GECCO'2012 Tutorial on Evolutionary Multiobjective Optimization

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Principles of Multiple Criteria Decision

A hypothetical problem: all solutions plotted
Principles of Multiple Criteria Decision

A hypothetical problem: all solutions plotted
**Observations:**

1. There is no single optimal solution, but
2. Some solutions (•) are better than others (○)

---

**Principles of Multiple Criteria Decision Analysis**

**Observations:**

1. There is no single optimal solution, but
2. Some solutions (•) are better than others (○)
Observations: 1. there is no single optimal solution, but
2. some solutions (●) are better than others (○)
Principles of Multiple Criteria Decision

**Observations:**

1. There is no single optimal solution, but
2. Some solutions (●) are better than others (○)
Possible Approach:

- supply more important than cost (ranking)
Possible Approach:

- supply more important than cost (ranking)
- cost must not exceed 2400 (constraint)
When to Make the Decision

Before Optimization:

- rank objectives,
- define constraints,…
- search for one (blue) solution
When to Make the Decision

Before Optimization:

- rank objectives, define constraints,…
- search for one (blue) solution
When to Make the Decision

Before Optimization:
- rank objectives, define constraints, …
- search for one (blue) solution

After Optimization:
- search for a set of (blue) solutions
- select one solution considering constraints, etc.
When to Make the Decision

Before Optimization:
- rank objectives, define constraints,…
- search for one (blue) solution

After Optimization:
- search for a set of (blue) solutions
- select one solution considering constraints, etc.

Focus: learning about a problem
- trade-off surface
- interactions among criteria
- structural information
Multiple Criteria Decision Making (MCDM)

Definition: MCDM

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process.

\[
\min_x [\mu_1(x), \mu_2(x), ..., \mu_n(x)]^T
\]

subject to:
\[
g(x) \leq 0
\]
\[
h(x) = 0
\]
\[
x_l \leq x \leq x_u
\]

Model

Trade-off surface

Decision making

(exact) optimization
Multiple Criteria Decision Making (MCDM)

Definition: MCDM

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process.

- non-linear objectives
- noisy objectives
- uncertain objectives
- expensive objectives (integrated simulations)
- many objectives
- huge search spaces
- many constraints

$$g(x) \leq 0$$
$$h(x) = 0$$
$$x_L \leq x \leq x_U$$

(exact) optimization
Multiple Criteria Decision Making (MCDM)

**Definition: MCDM**

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process.

- non-linear
- noisy
- non-differentiable
- uncertain
- expensive (integrated simulations)

Black box optimization

\[ x \in X \xrightarrow{f} (f_1(x), \ldots, f_k(x)) \]

- only mild assumptions
- many objectives

\[ g(x) \leq 0 \\
\bar{g}(x) = 0 \\
x_l \leq x \leq x_u \]
Evolutionary Multiobjective Optimization

Definition: EMO

EMO = evolutionary algorithms / randomized search algorithms
- applied to multiple criteria decision making (in general)
- used to approximate the Pareto-optimal set (mainly)

Mutation, survival, recombination, mating

Pareto set approximation
Some problems are easier to solve in a multiobjective scenario

example: TSP

\[\pi \in S_n \rightarrow f(\pi)\]

\[\pi \in S_n \rightarrow (f_1(\pi, a, b), f_2(\pi, a, b))\]

by addition of new “helper objectives” [Jensen 2004]

job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], theoretical (runtime) analyses [Brockhoff et al. 2009]

by decomposition of the single objective

TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], theoretical (runtime) analyses [Handl et al. 2008b]
Often innovative design principles among solutions are found

example:
clutch brake design

min. mass + stopping time

[Deb and Srinivasan 2006]
Often innovative design principles among solutions are found. Example: clutch brake design [Deb and Srinivasan 2006]

