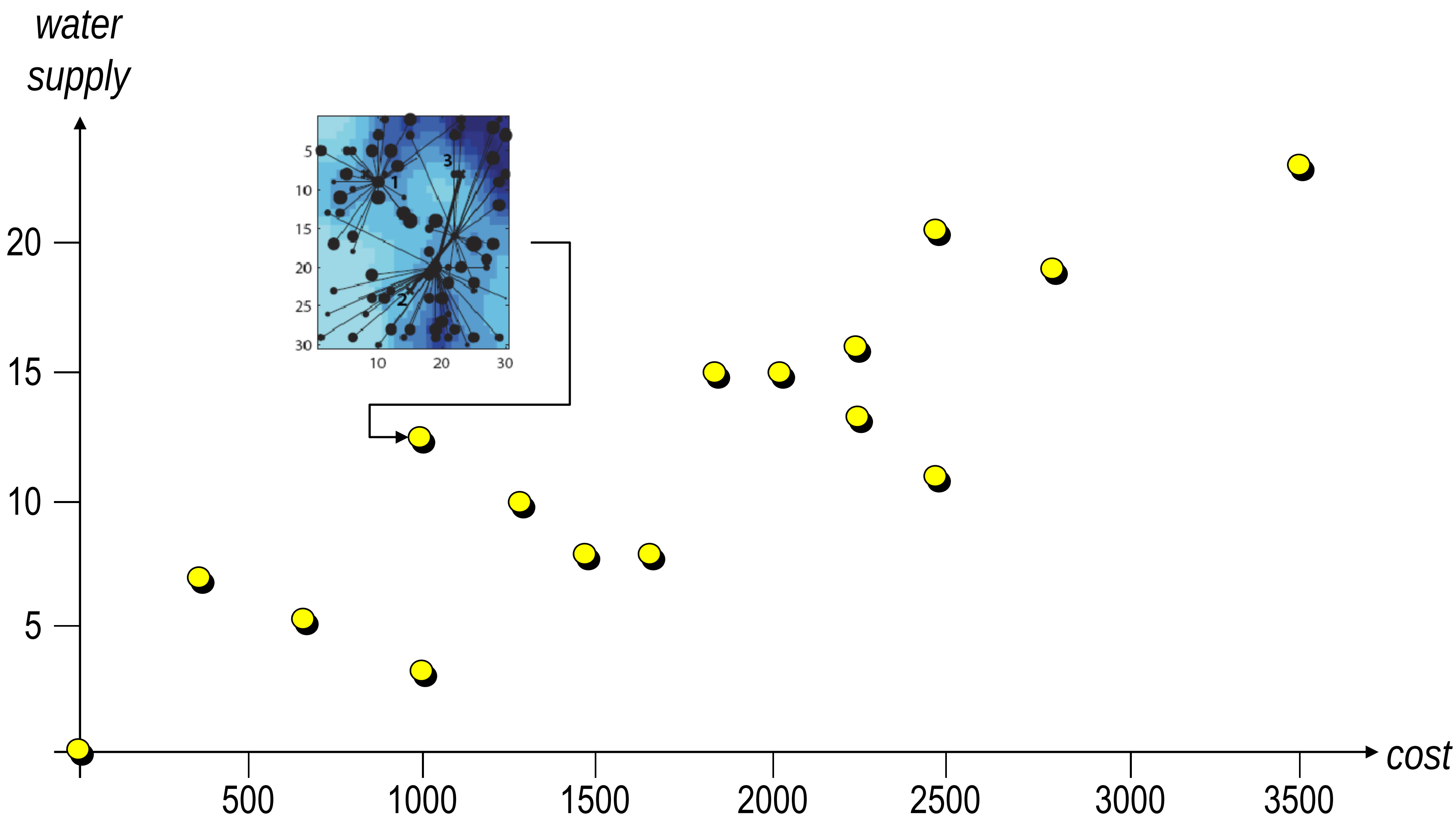
École des Ponts, December 3, 2009

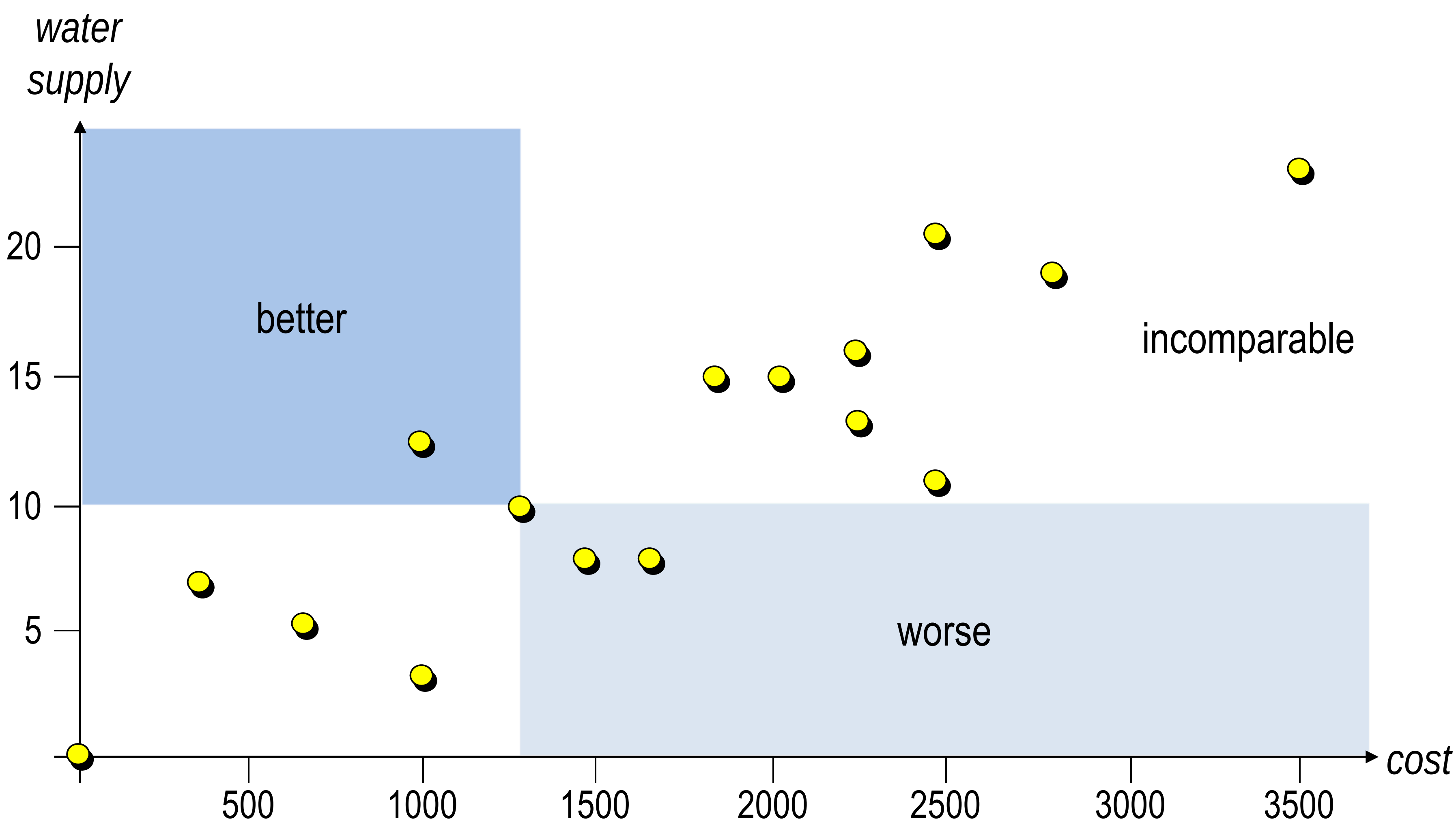
INSTITUT NATIONAL
DE RECHERCHE
EN INFORMATIQUE
ET EN AUTOMATIQUE

INRIA

A hypothetical problem: all solutions plotted



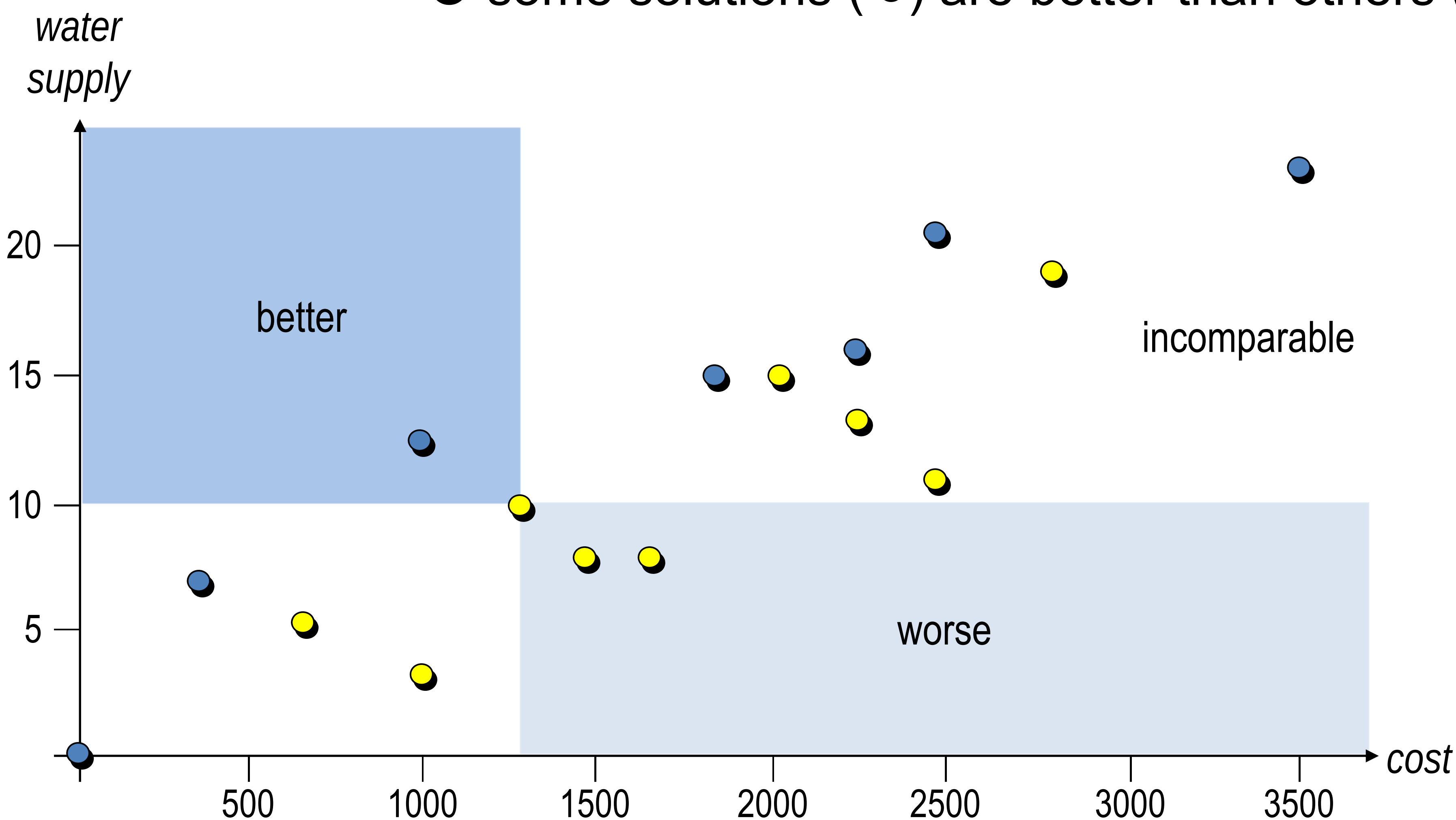
A hypothetical problem: all solutions plotted



Observations:

❶ there is no single optimal solution, but

❷ some solutions ( ) are better than others ( )



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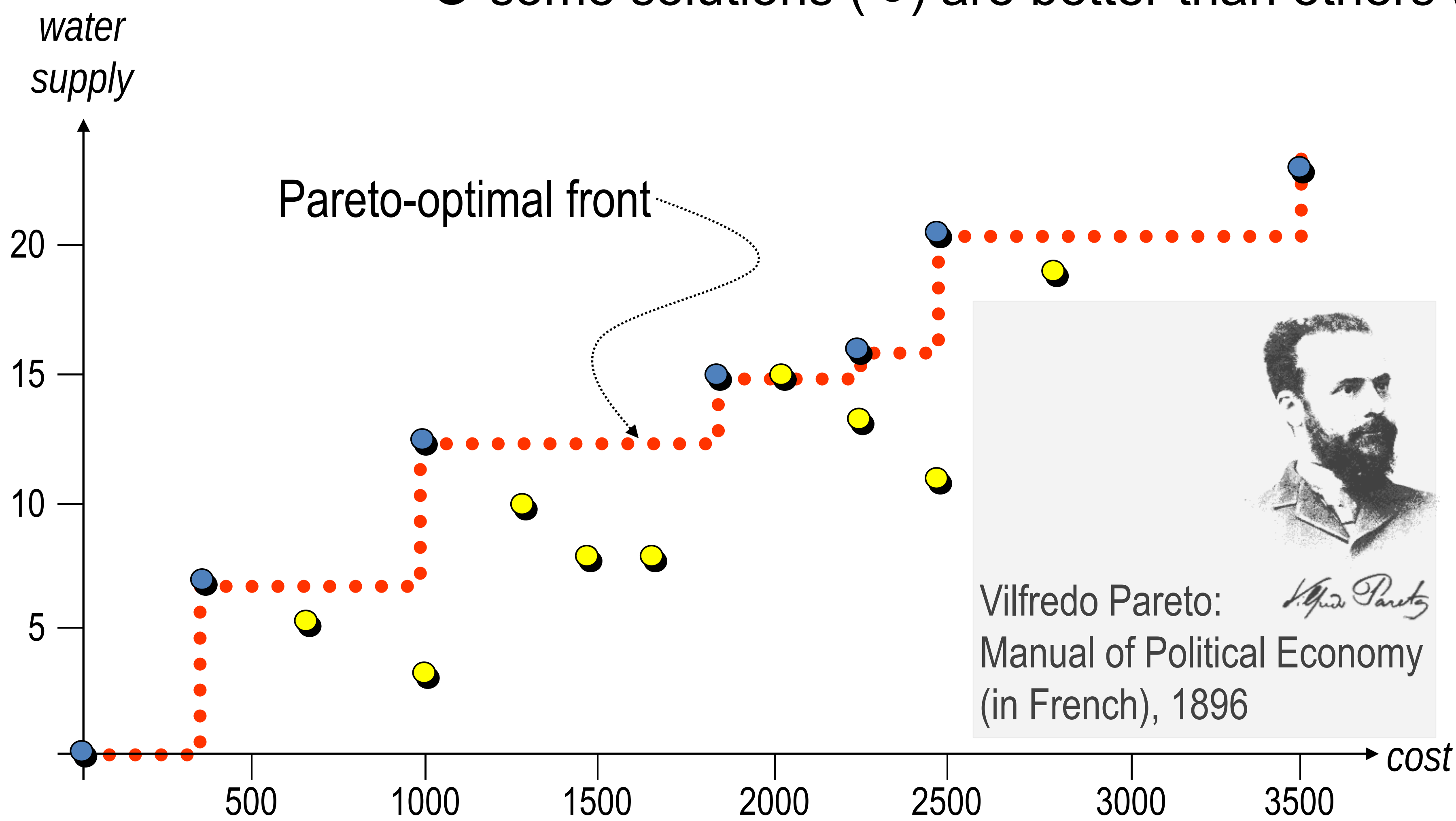

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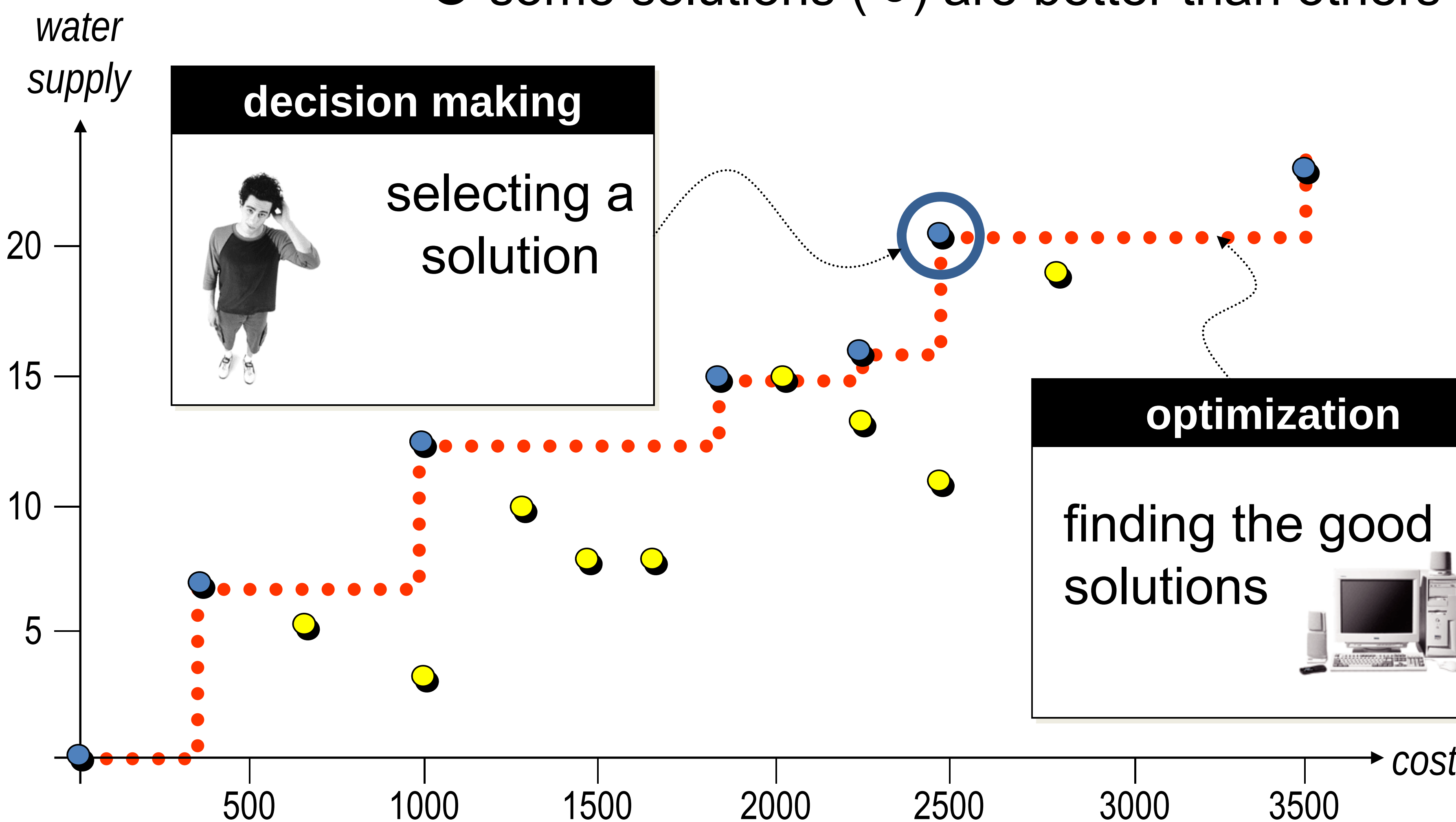
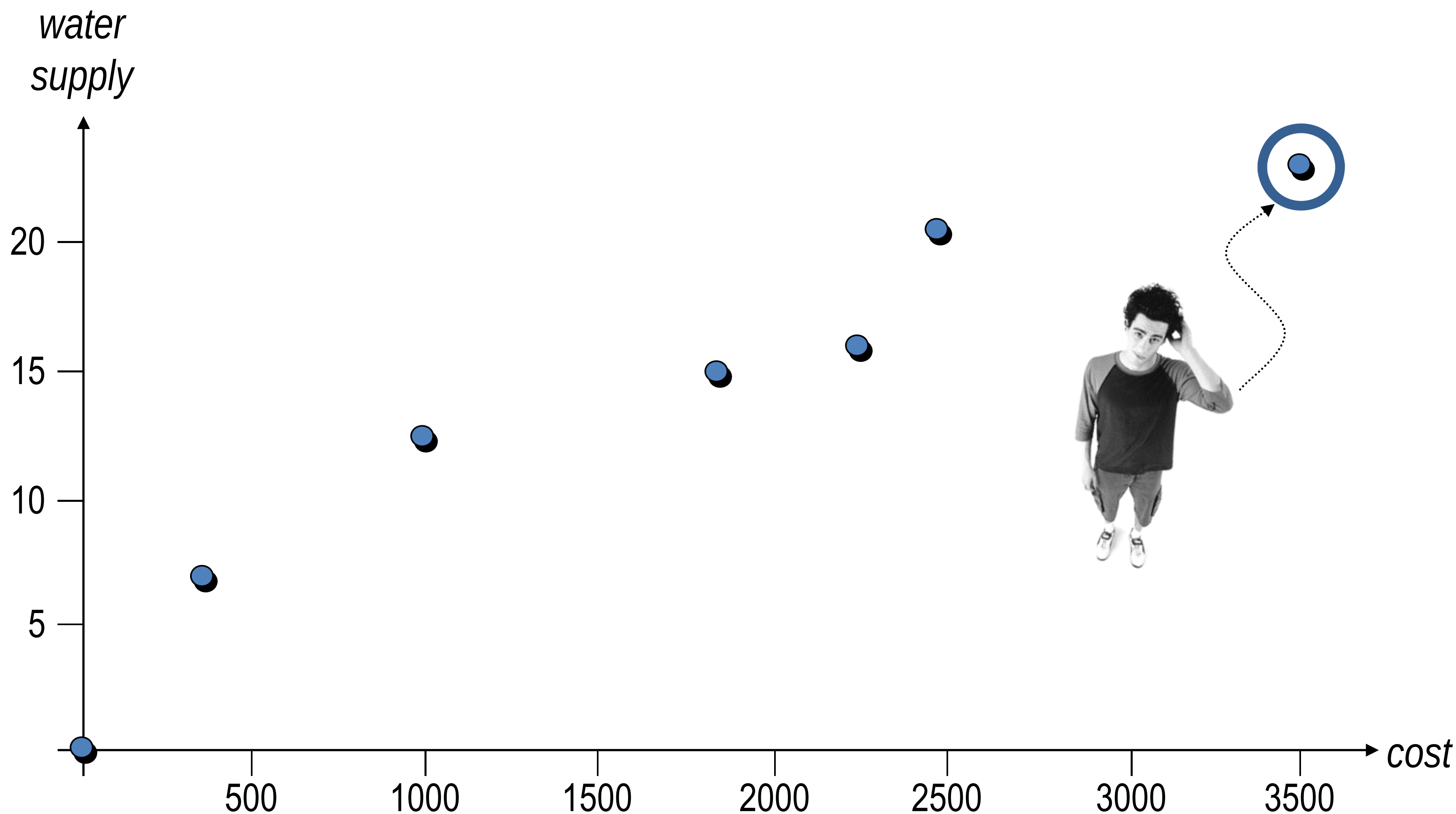


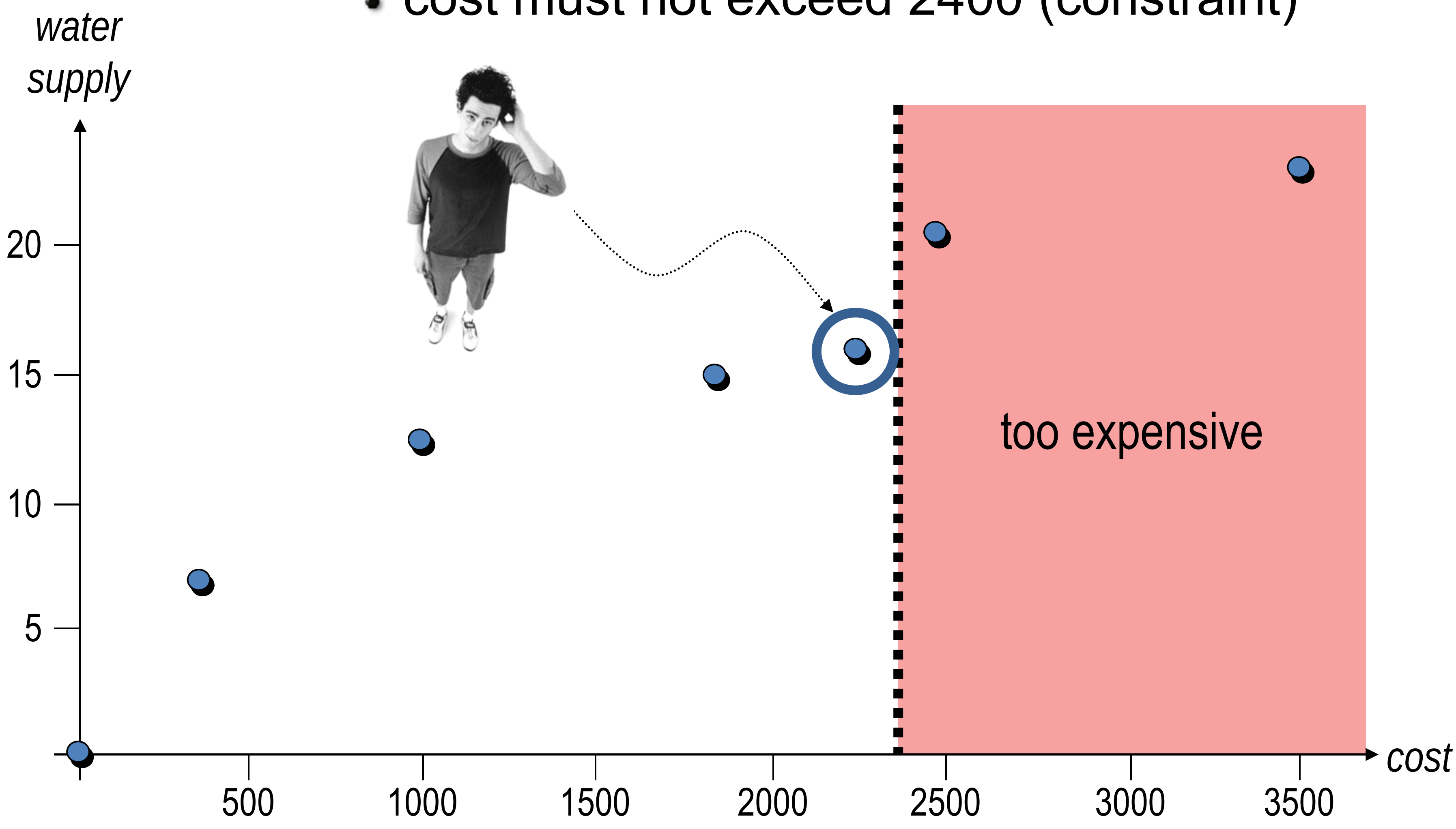
Decision Making: Selecting a Solution

Approaches: • supply more important than cost (ranking)



Decision Making: Selecting a Solution

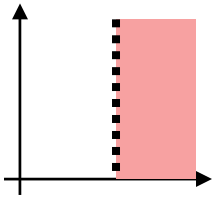
Approaches:
- supply more important than cost (ranking)
- cost must not exceed 2400 (constraint)



