GECCO'2014 Tutorial on Evolutionary Multiobjective Optimization

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INVENTORS FOR THE DIGITAL WORLD

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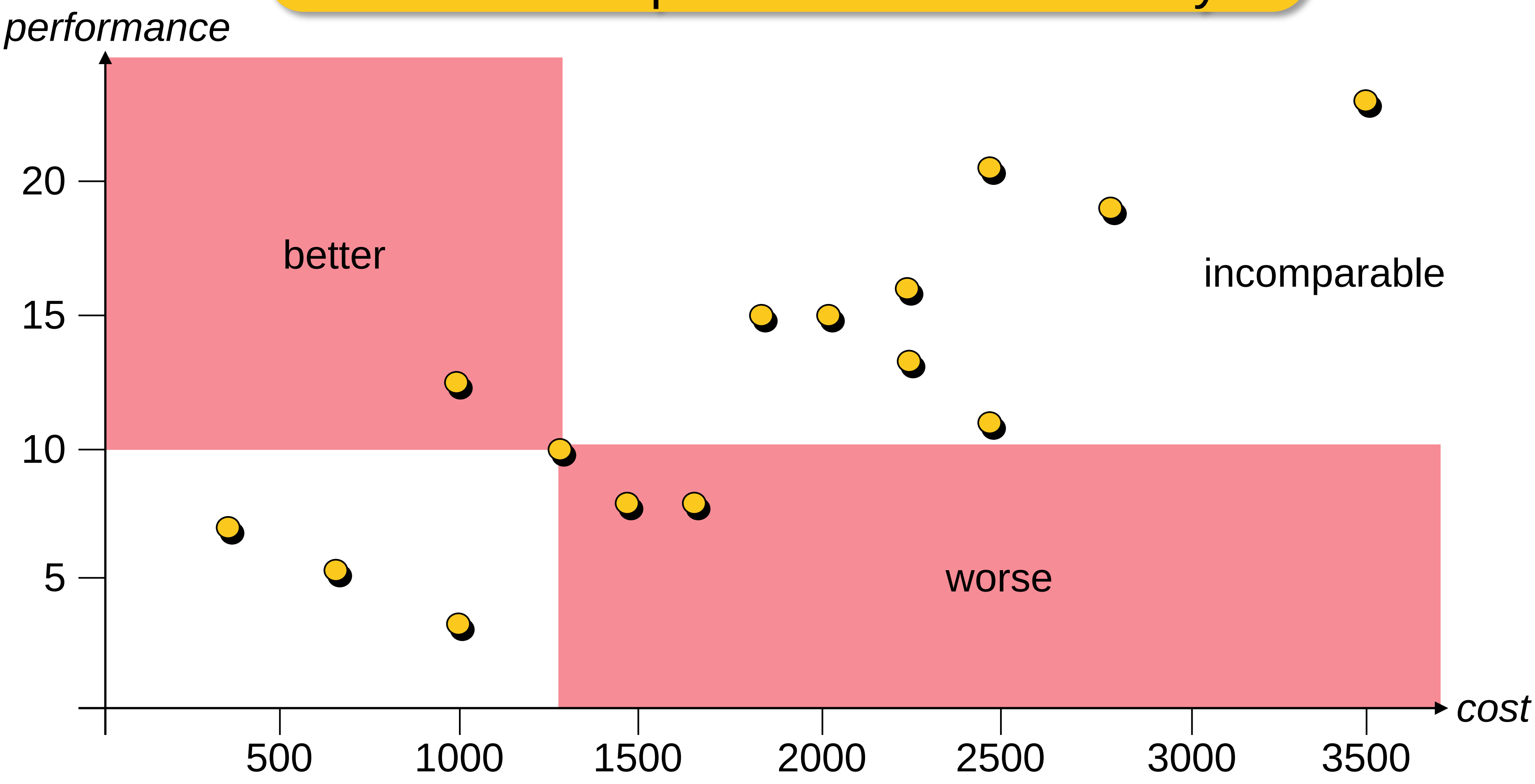
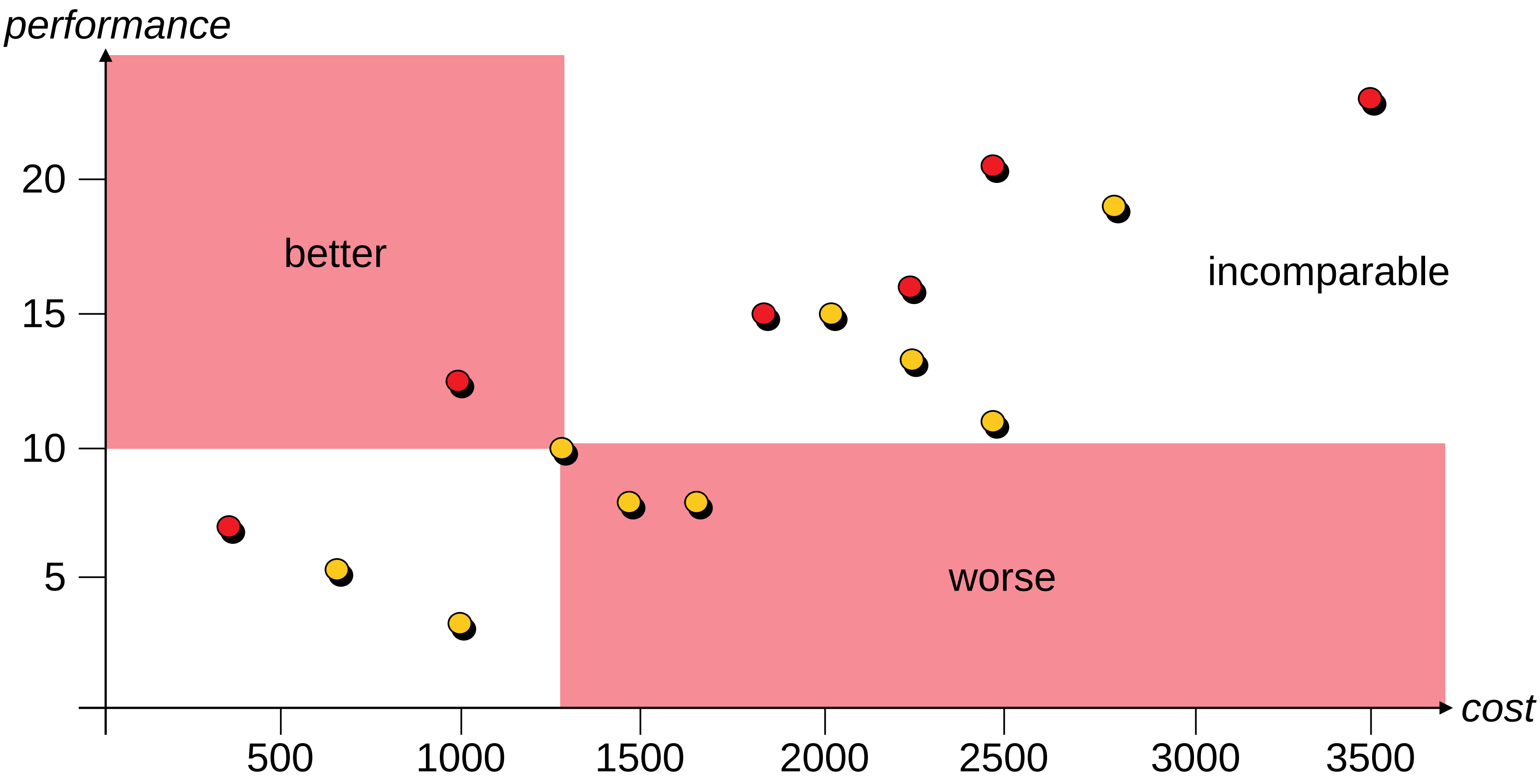
A Brief Introduction to Multiobjective Optimization

Multiobjective Optimization:
problems where multiple objectives have to be optimized simultaneously



Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



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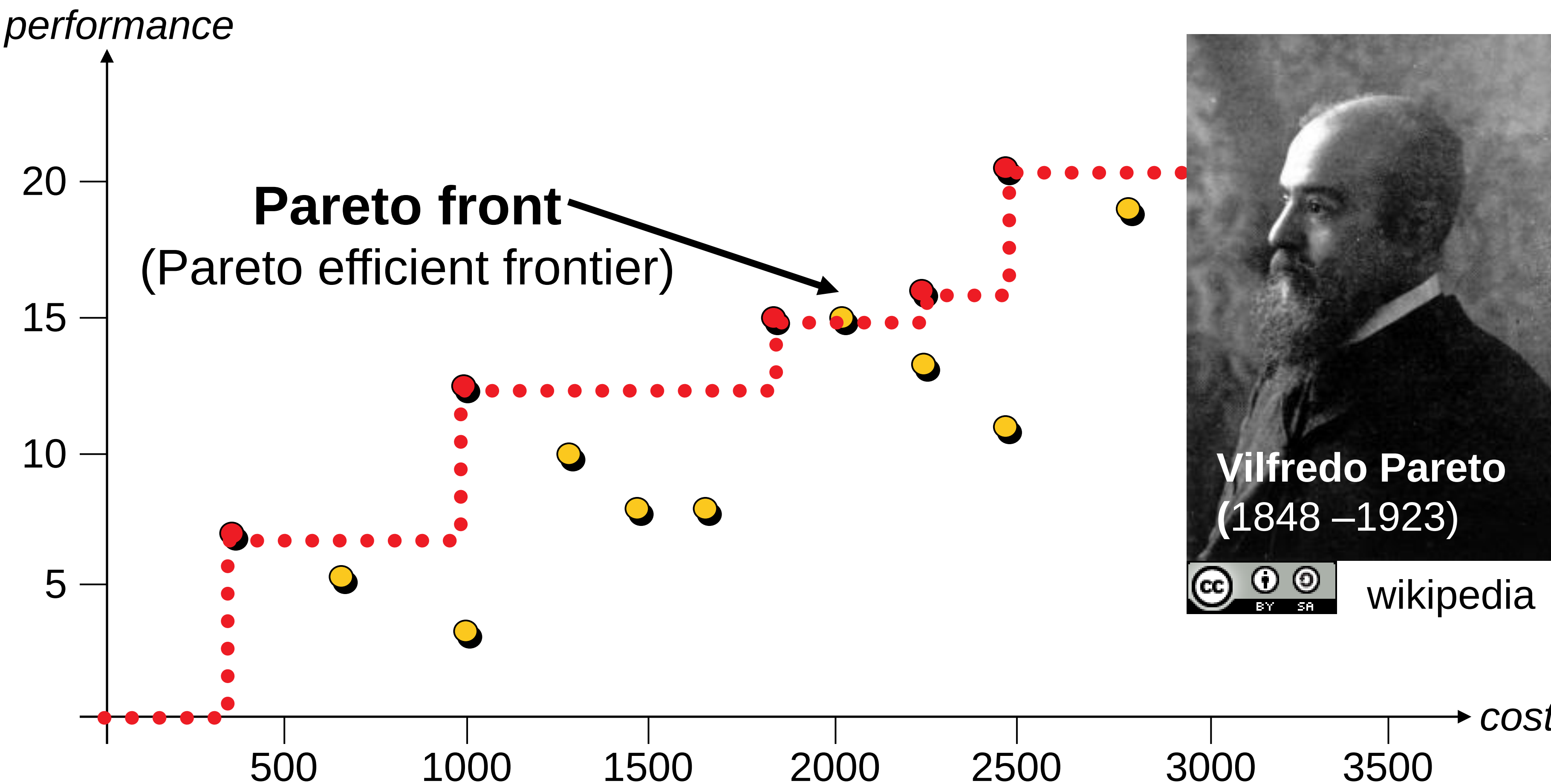

wikipedia

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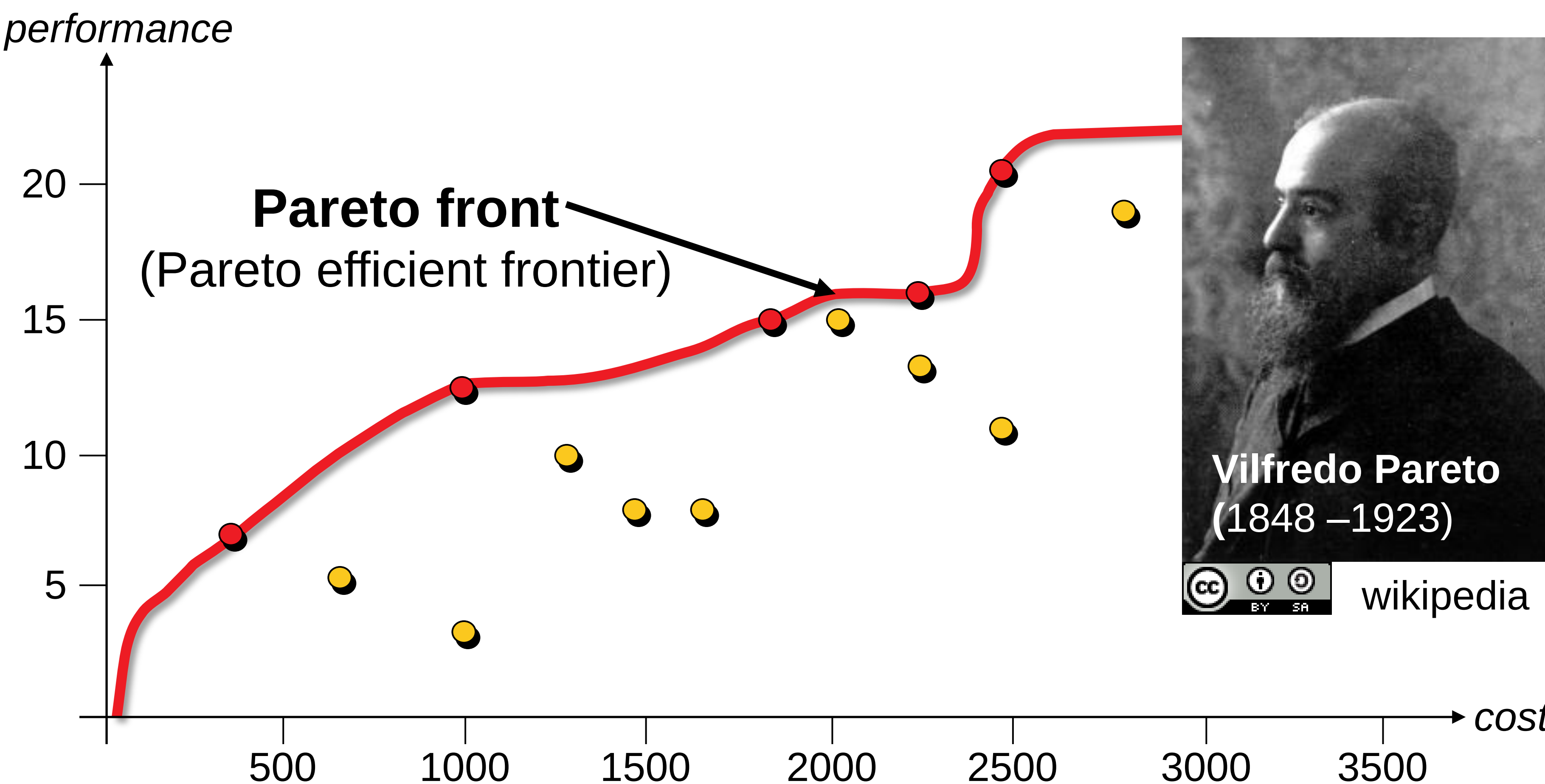

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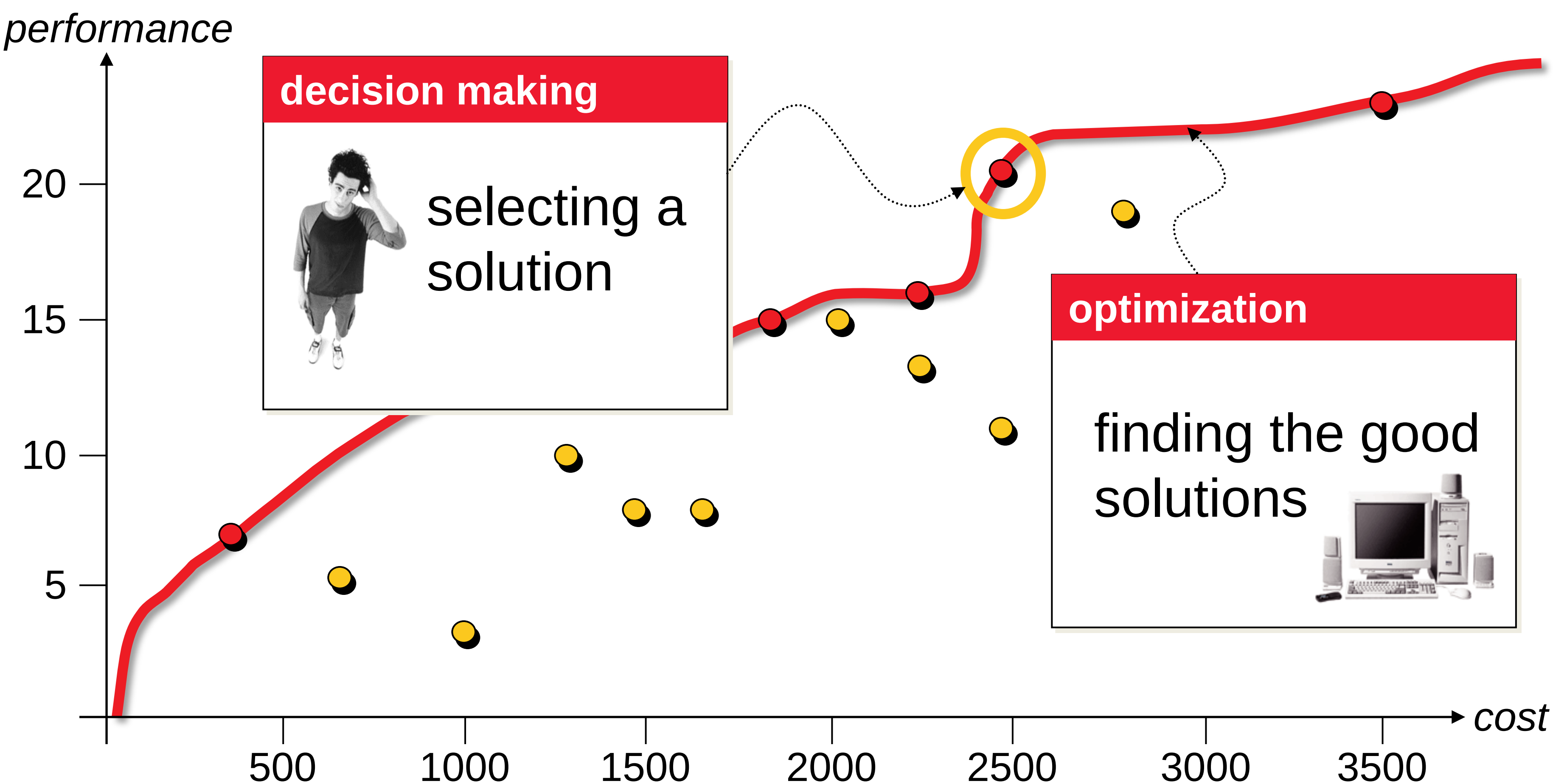
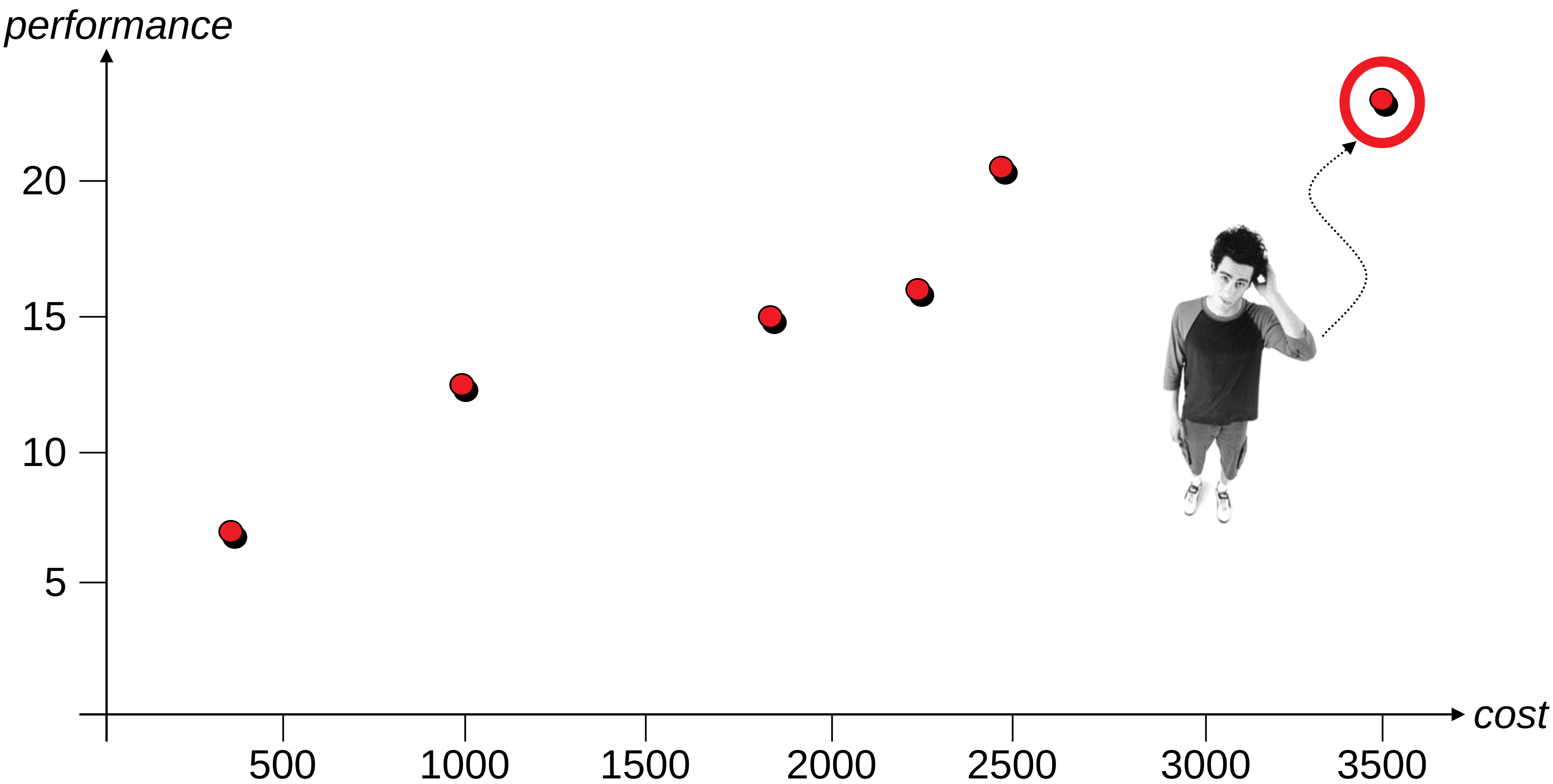