<table>
<thead>
<tr>
<th>Solution</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$f_1$</th>
<th>$f_2$</th>
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<tr>
<td>Min. $f_1$</td>
<td>70</td>
<td>90</td>
<td>1.5</td>
<td>1000</td>
<td>3</td>
<td>0.4704</td>
<td>11.7617</td>
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<tr>
<td>Min. $f_2$</td>
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<td>110</td>
<td>1.5</td>
<td>1000</td>
<td>9</td>
<td>2.0948</td>
<td>3.3505</td>
</tr>
</tbody>
</table>
Often innovative design principles among solutions are found

e.g., clutch brake design  
[Deb and Srinivasan 2006]

**Innovization**  
[Deb and Srinivasan 2006]

= using machine learning techniques to find new and innovative design principles among solution sets

= learning about a multiobjective optimization problem

**Other examples:**

- SOM for supersonic wing design  
[Obayashi and Sasaki 2003]

- biclustering for processor design and KP  
[Ulrich et al. 2007]
## The History of EMO At A Glance

<table>
<thead>
<tr>
<th>Year</th>
<th>Key Developments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>first EMO approaches</td>
</tr>
<tr>
<td>1990</td>
<td>dominance-based population ranking</td>
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<tr>
<td>1995</td>
<td>dominance-based EMO algorithms with diversity preservation techniques</td>
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<td>2000</td>
<td>attainment functions</td>
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<tr>
<td></td>
<td>elitist EMO algorithms</td>
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<td></td>
<td>test problem design</td>
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<tr>
<td></td>
<td>uncertainty and robustness</td>
</tr>
<tr>
<td></td>
<td>MCDM + EMO</td>
</tr>
<tr>
<td>2010</td>
<td>many-objective optimization</td>
</tr>
</tbody>
</table>
The History of EMO At A Glance

1984
- first EMO approaches
- dominance-based population ranking
- dominance-based EMO algorithms with diversity preservation techniques
- attainment functions
- elitist EMO algorithms
- preference articulation
- convergence proofs
- test problem design
- quantitative performance assessment
- multiobjectivization
- uncertainty and robustness
- running time analyses
- quality measure design
- MCDM + EMO
- EMO algorithms based on set quality measures
- high-dimensional objective spaces
- statistical performance assessment

2012

Overall: 6842 references by April 25, 2012

http://delta.cs.cinvestav.mx/~ccoello/EMOO/EMOOstatistics.html
The EMO Community

The EMO conference series:

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<td>Zurich</td>
<td>Faro</td>
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<td>Matsushima</td>
<td>Nantes</td>
<td>Ouro Peto</td>
<td>Sheffield</td>
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<td>Switzerland</td>
<td>Portugal</td>
<td>Mexico</td>
<td>Japan</td>
<td>France</td>
<td>Brazil</td>
<td>UK</td>
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</table>

45 / 87  56 / 100  59 / 115  65 / 124  39 / 72  42 / 83

Many further activities:

special sessions, special journal issues, workshops, tutorials, ...
Overview

The Big Picture

Basic Principles of Multiobjective Optimization
- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts
- indicator-based EMO
- preference articulation

A Few Examples From Practice
What makes evolutionary multiobjective optimization different from single-objective optimization?
A General (Multiobjective) Optimization

A multiobjective optimization problem: \((X, Z, f, g, \leq)\)

\[ X \]

search / parameter / decision space

\[ Z = \mathbb{R}^n \]

objective space

\[ f = (f_1, \ldots, f_n) \]

vector-valued objective function with

\[ f_i : X \rightarrow \mathbb{R} \]

\[ g = (g_1, \ldots, g_m) \]

vector-valued constraint function with

\[ g_i : X \rightarrow \mathbb{R} \]

\[ \leq \subseteq Z \times Z \]

binary relation on objective space

**Goal:** find decision vector(s) \(a \in X\) such that

1. for all \(1 \leq i \leq m\): \(g_i(a) \leq 0\) and
2. for all \(b \in X\): \(f(b) \leq f(a) \Rightarrow f(a) \leq f(b)\)
A Single-Objective Optimization Problem

- decision space
- objective space
- objective function
- total order

$$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$$
A Single-Objective Optimization Problem