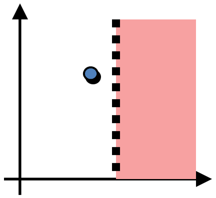
When to Make the Decision

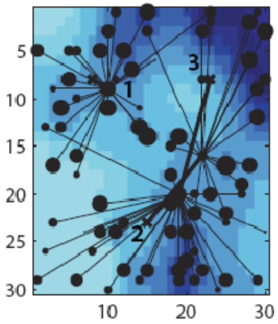
Before Optimization:

ranks objectives,
defines constraints,…



searches for one
(blue) solution





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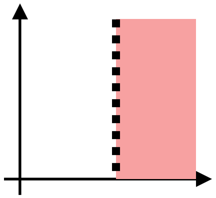

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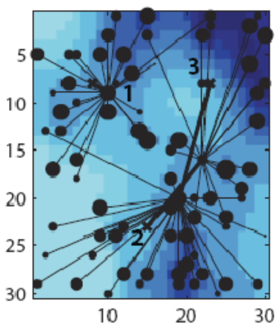

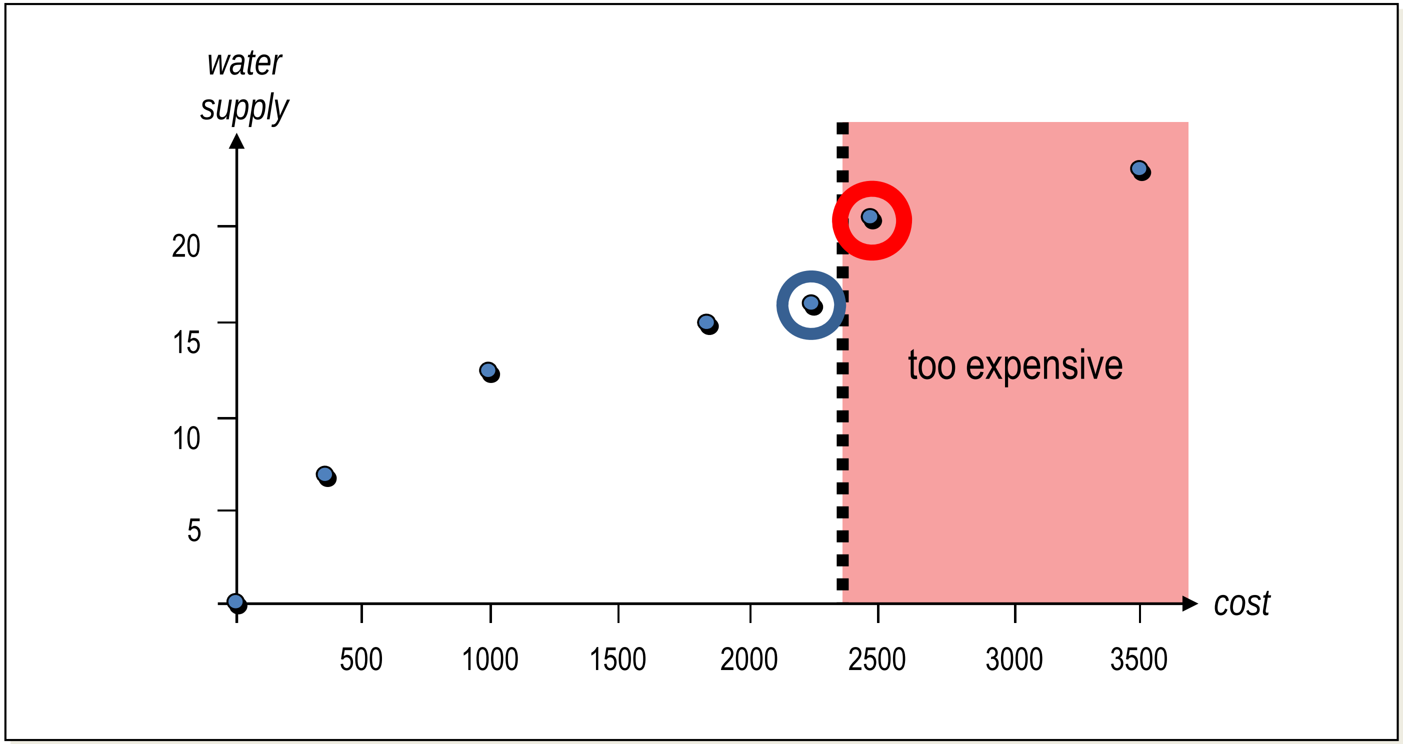

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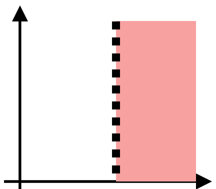

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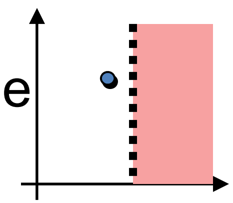
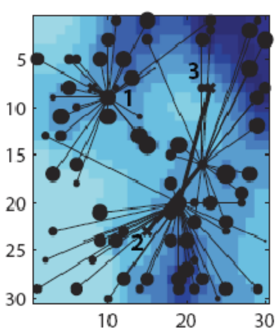
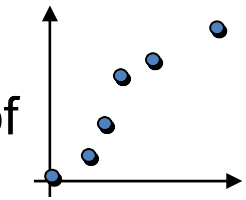


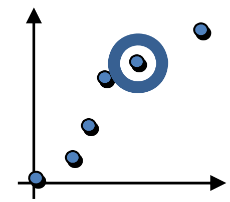


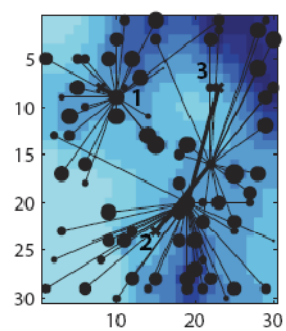
After Optimization:

searches for a set of
(blue) solutions



selects one solution
considering constraints, etc.





When to Make the Decision

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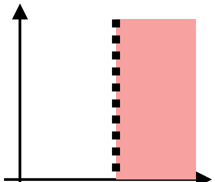

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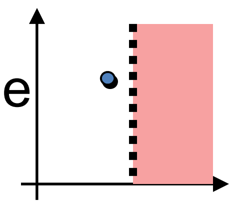

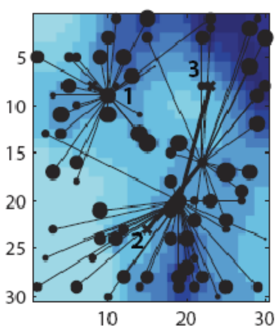

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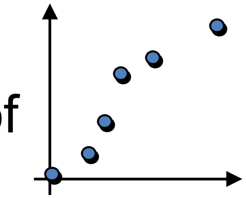

selects one solution
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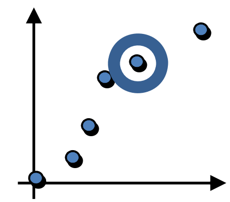


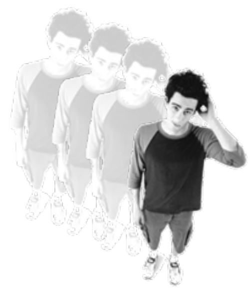
Focus: learning about a problem

- trade-off surface
- interactions among criteria
- structural information

Definition: MCDM

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process

International Society on Multiple Criteria Decision Making



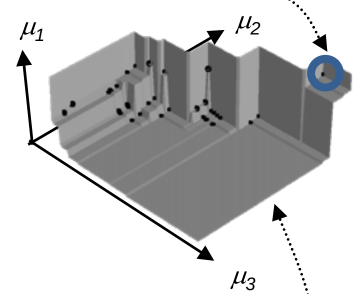
model

trade-off surface

$$\min_x \left[\mu_1(x), \mu_2(x), ..., \mu_n(x)\right]^T$$
$$s.t.$$
$$g(x) \leq 0$$
$$h(x) = 0$$
$$x_l \leq x \leq x_u$$

decision making

(exact) optimization



Multiple Criteria Decision Making (MCDM)

Definition: MCDM

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process

International Society on Multiple Criteria Decision Making

uncertainty

noise

non-linear objectives

non-differentiable objectives

integrated simulations

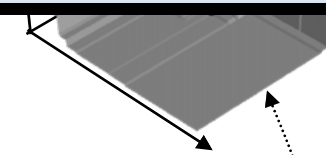
multiple objectives

many constraints / objectives

huge search spaces

$$g(x) \leq 0$$
$$h(x) = 0$$
$$x_l \leq x \leq x_u$$

(ex~~a~~ct) optimization



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International Society on Multiple Criteria Decision Making

noise

uncertainty

integrate

non-linear objectives

non-differentiable objectives

$g(x) \leq 0$

$h(x) = 0$

$x_l \leq x \leq x_u$

~~(exact)~~

Black box optimization

$$x \in X \longrightarrow \boxed{f} \longrightarrow (f_1(x), \ldots, f_k(x))$$

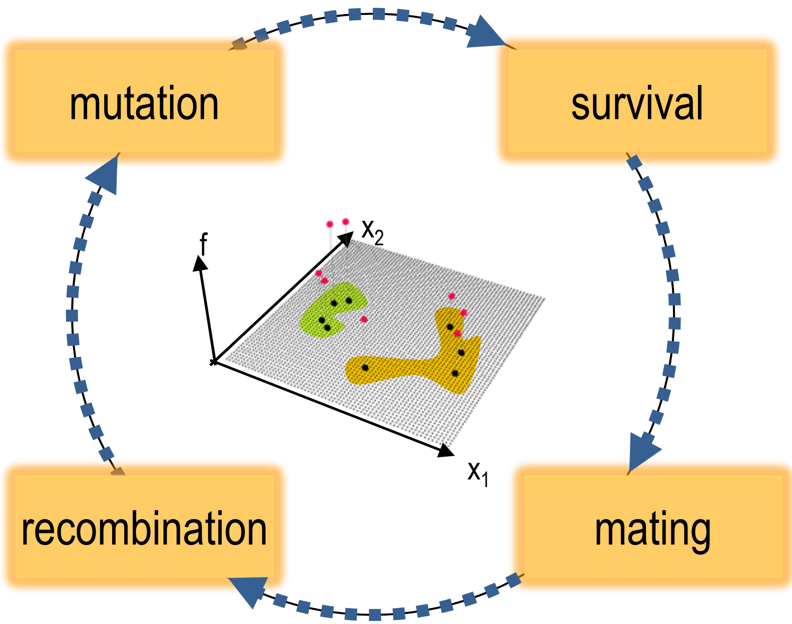
?

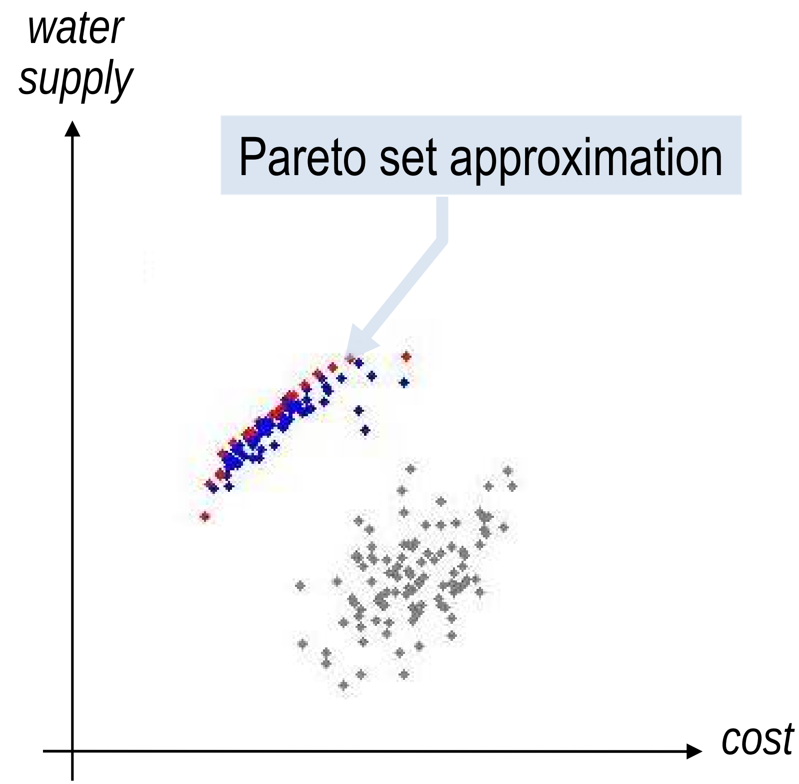
nonlinear function, simulation, experiment

Definition: EMO

EMO = **evolutionary algorithms** / randomized search algorithms

- applied to multiple criteria decision making (in general)
- used to approximate the Pareto-optimal set (mainly)





The History of EMO At A Glance

1984

first EMO approaches

dominance-based population ranking

1990

dominance-based EMO algorithms with diversity preservation techniques

1995

attainment functions

elitist EMO algorithms
preference articulation
convergence proofs

2000

test problem design
quantitative performance assessment
multiobjectivization

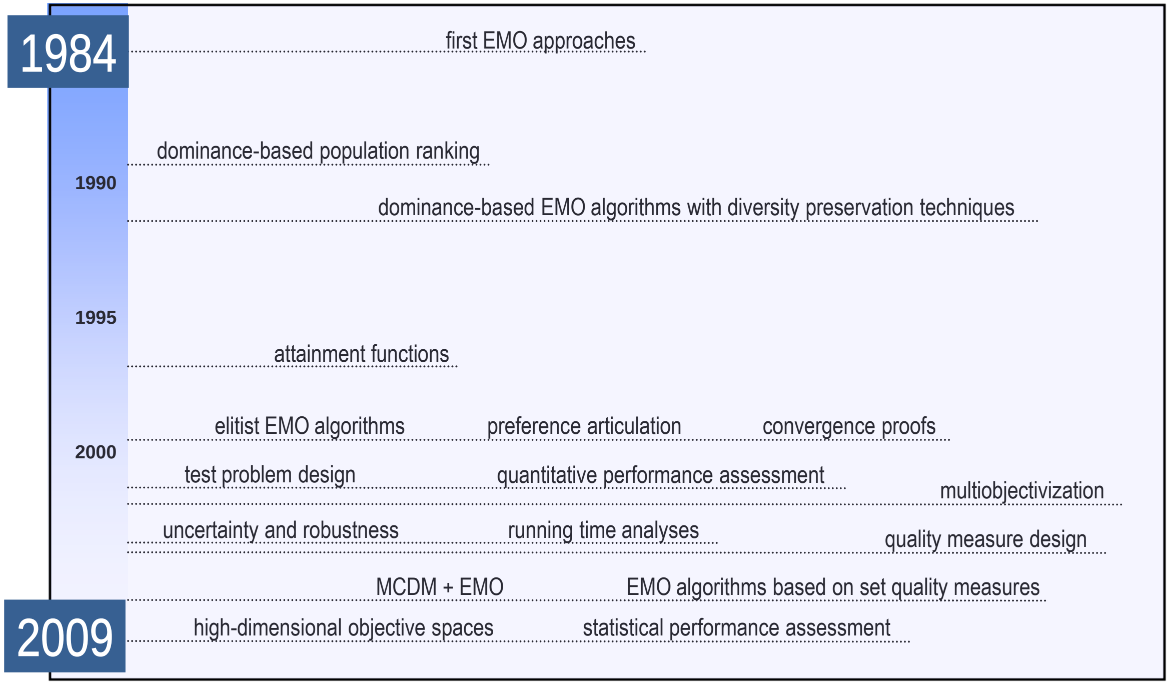
uncertainty and robustness
running time analyses
quality measure design

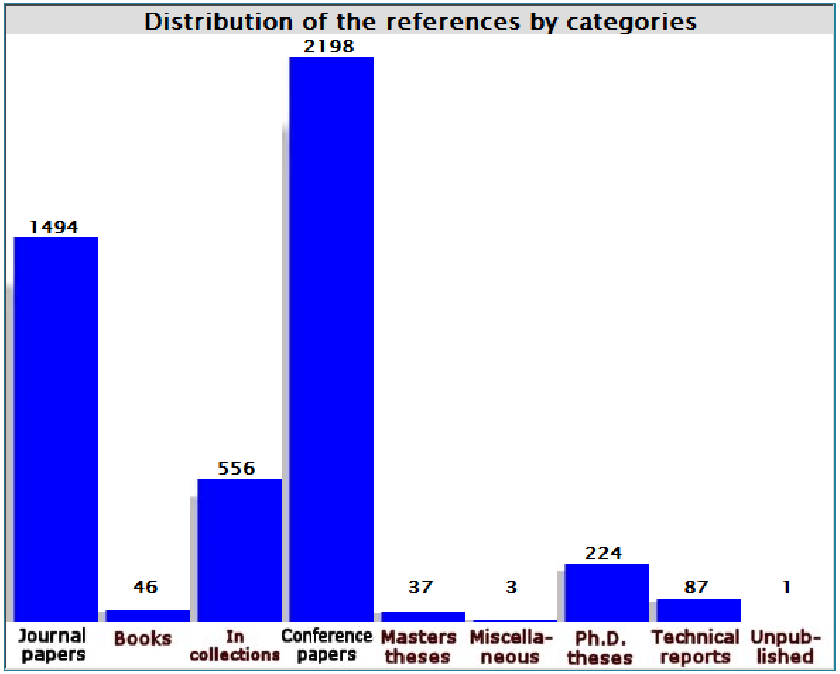
MCDM + EMO
EMO algorithms based on set quality measures

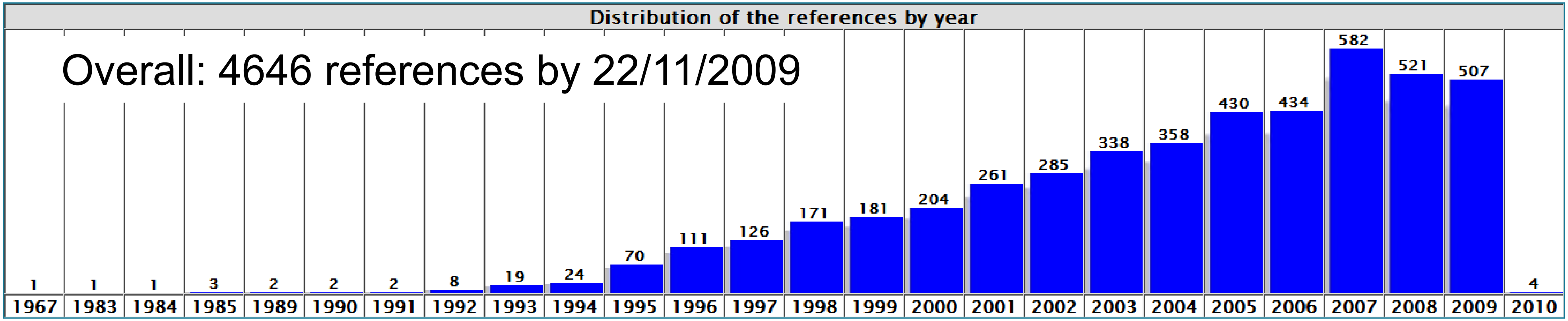
high-dimensional objective spaces
statistical performance assessment

2009

The History of EMO At A Glance







http://delta.cs.cinvestav.mx/~ccoello/EMOO/EMOOstatistics.html

The EMO Community

The EMO conference series:

EMO2001
Zurich
Switzerland

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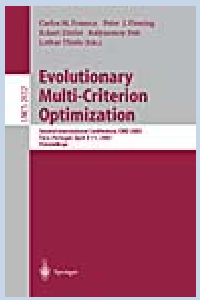
EMO2003
Faro
Portugal



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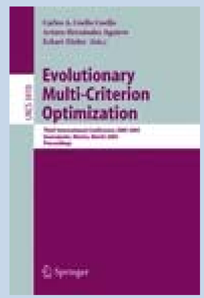
EMO2005
Guanajuato
Mexico



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EMO2007
Matsushima
Japan

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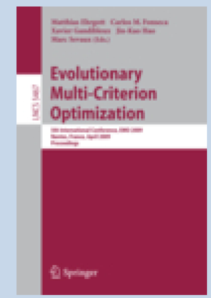
EMO2009
Nantes
France



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Many further activities:

special sessions, special journal issues, workshops, tutorials, ...

Overview

The Big Picture

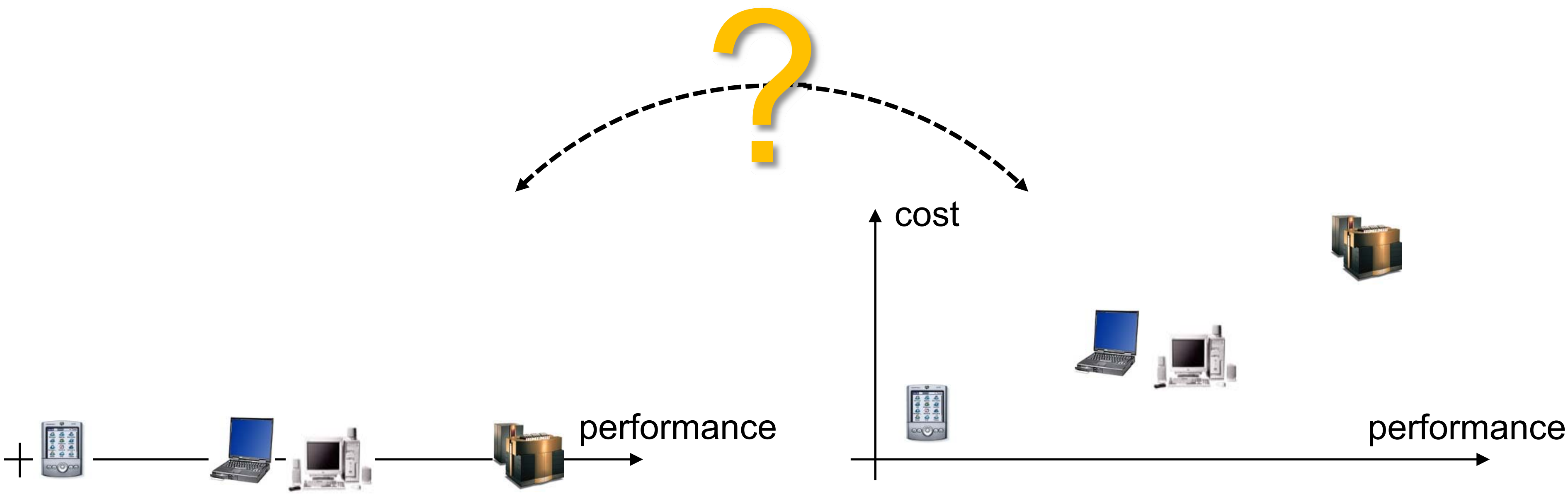
Basic Principles of Multiobjective Optimization

Algorithm Design Principles and Concepts

Performance Assessment

A Few Examples From Practice

What makes evolutionary multiobjective optimization different from single-objective optimization?



single objective **multiple objectives**

A General (Multiobjective) Optimization Problem

A *multiobjective optimization problem* is defined by a 5-tuple $(X, Z, \mathbf{f}, \mathbf{g}, \leqslant)$ where

- X is the decision space,
- $Z = \mathbb{R}^n$ is the objective space,
- $\mathbf{f} = (f_1, \ldots, f_n)$ is a vector-valued function consisting of n objective functions $f_i : X \mapsto \mathbb{R}$,
- $\mathbf{g} = (g_1, \ldots, g_m)$ is a vector-valued function consisting of m constraint functions $g_i : X \mapsto \mathbb{R}$, and
- $\leqslant \subseteq Z \times Z$ is a binary relation on the objective space.

The goal is to identify a decision vector $\mathbf{a} \in X$ such that (i) for all $1 \leq i \leq m$ holds $g_i(\mathbf{a}) \leq 0$ and (ii) for all $\mathbf{b} \in X$ holds $\mathbf{f}(\mathbf{b}) \leqslant \mathbf{f}(\mathbf{a}) \Rightarrow \mathbf{f}(\mathbf{a}) \leqslant \mathbf{f}(\mathbf{b})$.