Possible Approaches:

➊ **ranking:** performance more important than cost

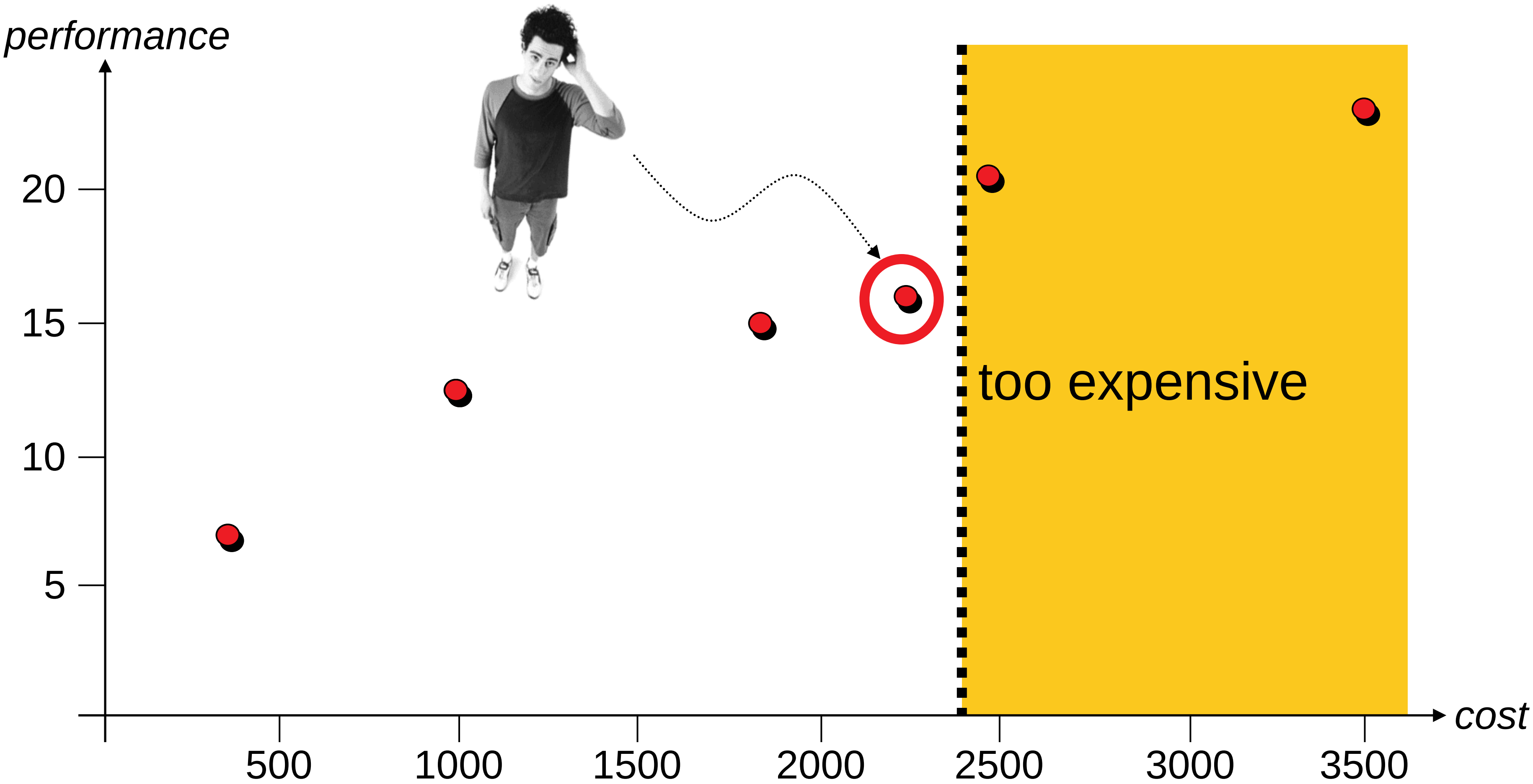


Selecting a Solution: Examples

Possible Approaches:

❶ ranking: performance more important than cost

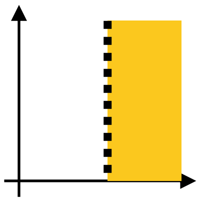
❷ **constraints:** cost must not exceed 2400



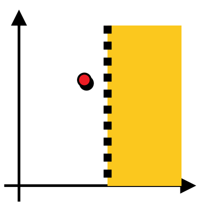
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…



search for one
(good) solution



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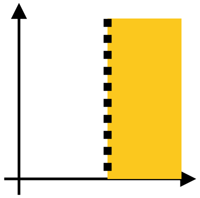

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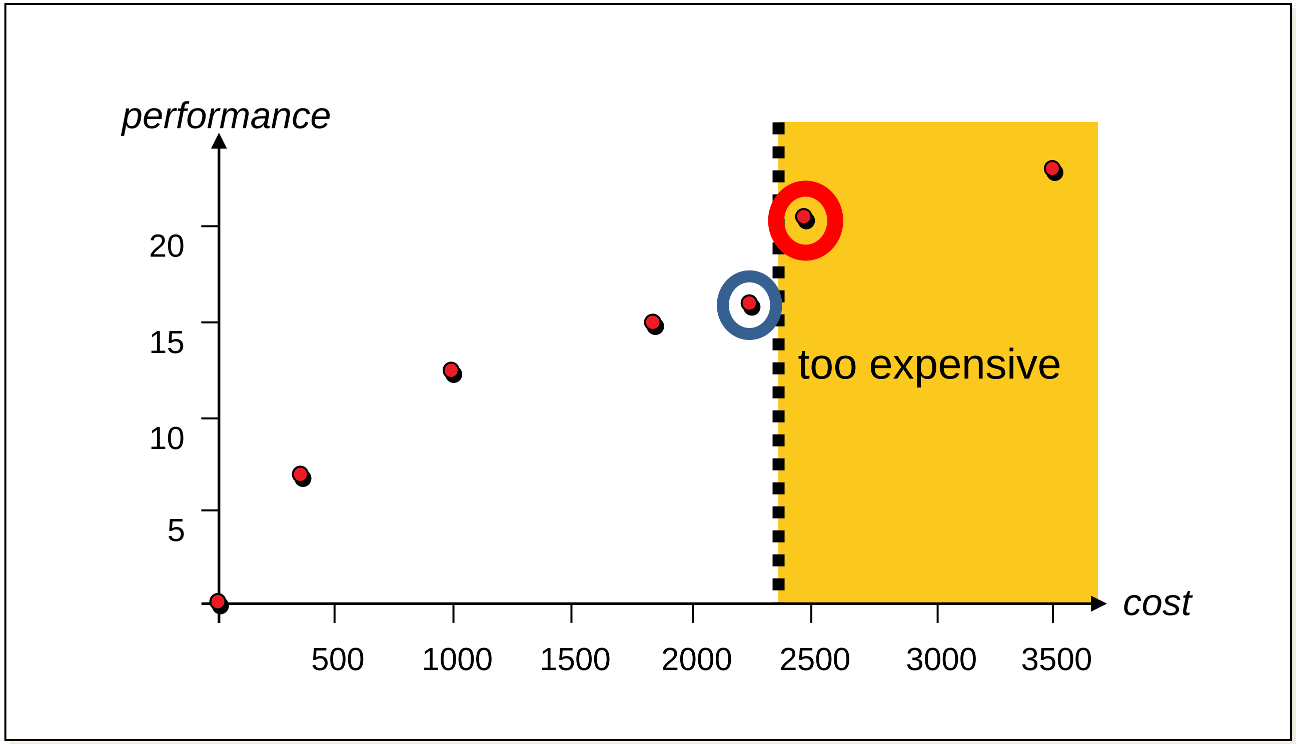

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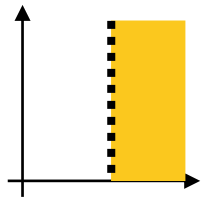

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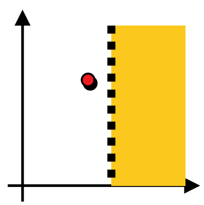
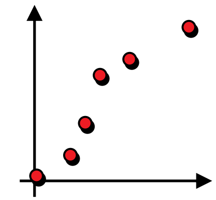
(good) solution

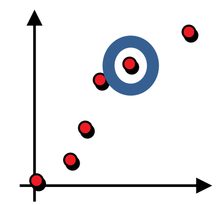


After Optimization:

search for a set of
(good) solutions



select one solution
considering
constraints, etc.



When to Make the Decision

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rank objectives,
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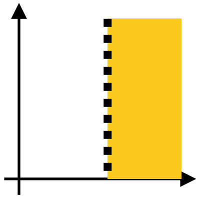

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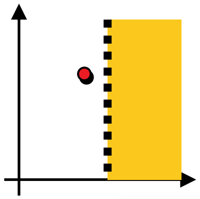

After Optimization:

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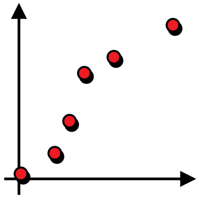

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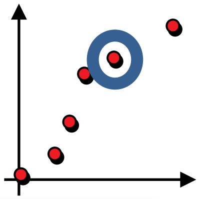


Focus: learning about a problem

- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

Two Communities...

International Society on Multiple Criteria Decision Making

- beginning in 1950s/1960s
- bi-annual conferences since 1975
- background in economics, math, management science
- both optimization and decision making

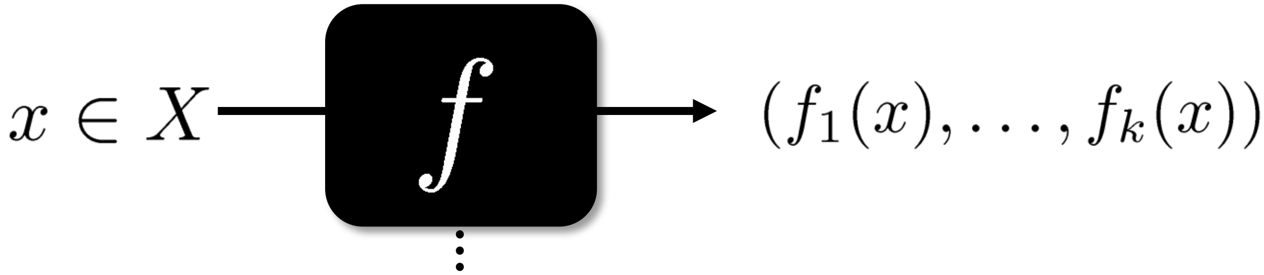
- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background evolutionary computation (applied math, computer science, engineering, ...)
- focus on optimization algorithms

...Slowly Merge Into One

International Society on
Multiple Criteria Decision Making

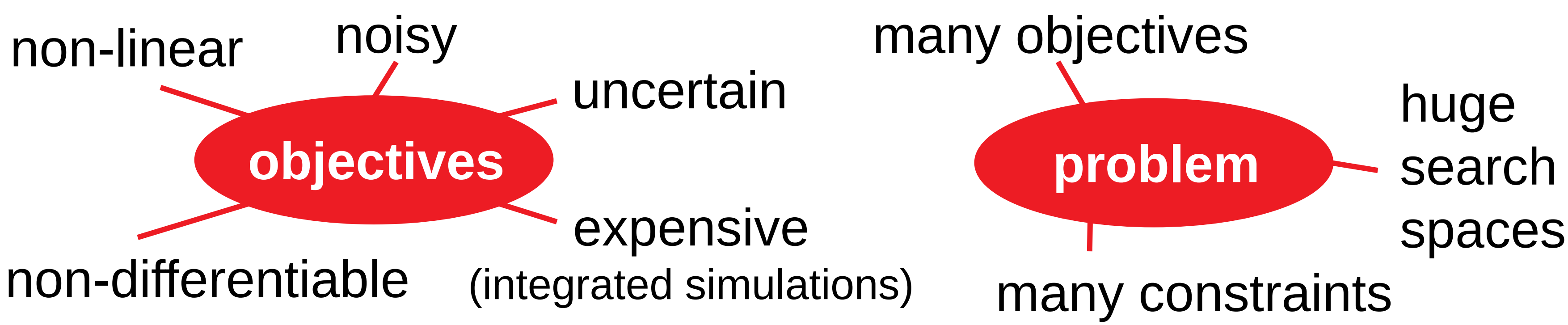
- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

Blackbox optimization



only mild assumptions

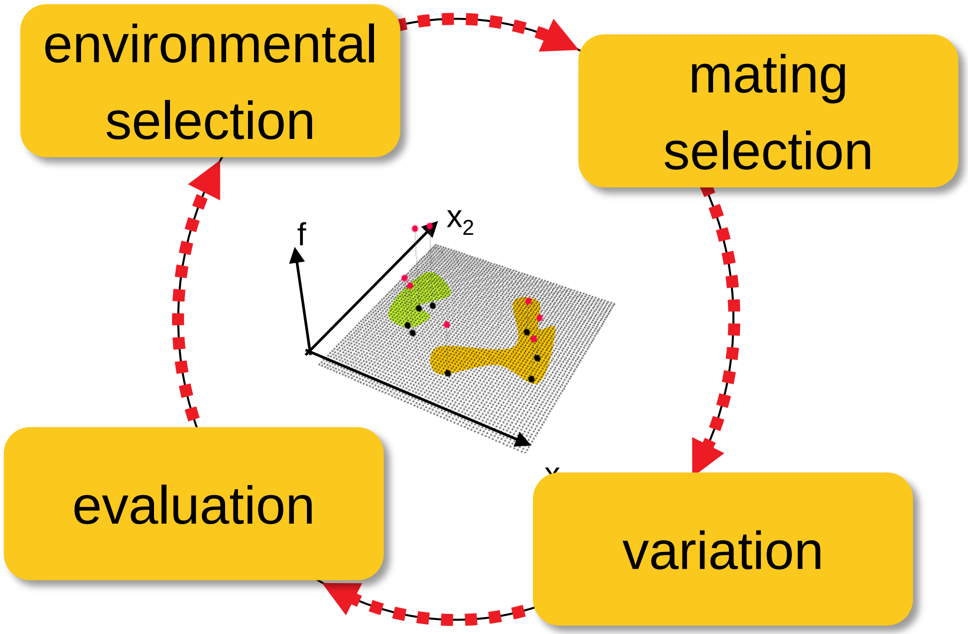
→ EMO therefore well-suited for real-world engineering problems

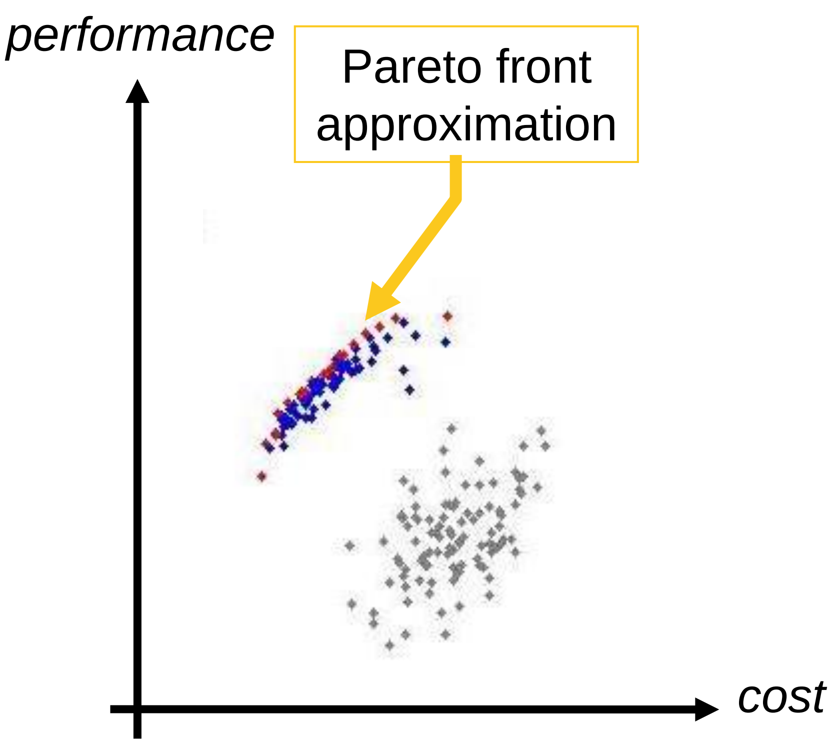


The Other Main Difference

Evolutionary Multiobjective Optimization

- set-based algorithms
- therefore possible to approximate the Pareto front in one run