- Decision space
- Objective space
- Objective function
- Total order
- Total preorder where

\[(X, Z, f: X \rightarrow Z, \text{rel} \subseteq Z \times Z)\]

\[(X, \text{prefrel})\]

\[a \text{ prefrel } b \iff f(a) \text{ rel } f(b)\]
Example: Leading Ones Problem

\[(X, Z, f: X \to Z, \text{rel} \subseteq Z \times Z)\]

\[
\begin{align*}
\{0, 1\}^n, \{0, 1, 2, \ldots, n\}, f_{\text{LO}} \mbox{, } \geq & \\
\text{where } f_{\text{LO}}(a) & = \sum_i (\prod_{j \leq i} a_j)
\end{align*}
\]
**Example:** $\geq$ (total order)

\[ a, b \in X \]

\[ a \geq b \]

\[ b \]

\[ a \]

**totally ordered**
Preference Relations

- Decision space
- Objective space
- Objective functions
- Partial order
- Preorder where
  \[ a \text{ prefrel } b : \iff f(a) \text{ rel } f(b) \]

\[ (X,\preceq_{\text{par}}) \]

\[ a \preceq_{\text{par}} b : \iff f(a) \preceq_{\text{par}} f(b) \]

*weak Pareto dominance*
A Multiobjective Optimization Problem

Example: Leading Ones Trailing Zeros Problem

($(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$)

$(X, prefrel)$
A Multiobjective Optimization Problem

Example: Leading Ones Trailing Zeros Problem

\[(X, Z, f: X \rightarrow Z, \text{rel} \subseteq Z \times Z)\]

\[(X, \text{prefrel})\]

\[
\begin{align*}
\{0,1\}^n \times \{0,1, 2, \ldots, n\} \times \{0,1, 2, \ldots, n\}, (f_{LO}, f_{TZ}), ?
\end{align*}
\]

\[
f_{LO}(a) = \sum_i (\prod_{j \leq i} a_j) \quad f_{TZ}(a) = \sum_i (\prod_{j \leq i} (1 - a_j))
\]
Pareto Dominance

(u₁, …, uₙ) weakly Pareto dominates (v₁, …, vₙ):

(u₁, …, uₙ) ≲ₚₑᵣₜ (v₁, …, vₙ) ⇔ ∀1 ≤ i ≤ n : uᵢ ≤ vᵢ

(u₁, …, uₙ) Pareto dominates (v₁, …, vₙ):

(u₁, …, uₙ) ≲ₚₑᵣₜ (v₁, …, vₙ) ∧ (v₁, …, vₙ) ≰ₚₑᵣₜ (u₁, …, uₙ)

water supply

dominated

dominating

incomparable

cost
Different Notions of Dominance

- $\varepsilon$-dominance
- Pareto dominance
- Cone dominance

Diagram showing points on a scatter plot with axes for water supply and cost.
The minimal set of a preordered set \((Y, \leq)\) is defined as

\[
\text{Min}(Y, \leq) := \{a \in Y \mid \forall b \in Y : b \leq a \Rightarrow a \leq b\}
\]
Visualizing Preference Relations

\[(f_{\text{cost}})\]

\[(f_{\text{water supply}})\]

\[(X, \preceq_{\text{par}})\]

\[(f_{\text{cost}}, f_{\text{water supply}})\]

optima
Remark: Properties of the Pareto Set

**Computational complexity:**
multiobjective variants can become NP- and #P-complete

**Size:** Pareto set can be exponential in the input length
(e.g. shortest path [Serafini 1986], MSP [Camerini et al. 1984])

![Diagram](https://example.com/diagram.png)
 Approaches To Multiobjective Optimization

A multiobjective problem is as such underspecified …because not any Pareto-optimum is equally suited!

Additional preferences are needed to tackle the problem:

**Solution-Oriented Problem Transformation:**
Induce a total order on the decision space, e.g., by aggregation.

**Set-Oriented Problem Transformation:**
First transform problem into a set problem and then define an objective function on sets.