A Single-Objective Optimization Problem

decision space | objective space | objective function

total order

$$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$$

A Single-Objective Optimization Problem

decision space, objective space, objective function

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

total order

$(X, prefrel)$

total preorder where

$a \; prefrel \; b :\Leftrightarrow f(a) \; rel \; f(b)$

A Single-Objective Optimization Problem

Example: Leading Ones Problem

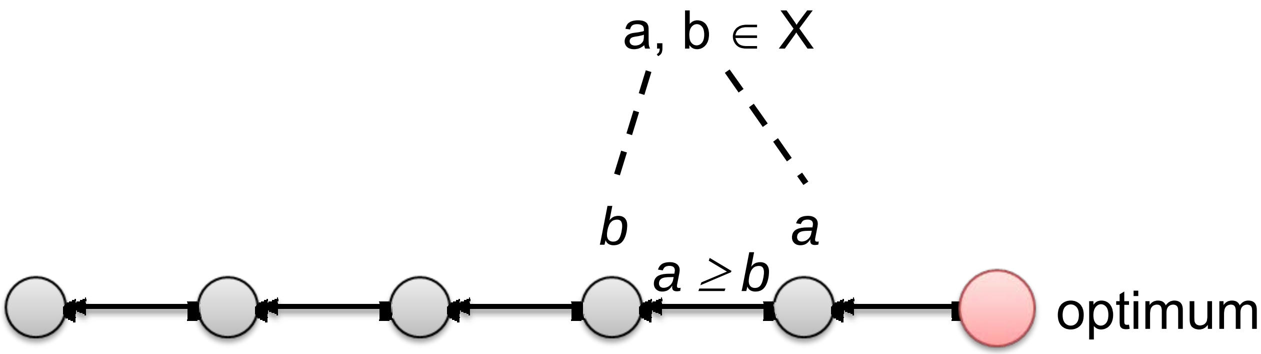
$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$

$(\{0,1\}^n, \{0,1, 2, \ldots, n\}, f_{LO}, \geq)$ where $f_{LO}(a) = \sum_i (\prod_{j \leq i} a_j)$

Simple Graphical Representation

Example: $\geq$ (total order)



totally ordered

Preference Relations

decision space objective space objective functions

partial order

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

preorder where
a prefrel b $:\Leftrightarrow$ *f(a) rel f(b)*

(X, prefrel)

$$(X, \preccurlyeq_{par})$$

$$\downarrow$$

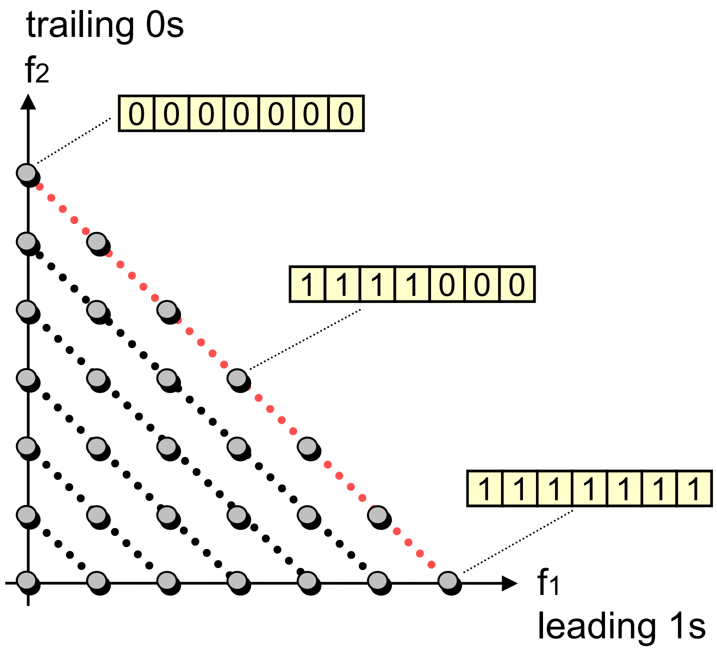
$$a \preccurlyeq_{par} b :\Leftrightarrow f(a) \leqslant_{par} f(b)$$

weak
Pareto dominance

Example: Leading Ones Trailing Zeros Problem

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$



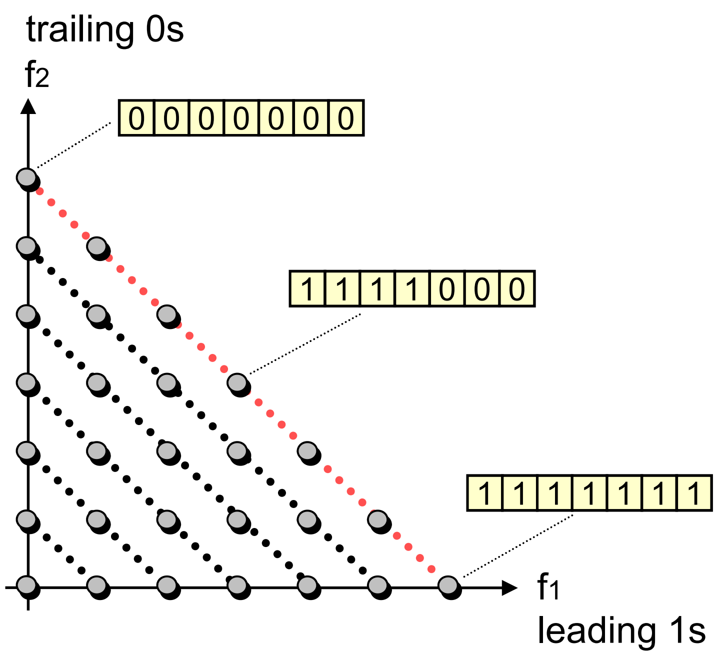
A Multiobjective Optimization Problem

Example: Leading Ones Trailing Zeros Problem

$(X, Z, f\!: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$

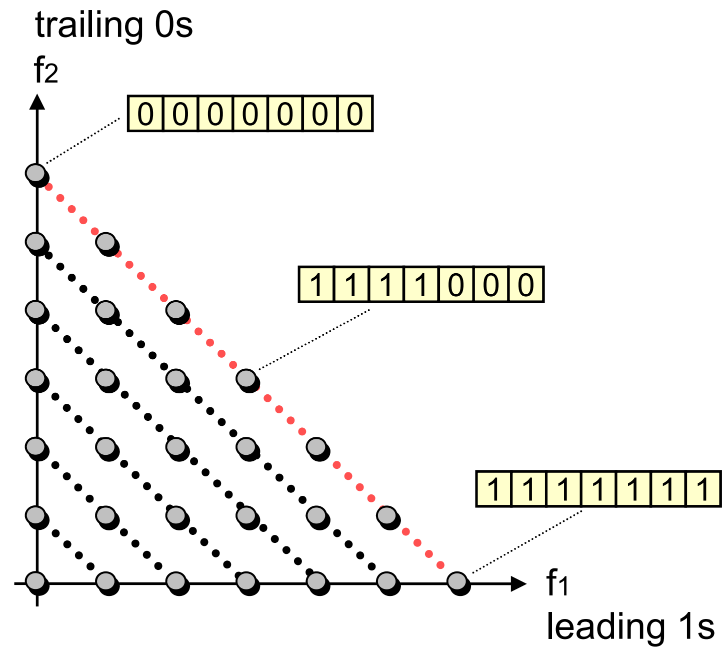
$(\{0,1\}^n,$



Example: Leading Ones Trailing Zeros Problem

$$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$$

$$(X, prefrel)$$



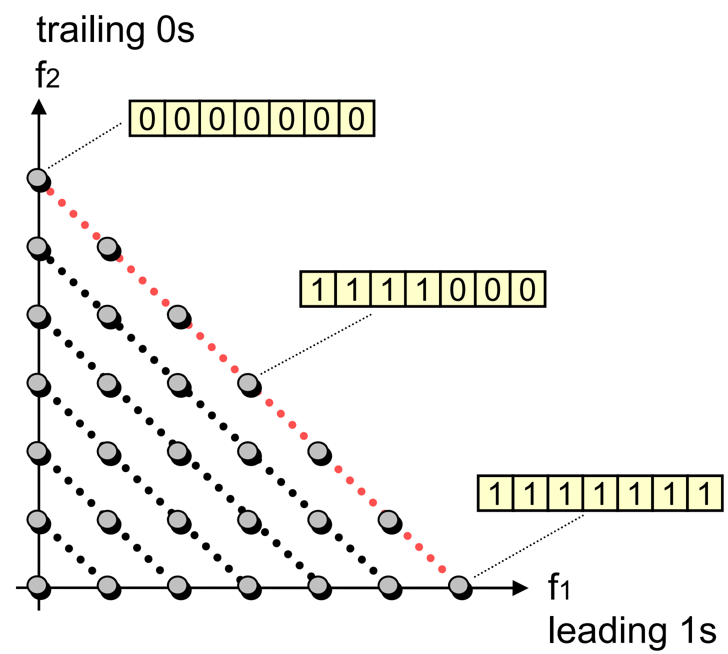
$$(\{0,1\}^n, \{0,1, 2, \ldots, n\} \times \{0,1, 2, \ldots, n\},$$

A Multiobjective Optimization Problem

Example: Leading Ones Trailing Zeros Problem

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$



$(\{0,1\}^n, \{0,1, 2, \ldots, n\} \times \{0,1, 2, \ldots, n\}, (f_{LO}, f_{TZ}),$

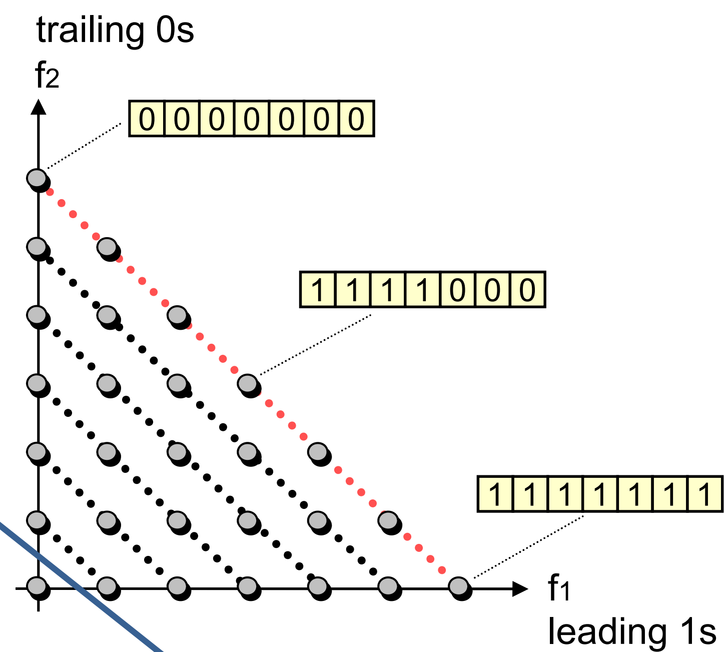
$f_{LO}(a) = \sum_i (\prod_{j\leq i} a_j)$ $\qquad$ $f_{TZ}(a) = \sum_i (\prod_{j\leq i} (1-a_j))$

A Multiobjective Optimization Problem

Example: Leading Ones Trailing Zeros Problem

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$



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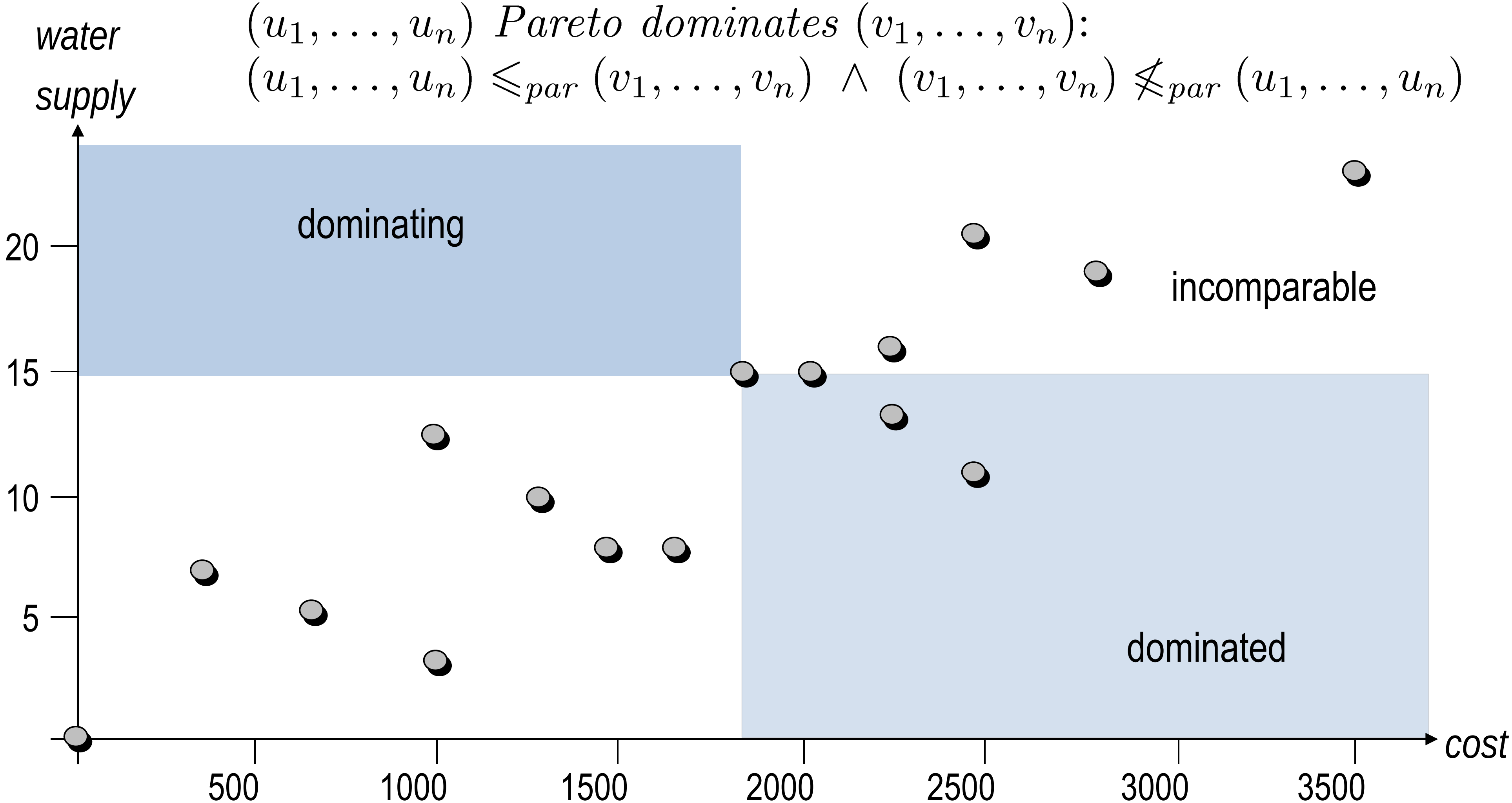
$f_{LO}(a) = \sum_i (\prod_{j \leq i} a_j)$ $\qquad$ $f_{TZ}(a) = \sum_i (\prod_{j \leq i} (1 - a_j))$

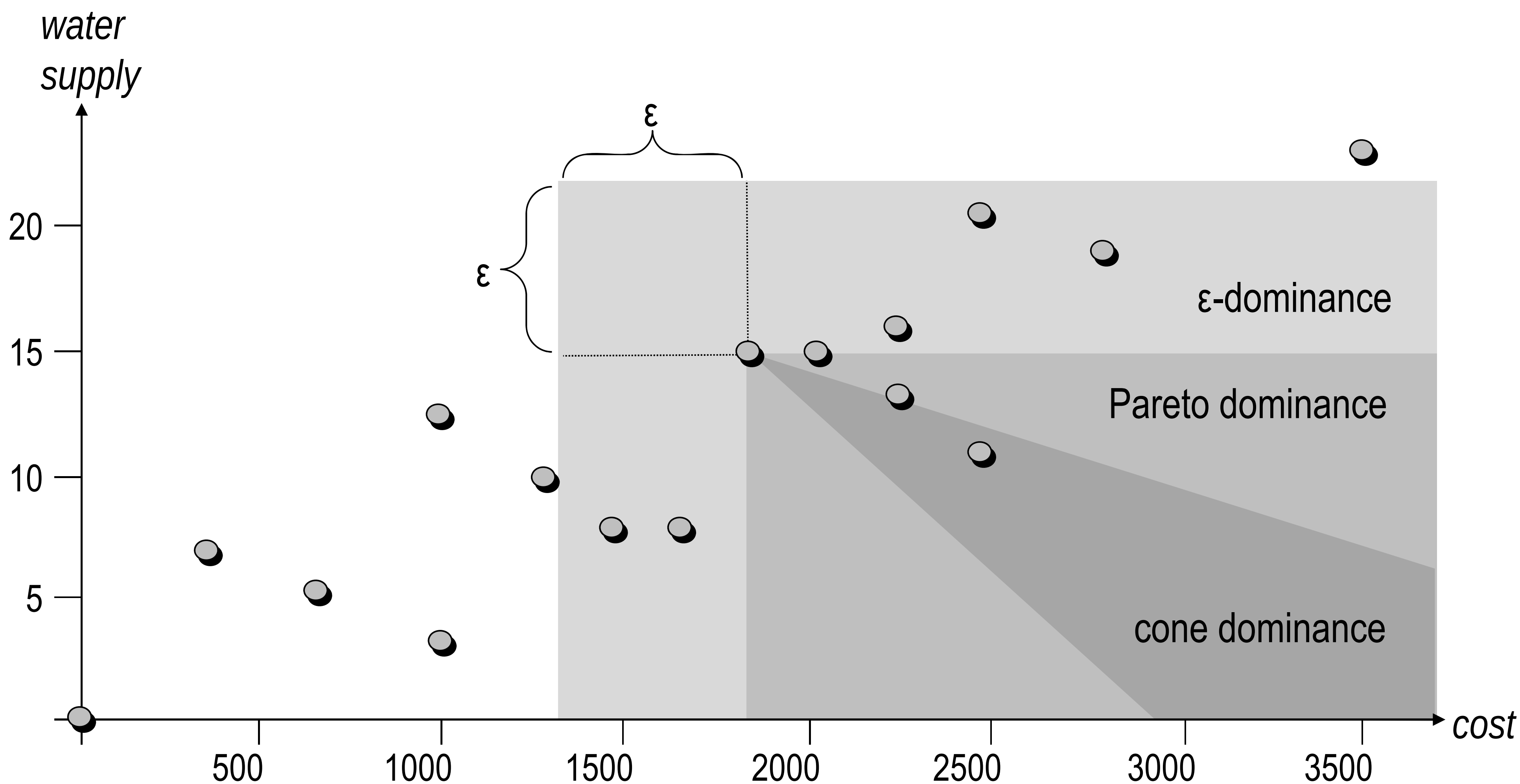
Pareto Dominance

$$(u_1, \ldots, u_n) \textit{ weakly Pareto dominates } (v_1, \ldots, v_n):$$
$$(u_1, \ldots, u_n) \leqslant_{par} (v_1, \ldots, v_n) :\Leftrightarrow \forall 1 \leq i \leq n : u_i \leq v_i$$

$$(u_1, \ldots, u_n) \textit{ Pareto dominates } (v_1, \ldots, v_n):$$
$$(u_1, \ldots, u_n) \leqslant_{par} (v_1, \ldots, v_n) \;\wedge\; (v_1, \ldots, v_n) \not\leqslant_{par} (u_1, \ldots, u_n)$$



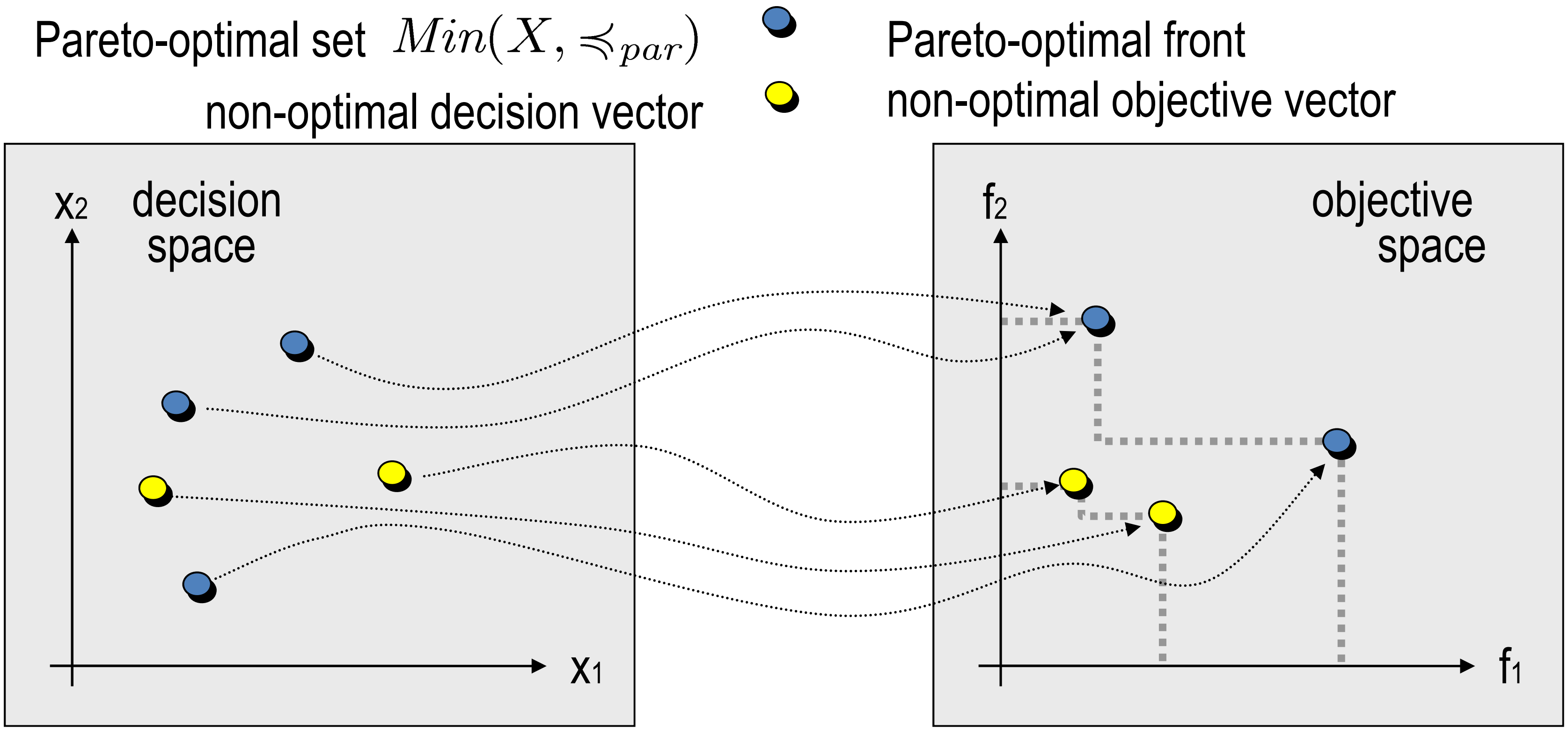
Different Notions of Dominance



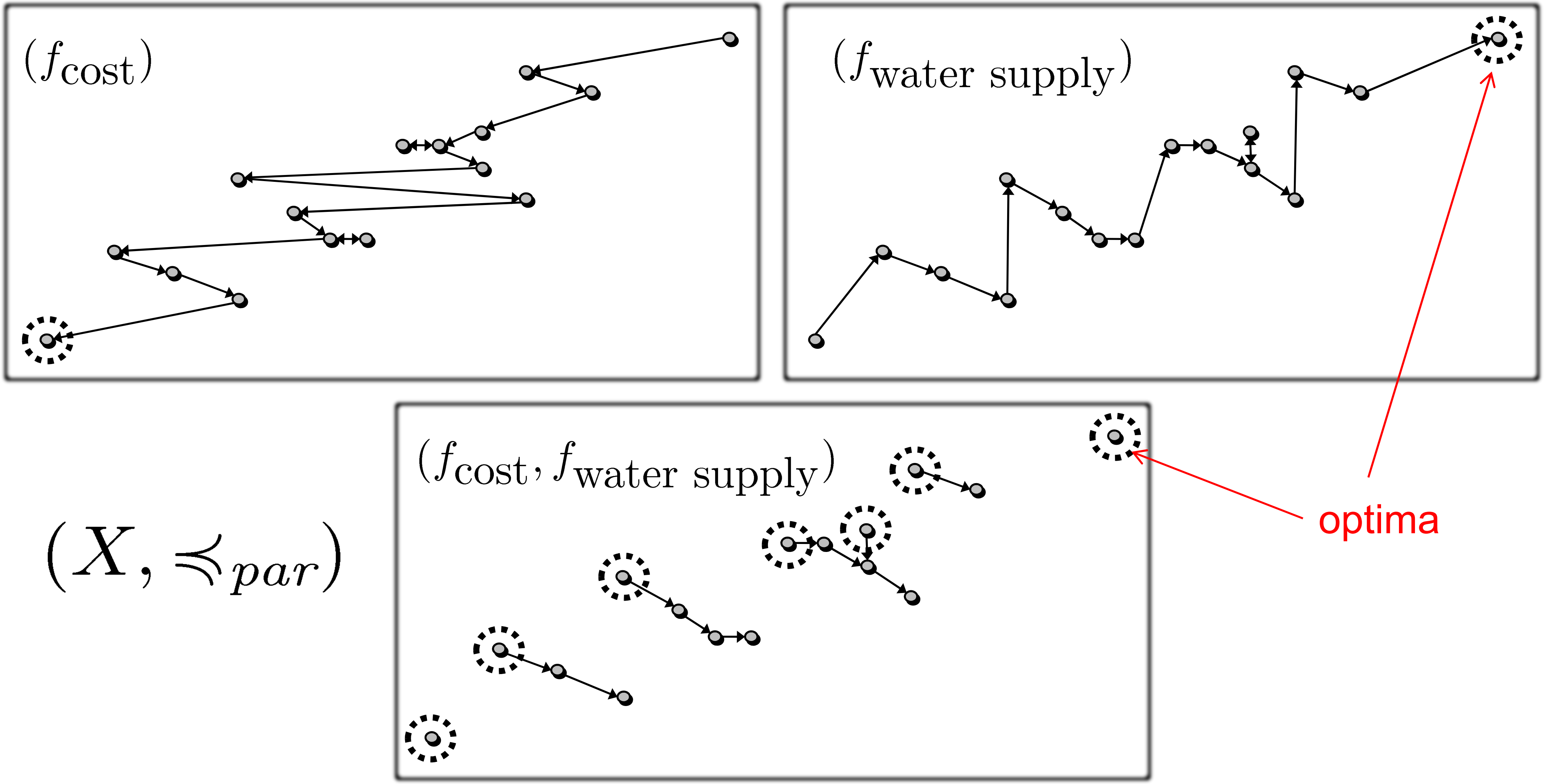
The Pareto-optimal Set

The *minimal set* of a preordered set $(Y, \leqq)$ is defined as

$$Min(Y, \leqq) := \{a \in Y \mid \forall b \in Y : b \leqq a \Rightarrow a \leqq b\}$$



Visualizing Preference Relations



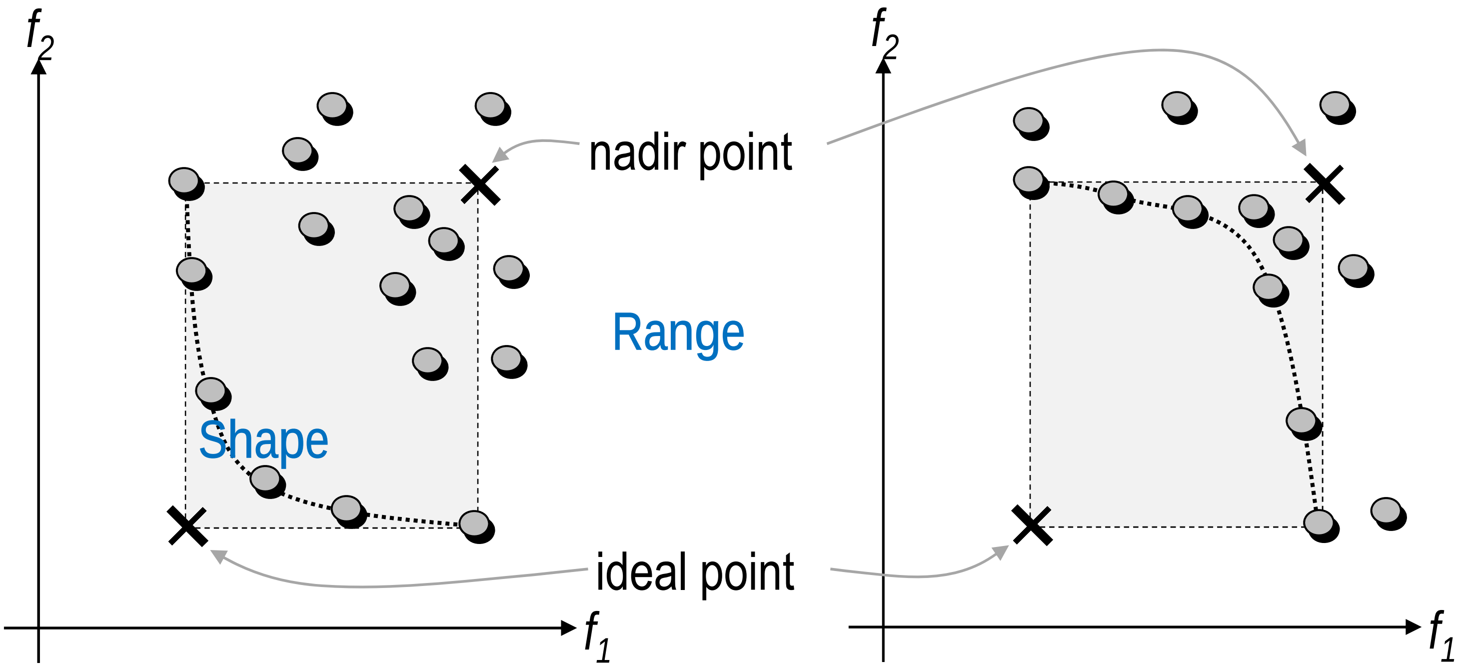
$$(X, \preccurlyeq_{par})$$

Remark: Properties of the Pareto-optimal Set

Computational complexity:

multiobjective variants can be become NP- and #P-complete

Size: Pareto set can be exponential in the input length (shortest path [Serafini 1986], MSP [Camerini et al. 1984])



Approaches To Multiobjective Optimization

A multiobjective problem is as such underspecified…

…because not any Pareto-optimum is equally suited!

Additional preferences are needed to tackle the problem:

- **Solution-Oriented Problem Transformation:**
  Induce a total order on the decision space, e.g., by aggregation.

- **Set-Oriented Problem Transformation:**
  First transform problem into a set problem and then define an objective function on sets.

Preferences are needed in any case, but the latter are weaker!