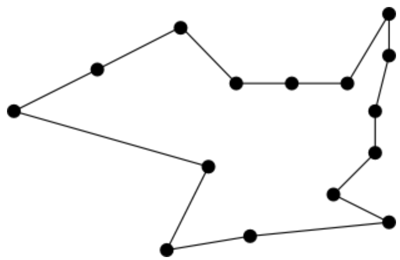


Multiobjectivization

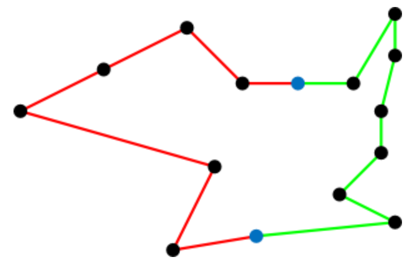
Some problems are easier to solve in a multiobjective scenario

example: TSP

[Knowles et al. 2001]



$\pi \in S_n \rightarrow f(\pi)$



$\pi \in S_n \rightarrow (f_1(\pi, a, b), f_2(\pi, a, b))$

Multiobjectivization

- by addition of new "helper objectives" [Jensen 2004]
  - job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], VRP [Watanabe and Sakakibara 2007], ...
- by decomposition of the single objective
  - TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], ...

also backed up by theory e.g. [Brockhoff et al. 2009, Handl et al. 2008b]

Innovization

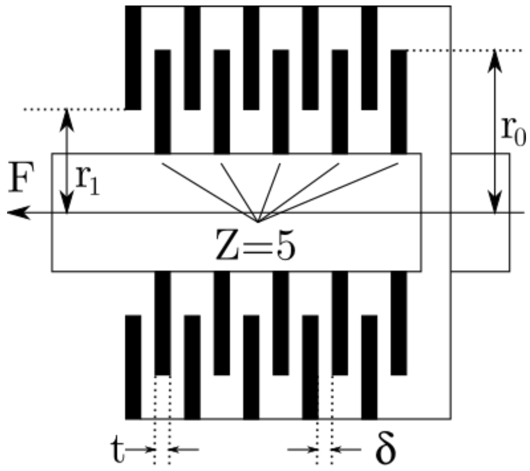
Often innovative design principles among solutions are found

example:

clutch brake design

[Deb and Srinivasan 2006]

min. mass +
stopping time



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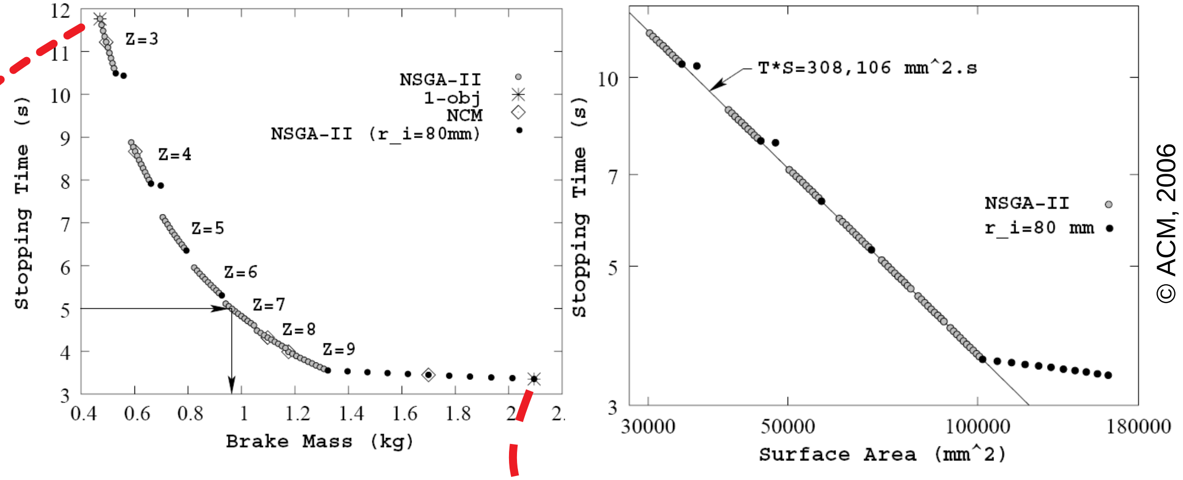

Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Often innovative design principles among solutions are found

example:

clutch brake design

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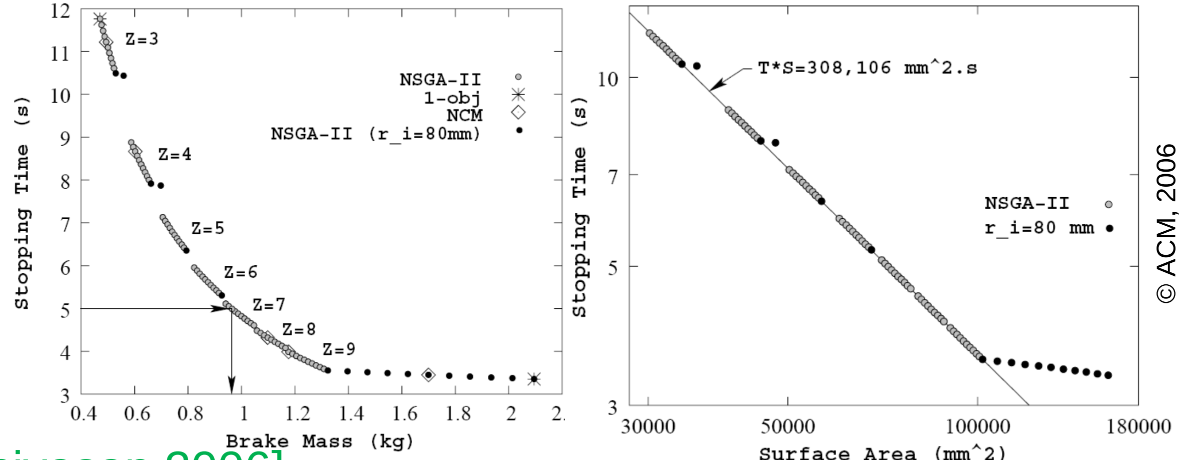

Innovization [Deb and Srinivasan 2006]

= using machine learning techniques to find new and innovative design principles among solution sets

= learning about a multiobjective optimization problem

Other examples:

- SOM for supersonic wing design [Obayashi and Sasaki 2003]
- biclustering for processor design and KP [Ulrich et al. 2007]

The History of EMO At A Glance

1984 — first EMO approaches

dominance-based population ranking

1990 — dominance-based EMO algorithms with diversity preservation techniques

1995

attainment functions

elitist EMO algorithms
preference articulation
convergence proofs

2000 — test problem design
quantitative performance assessment
multiobjectivization

uncertainty and robustness
running time analyses
quality measure design

MCDM + EMO
quality indicator based EMO algorithms

2010 — many-objective optimization
statistical performance assessment

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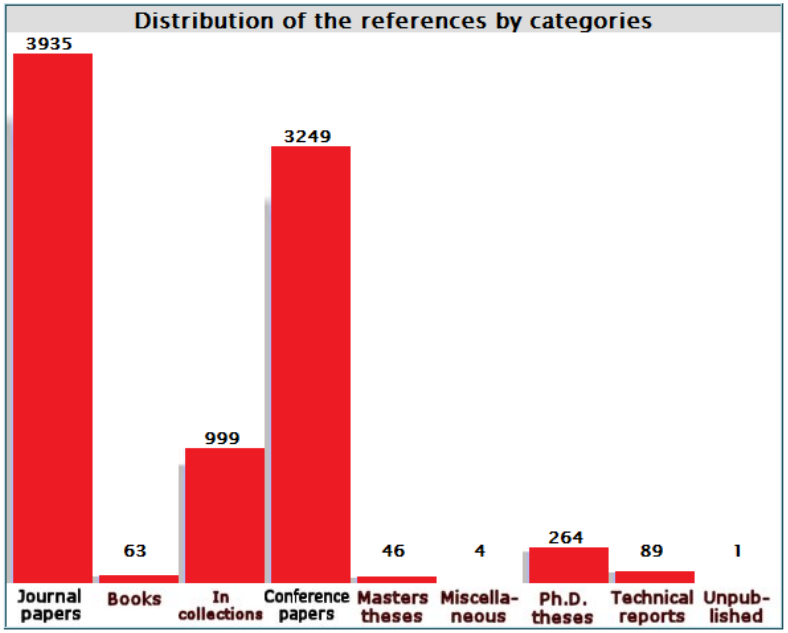
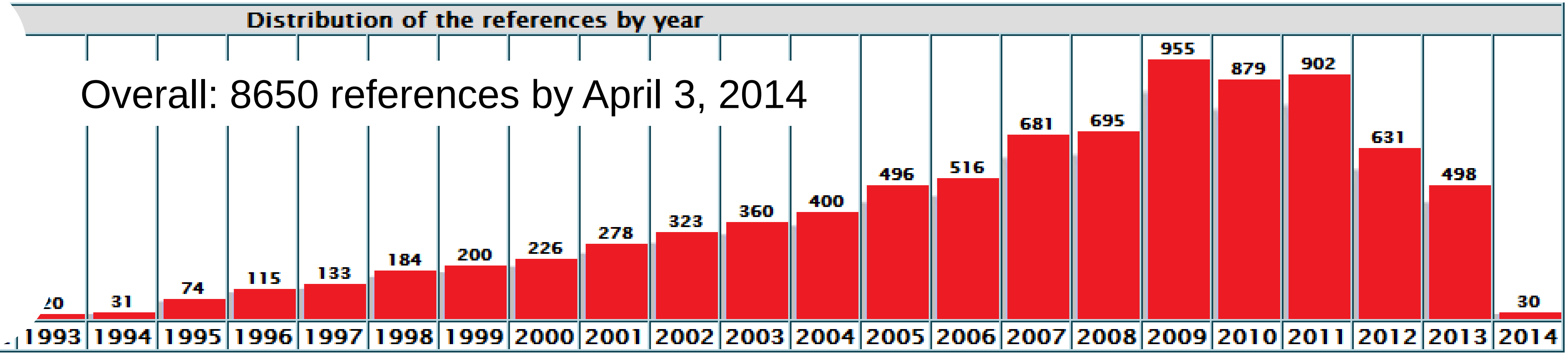
many-objective optimization — statistical performance assessment

2014





The EMO Community

from Google maps

The EMO conference series:

EMO 2013 Sheffield, GB

EMO 2015 Guimarães, PT

EMO 2009 Nantes, FR

EMO 2003 Faro, PT

EMO 2001 Zurich, CH

EMO 2007 Matsushima, JP

EMO 2005 Guanajuato, MX

EMO 2011 Ouro Preto, BR

Many further activities:

special sessions, special journal issues, workshops, tutorials, ...

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

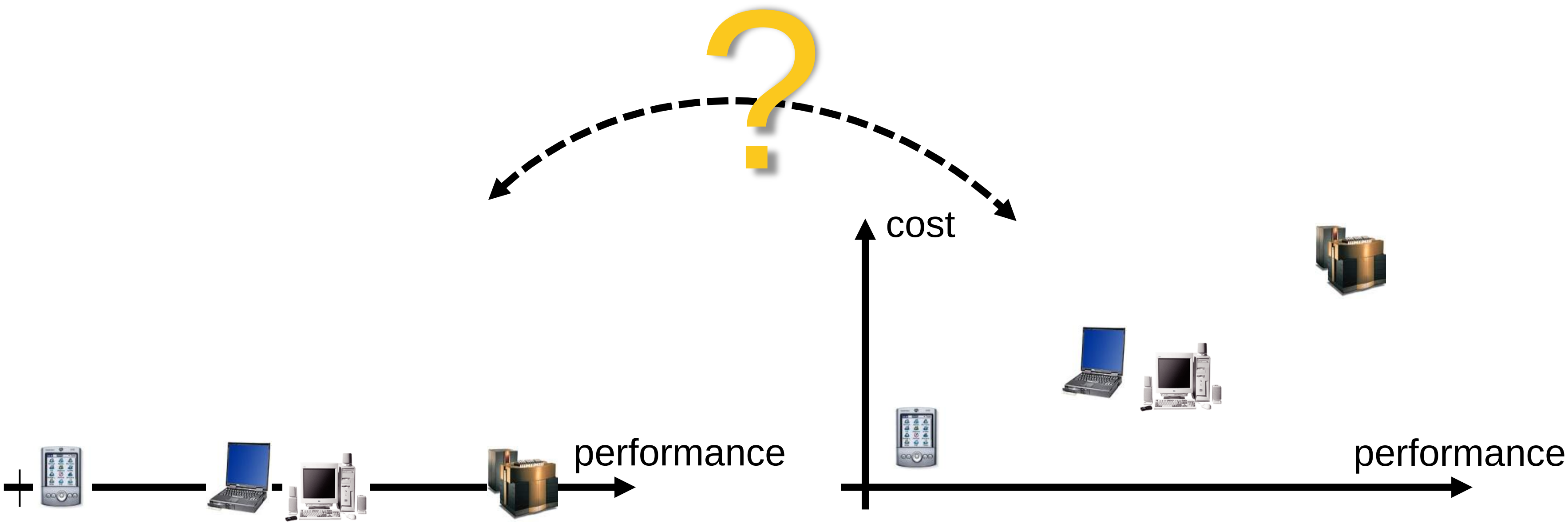
Selected Advanced Concepts

- indicator-based EMO
- preference articulation

A Few Examples From Practice

Starting Point

What makes evolutionary multiobjective optimization different from single-objective optimization?

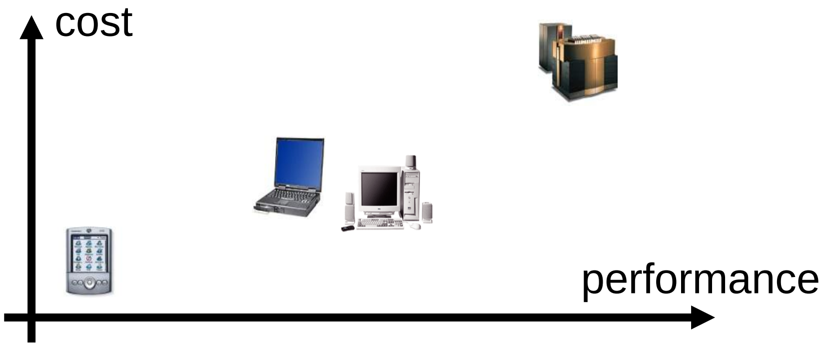


single objective

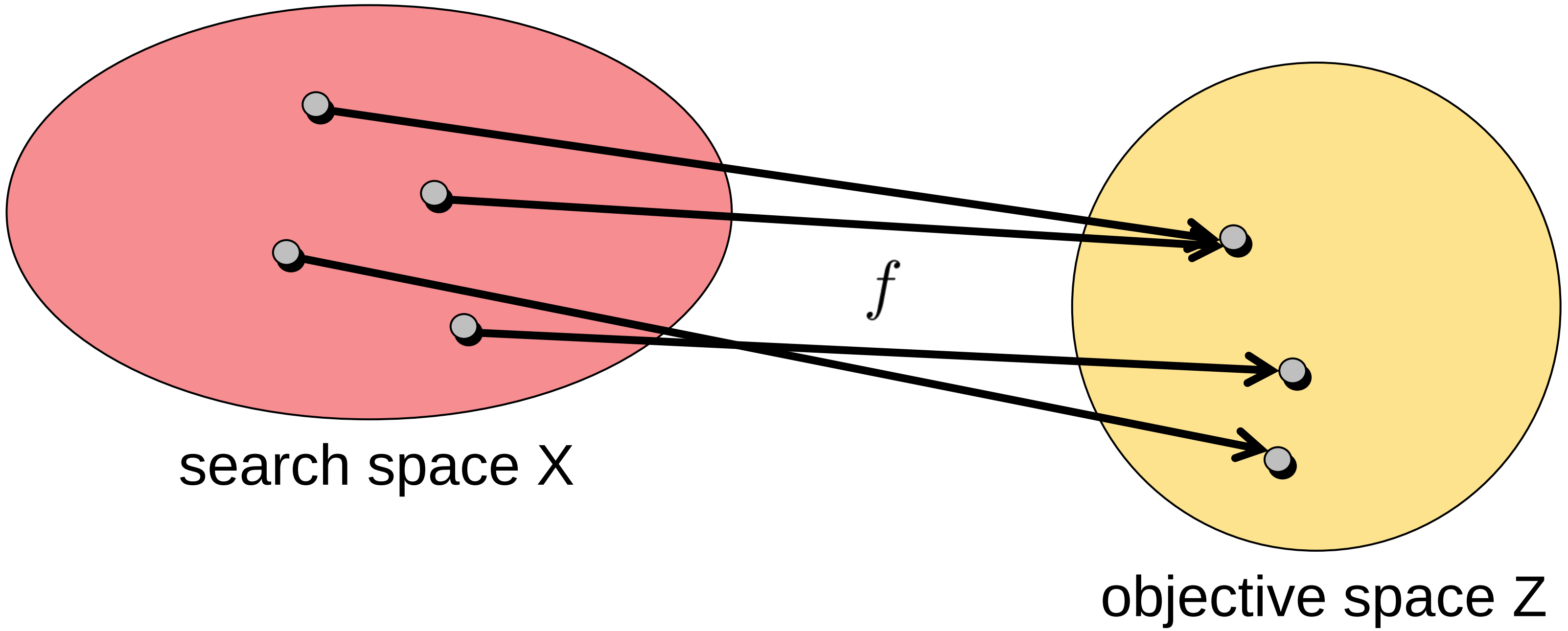
multiple objectives

Starting Point

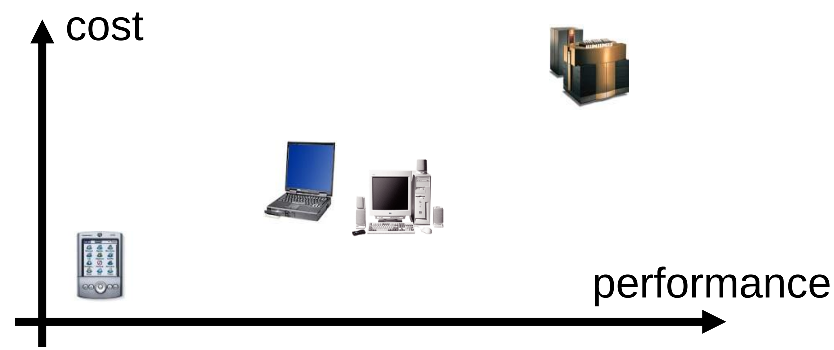
single objective



multiple objectives



The Main Difference



single objective

total order on $f(X) \subseteq \mathbb{R}$

total (pre-)order on X

where a better than b
if $f(a) \leq f(b)$

multiple objectives

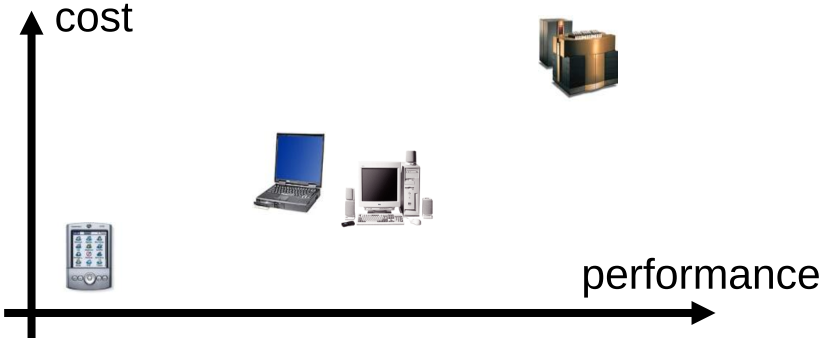
partial order on $f(X) \subseteq \mathbb{R}^k$