Preferences are needed in any case, but the latter are weaker!
Problem Transformations and Set Problems

single solution problem

set problem

search space

$$f(x) = (f_1(x), f_2(x), \ldots, f_k(x))$$

$$f^*(A) = \{ f(x) \mid x \in A \}$$

objective space

$$x \succeq y : \iff \forall_i f_i(x) \geq f_i(y)$$

$$A \succeq^* B : \iff \forall y \in B \exists x \in A x \succeq y$$

(partially) ordered set

$$(\mathbb{R}^k, \succeq)$$

$$(2\mathbb{R}^k, \succeq^*)$$

(totally) ordered set

$$(\mathbb{R}^K, \succeq)$$

$$(2\mathbb{R}^K, \succeq^*)$$

A scalarizing function $s$ is a function $s : Z \mapsto \mathbb{R}$ that maps each objective vector $(u_1, \ldots, u_n) \in Z$ to a real value $s(u_1, \ldots, u_n) \in \mathbb{R}$. 
Aggregation-Based Approaches

Example: weighting approach

\[ y = w_1y_1 + \ldots + w_ky_k \]

Other example: Tchebycheff

\[ y = \max w_i(u_i - z_i) \]
Set-Oriented Problem Transformations

For a multiobjective optimization problem \((X, Z, f, g, \preceq)\), the associated set problem is given by \((\Psi, \Omega, F, G, \preceq)\) where

- \(\Psi = 2^X\) is the space of decision vector sets, i.e., the powerset of \(X\),
- \(\Omega = 2^Z\) is the space of objective vector sets, i.e., the powerset of \(Z\),
- \(F\) is the extension of \(f\) to sets, i.e., \(F(A) := \{f(a) : a \in A\}\) for \(A \in \Psi\),
- \(G = (G_1, \ldots, G_m)\) is the extension of \(g\) to sets, i.e., \(G_i(A) := \max \{g_i(a) : a \in A\}\) for \(1 \leq i \leq m\) and \(A \in \Psi\),
- \(\preceq\) extends \(\preceq\) to sets where \(A \preceq B :\iff \forall b \in B \exists a \in A : a \preceq b\).
Pareto set approximation (algorithm outcome) = set of (usually incomparable) solutions

- **A weakly dominates B**: not worse in all objectives and sets not equal
- **C dominates D**: better in at least one objective
- **A strictly dominates C**: better in all objectives
- **B is incomparable to C**: neither set weakly better
What Is the Optimization Goal (Total Order)?

- Find all Pareto-optimal solutions?
  - Impossible in continuous search spaces
  - How should the decision maker handle 10000 solutions?

- Find a representative subset of the Pareto set?
  - Many problems are NP-hard
  - What does representative actually mean?

- Find a good approximation of the Pareto set?
  - What is a good approximation?
  - How to formalize intuitive understanding:
    1. close to the Pareto front
    2. well distributed
A (unary) quality indicator $I$ is a function $I : \Psi \rightarrow \mathbb{R}$ that assigns a Pareto set approximation a real value.
General Remarks on Problem

Idea:
Transform a preorder into a total preorder

Methods:
- Define single-objective function based on the multiple criteria
  *(shown on the previous slides)*
- Define any total preorder using a relation
  *(not discussed before)*

Question:
Is any total preorder ok resp. are there any requirements concerning the resulting preference relation?
⇒ Underlying dominance relation \( rel \) should be reflected
Refinements and Weak Refinements

1. \textbf{ref} \textbf{refines} a preference relation $\preceq$ iff

$$A \preceq B \land B \not\preceq A \implies A \preceq B \land B \not\preceq A$$ \hspace{1cm} (better $\Rightarrow$ better)

$\implies$ fulfills requirement

2. \textbf{ref} \textbf{weakly refines} a preference relation $\preceq$ iff

$$A \preceq B \land B \not\preceq A \implies A \preceq B$$ \hspace{1cm} (better $\Rightarrow$ weakly better)

$\implies$ does not fulfill requirement, but $\preceq$ does not contradict $\preceq$

...sought are total refinements...
Example: Refinements Using Indicators

\[ A \preceq B : \Leftrightarrow I(A) \geq I(B) \]

I(A) = volume of the weakly dominated area in objective space

\[ A \preceq B : \Leftrightarrow I(A,B) \leq I(B,A) \]