Problem Transformations and Set Problems

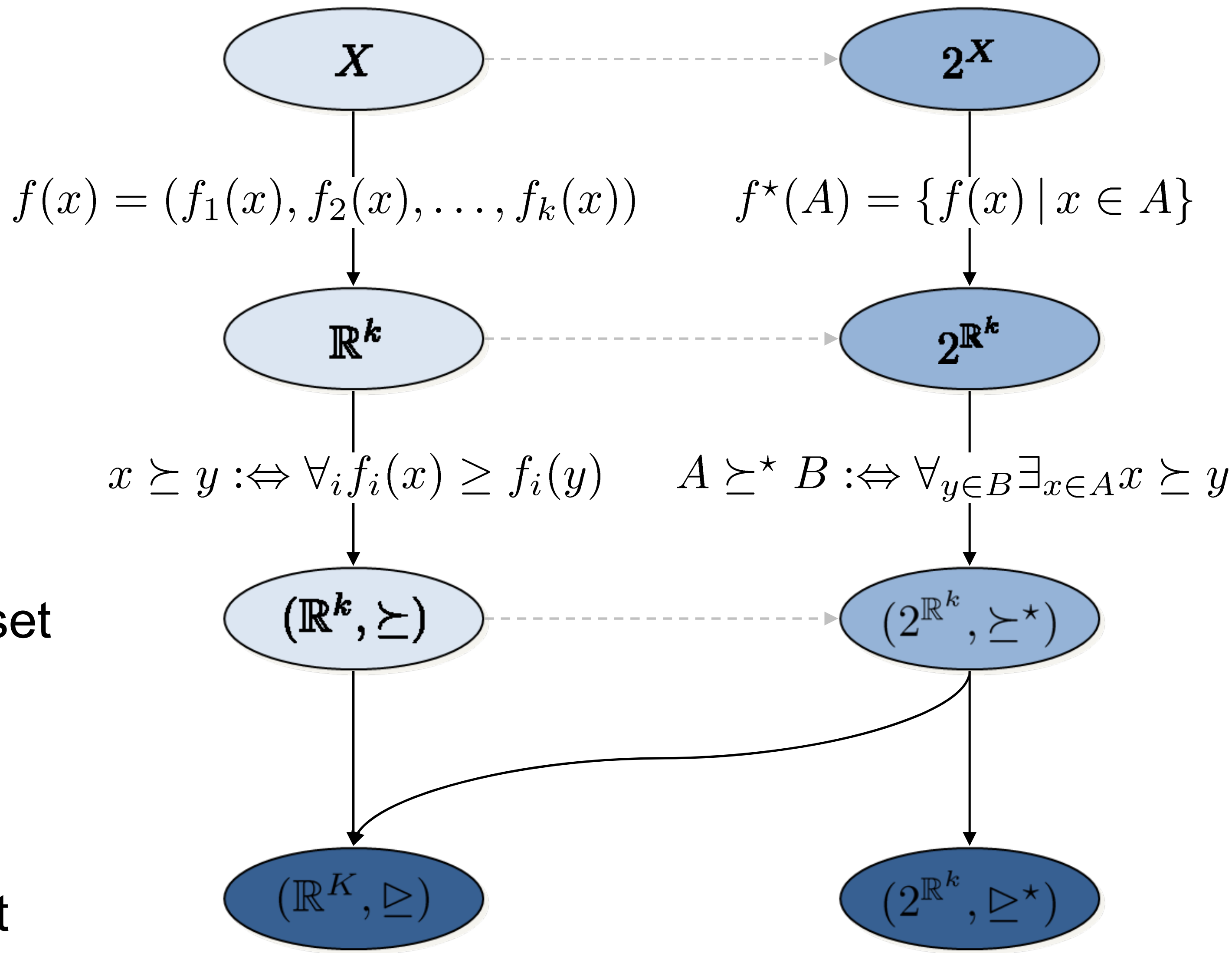
single solution problem | **set problem**

search space

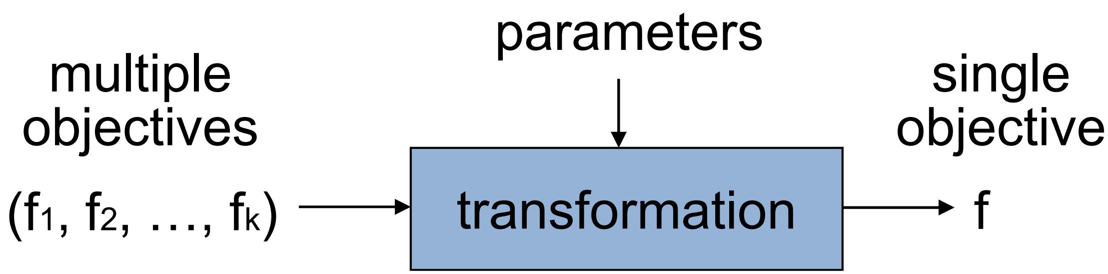
objective space

(partially) ordered set

(totally) ordered set



Solution-Oriented Problem Transformations

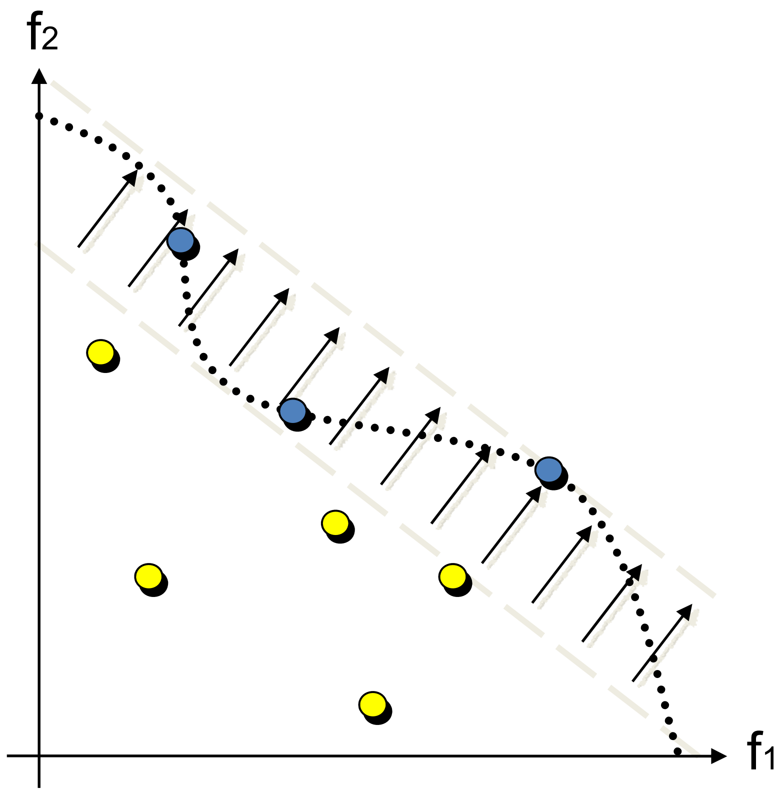


A *scalarizing function* s is a function $s : Z \mapsto \mathbb{R}$ that maps each objective vector $(u_1, \ldots, u_n) \in Z$ to a real value $s(u_1, \ldots, u_n) \in \mathbb{R}$.

Aggregation-Based Approaches

parameters

multiple objectives $(f_1, f_2, \ldots, f_k)$ → **transformation** → single objective f



Example: weighting approach

$(w_1, w_2, \ldots, w_k)$

$$y = w_1 y_1 + \ldots + w_k y_k$$

Other example: Tchebycheff

$$y = \max w_i(u_i - z_i)$$

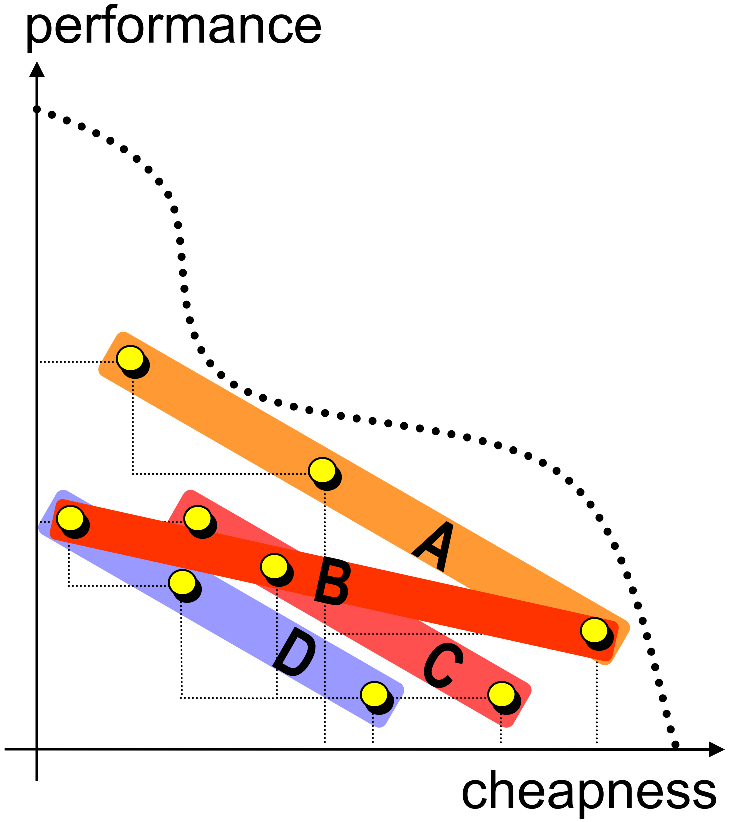
Set-Oriented Problem Transformations

For a multiobjective optimization problem $(X, Z, \mathbf{f}, \mathbf{g}, \leqslant)$, the associated *set problem* is given by $(\Psi, \Omega, F, \mathbf{G}, \preccurlyeq)$ where

- $\Psi = 2^X$ is the space of decision vector sets, i.e., the powerset of X,
- $\Omega = 2^Z$ is the space of objective vector sets, i.e., the powerset of Z,
- F is the extension of $\mathbf{f}$ to sets, i.e., $F(A) := \{\mathbf{f}(\mathbf{a}) : \mathbf{a} \in A\}$ for $A \in \Psi$,
- $\mathbf{G} = (G_1, \ldots, G_m)$ is the extension of $\mathbf{g}$ to sets, i.e., $G_i(A) := \max\{g_i(\mathbf{a}) : \mathbf{a} \in A\}$ for $1 \leq i \leq m$ and $A \in \Psi$,
- $\preccurlyeq$ extends $\leqslant$ to sets where $A \preccurlyeq B :\Leftrightarrow \forall \mathbf{b} \in B\ \exists \mathbf{a} \in A : \mathbf{a} \leqslant \mathbf{b}$.

Pareto Set Approximations

Pareto set approximation (algorithm outcome) =
set of (usually incomparable) solutions



A **weakly dominates** B
= not worse in all objectives
and sets not equal

C **dominates** D
= better in at least one objective

A **strictly dominates** C
= better in all objectives

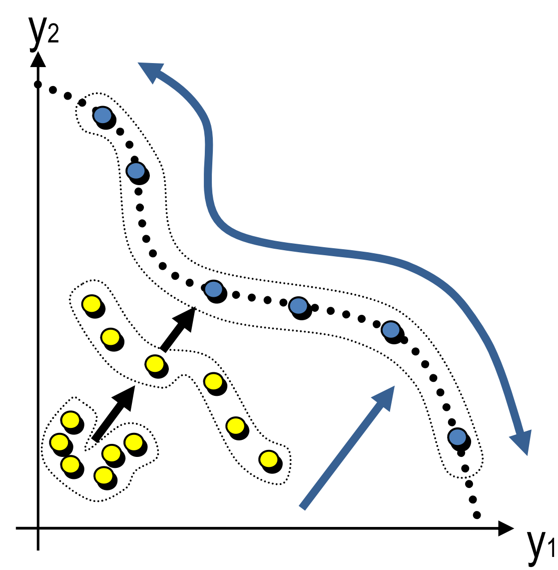
B is **incomparable** to C
= neither set weakly better

What Is the Optimization Goal (Total Order)?

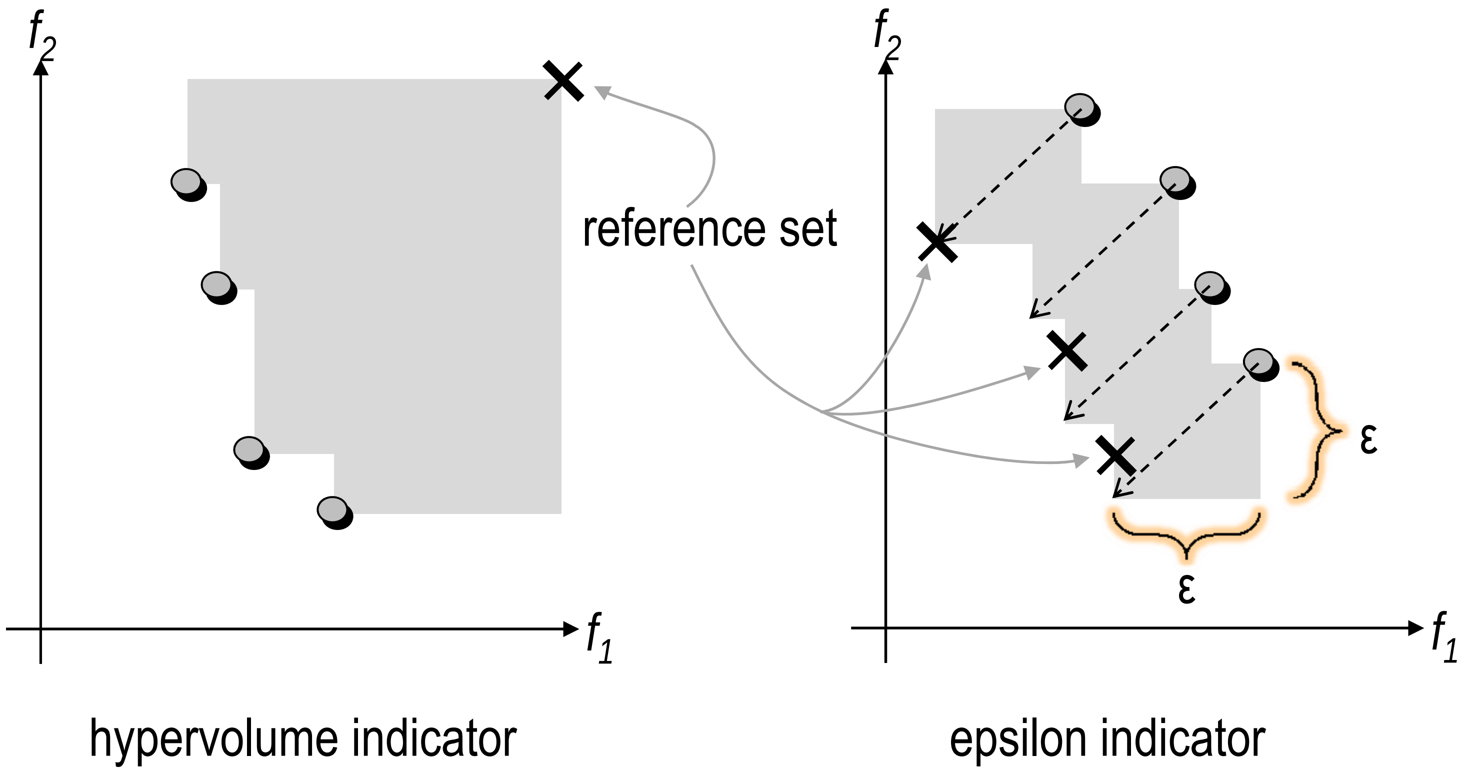
- Find all Pareto-optimal solutions?
  - Impossible in continuous search spaces
  - How should the decision maker handle 10000 solutions?
- Find a representative subset of the Pareto set?
  - Many problems are NP-hard
  - What does representative actually mean?
- Find a good approximation of the Pareto set?
  - What is a good approximation?
  - How to formalize intuitive understanding:

    ❶ close to the Pareto front

    ❷ well distributed



A (unary) *quality indicator* I is a function $I : \Psi \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.



General Remarks on Problem Transformations

Idea:

Transform a preorder into a total preorder

Methods:

- Define single-objective function based on the multiple criteria (shown on the previous slides)
- Define any total preorder using a relation (not discussed before)

Question:

Is any total preorder ok resp. are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation *rel* should be reflected

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

…sought are total refinements…

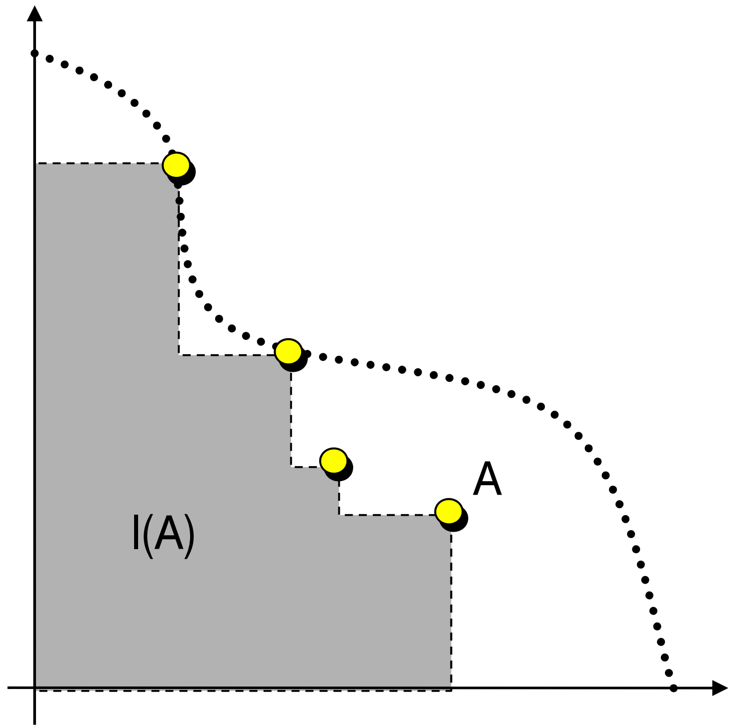
Example: Refinements Using Set Quality Measures

$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A) \geq I(B)$$

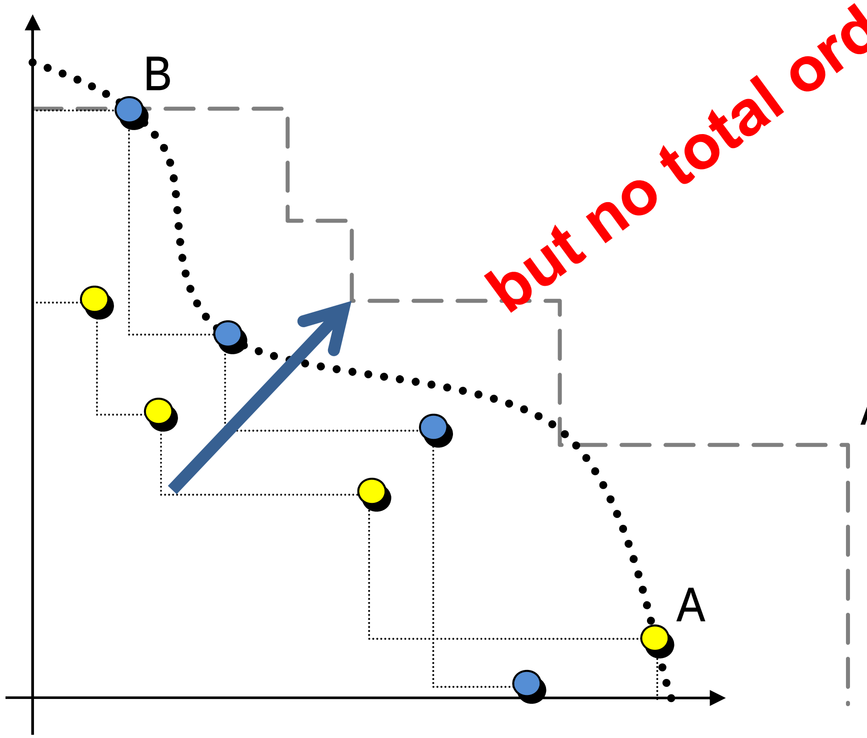
$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B



unary hypervolume indicator



but no total order

binary epsilon indicator

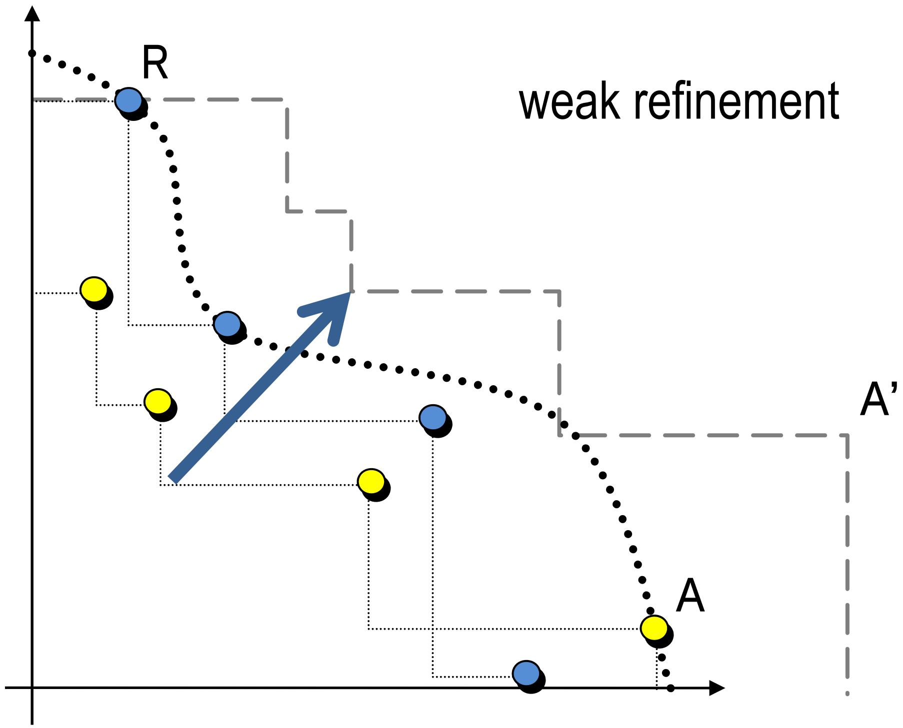
Example: Weak Refinement and No Refinement

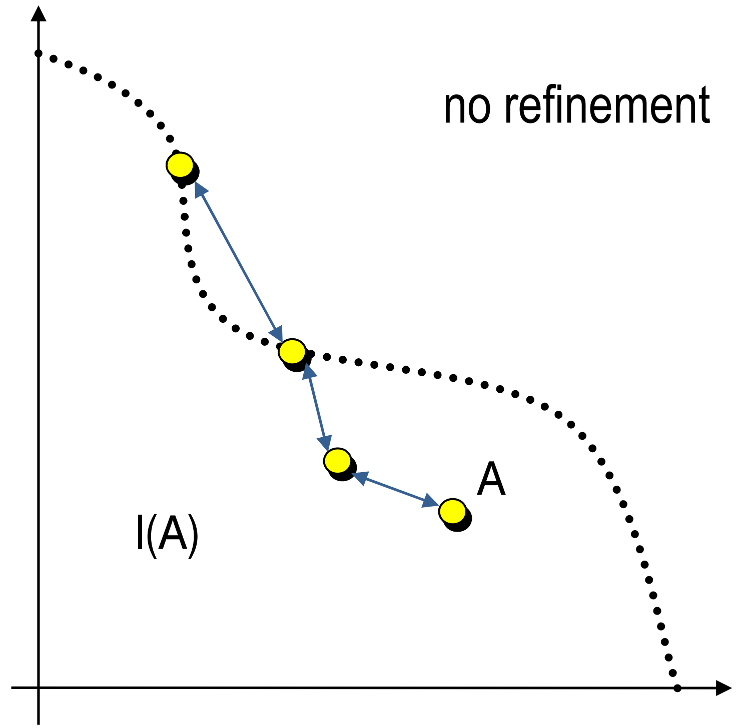
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,R) \leq I(B,R)$$

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \leq I(B)$$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances





unary epsilon indicator

unary diversity indicator

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

Algorithm Design Principles and Concepts

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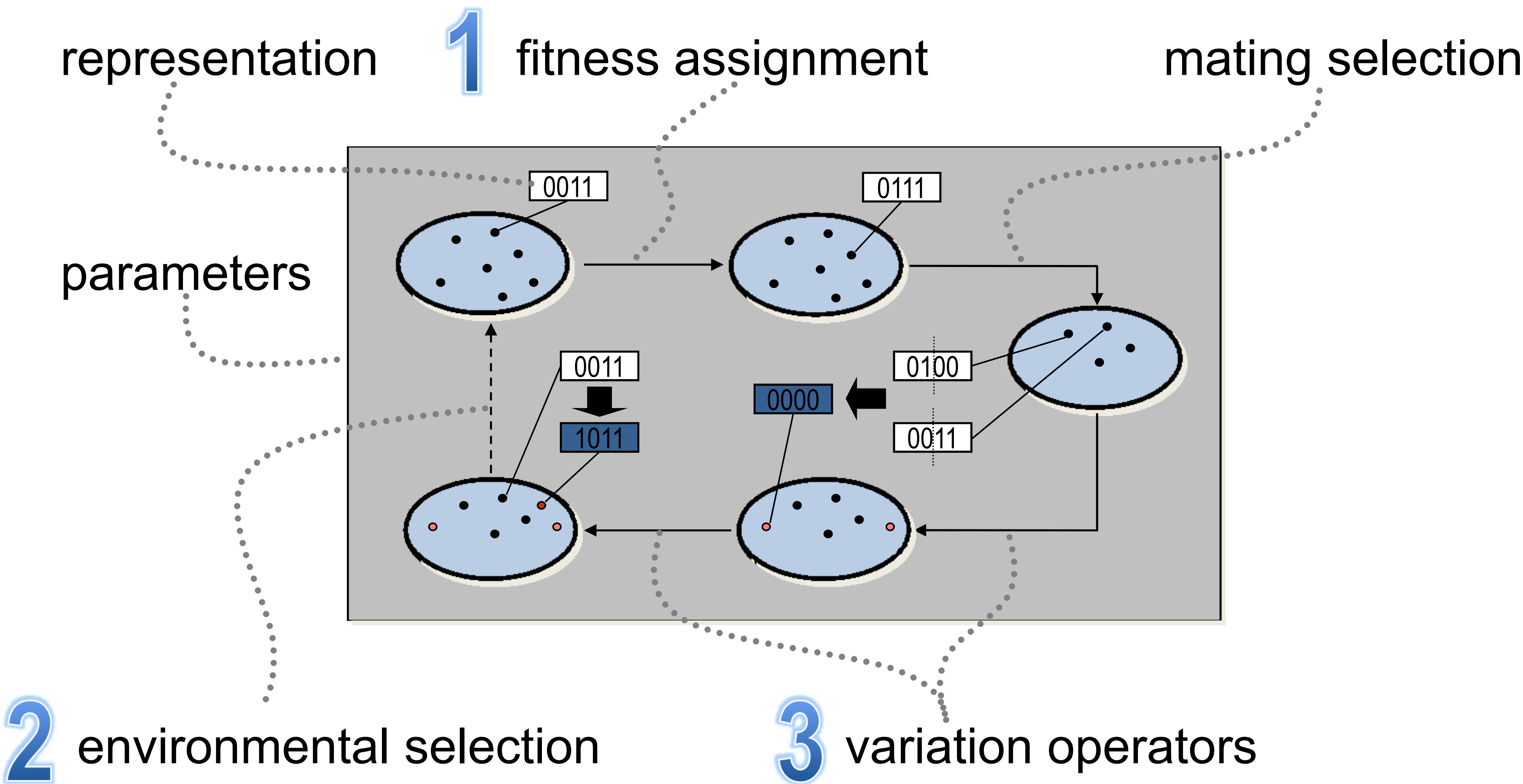
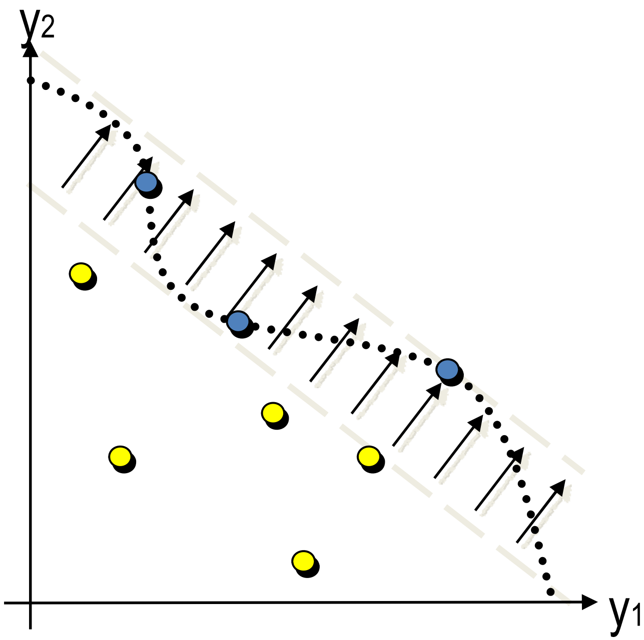
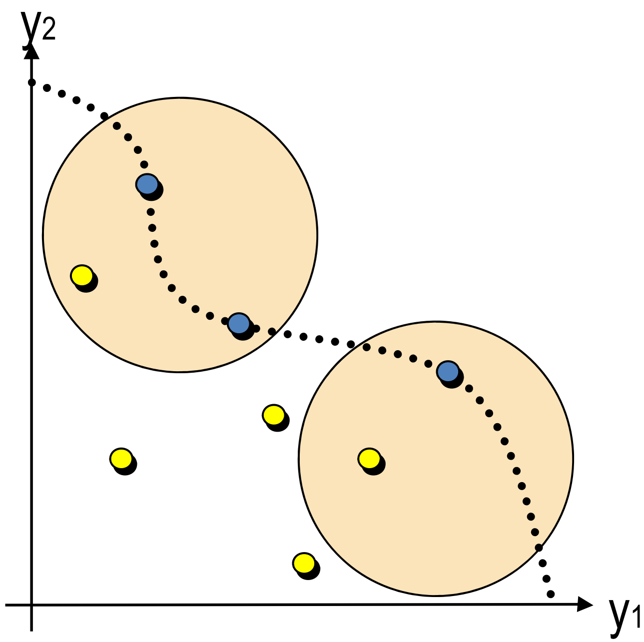
A Few Examples From Practice