preorder on X

where a better than b
if $f(a) \text{ prefrel } f(b)$

- Pareto dominance
- weak Pareto dominance
- ε-dominance
- cone dominance

The Main Difference



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multiple objectives

partial order on $f(X) \subseteq \mathbb{R}^k$

preorder on X

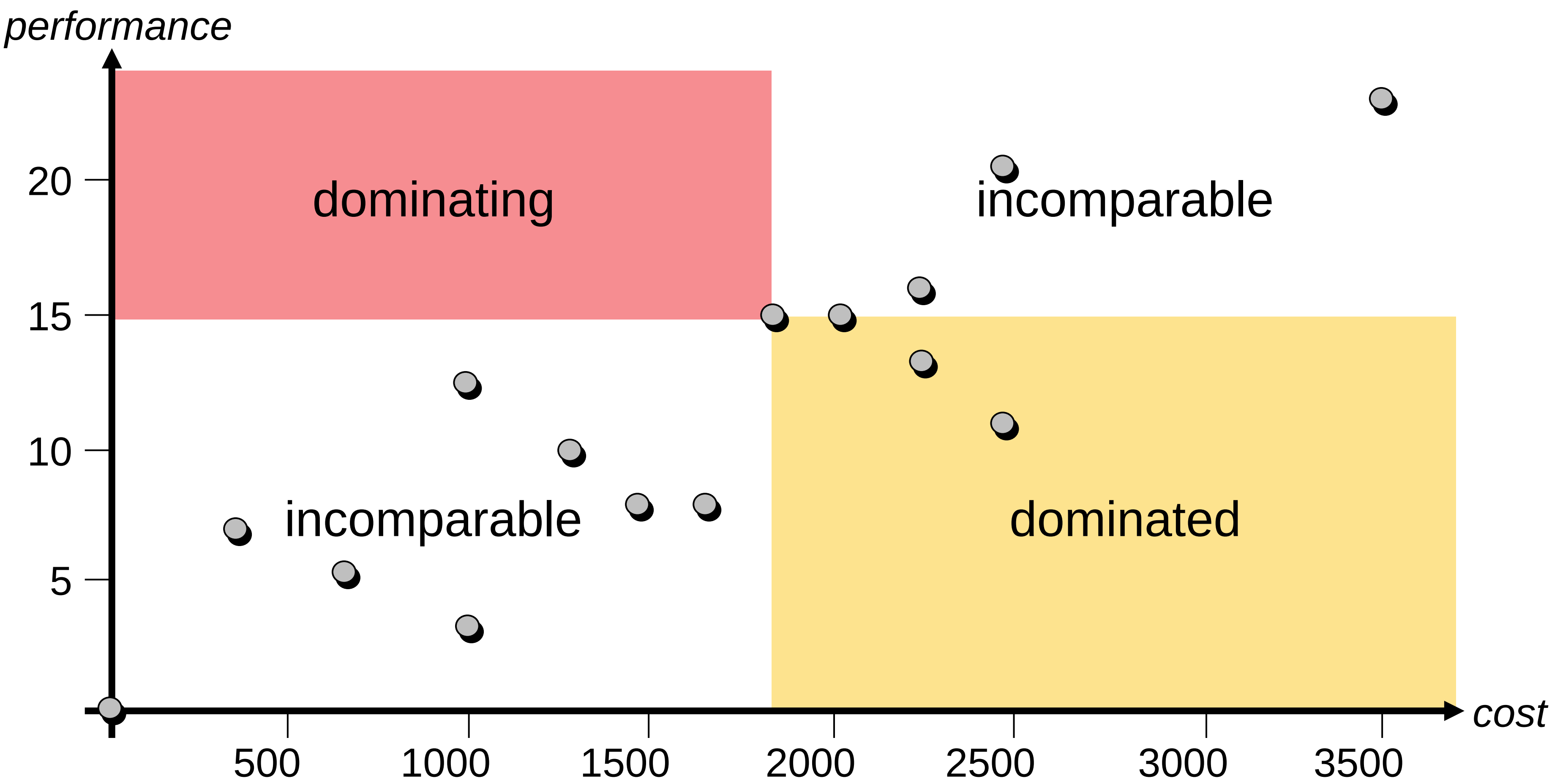
where a better than b
if $f(a)$ prefrel $f(b)$

even more complicated:
sought are **sets**!

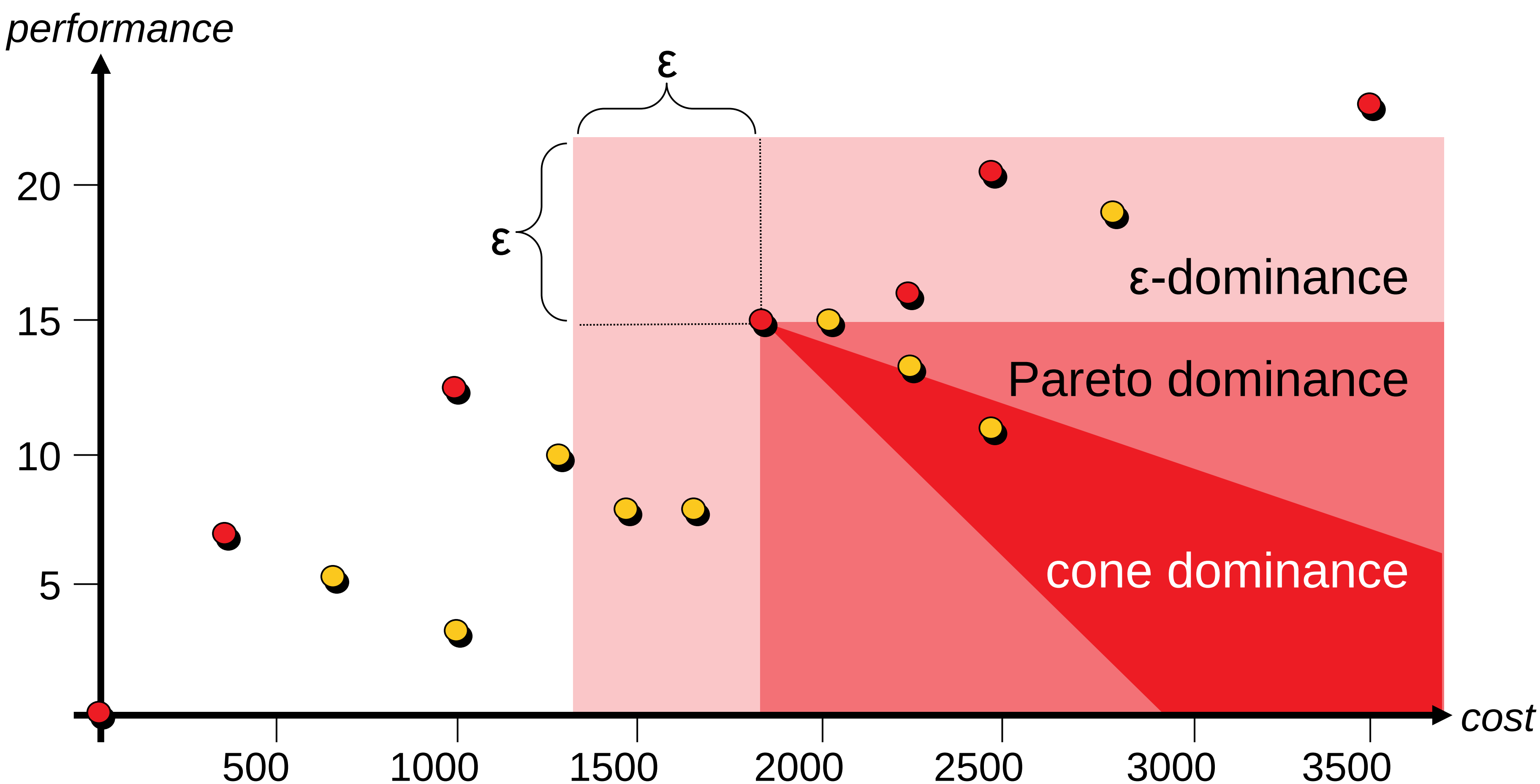
Most Common Example: Pareto Dominance

$$u \textit{ weakly Pareto dominates } v \ (u \preccurlyeq_{par} v): \quad \forall 1 \le i \le k : f_i(u) \le f_i(v)$$

$$u \textit{ Pareto dominates } v \ (u \prec_{par} v): \quad u \preccurlyeq_{par} v \ \wedge \ v \not\preccurlyeq_{par} u$$

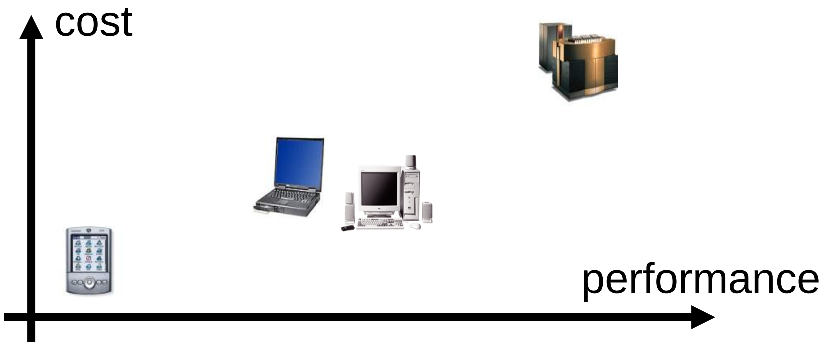


Different Notions of Dominance

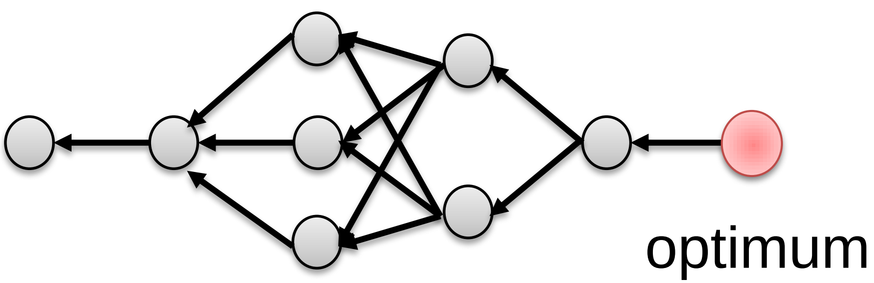


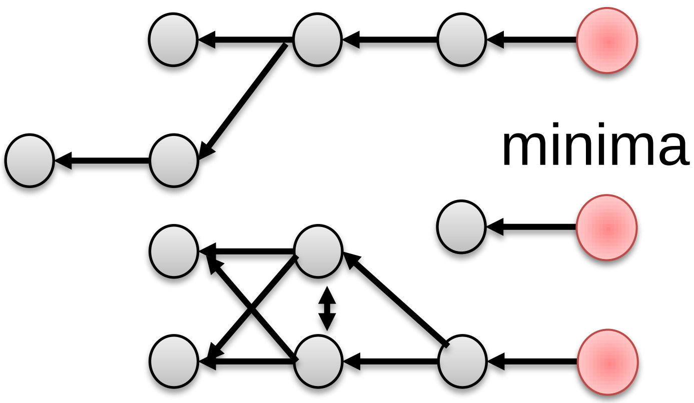
Visualizing Preference Relations

single objective



multiple objectives

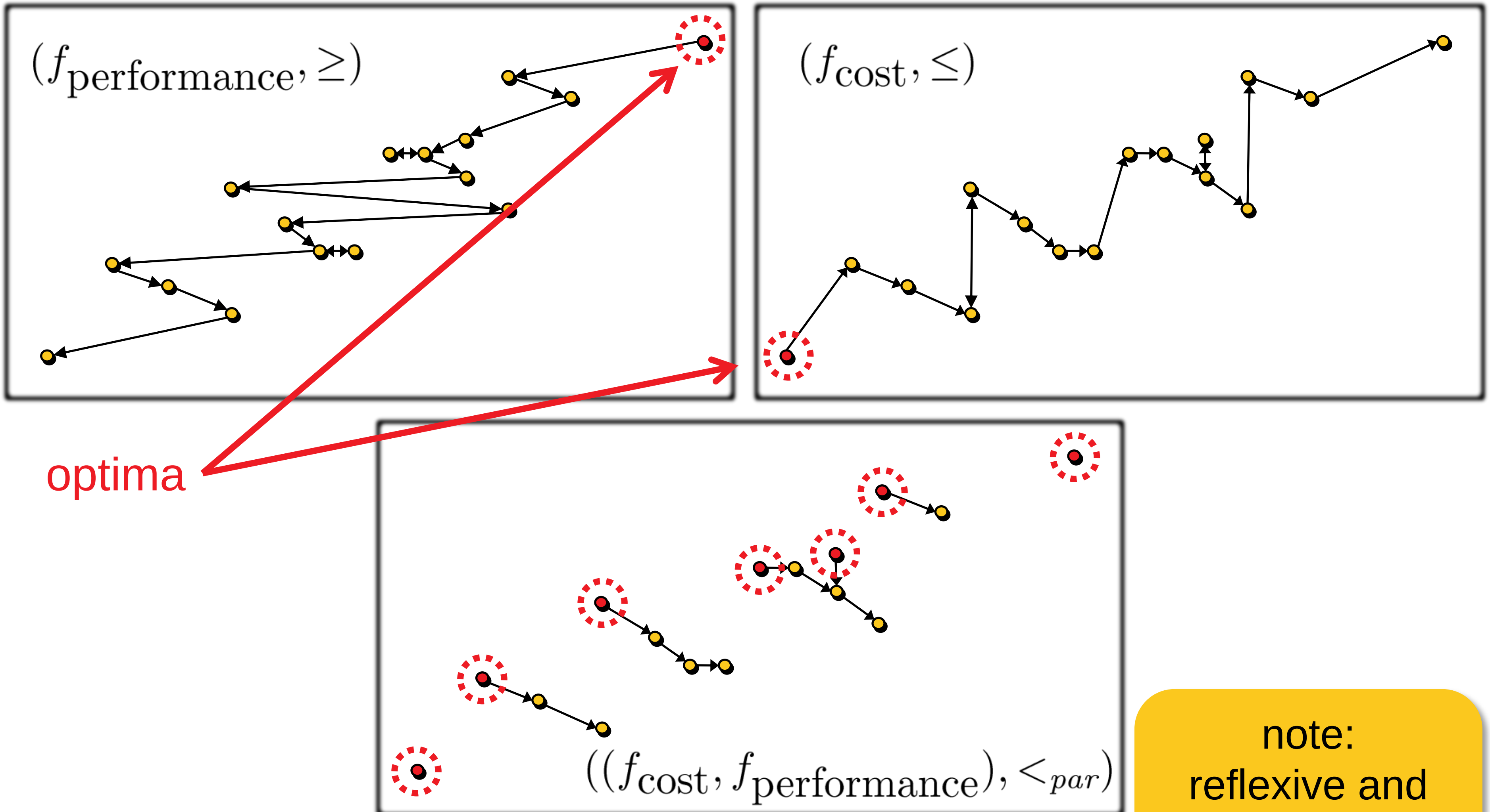




arrow from a to b if $f(a) < f(b)$

arrow from a to b if a weakly dominates b

Visualizing Preference Relations



note:
reflexive and
transitive edges
not shown

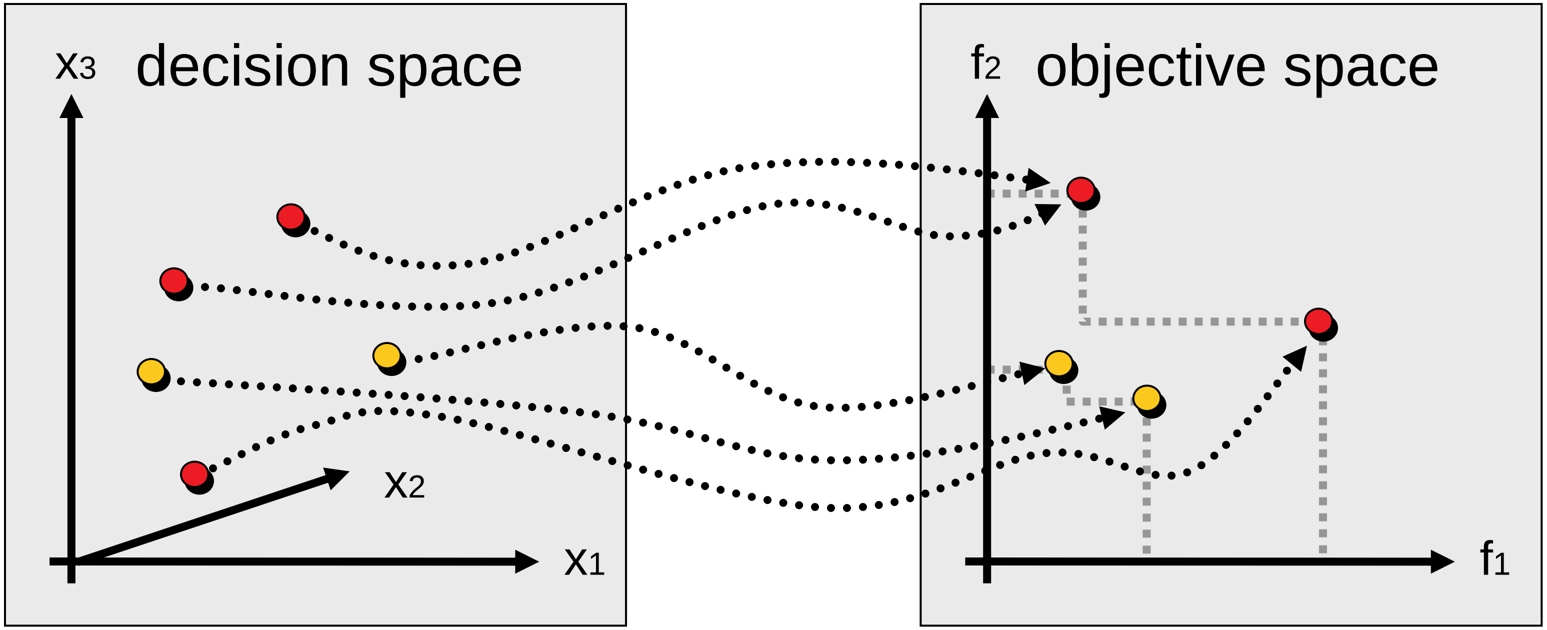
Pareto-optimal Set and Pareto(-optimal) Front

The *minimal set* of a preordered set $(Y, \leqq)$ is defined as

$$Min(Y, \leqq) := \{a \in Y \,|\, \forall b \in Y : b \leqq a \Rightarrow a \leqq b\}$$

Pareto-optimal set $Min(X, \preccurlyeq_{par})$ — Pareto-optimal front

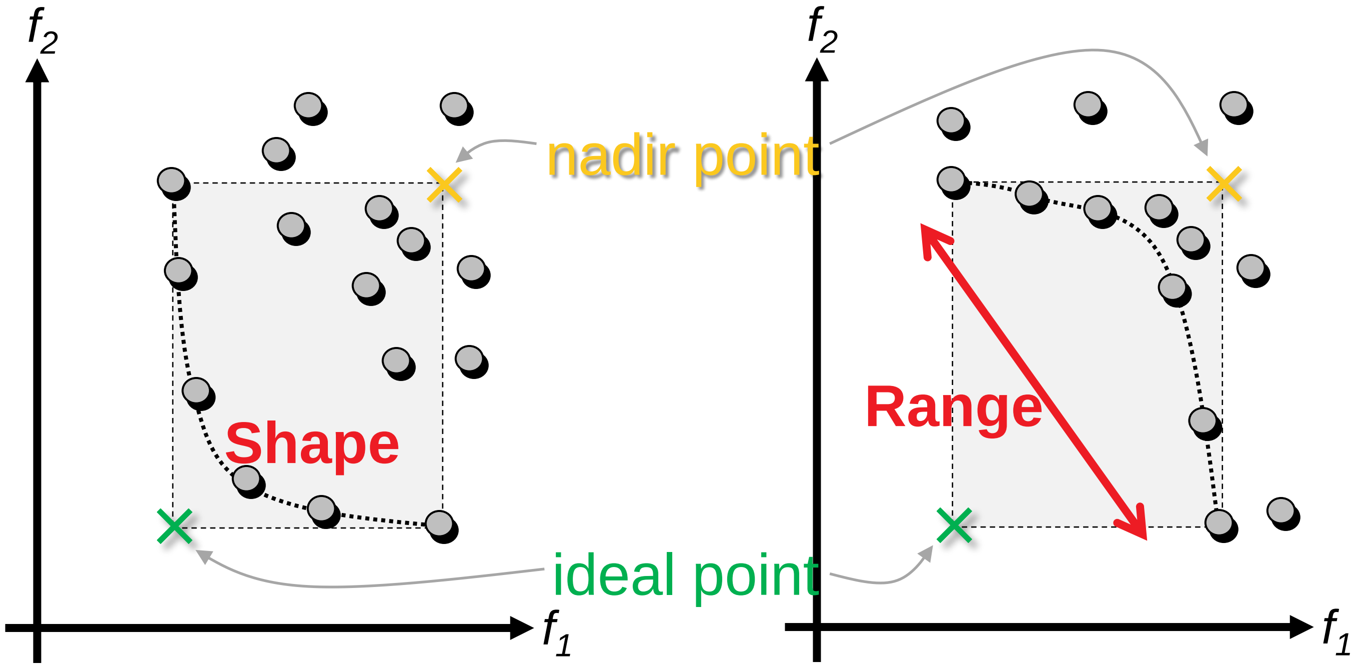
non-optimal decision vector — non-optimal objective vector



Other Related Definitions

Computational complexity for discrete problems:
multiobjective variants can become NP- and #P-complete

Size: Pareto set can be exponential in the input length
(e.g. shortest path [Serafini 1986], MSP [Camerini et al. 1984])



Approaches To Multiobjective Optimization

A multiobjective problem is as such underspecified
…because not any Pareto-optimum is equally suited!