I(A,B) = how much needs A to be moved to weakly dominate B

unary hypervolume indicator  binary epsilon indicator

but no total order
Example: Weak Refinement / No Refinement

ref

\[ A \preceq B :\Leftrightarrow I(A,R) \leq I(B,R) \]

\( I(A,R) = \) how much needs A to be moved to weakly dominate R

\[ A \preceq B :\Leftrightarrow I(A) \leq I(B) \]

\( I(A) = \) variance of pairwise distances

weak refinement

unary epsilon indicator

weak refinement no refinement

unary diversity indicator

Overview

The Big Picture

Basic Principles of Multiobjective Optimization
  - algorithm design principles and concepts
  - performance assessment

Selected Advanced Concepts
  - indicator-based EMO
  - preference articulation

A Few Examples From Practice
Algorithm Design: Particular Aspects

1. representation
2. environmental selection
3. variation operators

- fitness assignment
- mating selection
Fitness Assignment: Principal Approaches

**aggregation-based**
- weighted sum

**criterion-based**
- VEGA

**dominance-based**
- SPEA2

---

**parameter-oriented**
- scaling-dependent

**set-oriented**
- scaling-independent
Criterion-Based Selection: VEGA

select according to

shuffle

population k separate selections mating pool

Aggregation-Based: Multistart Constraint Method

Underlying concept:
- Convert all objectives except one into constraints
- Adaptively vary constraints

maximize $f_1$

feasible region constraint
Underlying concept:
- Convert all objectives except one into constraints
- Adaptively vary constraints

Aggregation-Based: Multistart Constraint Method
Aggregation-Based: Multistart Constraint Method

**Underlying concept:**
- Convert all objectives except of one into constraints
- Adaptively vary constraints
General Scheme of Dominance-Based EMO

mating selection (stochastic)

population (archiv)  offspring

environmental selection (greedy heuristic)

fitness assignment
partitioning into dominance classes

rank refinement within dominance classes

Note: good in terms of set quality = good in terms of search?
... goes back to a proposal by David Goldberg in 1989. ... is based on pairwise comparisons of the individuals only.

- **dominance rank**: by how many individuals is an individual dominated?  
  \[ \text{MOGA, NPGA} \]

- **dominance count**: how many individuals does an individual dominate?  
  \[ \text{SPEA, SPEA2} \]

- **dominance depth**: at which front is an individual located?  
  \[ \text{NSGA, NSGA-II} \]
Illustration of Dominance-based Partitioning

dominance rank

dominance depth
Refinement of Dominance Rankings

**Goal:** rank incomparable solutions within a dominance class

1. Density information (good for search, but **usually no refinements**)
   - **Kernel method**
     - density = function of the distances
   - **k-th nearest neighbor**
     - density = function of distance to k-th neighbor
   - **Histogram method**
     - density = number of elements within box

2. Quality indicator (good for set quality): soon...
Example: SPEA2 Dominance Ranking

**Basic idea:** the less dominated, the fitter...

**Principle:** first assign each solution a weight (strength), then add up weights of dominating solutions

\[ S (\text{strength}) = \text{#dominated solutions} \]

\[ R (\text{raw fitness}) = \sum \text{strengths of dominators} \]
Density Estimation

- k-th nearest neighbor method:
  - Fitness = \( R + \frac{1}{2 + D_k} \) < 1
  - \( D_k = \text{distance to the k-th nearest individual} \)
  - Usually used: \( k = 2 \)
Density Estimation

crowding distance:

- sort solutions wrt. each objective
- crowding distance to neighbors:

\[
d(i) = \sum_{\text{obj. } m} |f_m(i - 1) - f_m(i + 1)|
\]
Selection in SPEA2 and NSGA-II can result in \textit{deteriorative} cycles

non-dominated solutions already found can be lost
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …) use hypervolume indicator to guide the search: refinement!