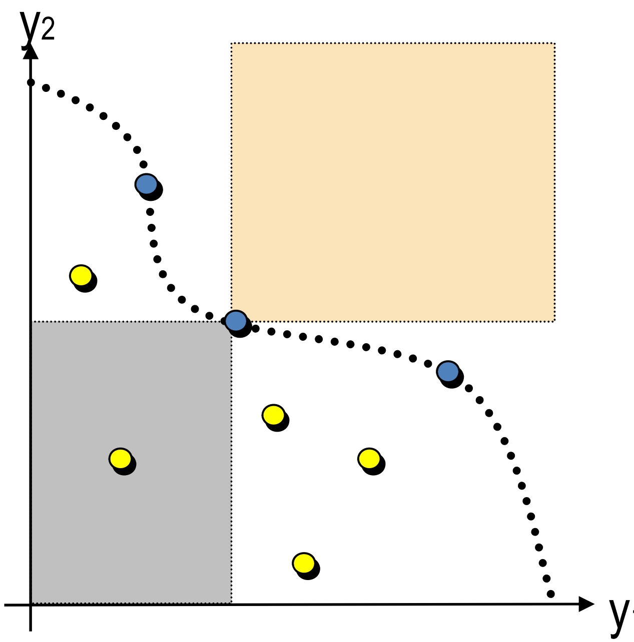
Algorithm Design: Particular Aspects



aggregation-based	**criterion-based**	**dominance-based**
weighted sum	*VEGA*	*SPEA2*







parameter-oriented
scaling-dependent

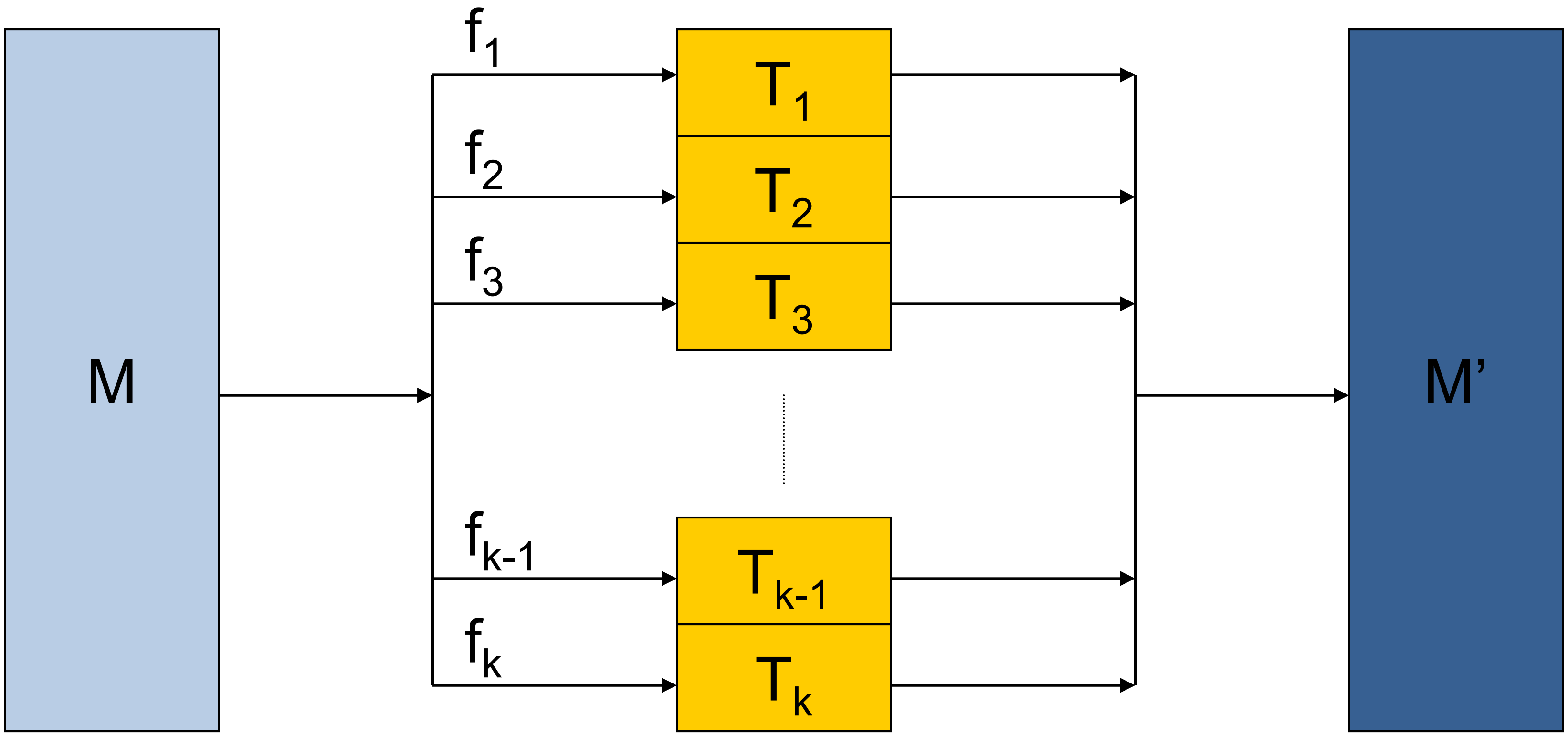
set-oriented
scaling-independent

Criterion-Based Selection: VEGA

select according to

shuffle

[Schaffer 1985]



population

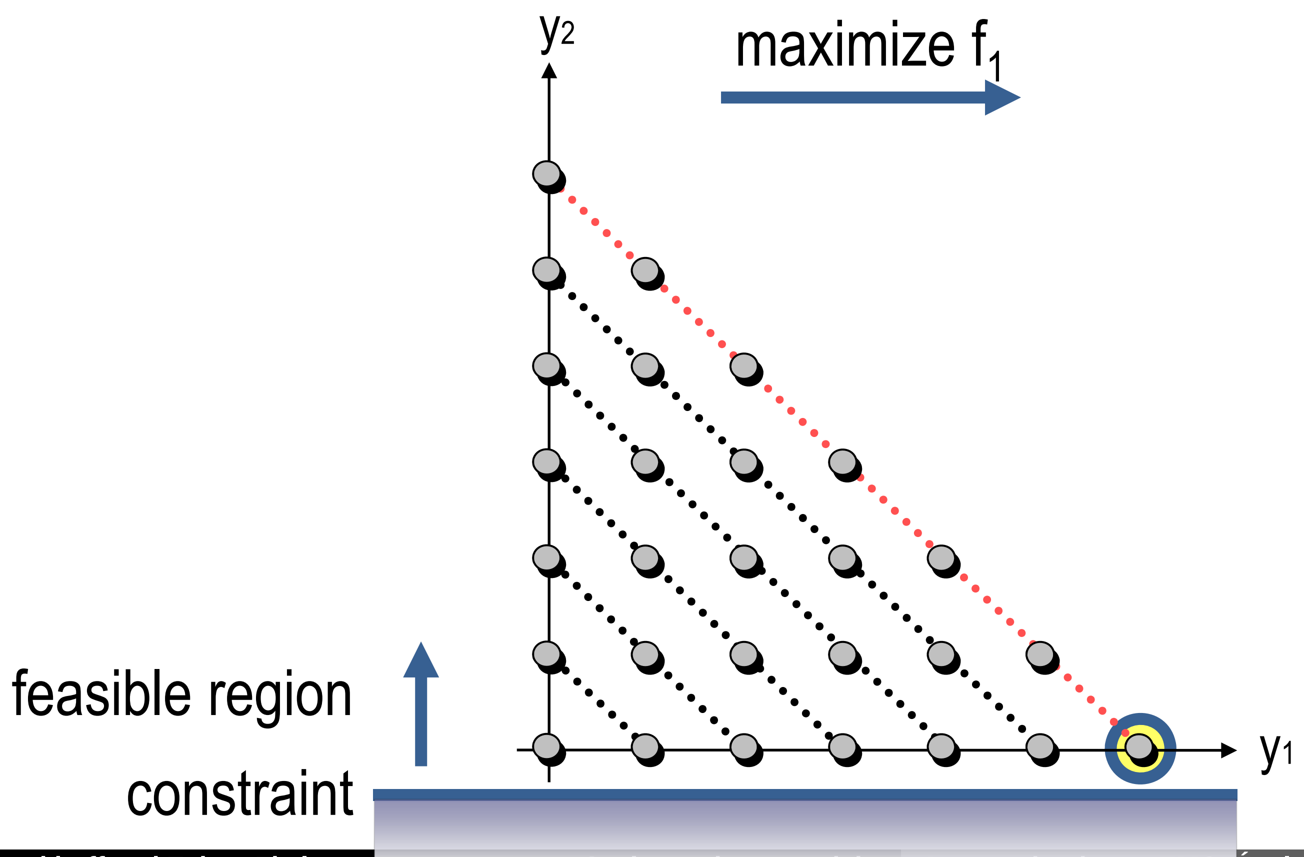
k separate selections

mating pool

Aggregation-Based: Multistart Constraint Method

Underlying concept:

- Convert all objectives except of one into constraints
- Adaptively vary constraints



Aggregation-Based: Multistart Constraint Method

Underlying concept:

- Convert all objectives except of one into constraints
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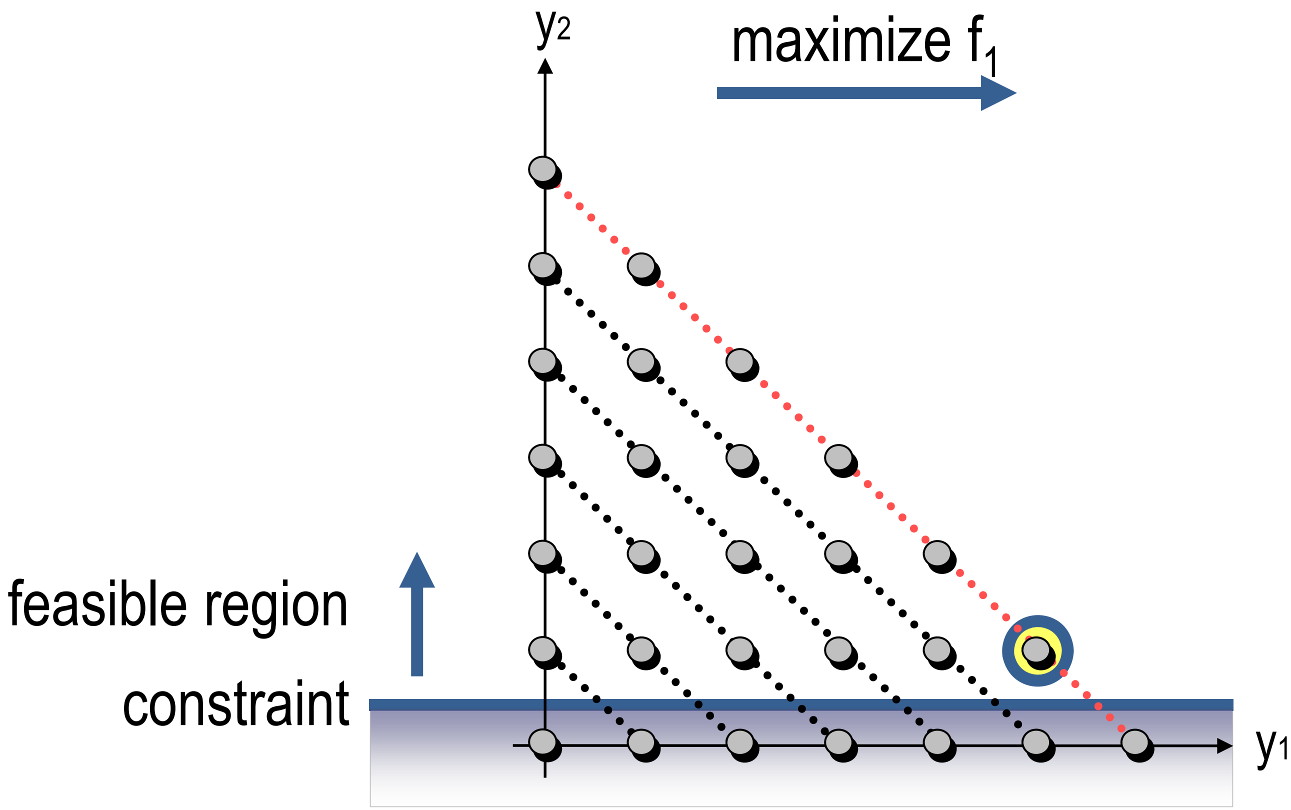

Aggregation-Based: Multistart Constraint Method

Underlying concept:

- Convert all objectives except of one into constraints
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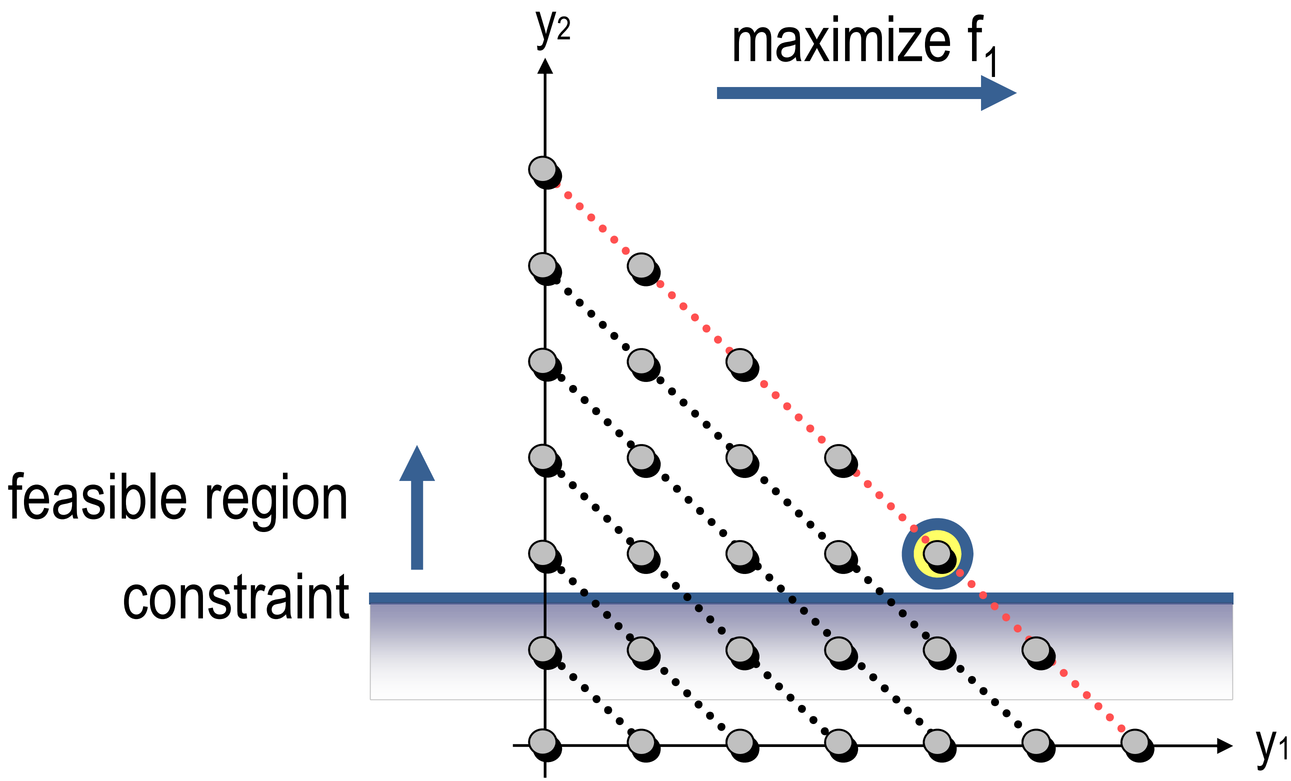
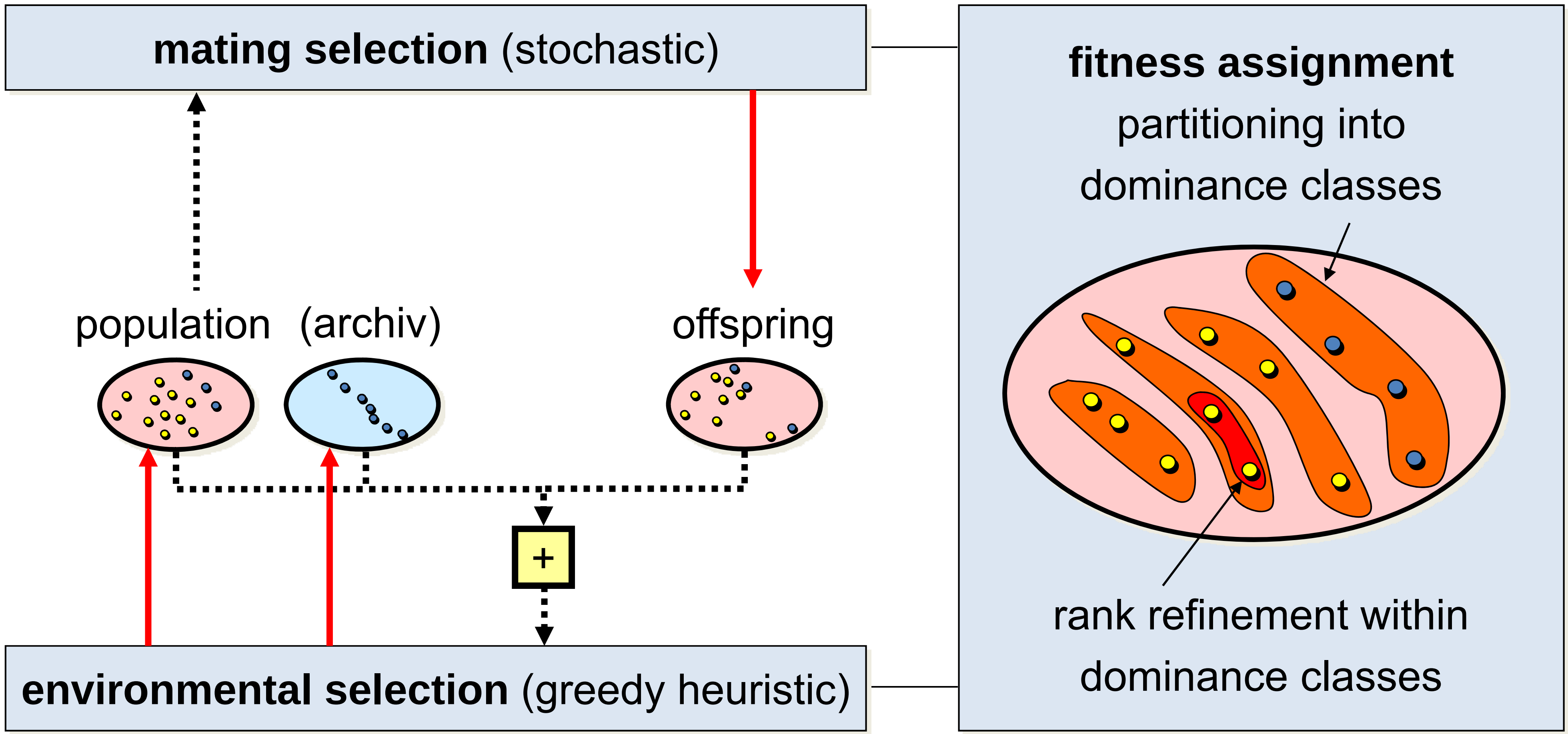


A General Scheme of a Dominance-Based MOEA



Note: good in terms of set quality = good in terms of search?

Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II

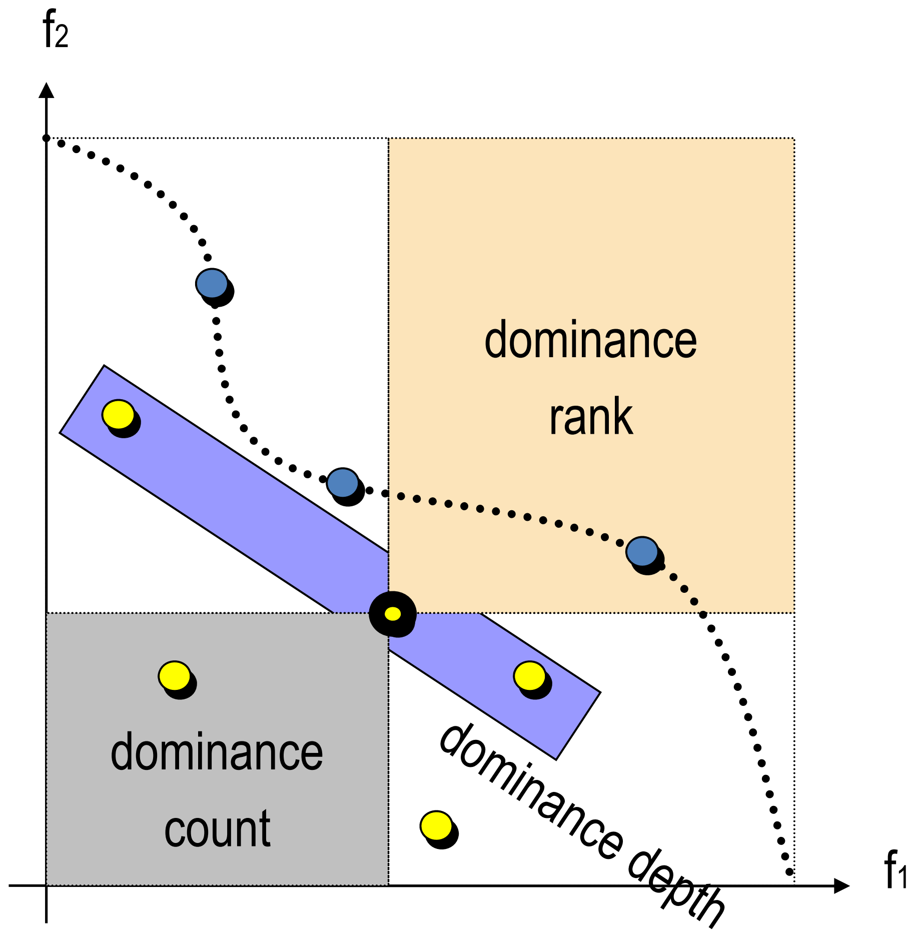
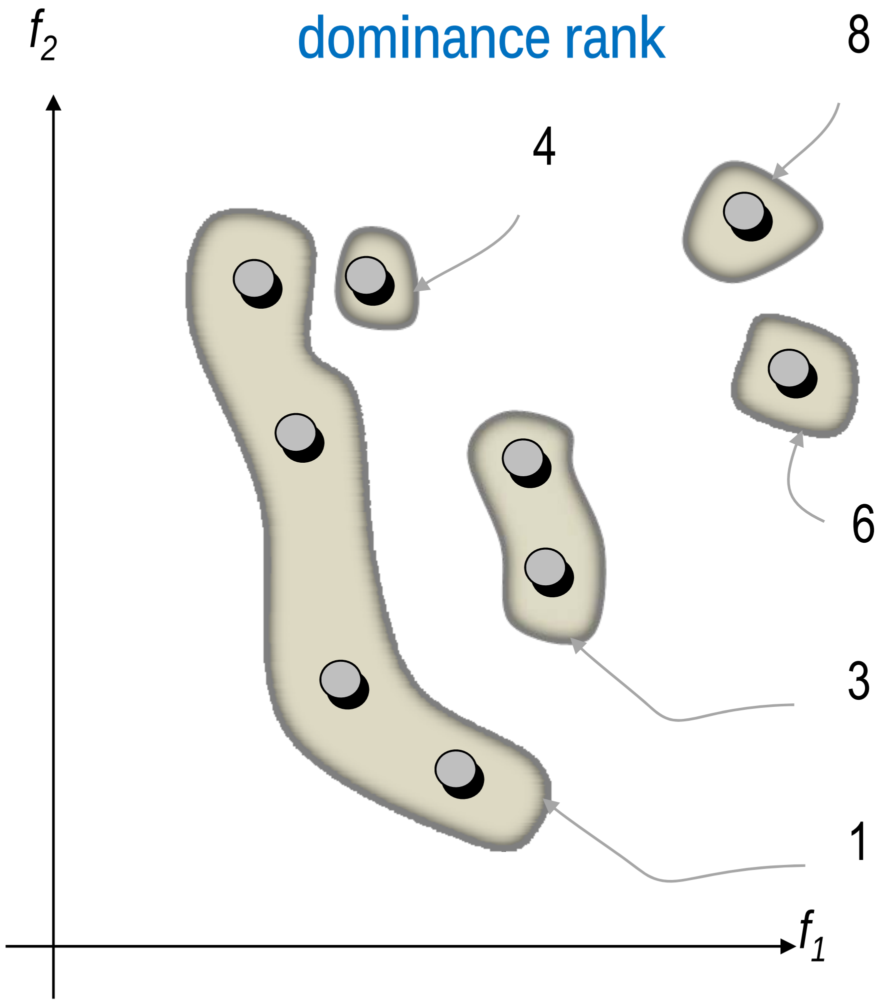
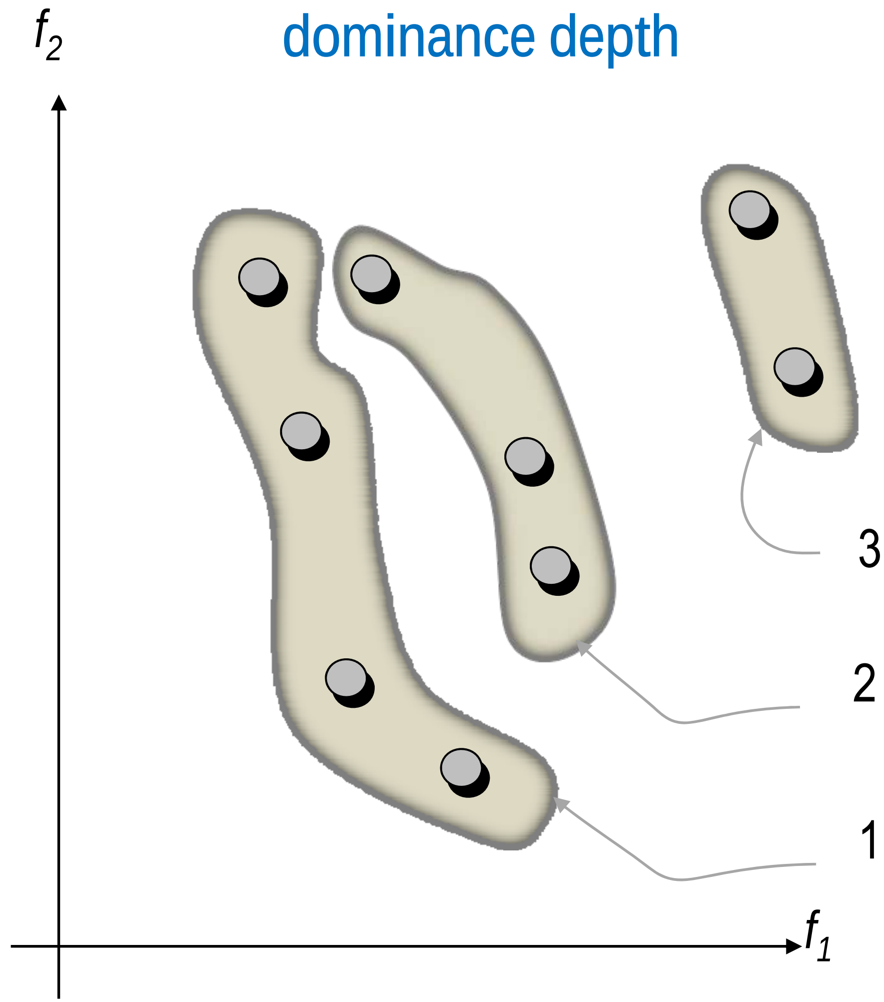