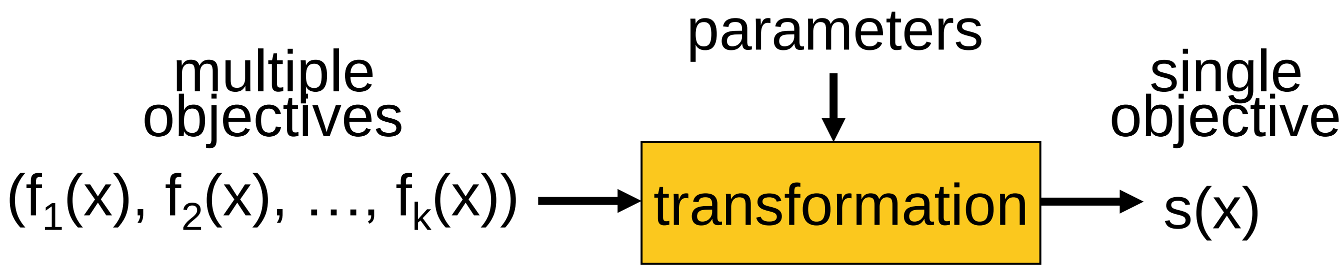
Additional preferences are needed to tackle the problem:

Solution-Oriented Problem Transformation:
Classical approach: Induce a total order on the decision space, e.g., by aggregation

Set-Oriented Problem Transformation:
Recent view on EMO: First transform problem into a set problem and then define an objective function on sets [Zitzler et al. 2010]

Preferences are needed in both cases, but the latter are weaker!

Solution-Oriented Problem Transformations

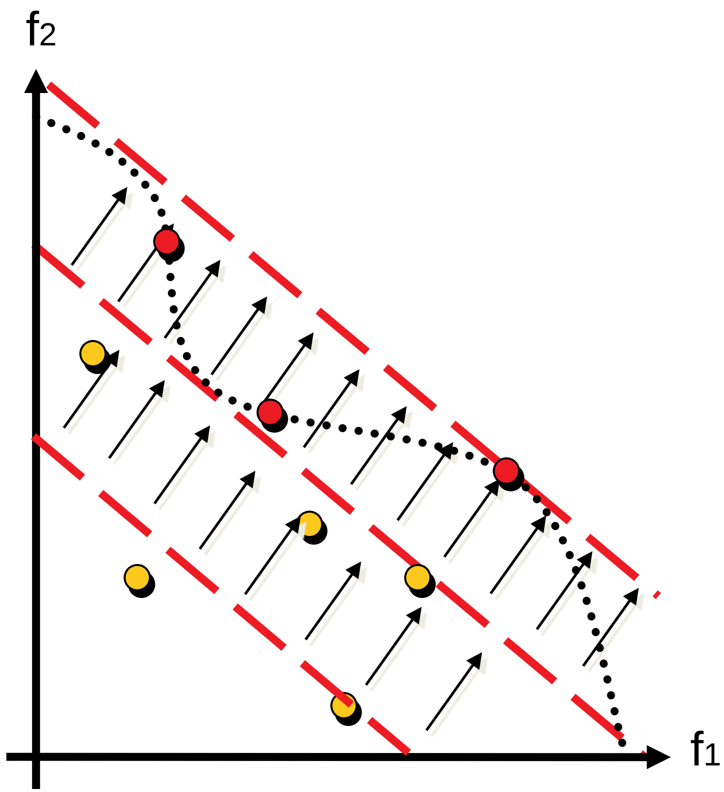


A scalarizing function s is a function $s: Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

Solution-Oriented Problem Transformations

parameters

multiple objectives $(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** → single objective $s(x)$



Example 1: weighted sum approach

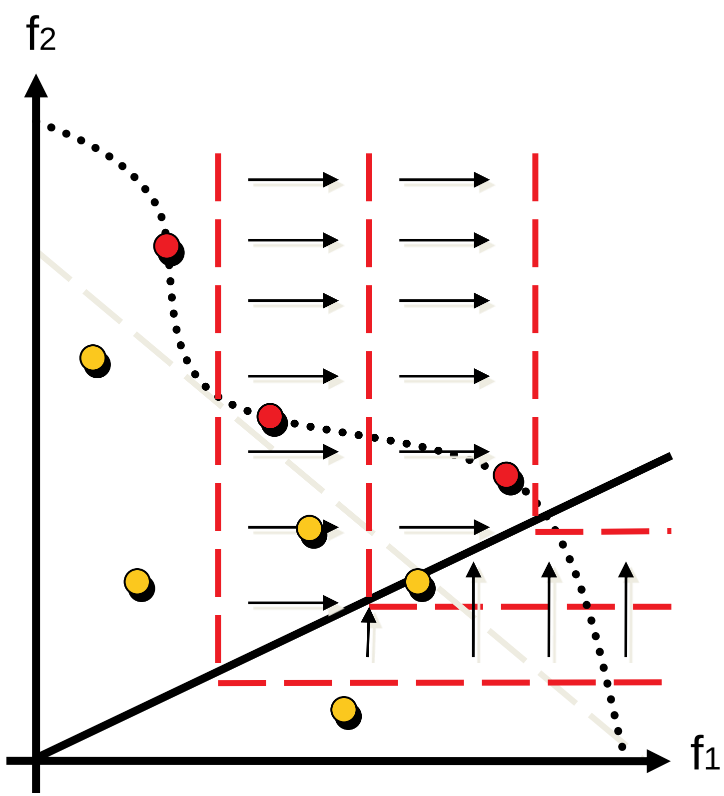
$(w_1, w_2, \ldots, w_k)$

→ $y = w_1 y_1 + \ldots + w_k y_k$ →

Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex

Solution-Oriented Problem Transformations

multiple objectives — $(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** (parameters) → single objective — $s(x)$



Example 2: weighted Tchebycheff

$(\lambda_1, \lambda_2, \ldots, \lambda_k)$

$$y = \max_i |\lambda_i(u_i - z_i)|$$

Several other scalarizing functions are known, see e.g. [Miettienen 1999]

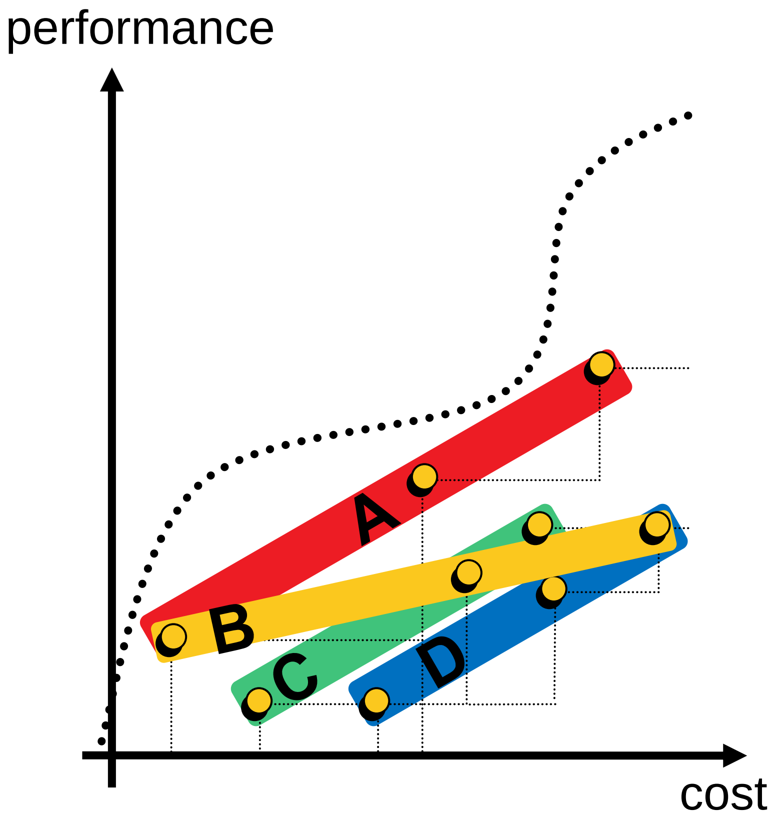
Set-Oriented Problem Transformations

For a multiobjective optimization problem $(X, Z, \mathbf{f}, \mathbf{g}, \leqslant)$, the associated *set problem* is given by $(\Psi, \Omega, F, \mathbf{G}, \preccurlyeq)$ where

- $\Psi = 2^X$ is the space of decision vector sets, i.e., the powerset of X,
- $\Omega = 2^Z$ is the space of objective vector sets, i.e., the powerset of Z,
- F is the extension of $\mathbf{f}$ to sets, i.e., $F(A) := \{\mathbf{f}(\mathbf{a}) \,:\, \mathbf{a} \in A\}$ for $A \in \Psi$,
- $\mathbf{G} = (G_1, \ldots, G_m)$ is the extension of $\mathbf{g}$ to sets, i.e., $G_i(A) := \max\{g_i(\mathbf{a}) \,:\, \mathbf{a} \in A\}$ for $1 \leq i \leq m$ and $A \in \Psi$,
- $\preccurlyeq$ extends $\leqslant$ to sets where $A \preccurlyeq B :\Leftrightarrow \forall \mathbf{b} \in B\ \exists \mathbf{a} \in A : \mathbf{a} \leqslant \mathbf{b}$.

Pareto Set Approximations

Pareto set approximation (algorithm outcome) =
set of (usually incomparable) solutions



A weakly dominates B
= not worse in all objectives
and sets not equal

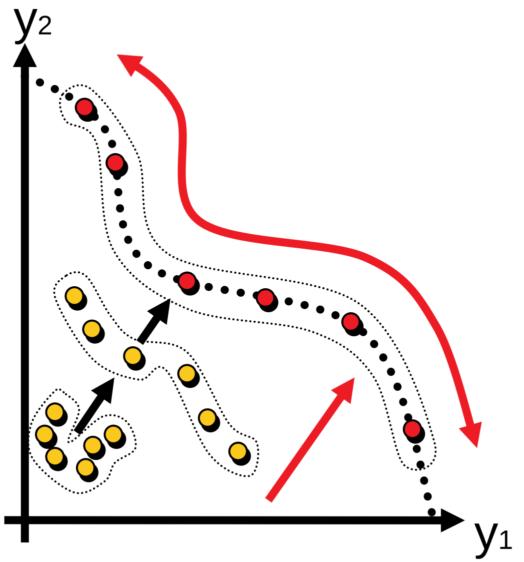
C dominates D
= better in at least one objective

A strictly dominates C
= better in all objectives

B is **incomparable** to C
= neither set weakly better

What Is the Optimization Goal of a Set Problem?

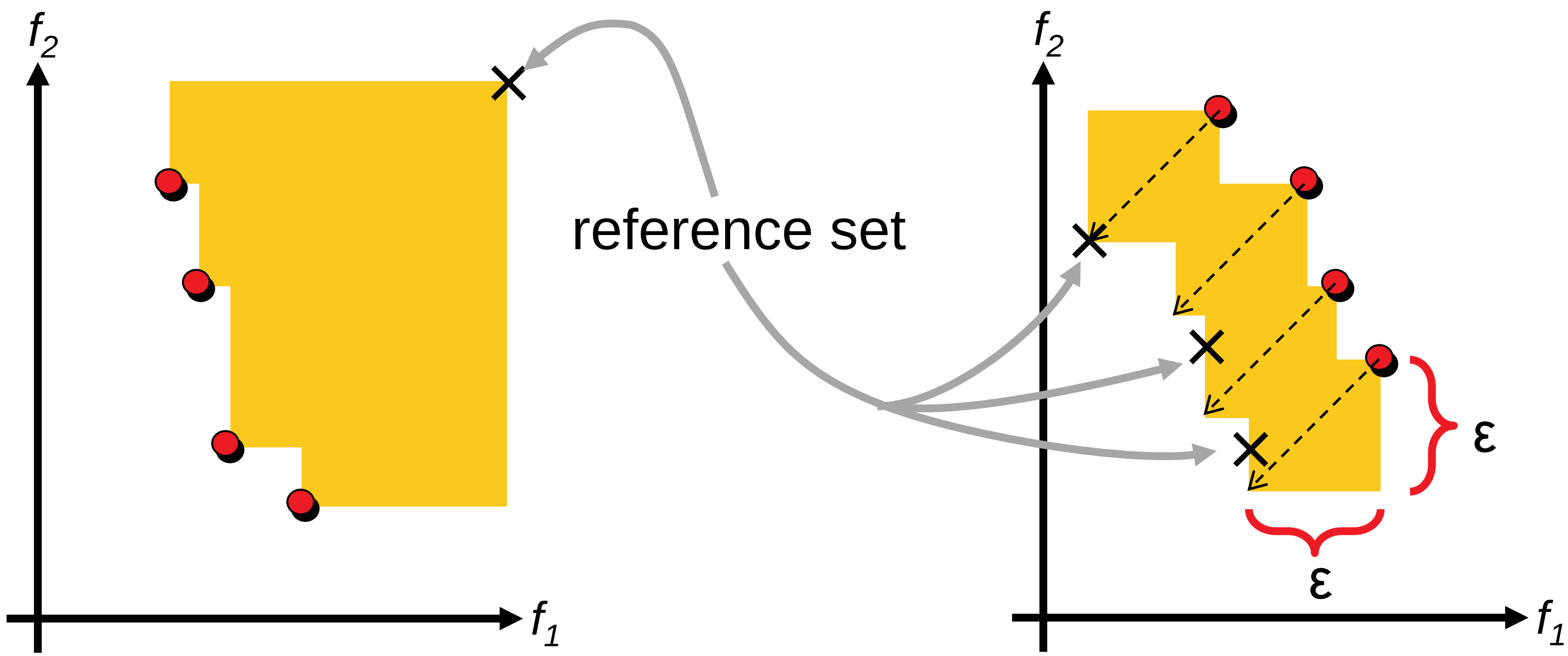
- Find all Pareto-optimal solutions?
  - Impossible in continuous search spaces
  - How should the decision maker handle 10000 solutions?
- Find a representative subset of the Pareto set?
  - Many problems are NP-hard
  - What does representative actually mean?
- Find a good approximation of the Pareto set?
  - What is a good approximation?
  - How to formalize intuitive understanding:
    - ❶ close to the Pareto front
    - ❷ well distributed



Most common: use of **quality indicators**

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

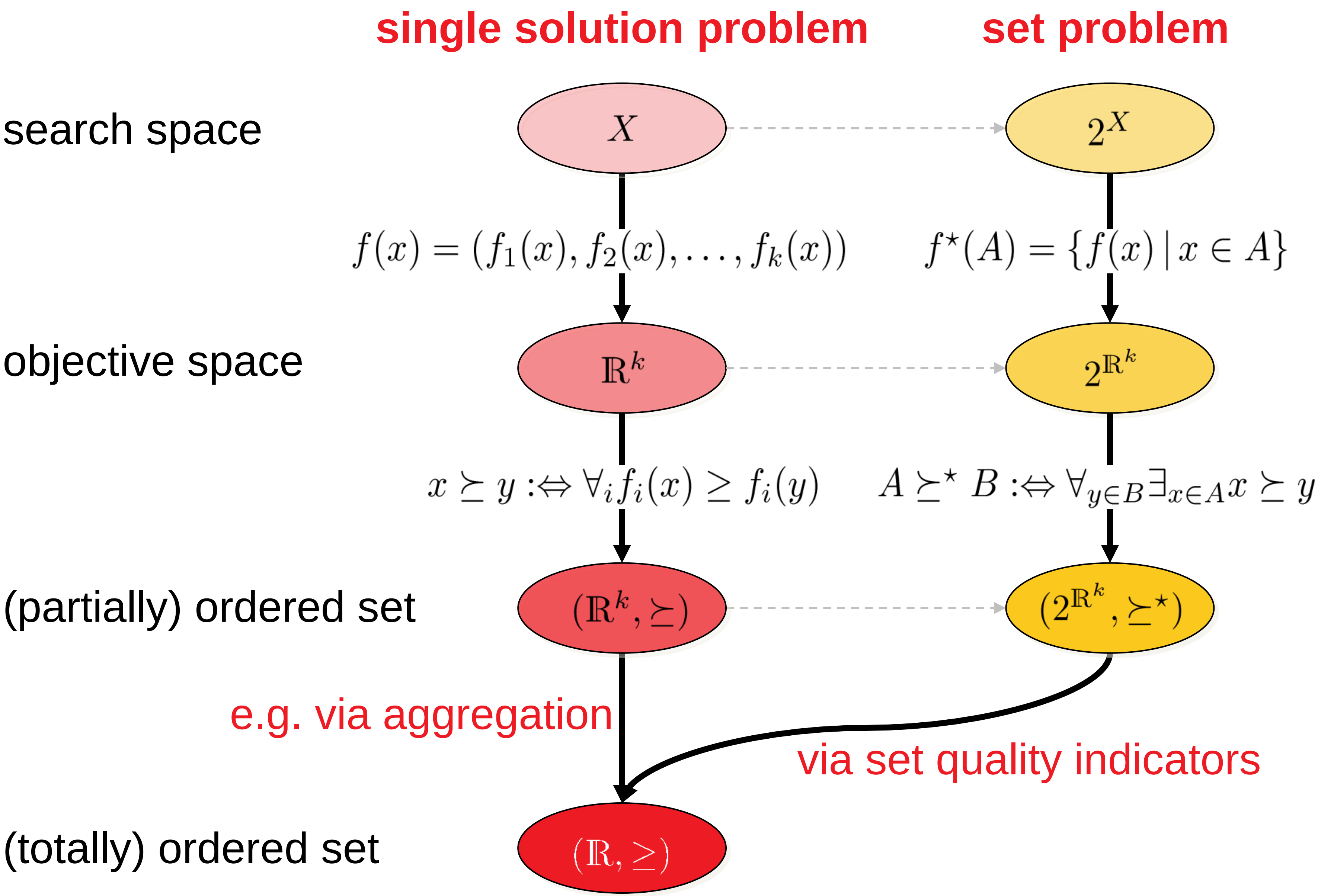
well-known examples:



hypervolume indicator

epsilon indicator

Problem Transformations and Set Problems



General Remarks on Problem Transformations

Main Goal:

Transform a preorder into a total preorder on X

Methods:

- Define single-objective function based on the multiple criteria *(e.g. via aggregation)*
- Define total preorder on sets by using a quality indicator *(e.g. via hypervolume indicator)*

Question:

Is any total preorder okay or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \prec B \wedge B \not\prec A \Rightarrow A \overset{\text{ref}}{\prec} B \wedge B \overset{\text{ref}}{\not\prec} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \prec B \wedge B \not\prec A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

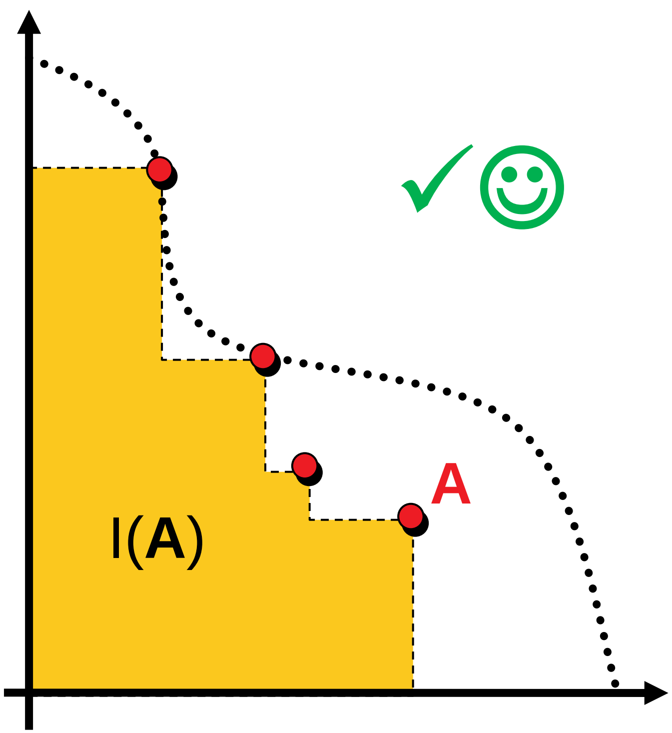
Example: Refinements Using Indicators

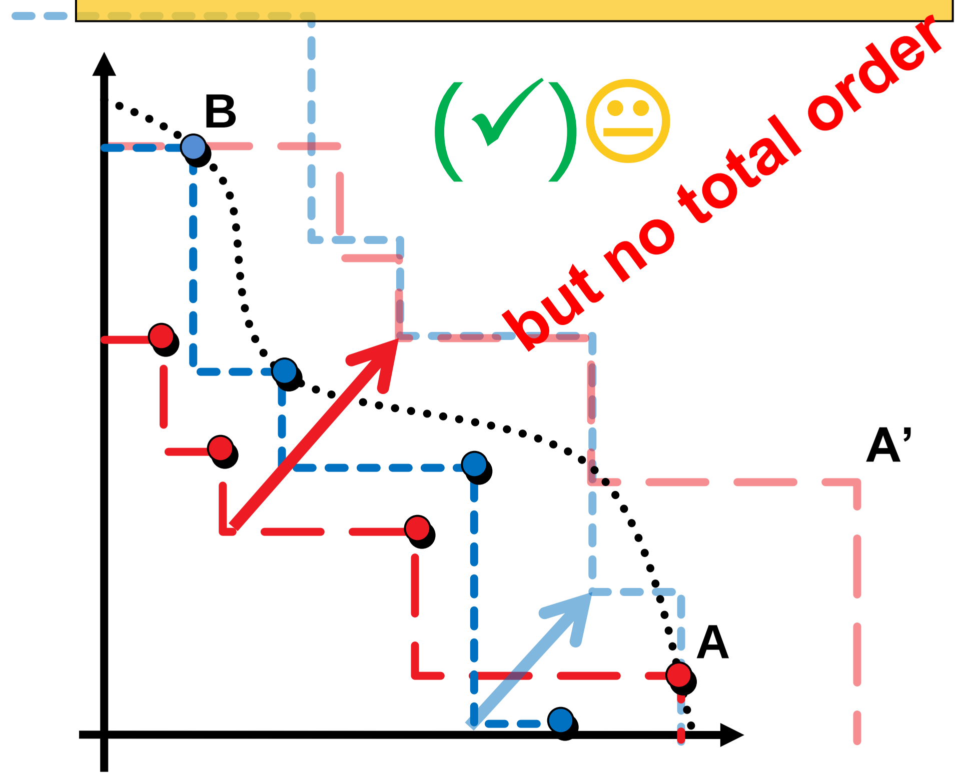
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B





unary hypervolume indicator

binary epsilon indicator

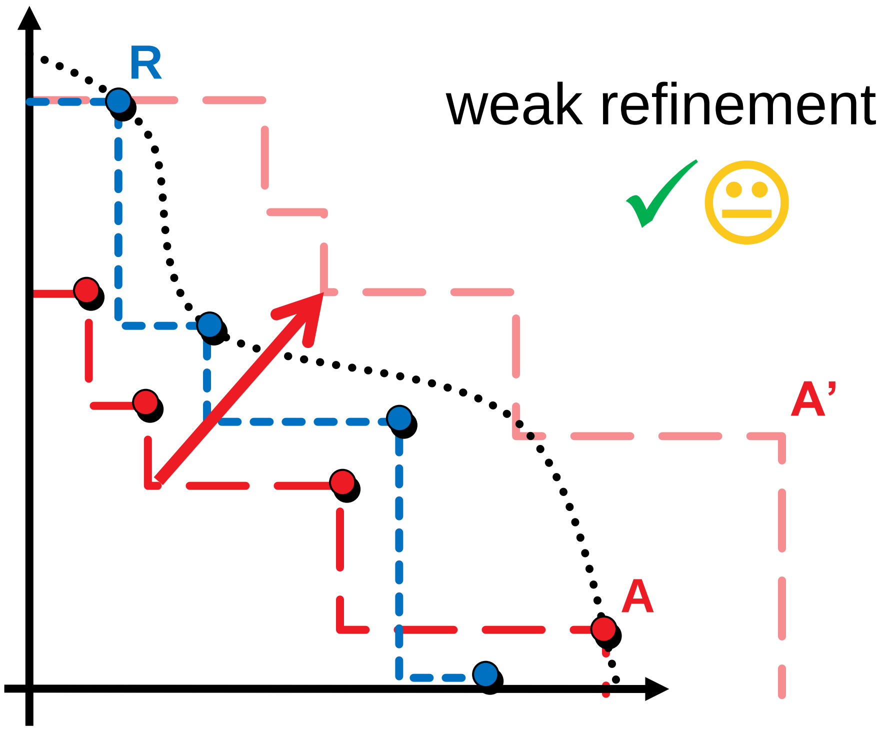
Example: Weak Refinement / No Refinement

$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A,R) \leq I(B,R)$$

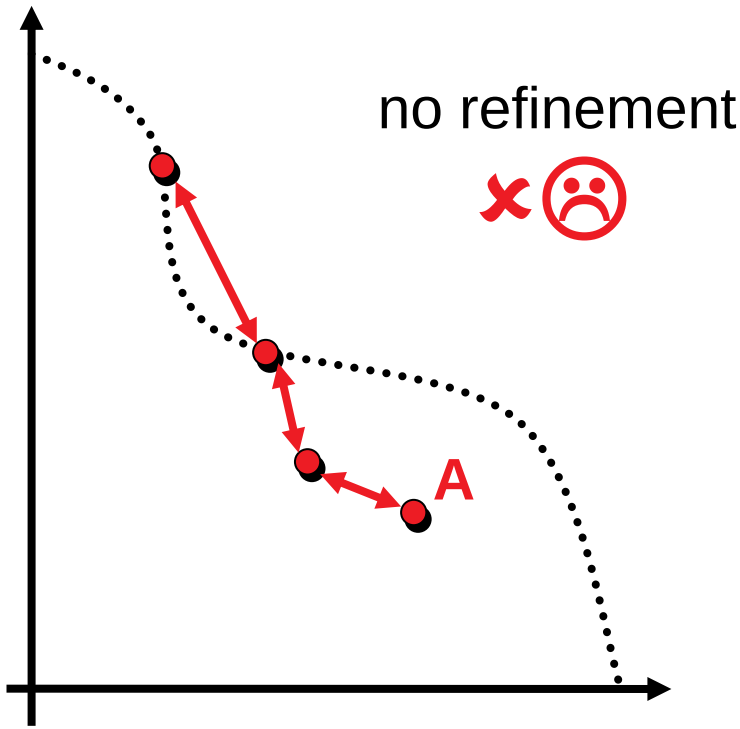
$$A \preccurlyeq^{\text{ref}} B :\Leftrightarrow I(A) \leq I(B)$$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



unary epsilon indicator



unary diversity indicator

Overview

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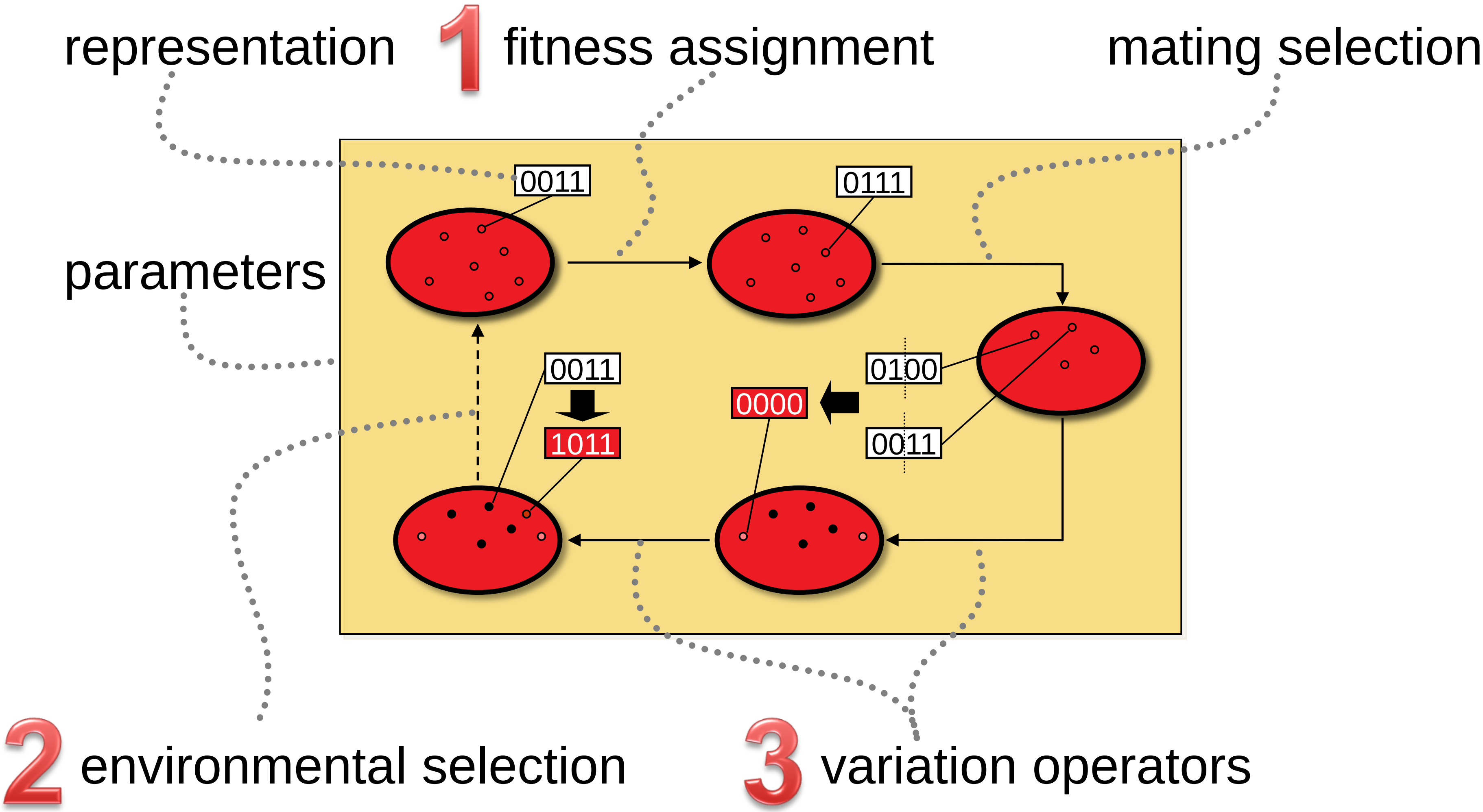
- indicator-based EMO
- preference articulation

A Few Examples From Practice

Algorithm Design: Particular Aspects

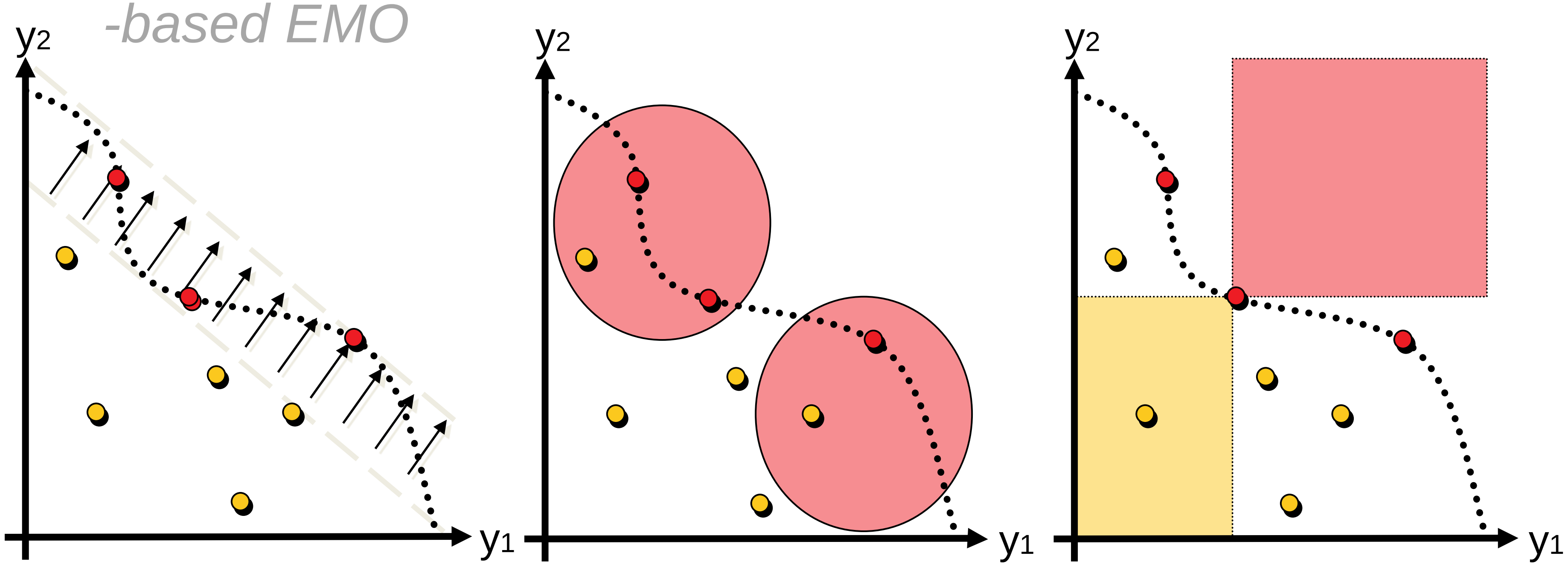


Fitness Assignment: Principal Approaches

aggregation-based
weighted sum
but also decomposition-based EMO

criterion-based
VEGA

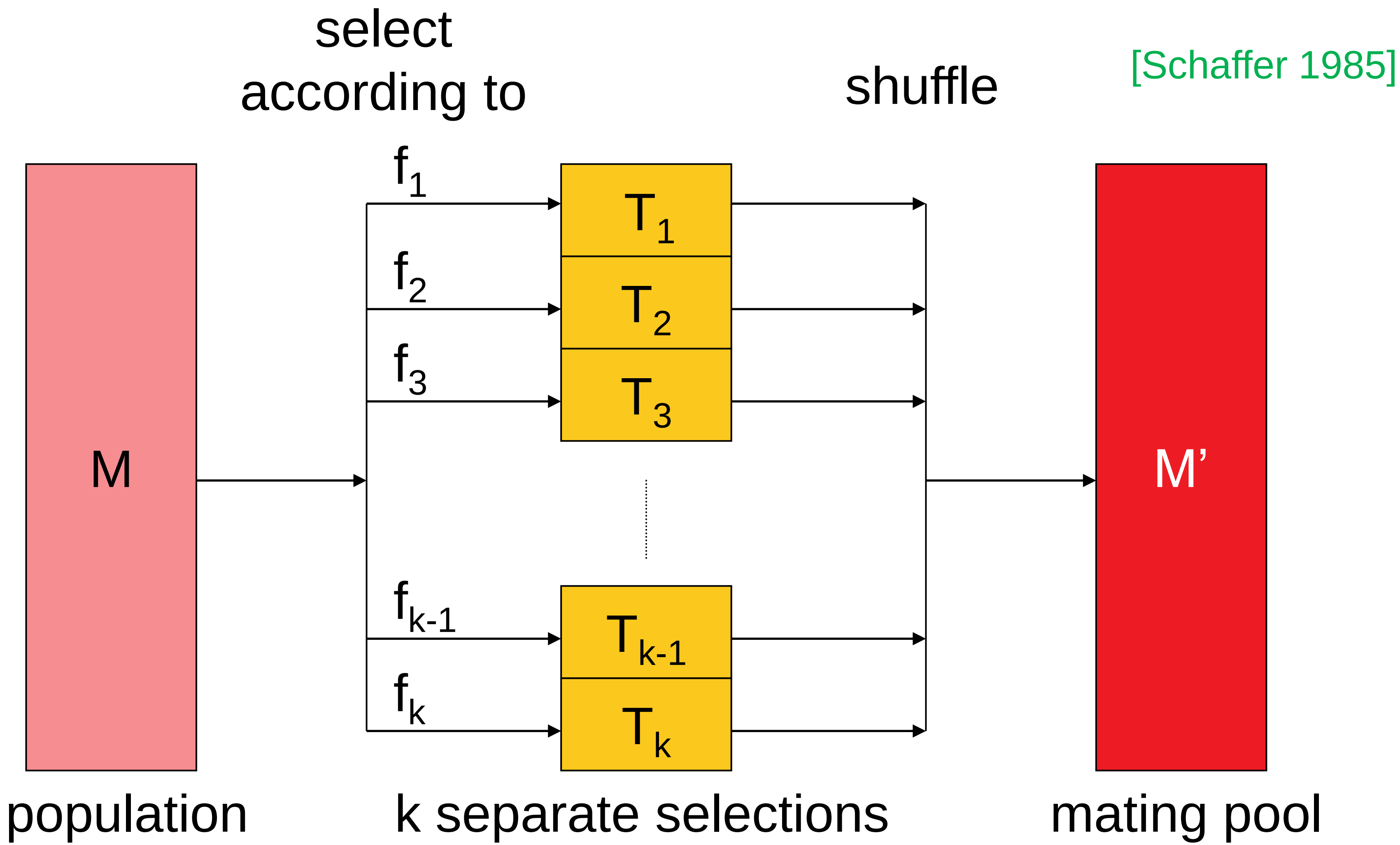
dominance-based
SPEA2, NSGA-II
“modern” EMOA