Main idea
Delete solutions with the smallest hypervolume loss
\[ d(s) = I_H(P) - I_H(P / \{s\}) \]
iteratively

But: can also result in cycles [Judt et al. 2011]
and is expensive [Bringmann and Friedrich 2009]

Moreover: HypE [Bader and Zitzler 2011]
Sampling + Contribution if more than 1 solution deleted
MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:
- Optimize N scalarizing functions in parallel
- Use only best solutions of “neighbored scalarizing function” for mating
- keep the best solutions for each scalarizing function
- use external archive for non-dominated solutions
At first sight not different from single-objective optimization
Most algorithm design effort on selection until now
But: convergence to a set ≠ convergence to a point

Open Question:
- how to achieve fast convergence to a set?

Related work:
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
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A Few Examples From Practice
Once Upon a Time...

... multiobjective EAs were mainly compared visually:

ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II
...And Even Today!

(found in a paper from 2009)
Two Approaches for Empirical Studies

Attainment function approach:

- Applies statistical tests directly to the samples of approximation sets
- Gives detailed information about how and where performance differences occur

Quality indicator approach:

- First, reduces each approximation set to a single value of quality
- Applies statistical tests to the samples of quality values

see e.g. [Zitzler et al. 2003]
Empirical Attainment Functions

three runs of two multiobjective optimizers

frequency of attaining regions
50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)

latest implementation online at http://eden.dei.uc.pt/~cmfonsec/software.html
see [Fonseca et al. 2011]
Attainment Plots

latest implementation online at
http://eden.dei.uc.pt/~cmfonsec/software.html
see [Fonseca et al. 2011]
Quality Indicator Approach

**Goal:** compare two Pareto set approximations A and B

![Diagram showing Pareto fronts A and B with quality indicators]

<table>
<thead>
<tr>
<th>Quality Indicator</th>
<th>A</th>
<th>B</th>
</tr>
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<tbody>
<tr>
<td>hypervolume</td>
<td>432.34</td>
<td>420.13</td>
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<tr>
<td>distance</td>
<td>0.3308</td>
<td>0.4532</td>
</tr>
<tr>
<td>diversity</td>
<td>0.3637</td>
<td>0.3463</td>
</tr>
<tr>
<td>spread</td>
<td>0.3622</td>
<td>0.3601</td>
</tr>
<tr>
<td>cardinality</td>
<td>6</td>
<td>5</td>
</tr>
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</table>

Comparison method $C = \text{quality measure(s)} + \text{Boolean function}$

A, B $\xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$

Reduction and interpretation
Example: Box Plots

epsilon indicator  hypervolume  R indicator

DTLZ2

Knapsack

ZDT6
## Statistical Assessment (Kruskal Test)

<table>
<thead>
<tr>
<th></th>
<th>ZDT6 Epsilon</th>
<th>DTLZ2 R</th>
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<tr>
<td><strong>is better than</strong></td>
<td><strong>IBEA</strong></td>
<td><strong>NSGA2</strong></td>
</tr>
<tr>
<td><strong>IBEA</strong></td>
<td>~0 😊</td>
<td>~0 😊</td>
</tr>
<tr>
<td><strong>NSGA2</strong></td>
<td>1</td>
<td>~0 😊</td>
</tr>
<tr>
<td><strong>SPEA2</strong></td>
<td>1</td>
<td>1</td>
</tr>
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</table>

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

**Knapsack/Hypervolume:** $H_0 = \text{No significance of any differences}$

---

Problems With Non-Compliant Indicators

<table>
<thead>
<tr>
<th>Indicator</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generational distance</td>
<td>3.46396</td>
<td>2.37411</td>
</tr>
<tr>
<td>Spacing (Schott)</td>
<td>0.26476</td>
<td>0.19989</td>
</tr>
<tr>
<td>Max Pareto front error</td>
<td>3.35489</td>
<td>3.51314</td>
</tr>
<tr>
<td>Extent</td>
<td>3.56039</td>
<td>3.57319</td>
</tr>
</tbody>
</table>
What Are Good Set Quality Measures?

There are **three aspects** [Zitzler et al. 2000]

Comparing different optimization techniques experimentally always involves the notion of performance. In the case of multiobjective optimization, the definition of quality is substantially more complex than for single-objective optimization problems, because the optimization goal itself consists of multiple objectives:

- The **distance** of the resulting nondominated set to the Pareto-optimal front should be minimized.
- A good (in most cases uniform) **distribution** of the solutions found is desirable. The assessment of this criterion might be based on a certain distance metric.
- The **extent** of the obtained nondominated front should be maximized, i.e., for each objective, a wide range of values should be covered by the nondominated solutions.