Illustration of Dominance-based Partitioning





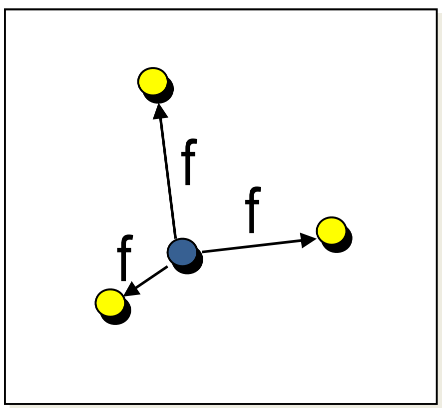
Refinement of Dominance Rankings

Goal: rank incomparable solutions within a dominance class

❶ Density information (good for search, but usually no refinements)

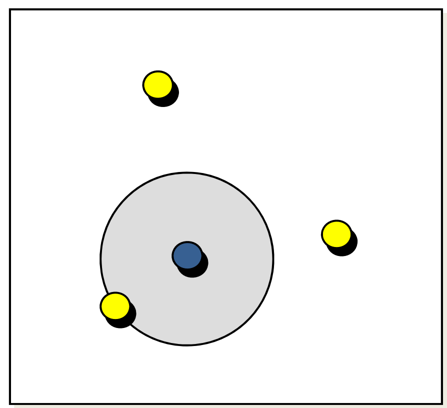
Kernel method

density =
function of the
distances



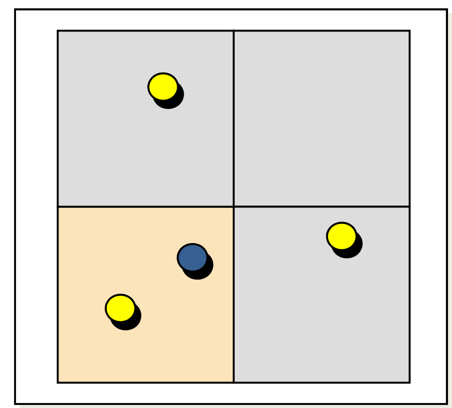
k-th nearest neighbor

density =
function of distance
to k-th neighbor



Histogram method

density =
number of elements
within box

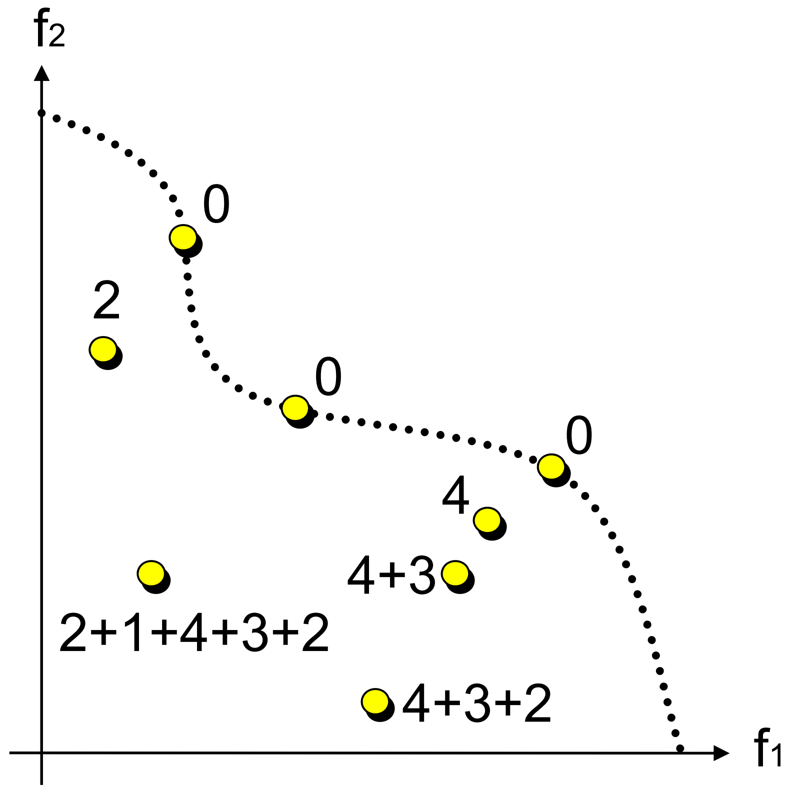


❷ Quality indicator (good for set quality): soon...

Example: SPEA2 Dominance Ranking

Basic idea: the less dominated, the fitter...

Principle: first assign each solution a weight (strength), then add up weights of dominating solutions



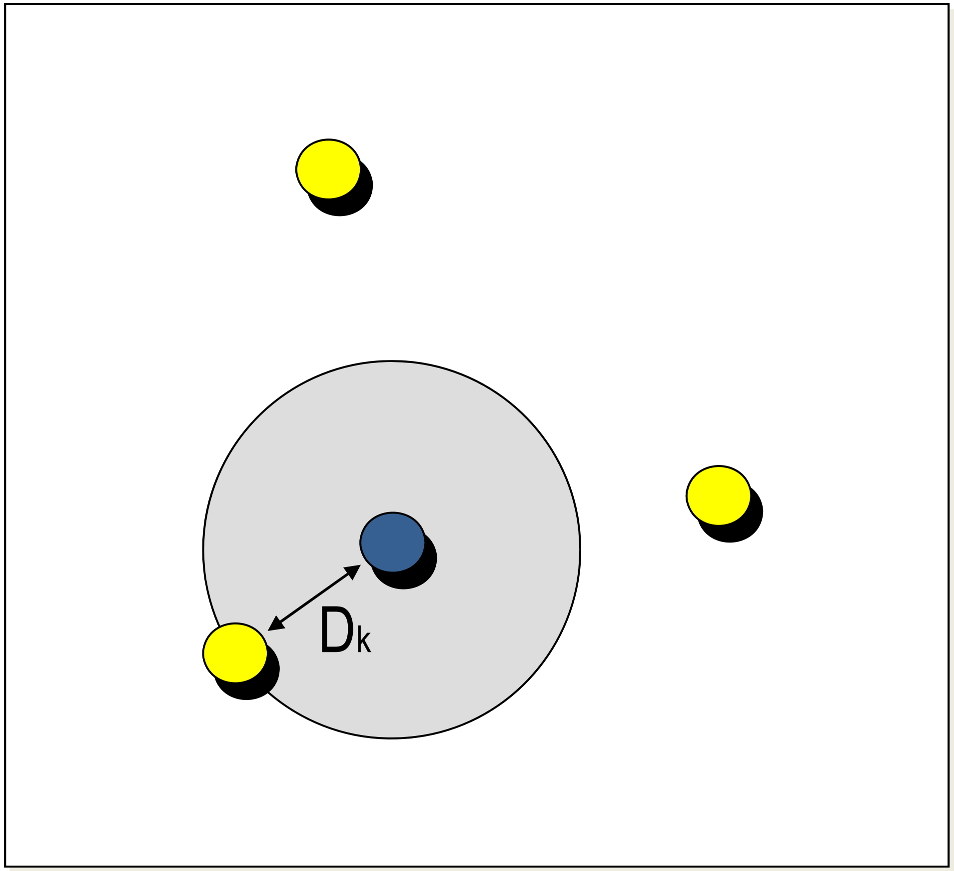
- S (strength) = #dominated solutions
- R (raw fitness) = $\sum$ strengths of dominators

Example: SPEA2 Diversity Preservation

Density Estimation

k-th nearest neighbor method:

- Fitness = R + 1 / $\underbrace{(2 + D_k)}_{< 1}$
- D_k = distance to the k-th nearest individual
- Usually used: k = 2



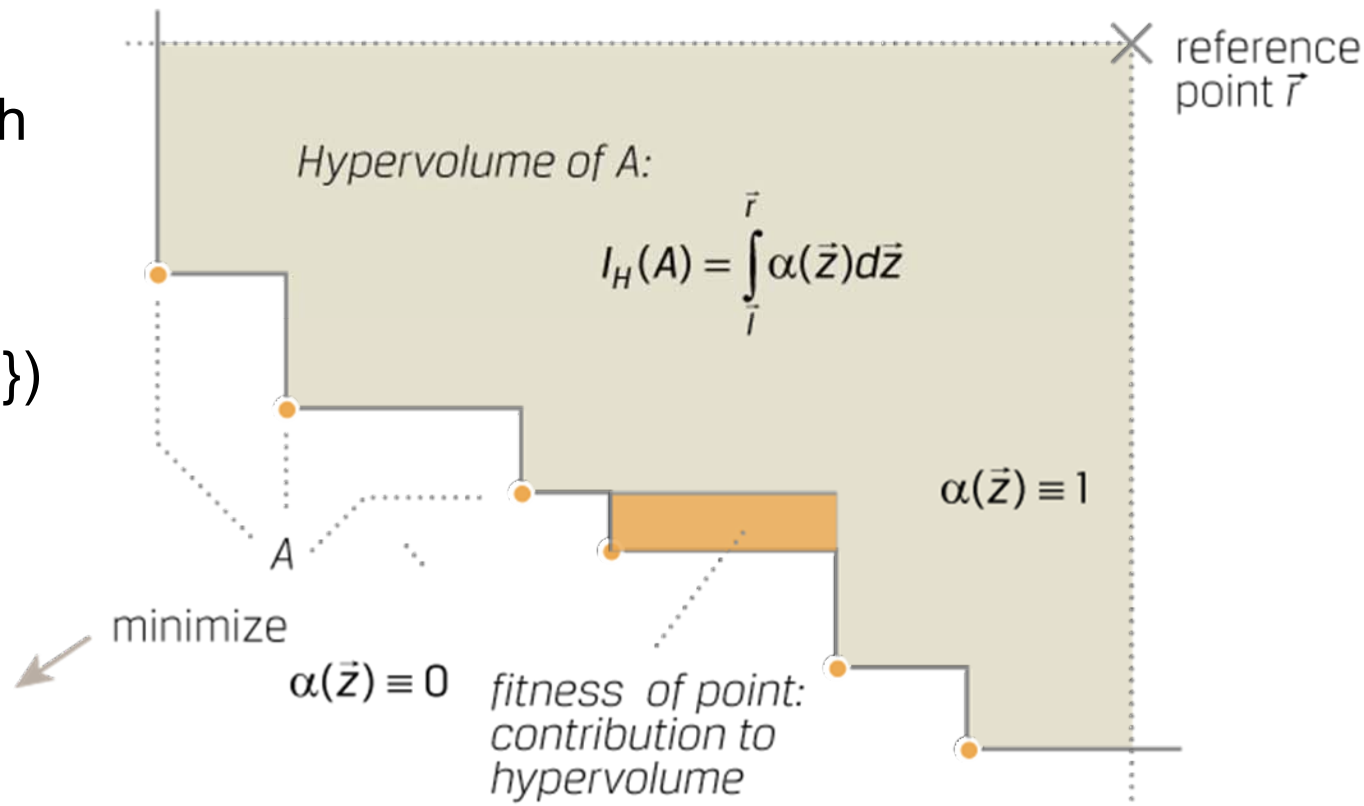
Problem of many secondary selection criterions: no refinement

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refinement!

Main idea

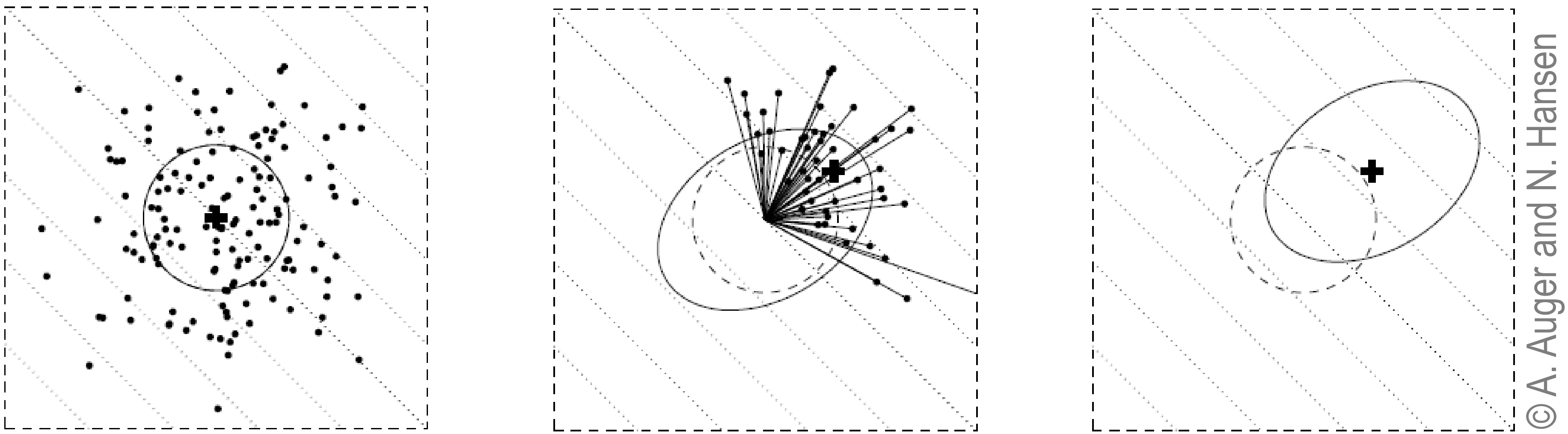
Delete solutions with the smallest hypervolume loss $d(s) = I_H(P) - I_H(P / \{s\})$ iteratively



Sampling New Points: Covariance Matrix Adaptation

Concept

- use single-objective mutation of CMA-ES for each individual [ihr2007a]
- Sample multivariate normal distribution m+σ N(0,C)
- m, σ, and C are updated every generation depending on success



Procedure `updateStepSize` $(a = [x, \overline{p}_{\text{succ}}, \sigma, p_c, C], p_{\text{succ}})$

$\overline{p}_{\text{succ}} \leftarrow (1 - c_p)\,\overline{p}_{\text{succ}} + c_p p_{\text{succ}}$

$$\sigma \leftarrow \sigma \cdot \exp\left(\frac{1}{d}\,\frac{\overline{p}_{\text{succ}} - p_{\text{succ}}^{\text{target}}}{1 - p_{\text{succ}}^{\text{target}}}\right)$$

Procedure `updateCovariance` $(a = [x, \overline{p}_{\text{succ}}, \sigma, p_c, C], x_{\text{step}} \in \mathbb{R}^n)$

if $\overline{p}_{\text{succ}} < p_{\text{thresh}}$ **then**

$\quad p_c \leftarrow (1 - c_c)p_c + \sqrt{c_c(2 - c_c)}\; x_{\text{step}}$

$\quad C \leftarrow (1 - c_{\text{cov}})C + c_{\text{cov}} \cdot p_c {p_c}^{\mathrm{T}}$

else

$\quad p_c \leftarrow (1 - c_c)p_c$

$\quad C \leftarrow (1 - c_{\text{cov}})C + c_{\text{cov}} \cdot \left(p_c {p_c}^{\mathrm{T}} + c_c(2 - c_c)C\right)$

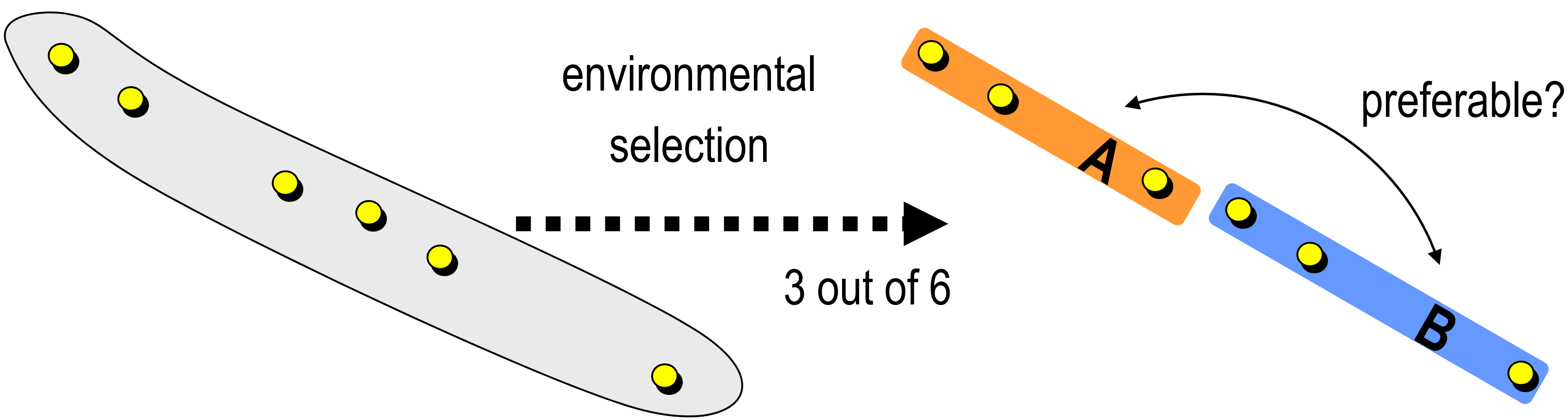
Articulating User Preferences During Search

What we thought: EMO is preference-less

> given by the DM.
>
> **Search before decision making:** Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.
>
> **Decision making during search:** The DM can articulate preferences during

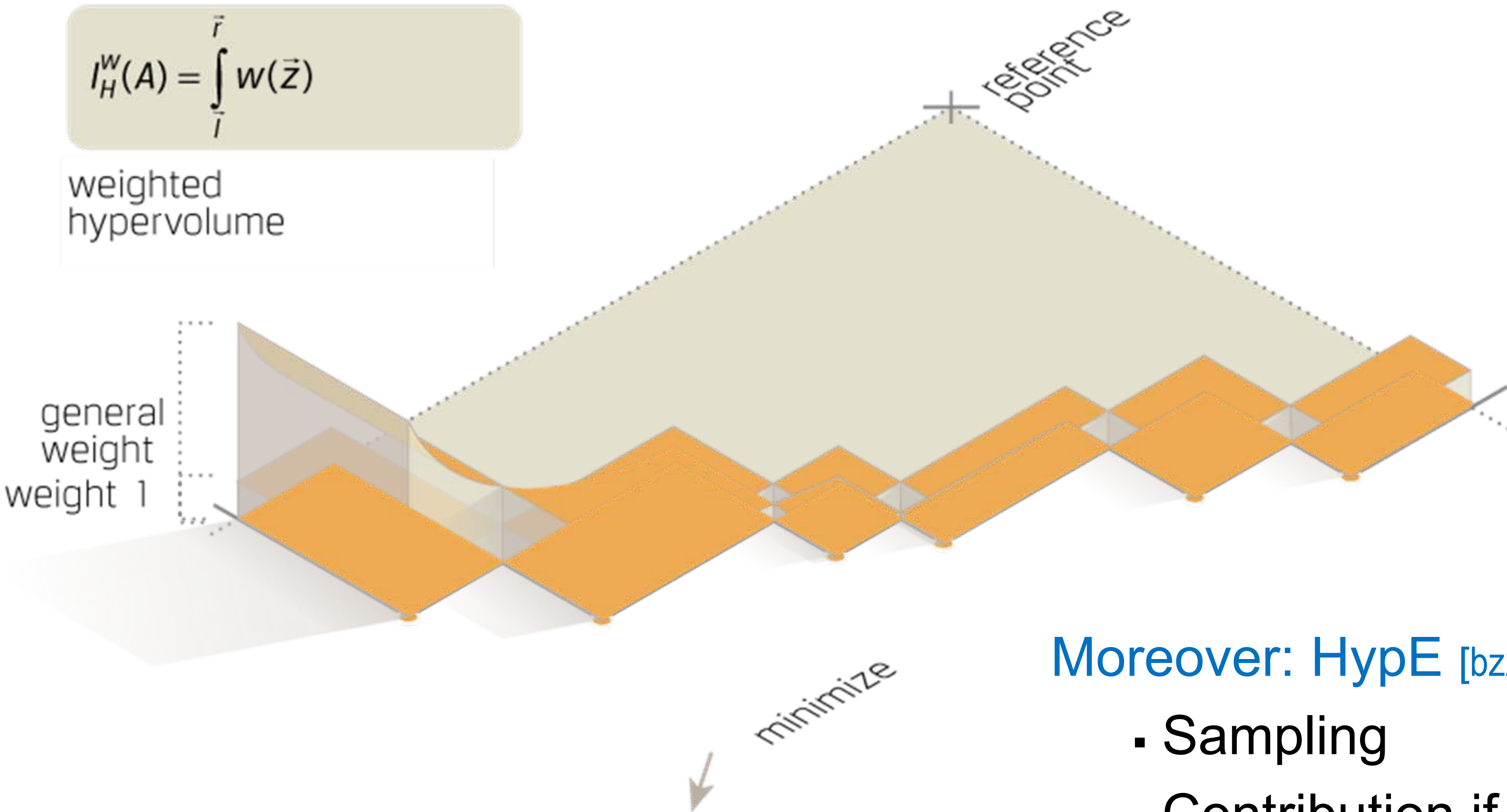
[Zitzler 1999]

What we learnt: EMO just uses weaker preference information



⇒ (almost) all MOEAs implicitly implement specific preferences

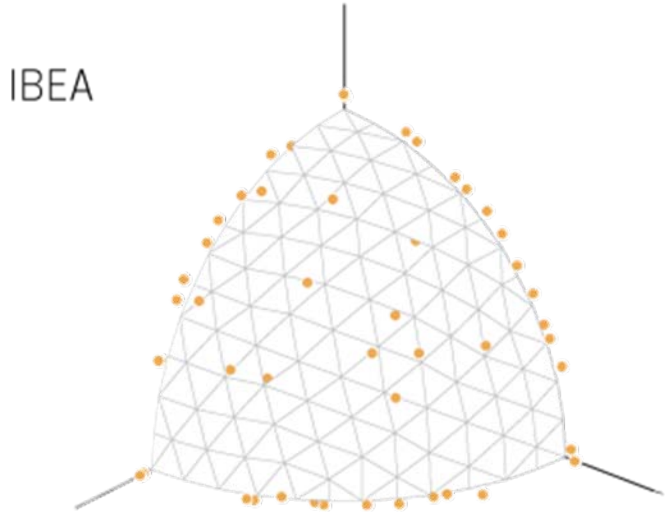
Example: Weighted Hypervolume Indicator

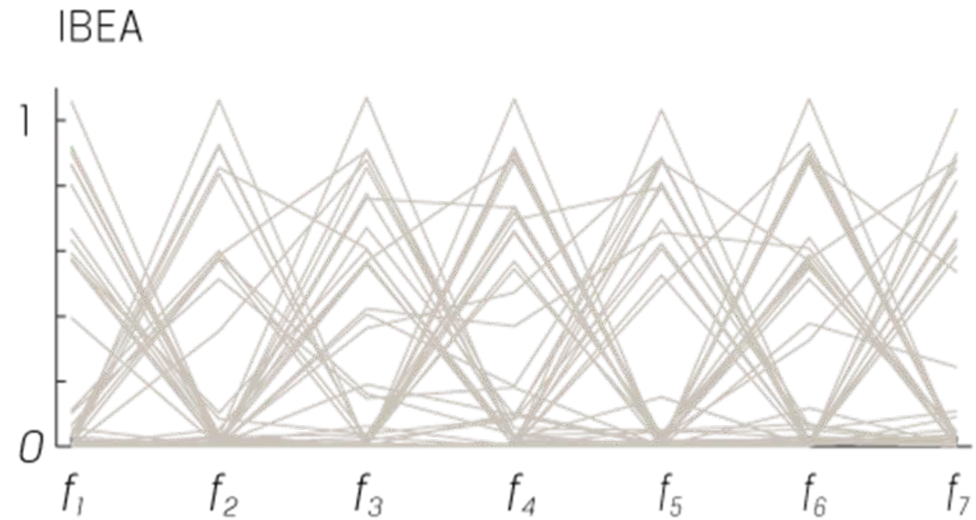


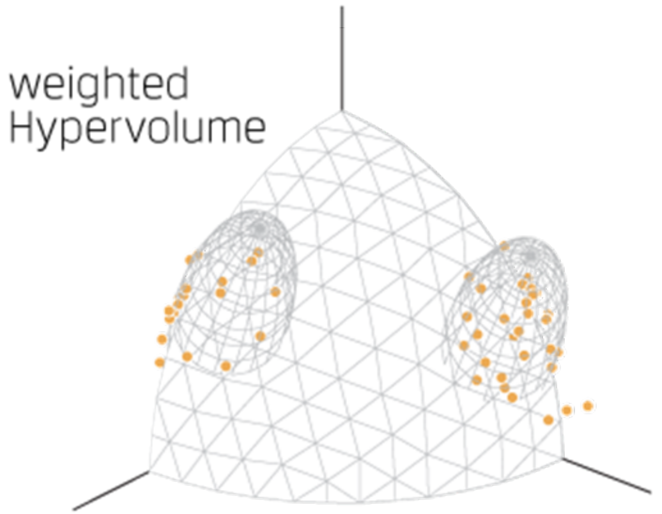
Moreover: HypE [bz2009d]