parameter-oriented
scaling-dependent

set-oriented
scaling-independent

Criterion-Based Selection: VEGA

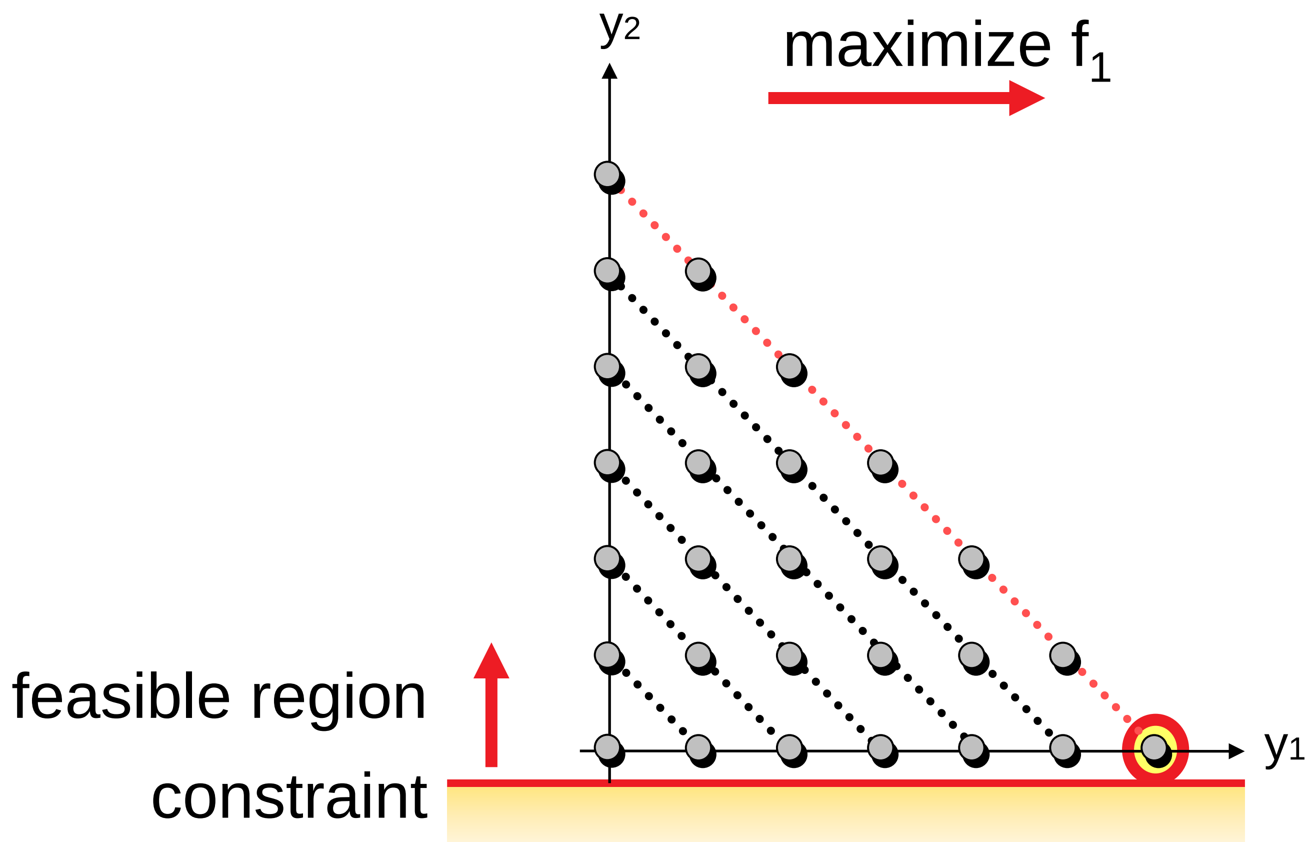


Drawback: only allows to find extremes of the Pareto front

Aggregation-Based: Multistart Constraint Method

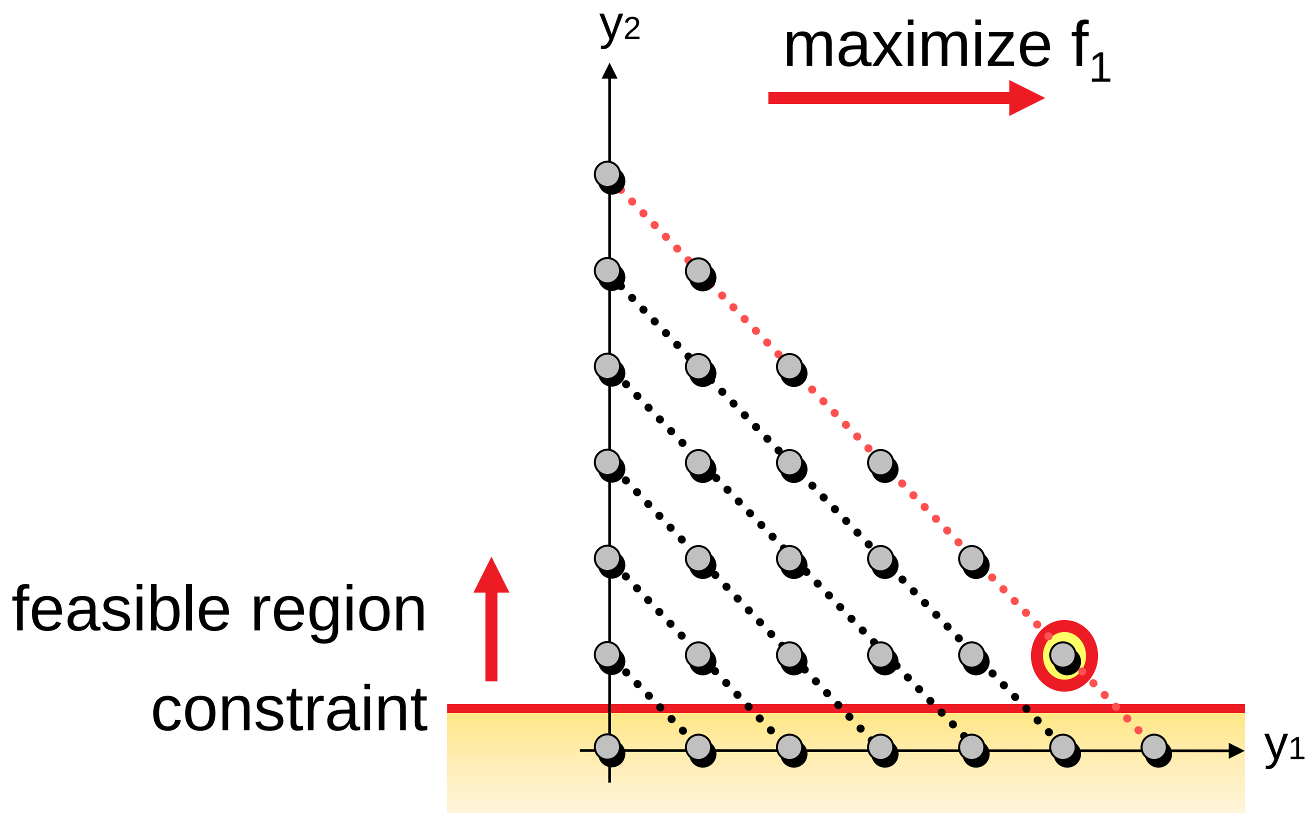
Underlying concept:

- Convert all objectives except of one into constraints
- Adaptively vary constraints



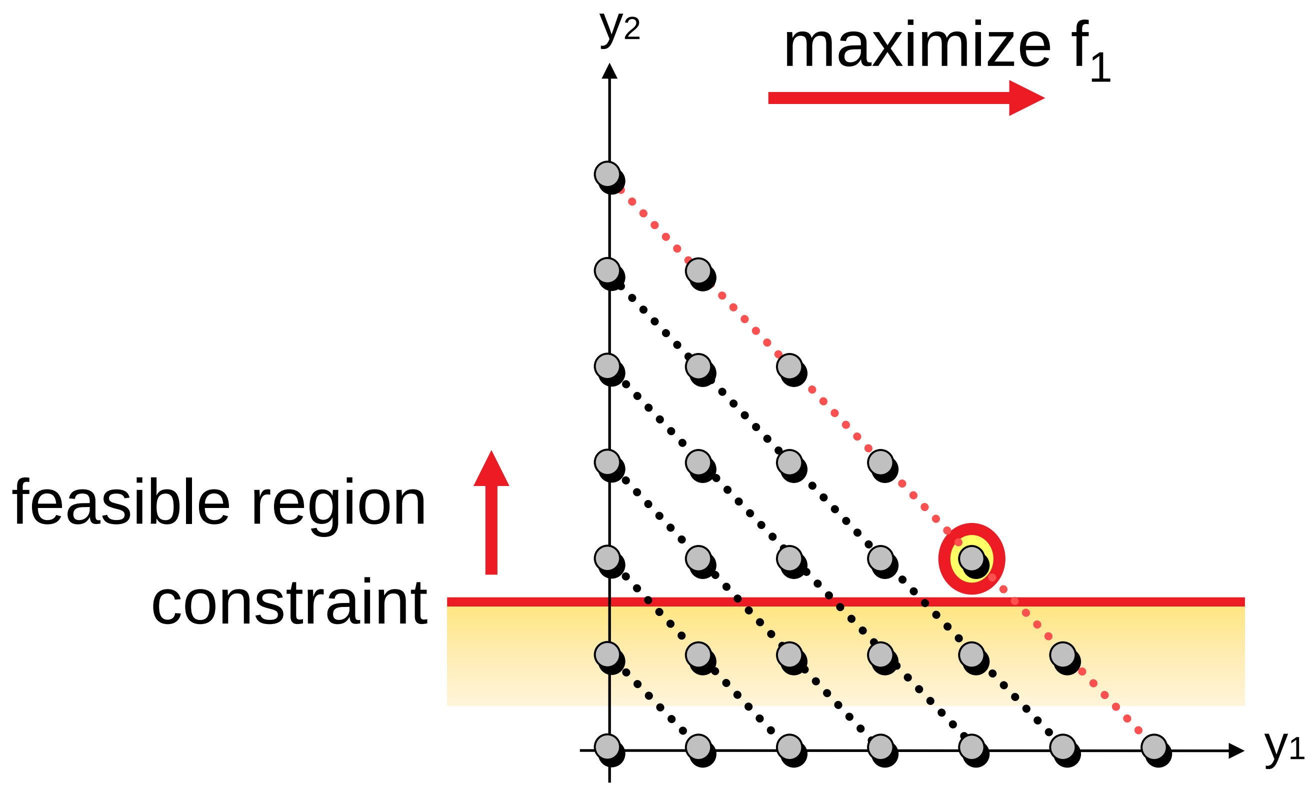
Underlying concept:

- Convert all objectives except of one into constraints
- Adaptively vary constraints

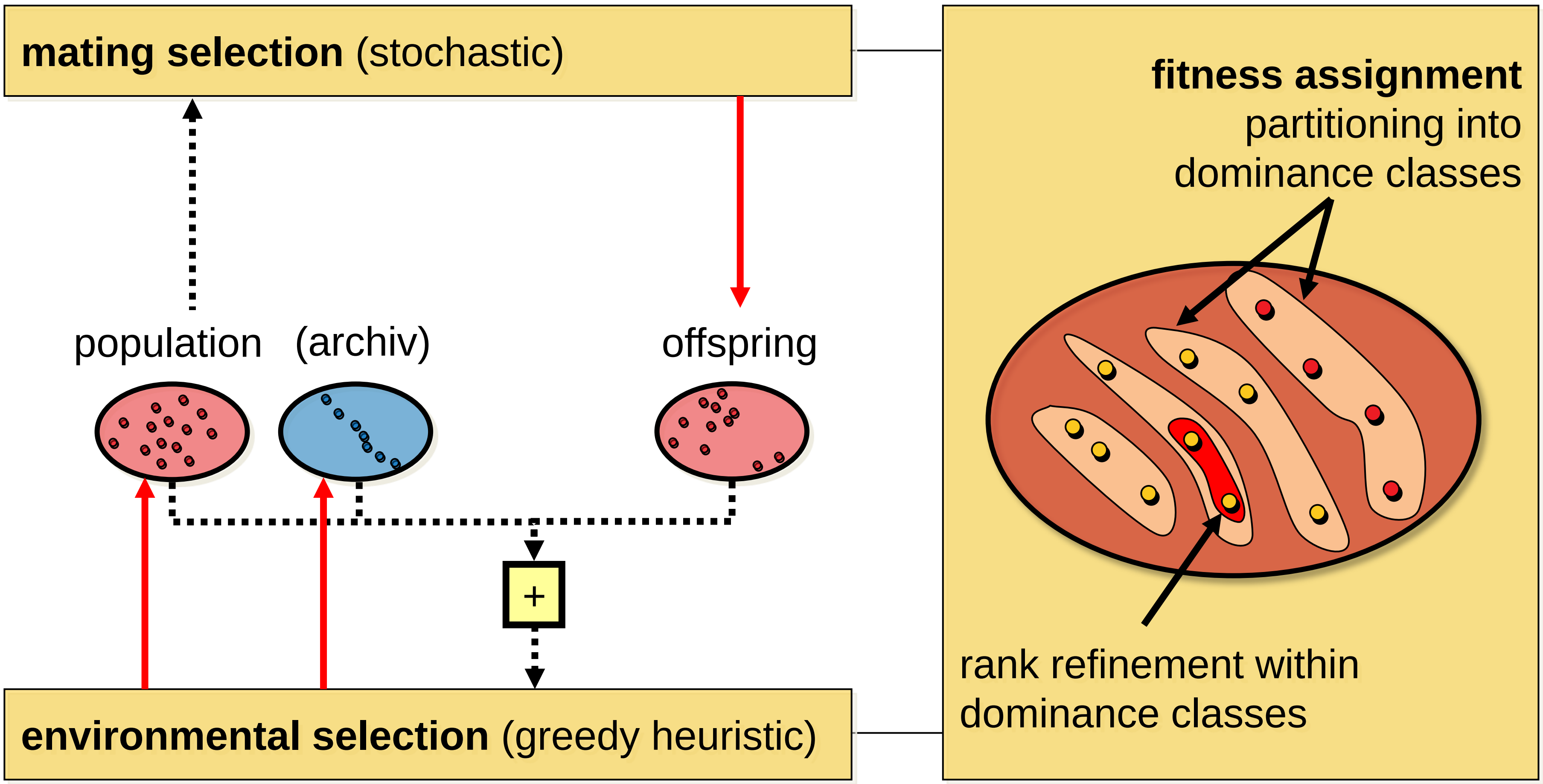


Underlying concept:

- Convert all objectives except of one into constraints
- Adaptively vary constraints



General Scheme of Most Dominance-Based EMO



Note: good in terms of set quality = good in terms of search?

Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms

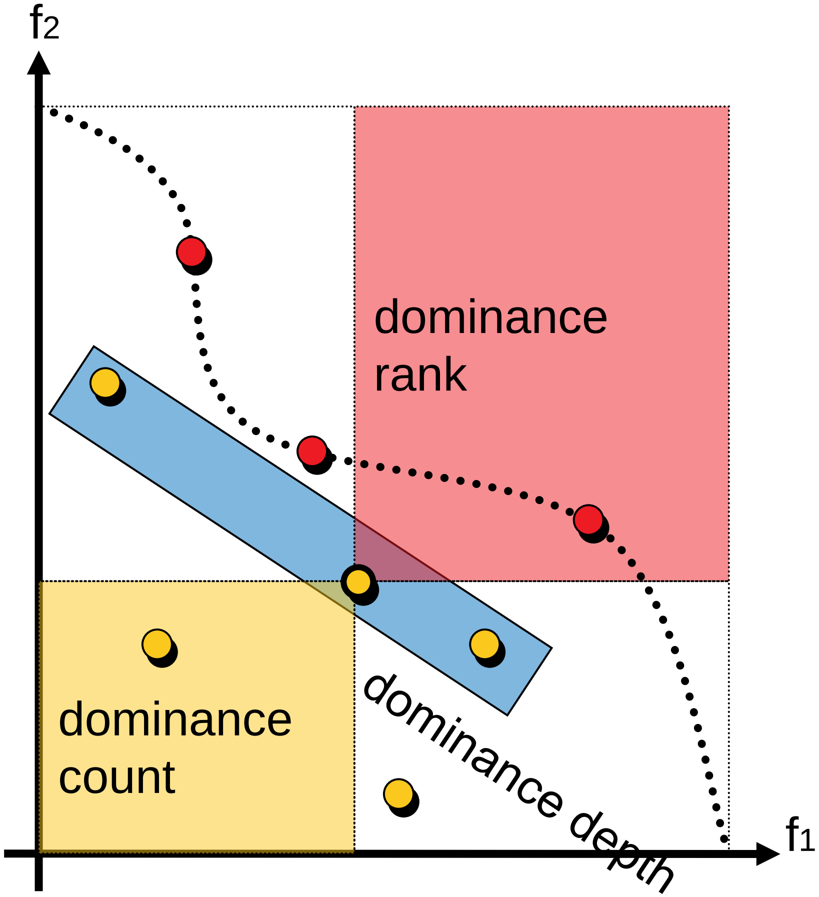
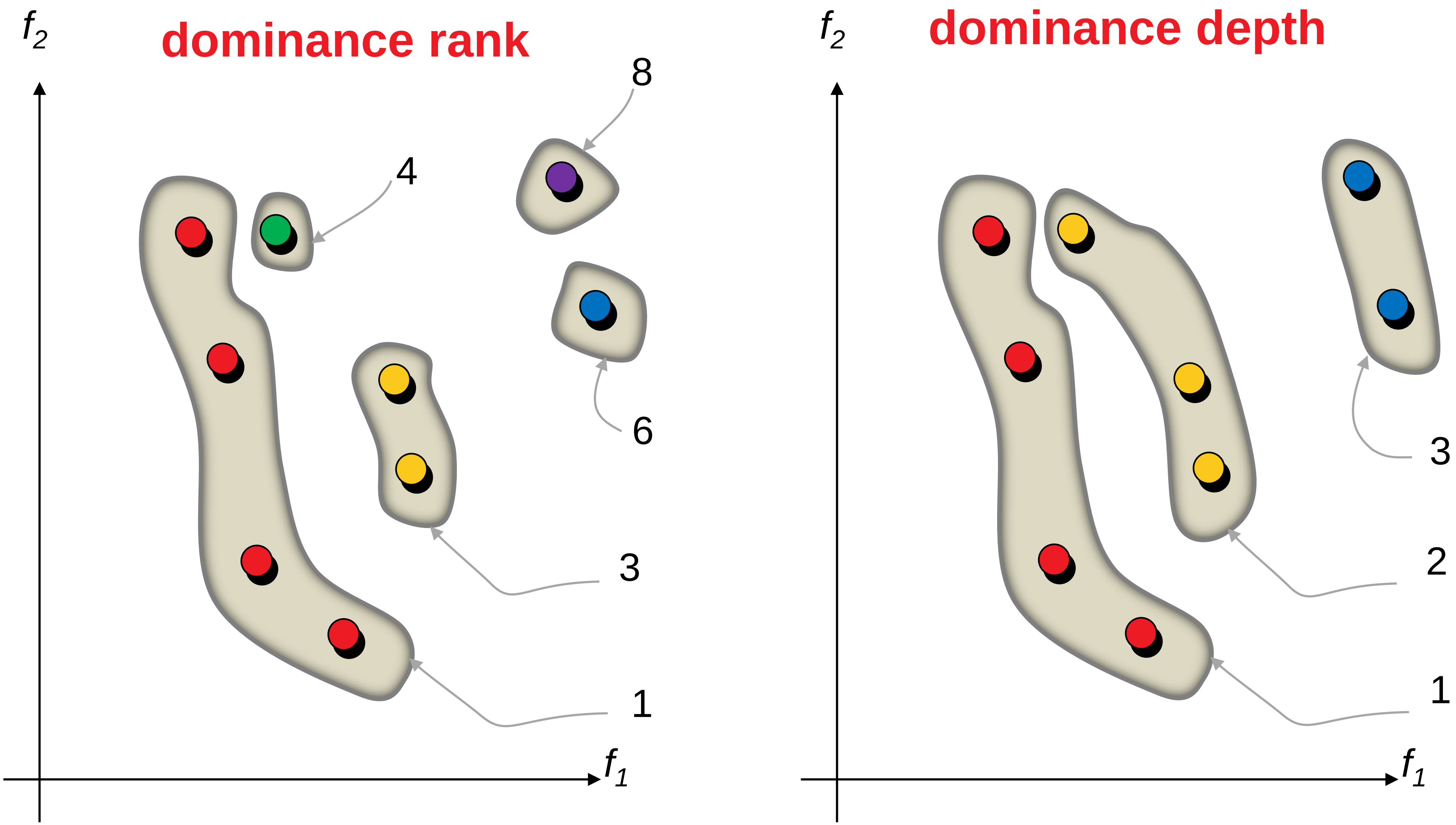