In the literature, some attempts can be found to formalize the above definition (for parts...

**Wrong!** [Zitzler et al. 2003]

An infinite number of unary set measures is needed to detect in general whether A is better than B
Open Questions:

- how to design a good benchmark suite?
- are there other unary indicators that are (weak) refinements?
- how to achieve good indicator values?
Overview

The Big Picture

Basic Principles of Multiobjective Optimization
- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts
- indicator-based EMO
- preference articulation

A Few Examples From Practice
When the goal is to maximize a unary indicator…

- we have a single-objective set problem to solve
- but what is the optimum?
- important: population size $\mu$ plays a role!

### Optimal $\mu$-Distribution:

A set of $\mu$ solutions that maximizes a certain unary indicator $I$ among all sets of $\mu$ solutions is called optimal $\mu$-distribution for $I$.

[Auger et al. 2009a]
Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

\[ \Rightarrow \text{most results on optimal } \mu\text{-distributions for hypervolume} \]

Optimal μ-Distributions (example results)

-[Auger et al. 2009a]:
- contain equally spaced points iff front is linear
- density of points \( \propto \sqrt{-f'(x)} \) with \( f' \) the slope of the front

-[Friedrich et al. 2011]:
optimal \( \mu \)-distributions for the hypervolume correspond to \( \varepsilon \)-approximations of the front

\[
\begin{align*}
\text{OPT} & \quad 1 + \frac{\log(\min\{A/a, B/b\})}{n} \\
\text{HYP} & \quad 1 + \frac{\sqrt{A/a} + \sqrt{B/b}}{n - 4} \\
\log\text{HYP} & \quad 1 + \frac{\sqrt{\log(A/a) \log(B/b)}}{n - 2}
\end{align*}
\]

! (probably) does not hold for > 2 objectives
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A Few Examples From Practice
Articulating User Preferences During Search

What we thought: EMO is preference-less

Search before decision making: Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

Decision making during search: The DM can articulate preferences during

What we learnt: EMO just uses weaker preference information

environmental selection

preferable?

3 out of 6
Incorporation of Preferences During Search

Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is too large

[Branke 2008], [Rachmawati and Srinivasan 2006], [Coello Coello 2000]

1. **Refine/modify dominance relation, e.g.:**
   - using goals, priorities, constraints
     [Fonseca and Fleming 1998a,b]
   - using different types of cones
     [Branke and Deb 2004]

2. **Use quality indicators, e.g.:**
   - based on reference points and directions
     [Deb and Sundar 2006, Deb and Kumar 2007]
   - based on binary quality indicators
     [Zitzler and Künzli 2004]
   - based on the hypervolume indicator (now)
     [Zitzler et al. 2007]
Example: Weighted Hypervolume Indicator

\[ I_H^w(A) = \int w(\tilde{z}) \]

[Zitzler et al. 2007]
Weighted Hypervolume in Practice

[Auger et al. 2009b]
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A Few Examples From Practice
Application: Design Space Exploration

Specification → Optimization → Evaluation → Implementation
Application: Design Space Exploration

Truss Bridge Design
[Bader 2010]

[Diagram of a truss bridge with annotations for fixed nodes, initial bridge, woven truss, right side mirrored, n decks, load L, water level, and no robustness vs. HypE_001a comparison.]

Implementation
Application: Design Space Exploration

Truss Bridge Design
[Bader 2010]

Network Processor Design
[Thiele et al. 2002]

Application: Design Space Exploration

**Truss Bridge Design**
[Bader 2010]

**Network Processor Design**
[Thiele et al. 2002]

**Water resource management**
[Siegfried et al. 2009]
Find group of genes wrt different data types:

- similarity of gene expression profiles
- overlap of protein interaction partners
- metabolic pathway map distances
Conclusions: EMO as Interactive Decision Support

- Problem
  - Specification
  - Optimization
  - Preference
  - Articulation
- Solution
  - Adjustment
  - Analysis
  - Visualization
- Decision Making
  - Modeling
The EMO Community