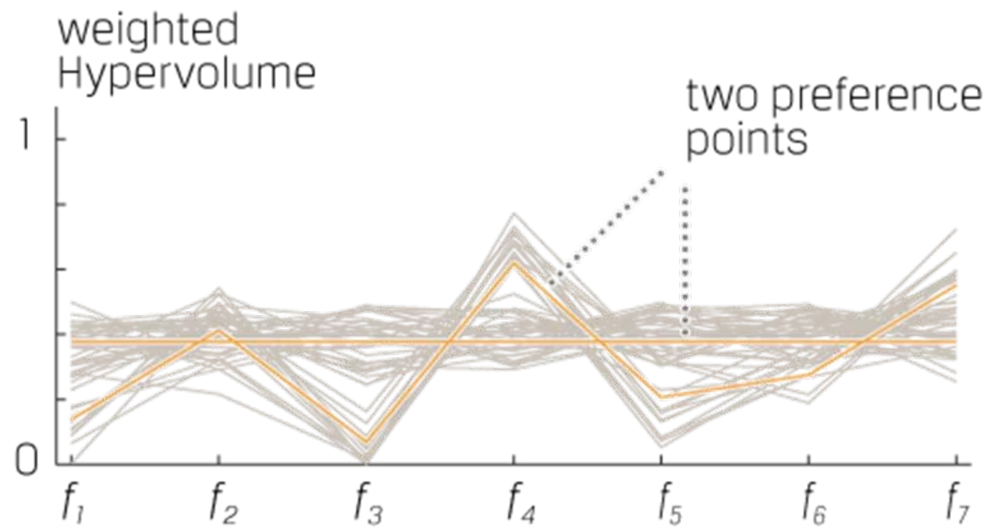
- Sampling
- Contribution if >1 solution deleted

Weighted Hypervolume in Practice









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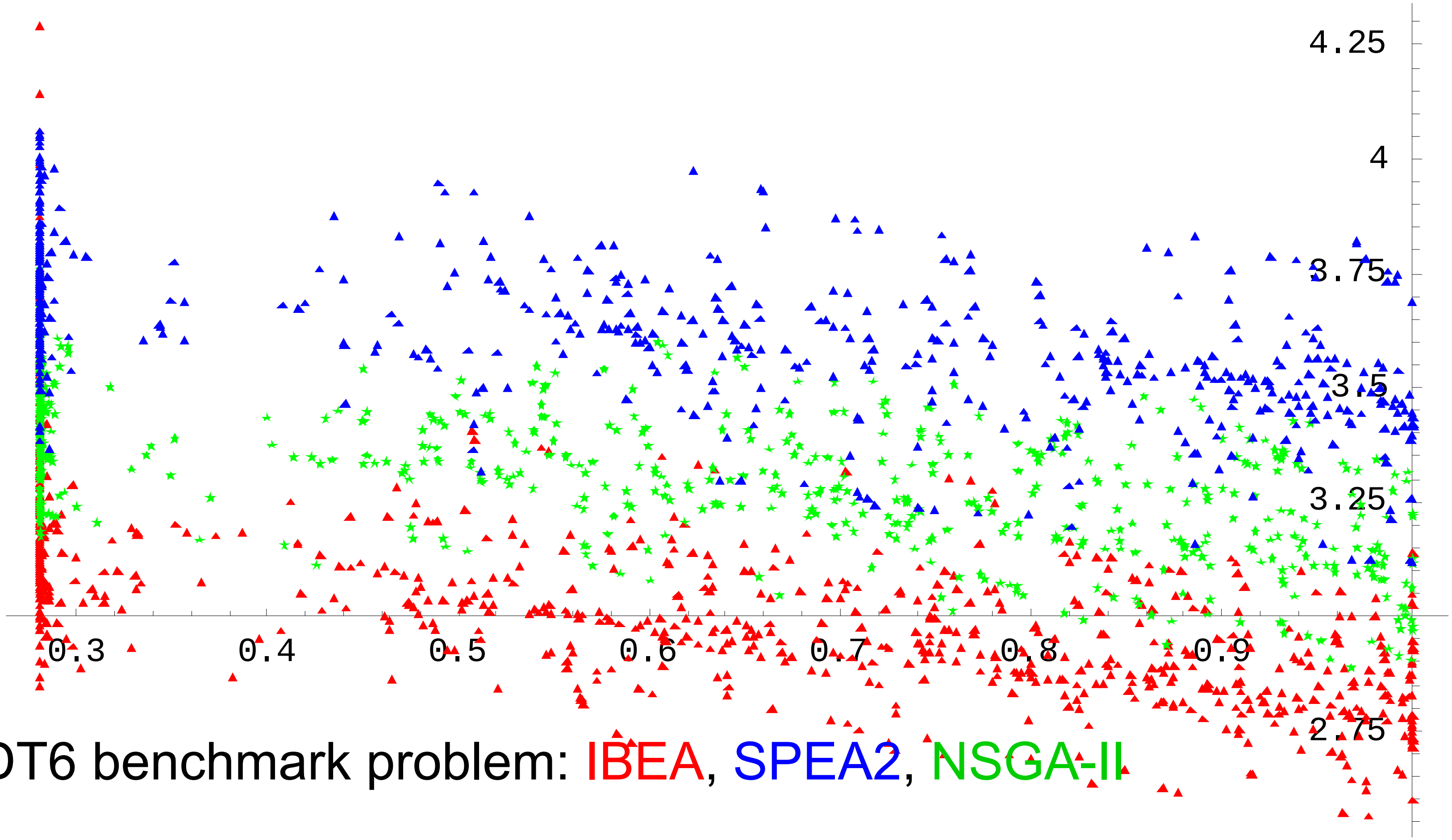
Once Upon a Time...

... multiobjective EAs were mainly compared visually:

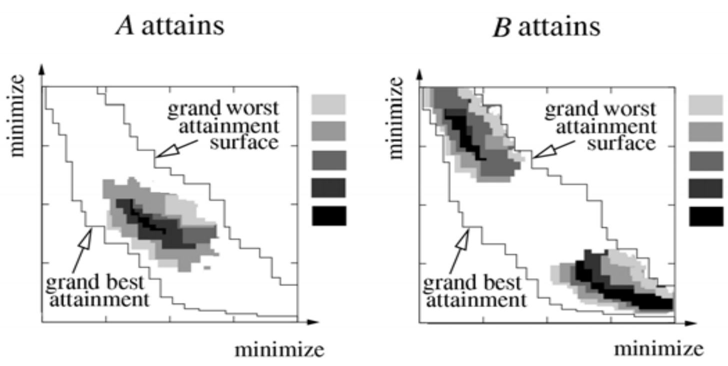


ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

Two Approaches for Empirical Studies

Attainment function approach:

- Applies statistical tests directly to the samples of approximation sets
- Gives detailed information about how and where performance differences occur



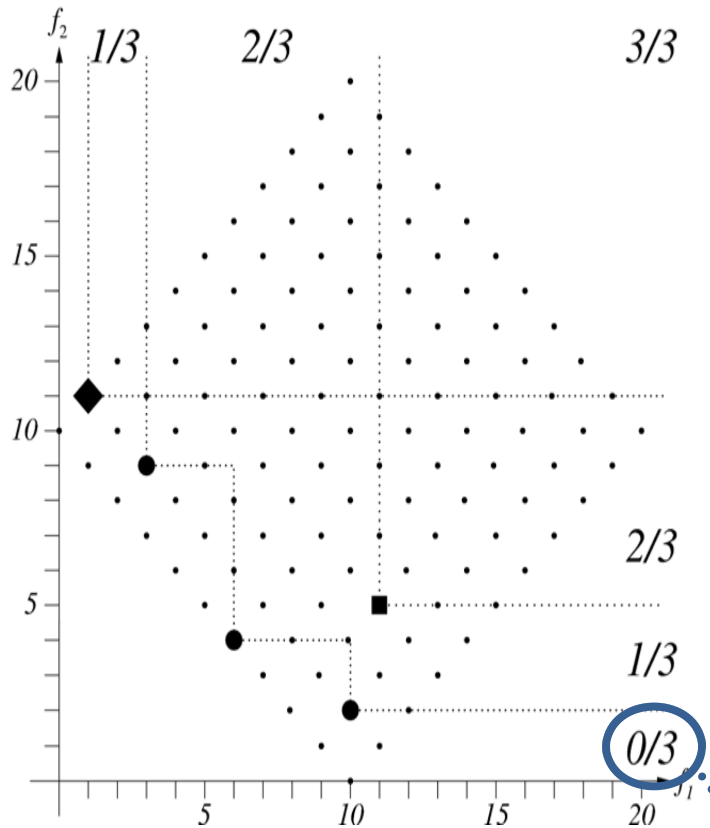
Quality indicator approach:

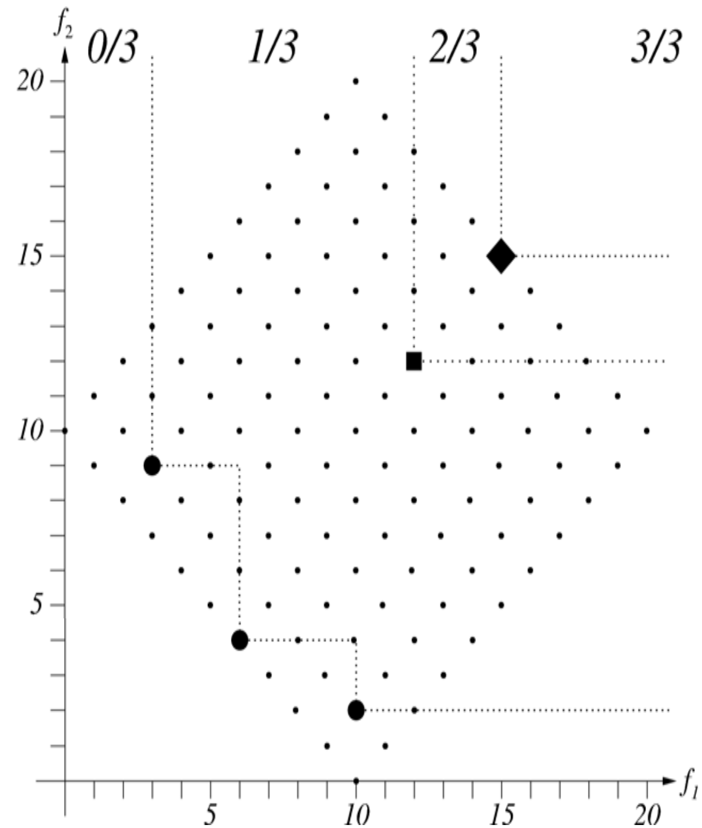
- First, reduces each approximation set to a single value of quality
- Applies statistical tests to the samples of quality values

Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

Empirical Attainment Functions

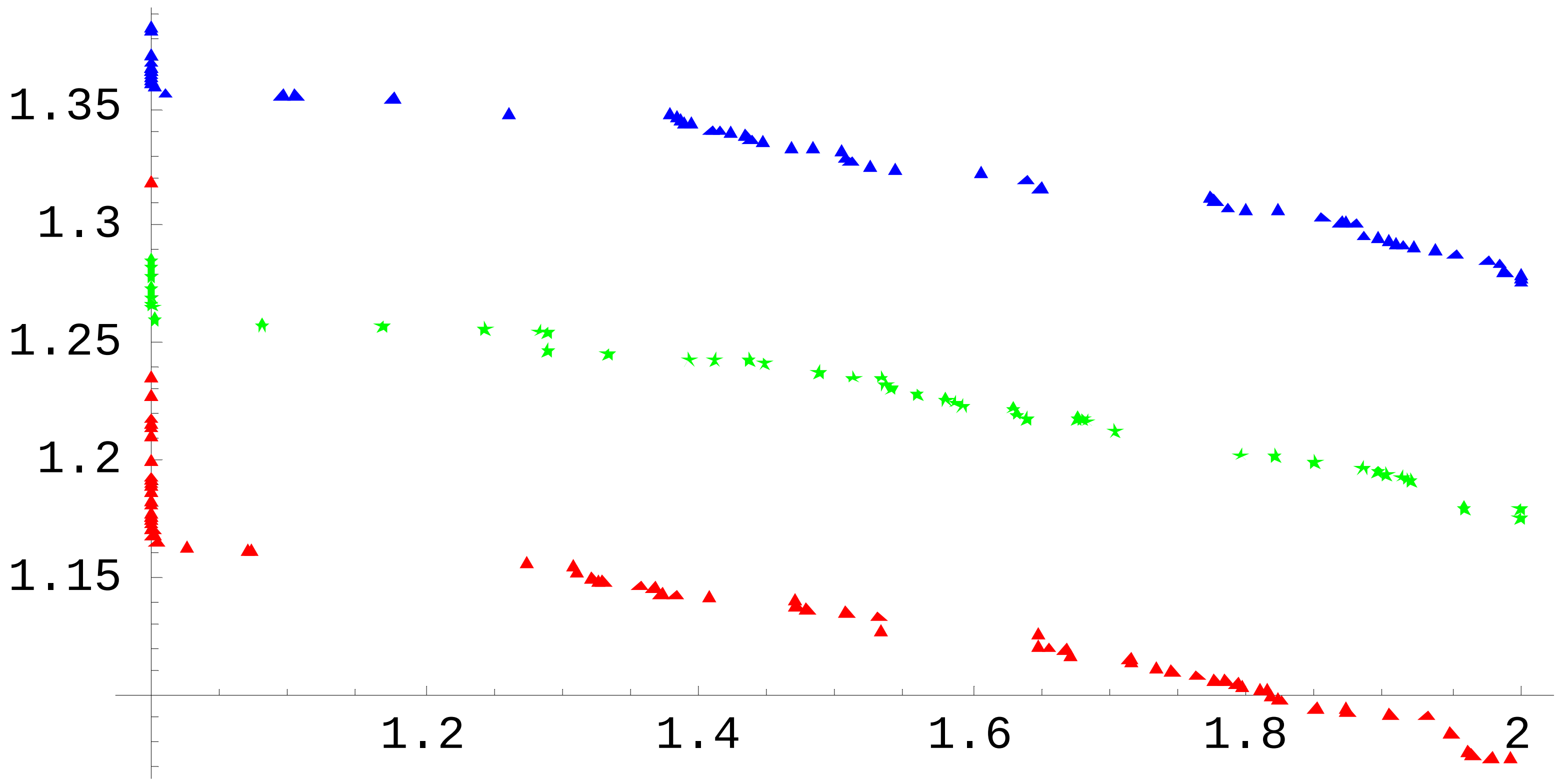
three runs of two multiobjective optimizers





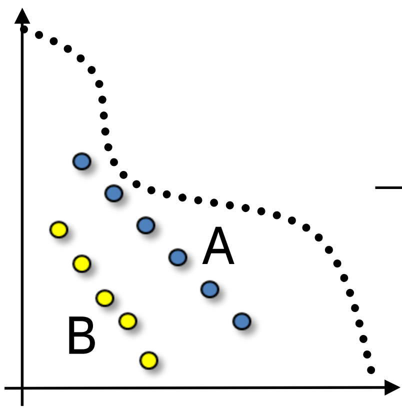
frequency of attaining regions

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



→

	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

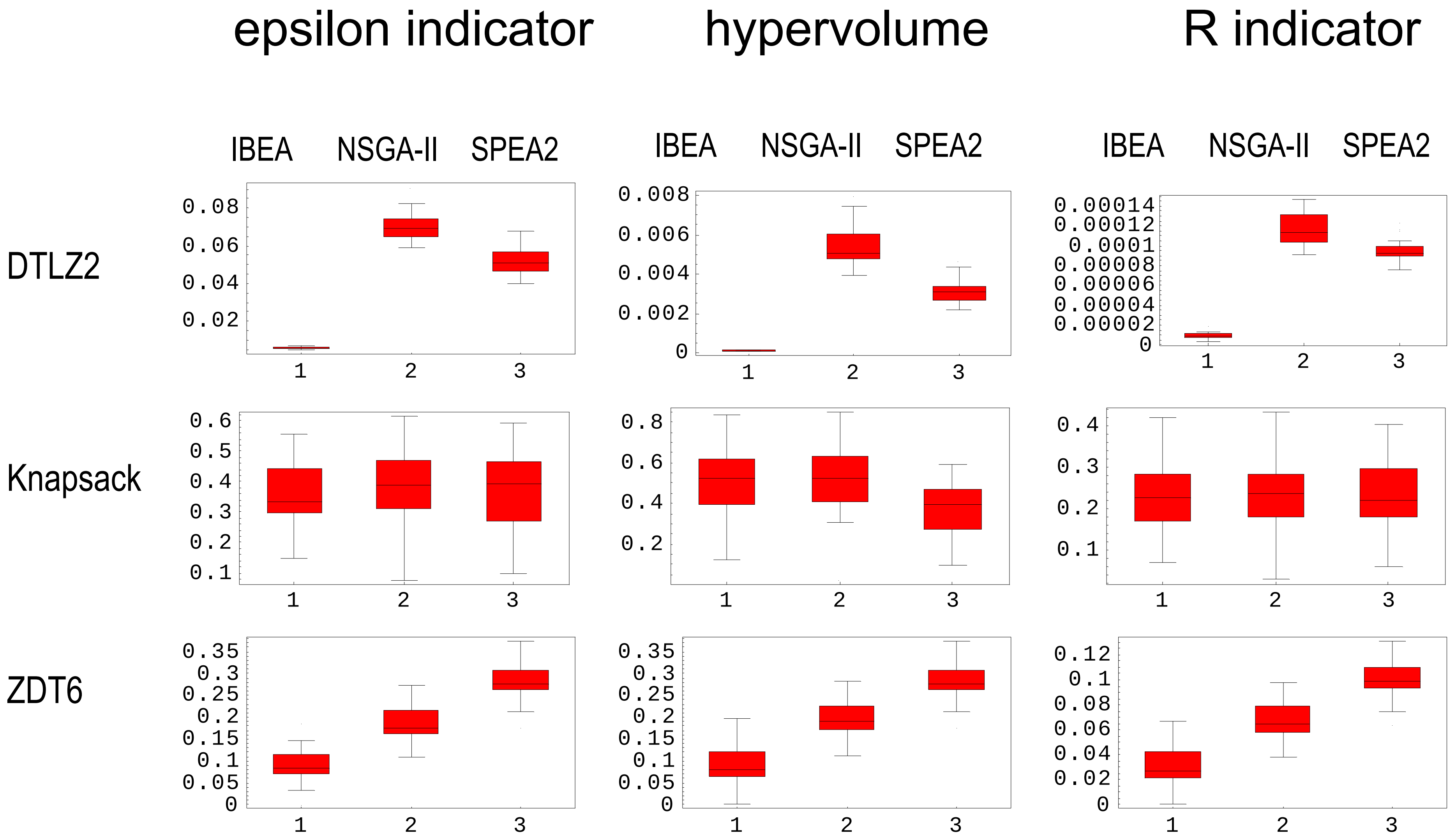
→ "A better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$$

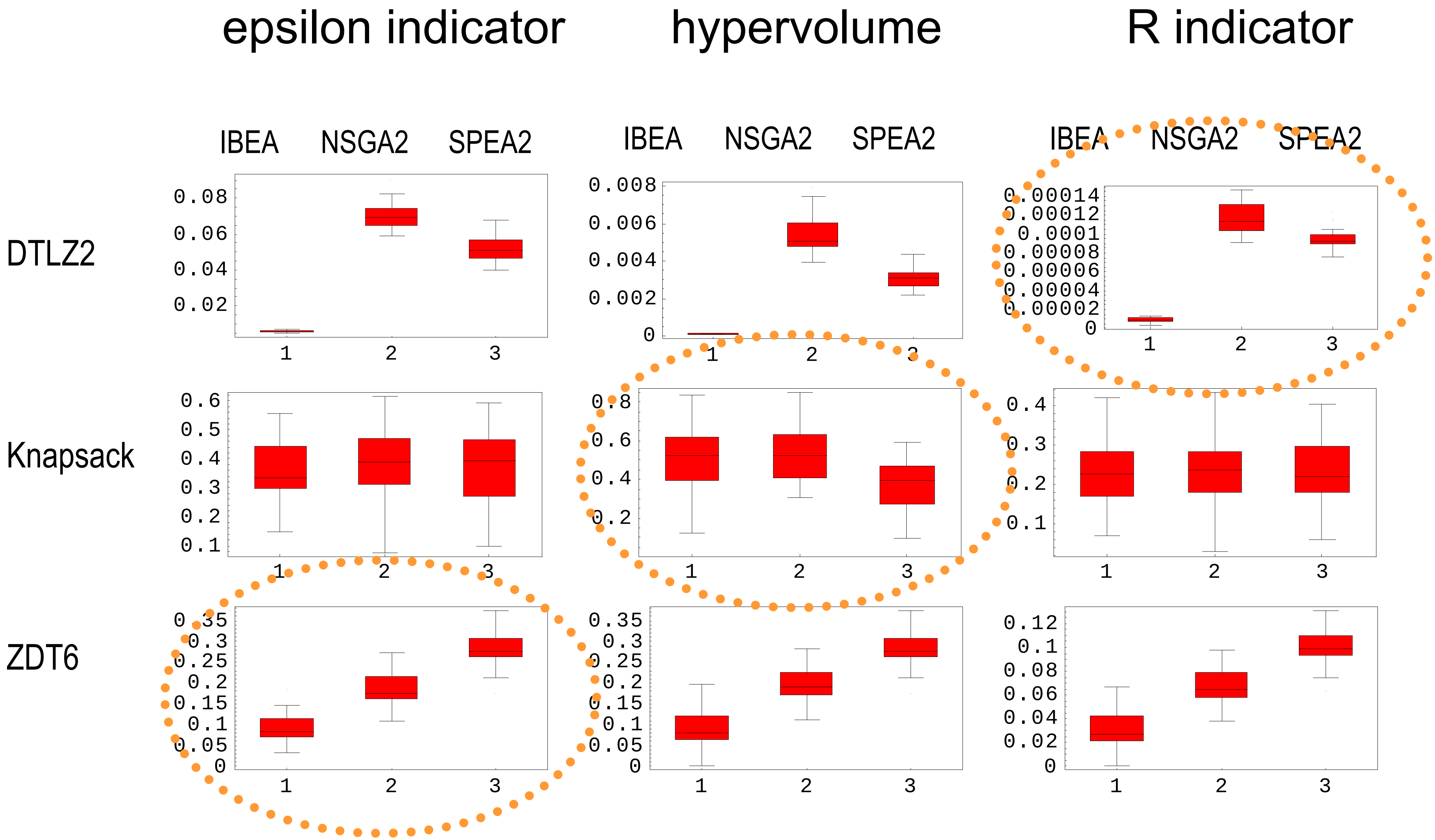
reduction **interpretation**

Example: Box Plots



[Fonseca et al. 2005]

Example: Box Plots



[Fonseca et al. 2005]

Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than

	IBEA	NSGA2	SPEA2
IBEA		~0	~0
NSGA2	1		~0
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

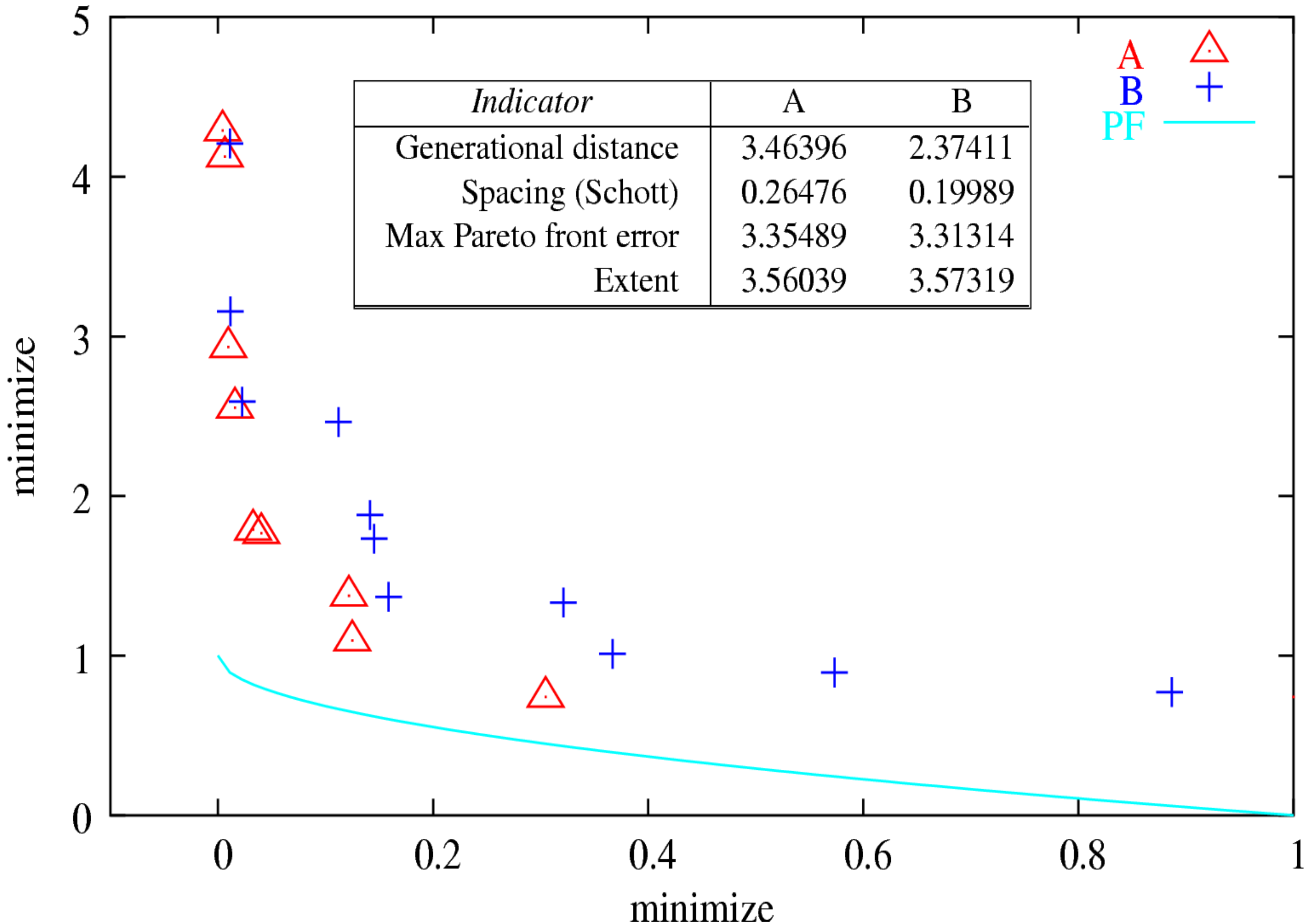
is better than

	IBEA	NSGA2	SPEA2
IBEA		~0	~0
NSGA2	1		1
SPEA2	1	~0	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H0 = No significance of any differences

Problems With Non-Compliant Indicators

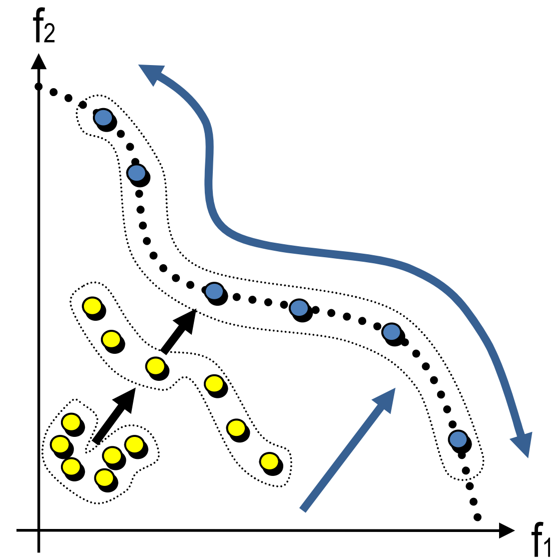


Indicator	A	B
Generational distance	3.46396	2.37411
Spacing (Schott)	0.26476	0.19989
Max Pareto front error	3.35489	3.31314
Extent	3.56039	3.57319

What Are Good Set Quality Measures?

There are **three aspects** [Zitzler et al. 2000]

> Comparing different optimization techniques experimentally always involves the notion of performance. In the case of multiobjective optimization, the definition of quality is substantially more complex than for single-objective optimization problems, because the optimization goal itself consists of multiple objectives:
>
> - The distance of the resulting nondominated set to the Pareto-optimal front should be minimized.
> - A good (in most cases uniform) distribution of the solutions found is desirable. The assessment of this criterion might be based on a certain distance metric.
> - The extent of the obtained nondominated front should be maximized, i.e., for each objective, a wide range of values should be covered by the nondominated solutions.
>
> In the literature, some attempts can be found to formalize the above definition (or parts



Wrong! [Zitzler et al. 2003]

An infinite number of unary set measures is needed to detect in general whether A is better than B

Overview

The Big Picture

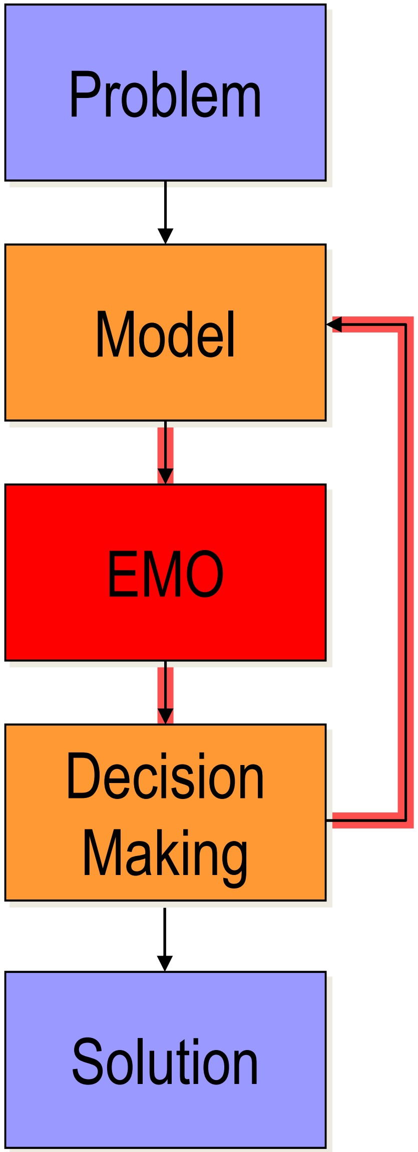
Basic Principles of Multiobjective Optimization

Algorithm Design Principles and Concepts

Performance Assessment

A Few Examples From Practice

EMO Provides Information About a Problem



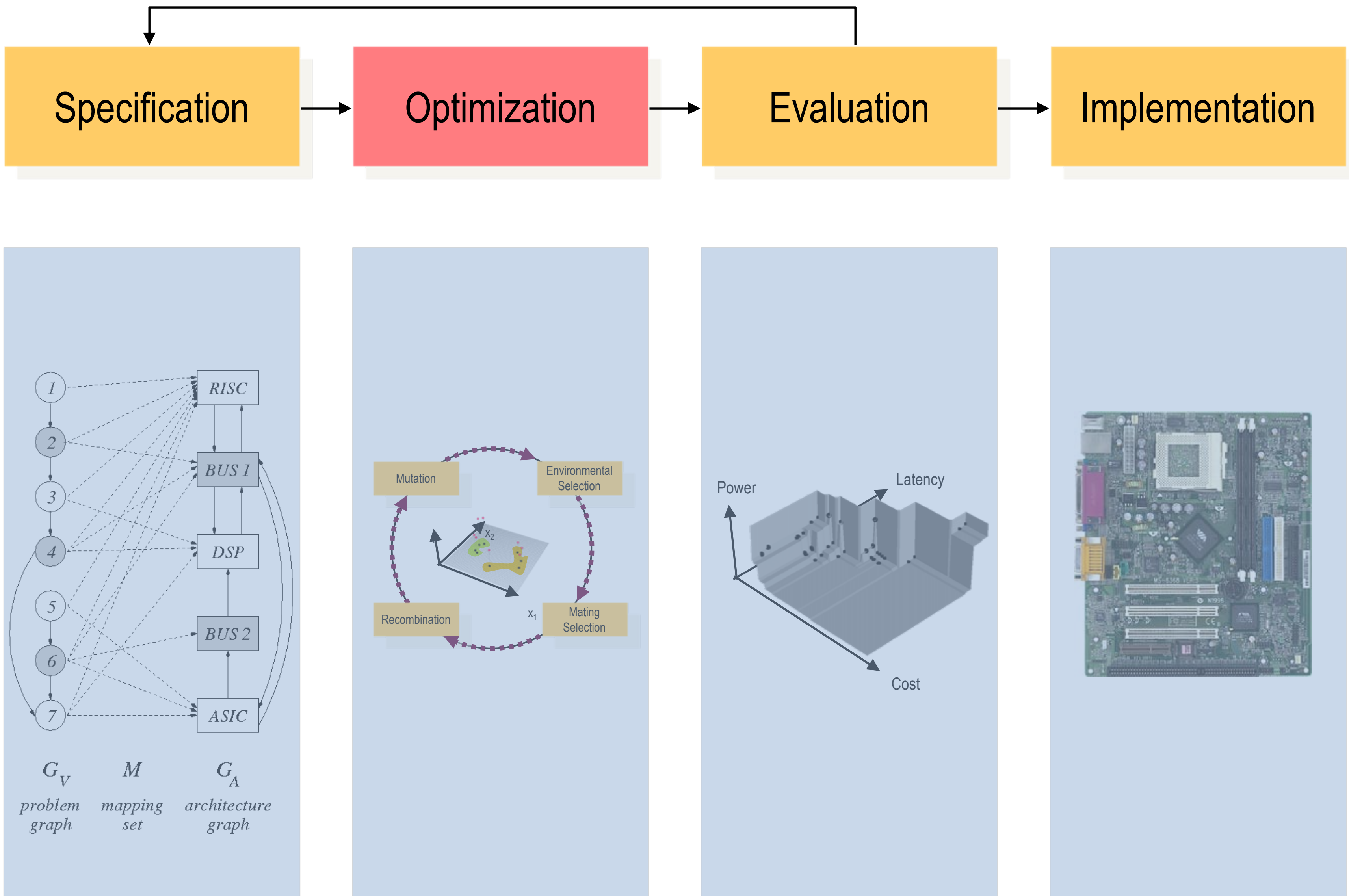
The question:

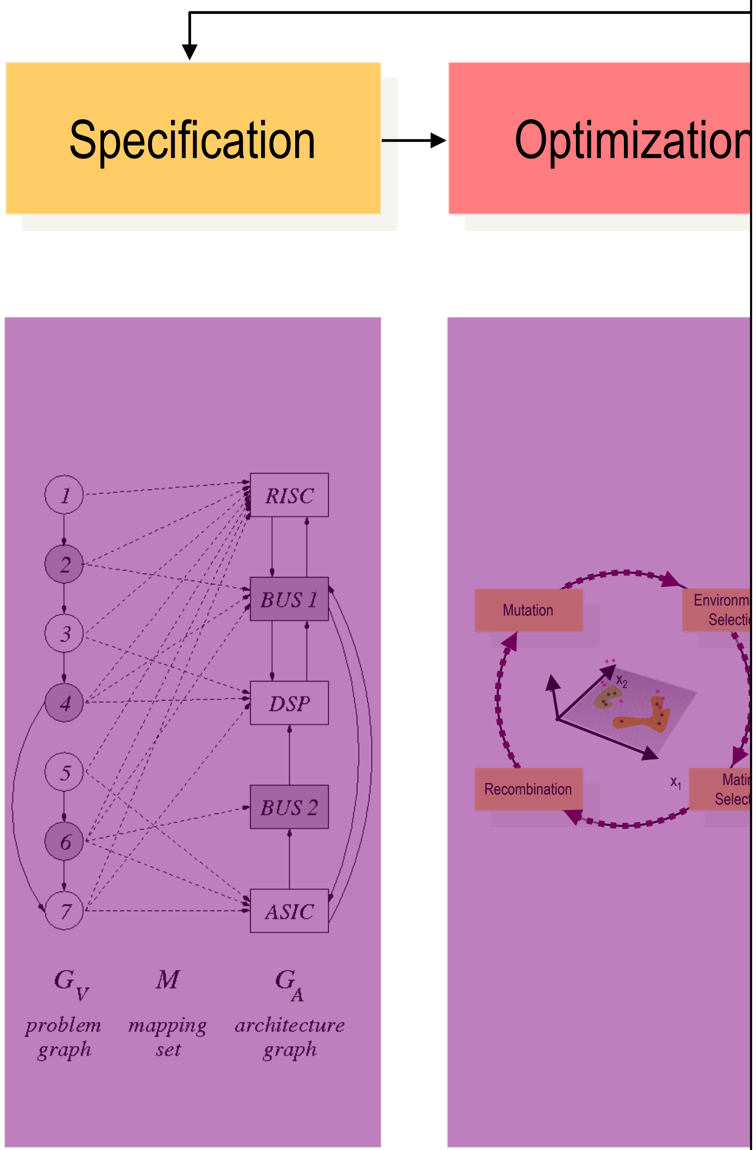
Why at all should one try to approximate the entire Pareto-optimal set?

An answer:

Because it provides useful information about the problem...

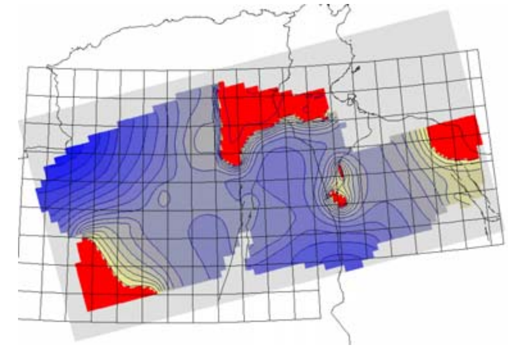
Application: Design Space Exploration

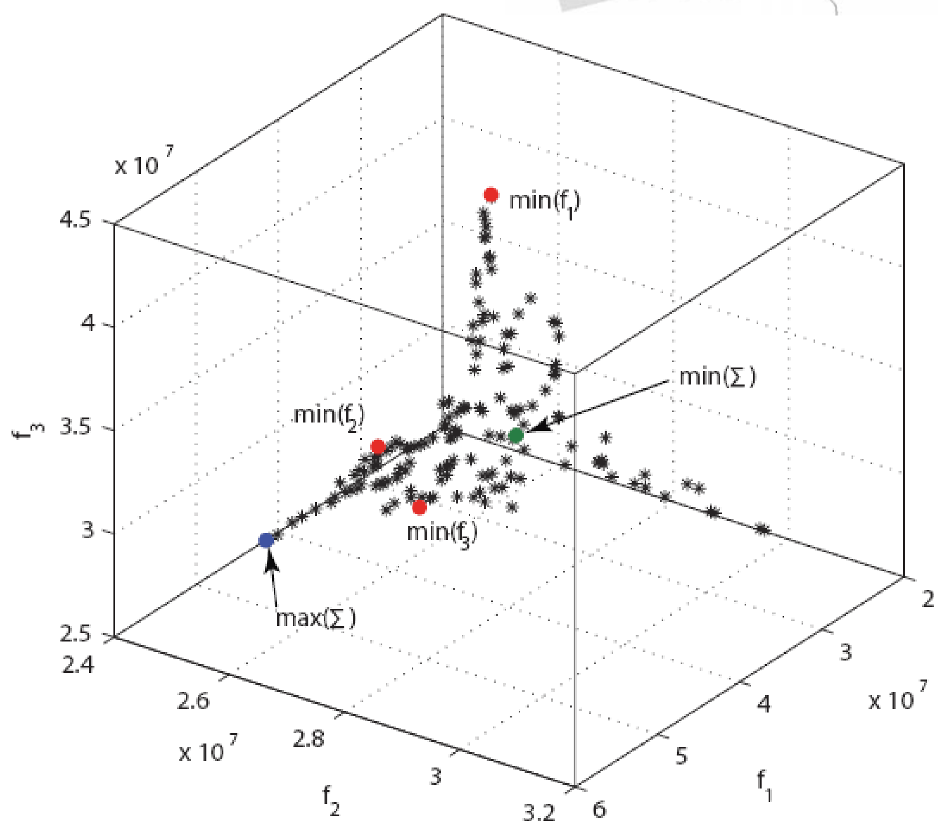




Water resource management

[Siegfried et al. 2009]

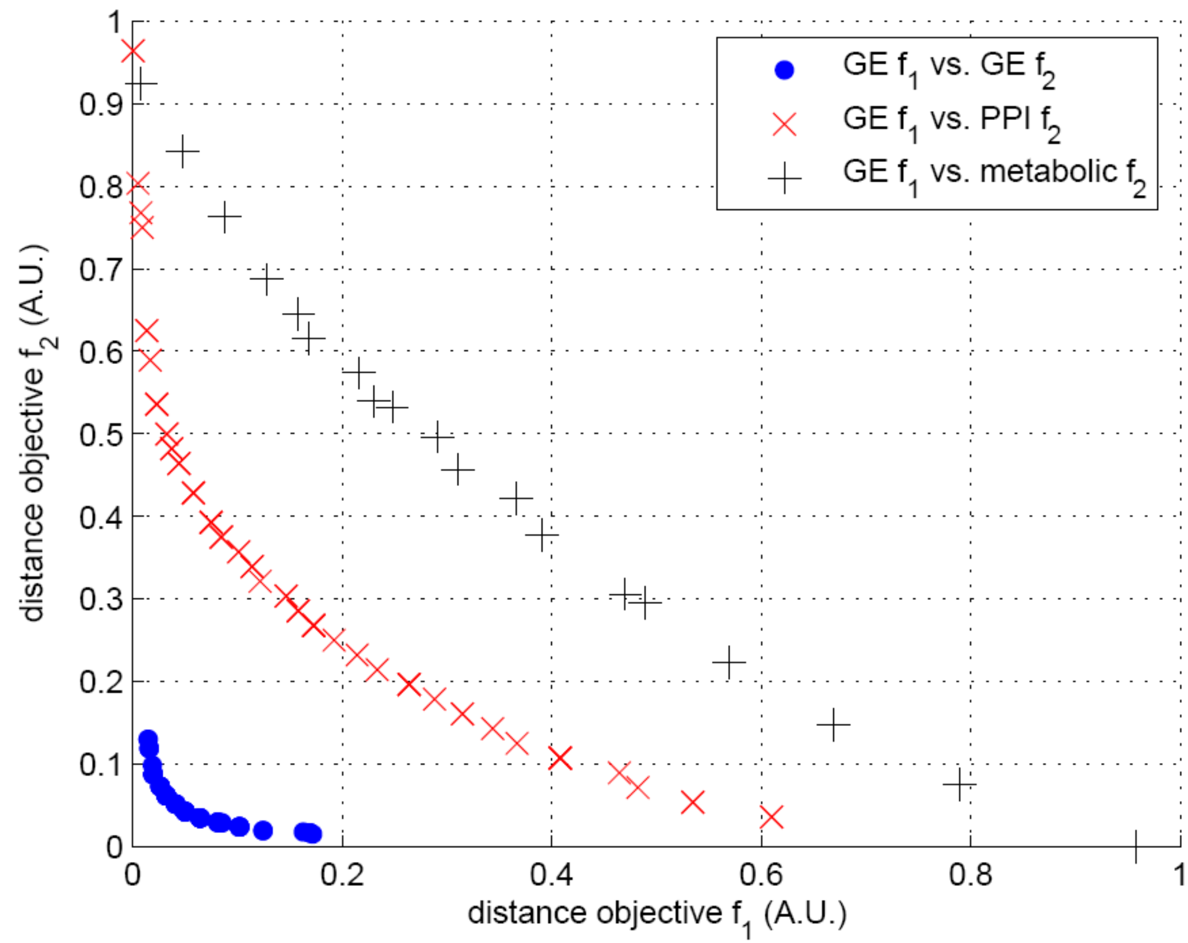




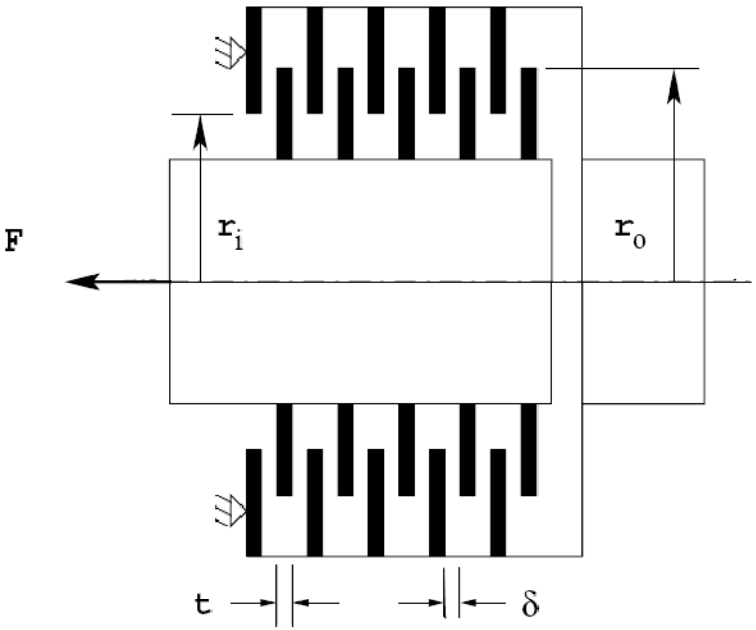
Module identification from biological data [Calonder et al. 2006]

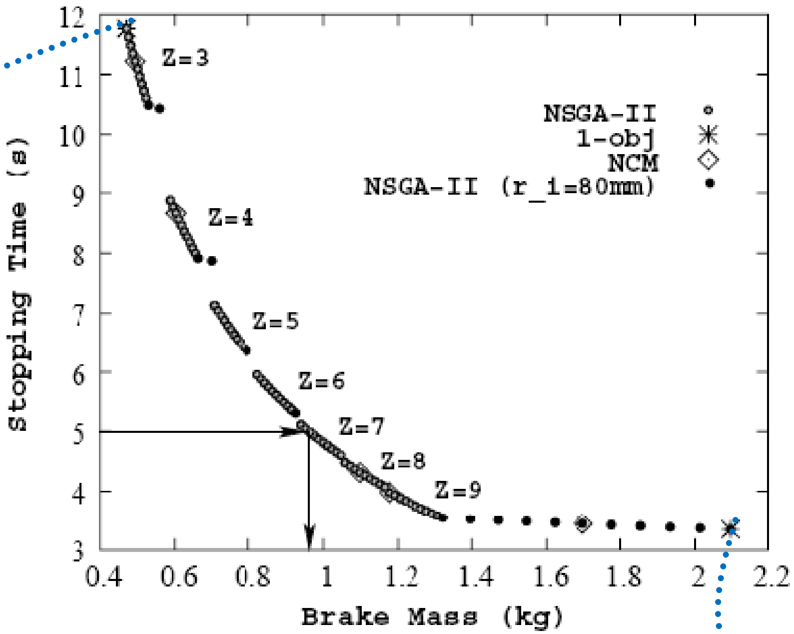
Find group of genes wrt different data types:

- similarity of gene expression profiles
- overlap of protein interaction partners
- metabolic pathway map distances



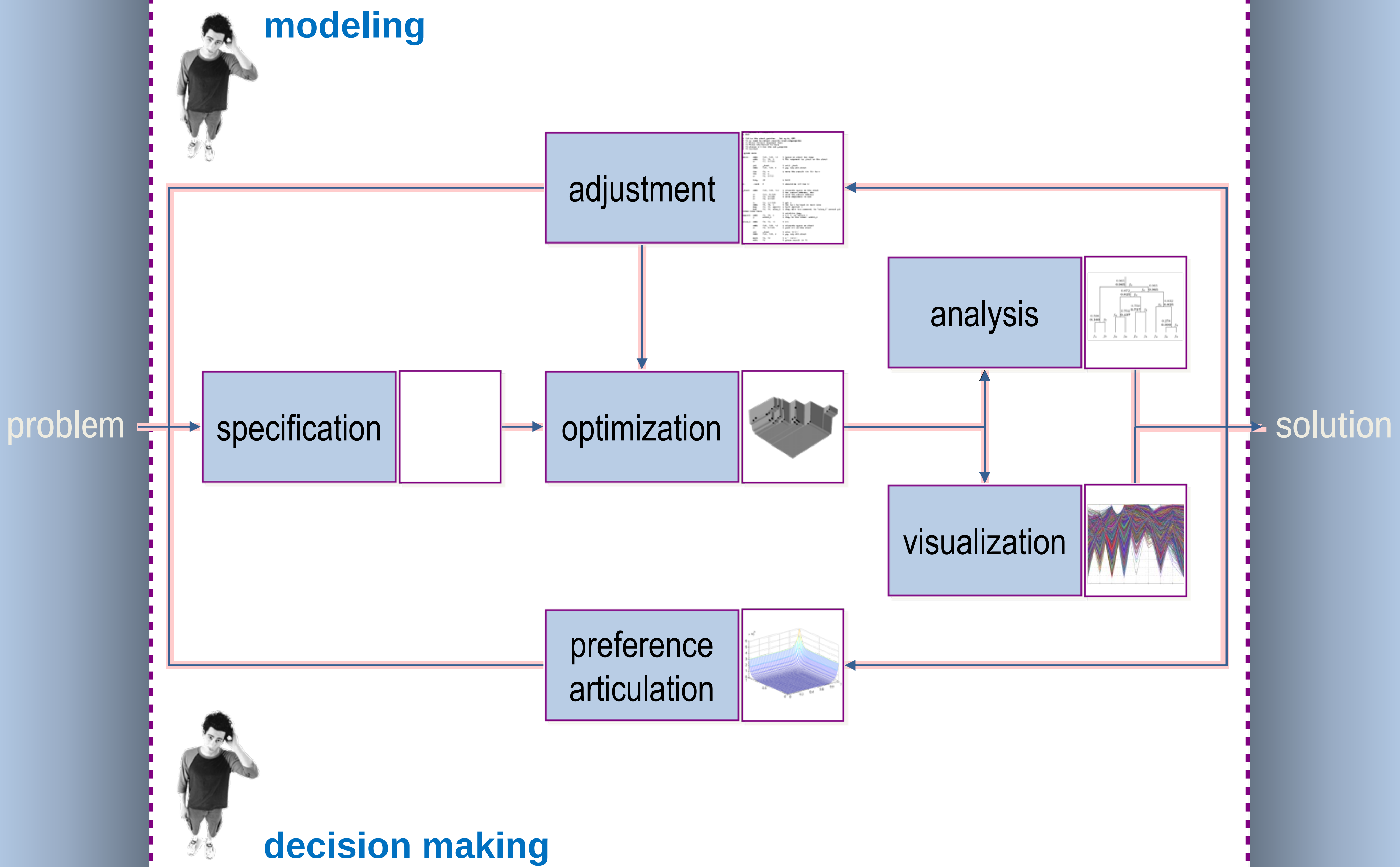
Multiple disk clutch brake design [Deb, Srinivasan 2006]





Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Conclusions: EMO as Interactive Decision Support



The EMO Community

Links:

- EMO mailing list:
  http://w3.ualg.pt/lists/emo-list/
- EMO bibliography:
  http://www.lania.mx/~ccoello/EMOO/

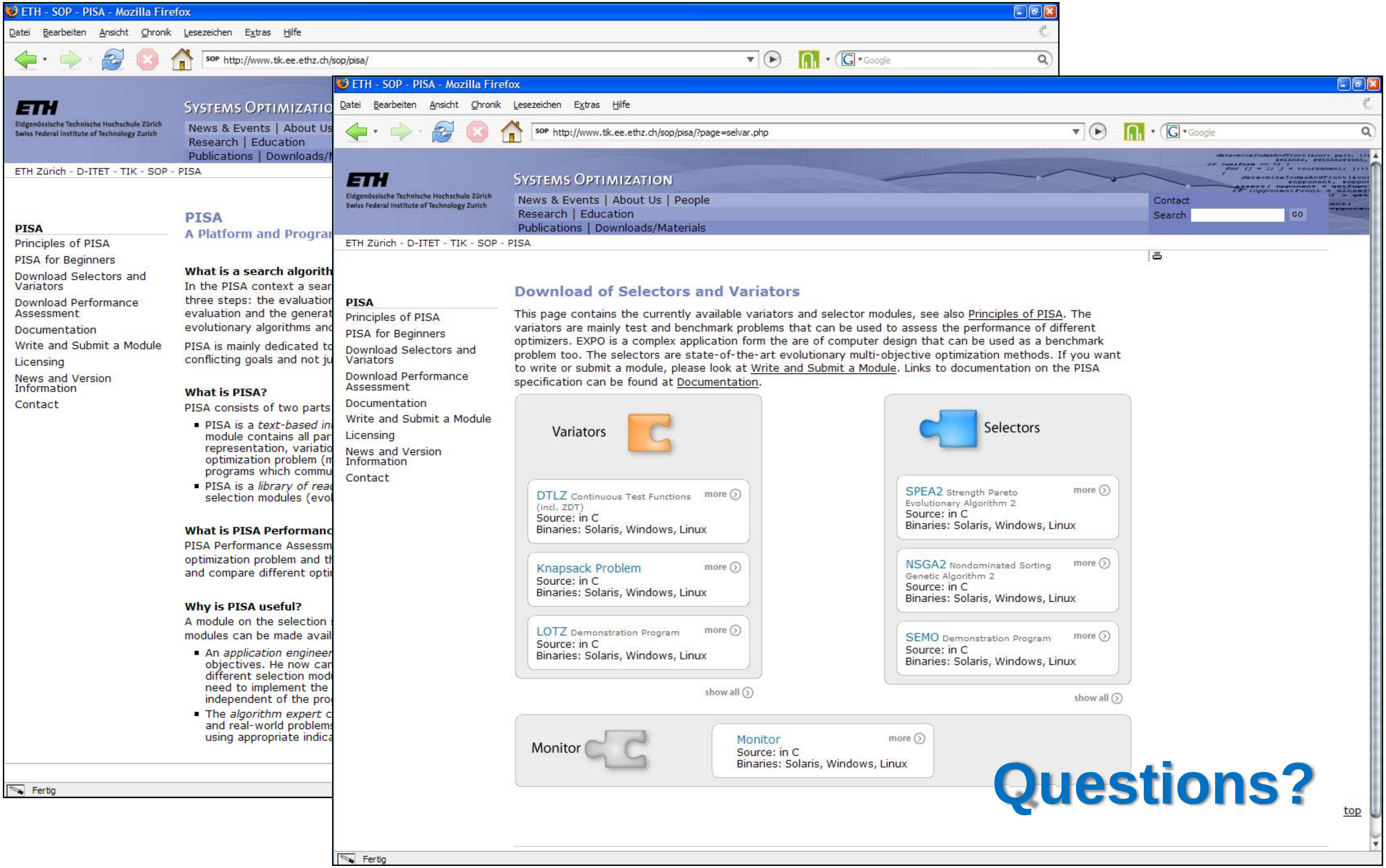
Events:

- Conference on Evolutionary Multi-Criterion Optimization

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2006
- and more…

PISA: http://www.tik.ee.ethz.ch/sop/pisa/



Questions?