Links:
- EMO mailing list: http://w3.ualg.pt/lists/emo-list/
- EMO bibliography: http://www.lania.mx/~ccoello/EMOO/
- EMO conference series: http://www.shef.ac.uk/emo2013/

Books:
- Multi-Objective Optimization using Evolutionary Algorithms
  Kalyanmoy Deb, Wiley, 2001
- Evolutionary Algorithms for Solving Multi Evolutionary Algorithms
  for Solving Multi-Objective Problems Objective Problems,
  Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont,
  Kluwer, 2nd Ed. 2007
- Multiobjective Optimization—Interactive and Evolutionary
  Approaches, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors,
  volume 5252 of LNCS. Springer, 2008 [many open questions!]
- and more…
### Optimization Problems (variant)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Source</th>
<th>Binaries</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>GWLAB - Multi-Objective Groundwater Management</td>
<td>in Matlab</td>
<td></td>
<td><a href="#">more...</a></td>
</tr>
<tr>
<td>LOTZ - Demonstration Program</td>
<td>in C</td>
<td>Solaris, Windows, Linux</td>
<td><a href="#">more...</a></td>
</tr>
<tr>
<td>LOTZ2 - Leading Ones Trailing Zeros</td>
<td>in C</td>
<td>Solaris, Windows, Linux</td>
<td><a href="#">more...</a></td>
</tr>
<tr>
<td>LOTZ2 - Java Example Variator</td>
<td>in Java</td>
<td>Windows, Linux</td>
<td><a href="#">more...</a></td>
</tr>
</tbody>
</table>

### Optimization Algorithms (selector)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Source</th>
<th>Binaries</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPAM - Set Preference Algorithm for Multiobjective Optimization</td>
<td>in C</td>
<td>Windows, Linux 32bit, Linux 64bit</td>
<td><a href="#">more...</a></td>
</tr>
<tr>
<td>SIV - Sampling-based HyperVolume-oriented Algorithm</td>
<td>in C</td>
<td>Windows, Linux 32bit, Linux 64bit</td>
<td><a href="#">more...</a></td>
</tr>
<tr>
<td>SIBEA - Simple Indicator Based Evolutionary Algorithm</td>
<td>in Java as jar or zip</td>
<td>as jar, as zip or as tar.gz</td>
<td><a href="#">more...</a></td>
</tr>
<tr>
<td>HypF - HyperVolume Estimation Algorithm for Multiobjective Optimization</td>
<td>in C</td>
<td>Windows, Linux 32bit, Linux 64bit</td>
<td><a href="#">more...</a></td>
</tr>
</tbody>
</table>

### Questions?

Additional Slides
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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at INRIA Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011 he has been a junior researcher (CR2) at INRIA Lille - Nord Europe in Villeneuve d'Ascq, France. His research interests are focused on evolutionary multiobjective optimization (EMO), in particular on many-objective optimization, benchmarking, and theoretical aspects of indicator-based search.
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He holds Deva Raj Chair Professor at Indian Institute of Technology Kanpur in India. He is the recipient of the prestigious MCDM Edgeworth-Pareto award by the Multiple Criterion Decision Making (MCDM) Society, one of the highest awards given in the field of multi-criterion optimization and decision making. He has also received prestigious Shanti Swarup Bhatnagar Prize in Engineering Sciences for the year 2005 from Govt. of India.

He has also received the `Thomson Citation Laureate Award' from Thompson Scientific for having highest number of citations in Computer Science during the past ten years in India. He is a fellow of Indian National Academy of Engineering (INAE), Indian National Academy of Sciences, and International Society of Genetic and Evolutionary Computation (ISGEC). He has received Fredrick Wilhelm Bessel Research award from Alexander von Humboldt Foundation in 2003. His main research interests are in the area of computational optimization, modeling and design, and evolutionary algorithms. He has written two text books on optimization and more than 240 international journal and conference research papers. He has pioneered and a leader in the field of evolutionary multi-objective optimization. He is associate editor of two major international journals and an editorial board members of five major journals.
References


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