Illustration of Dominance-based Partitioning



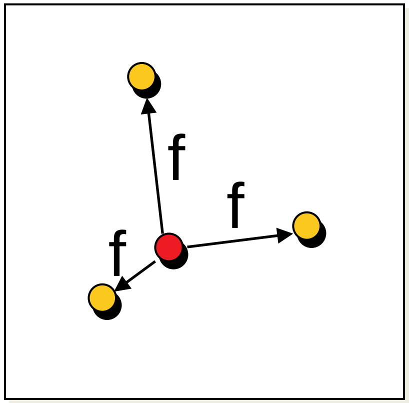
Refinement of Dominance Rankings

Goal: rank incomparable solutions within a dominance class

❶ Density information (good for search, but usually no refinements)

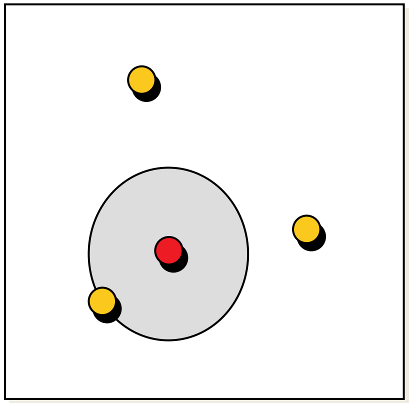
Kernel method

density =
function of the
distances



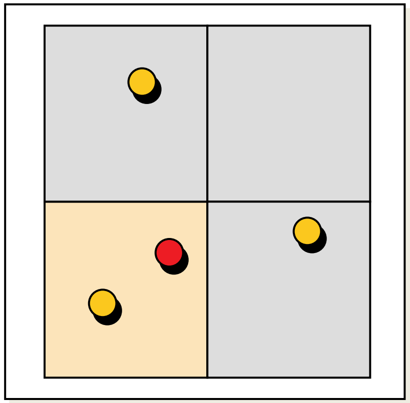
k-th nearest neighbor

density =
function of distance
to k-th neighbor



Histogram method

density =
number of elements
within box



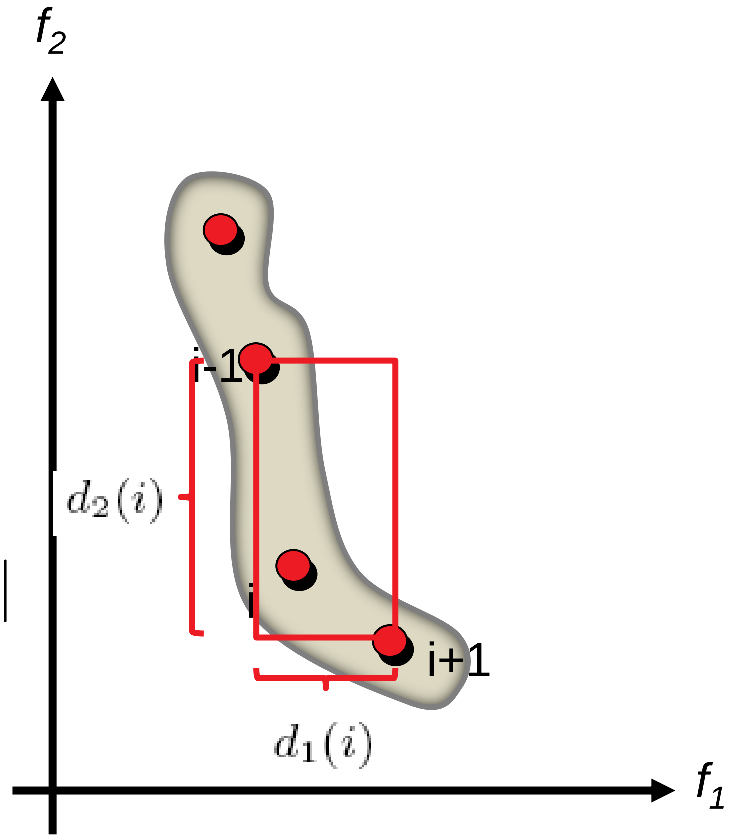
❷ Quality indicator (good for set quality): soon...

Density Estimation

crowding distance:

- sort solutions wrt. each objective
- crowding distance to neighbors:

$$d(i) - \sum_{\text{obj. } m} |f_m(i-1) - f_m(i+1)|$$

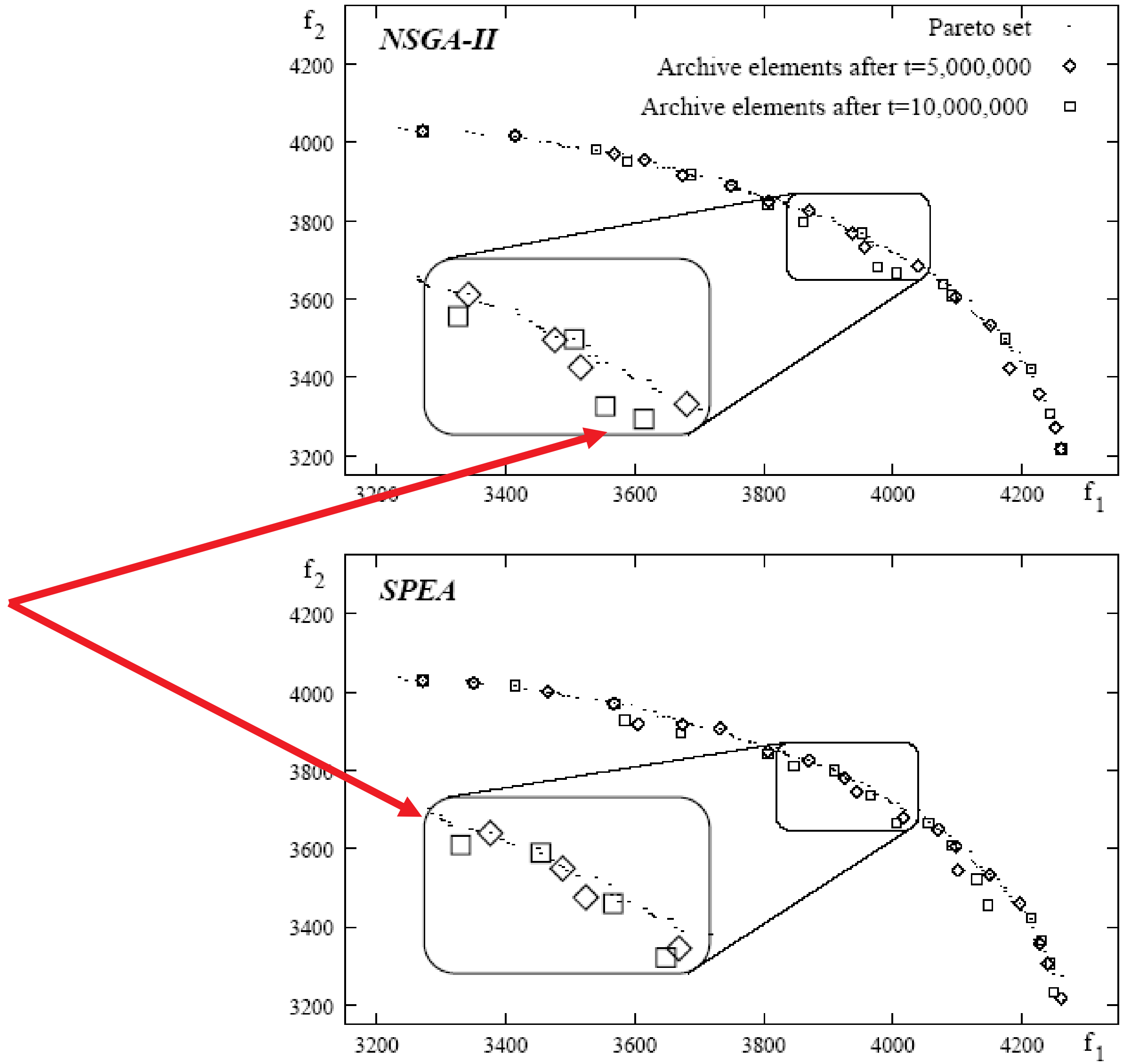


SPEA2 and NSGA-II: Cycles in Optimization

Selection in SPEA2 and NSGA-II can result in

deteriorative cycles

non-dominated solutions already found can be lost



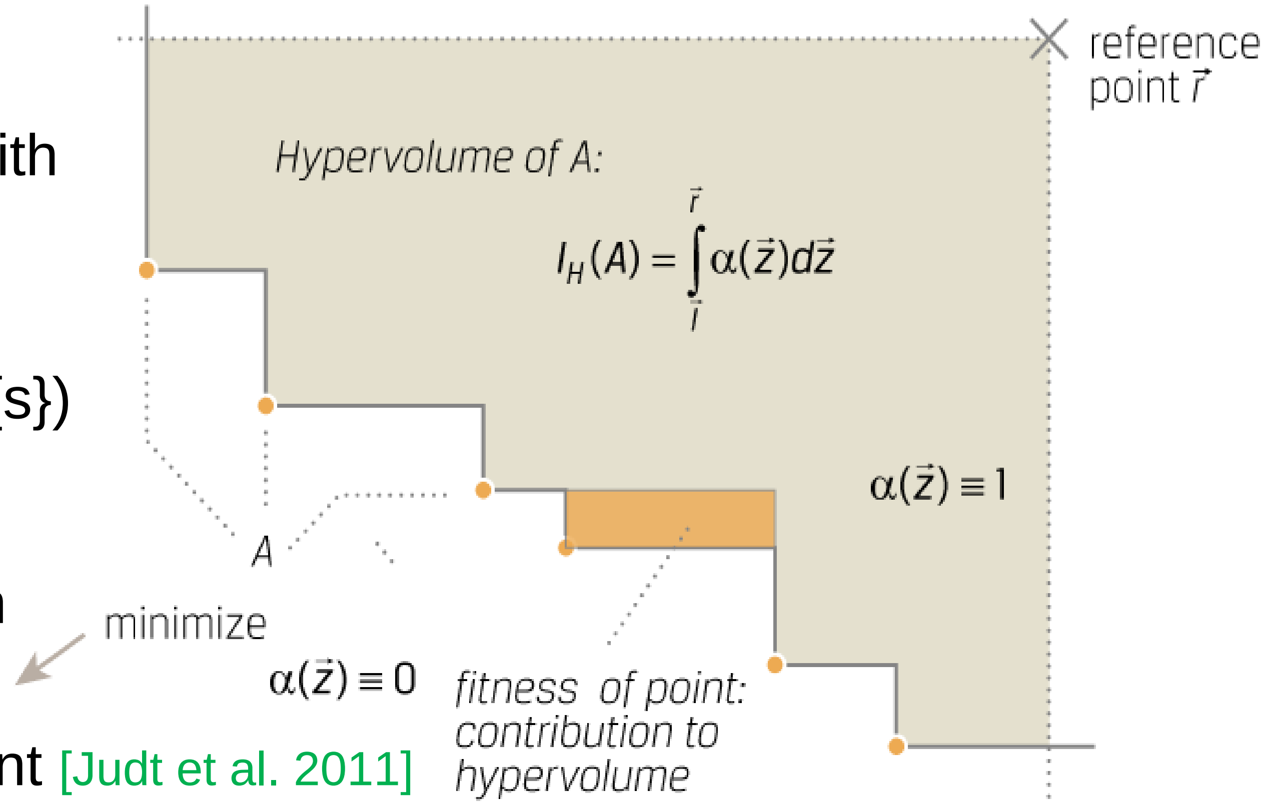
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refinement!

Main idea

Delete solutions with the smallest hypervolume loss $d(s) = I_H(P)-I_H(P / \{s\})$ iteratively



But: can also result in cycles if reference point is not constant [Judt et al. 2011] and is expensive to compute exactly [Bringmann and Friedrich 2009]

Moreover: HypE [Bader and Zitzler 2011]

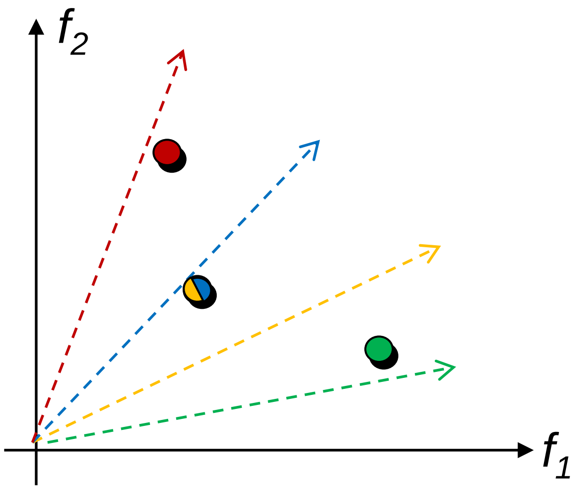
Sampling + Contribution if more than 1 solution deleted

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- Optimize N scalarizing functions in parallel
- Use best solutions of “neighbored scalarizing function” for mating
- keep the best solutions for each scalarizing function
- eventually replace neighbors
- use external archive for non-dominated solutions
- several improved versions recently



Scalarizing Approaches

Open Questions:

- how to choose “the right” scalarization even if the direction in objective space is given by the DM?
- combinations/adaptation of scalarization functions
- independent optimization vs. cooperation between single-objective optimization

Variation in EMO

- At first sight not different from single-objective optimization
- Most algorithm design effort on selection until now
- But: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a set?

Related work:

- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]

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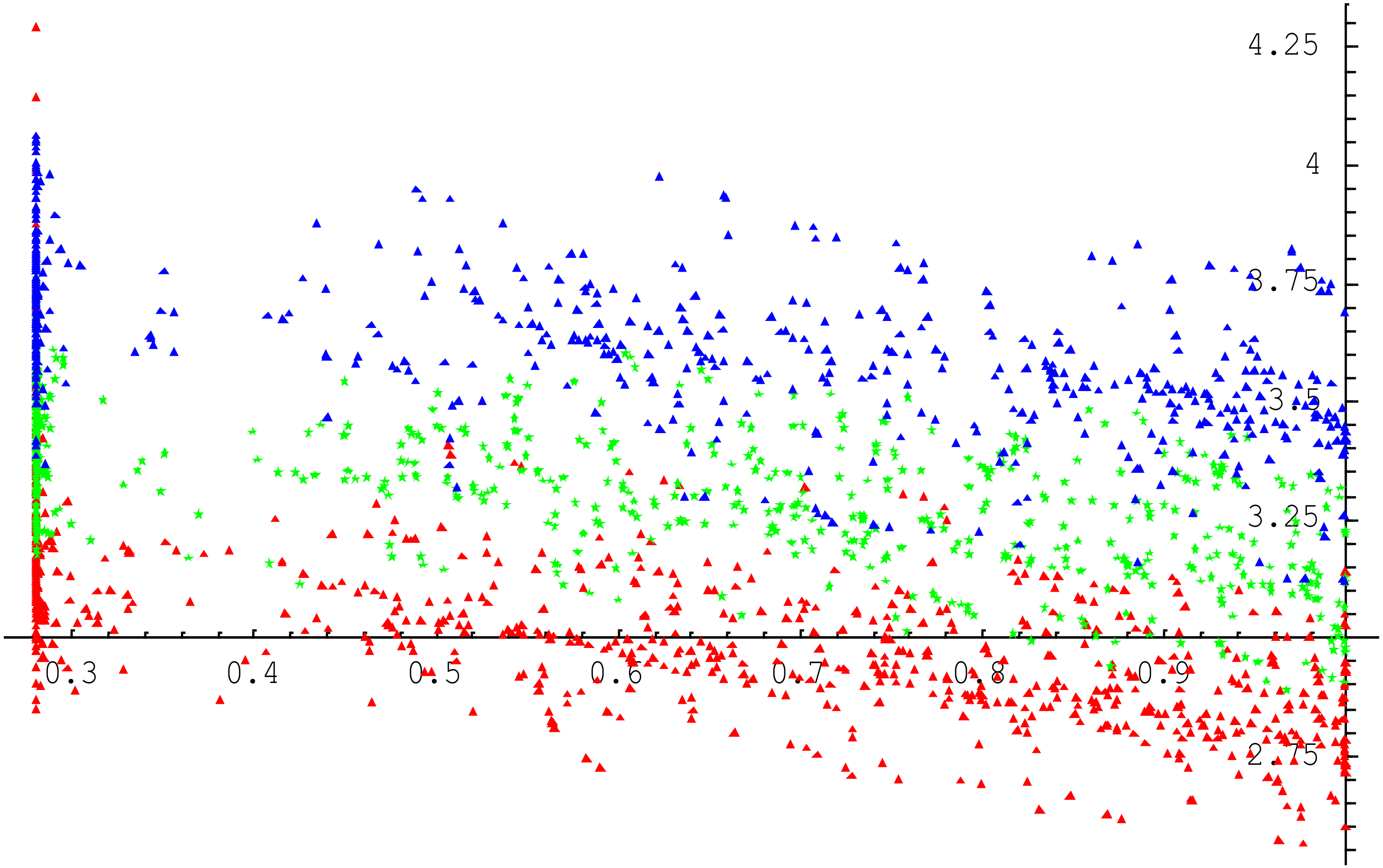
A Few Examples From Practice

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

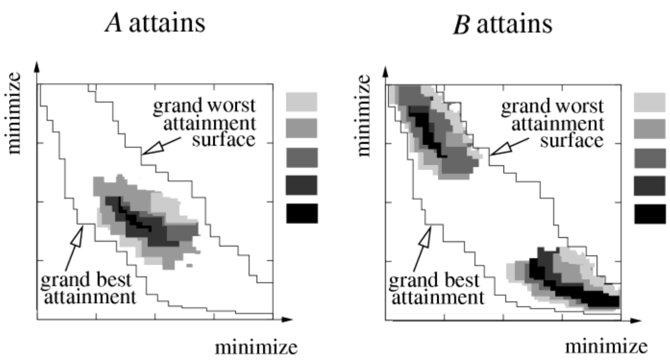
Two Approaches for Empirical Studies

Attainment function approach:

- Applies statistical tests directly to the samples of approximation sets
- Gives detailed information about how and where performance differences occur

Quality indicator approach:

- First, reduces each approximation set to a single value of quality
- Applies statistical tests to the samples of quality values

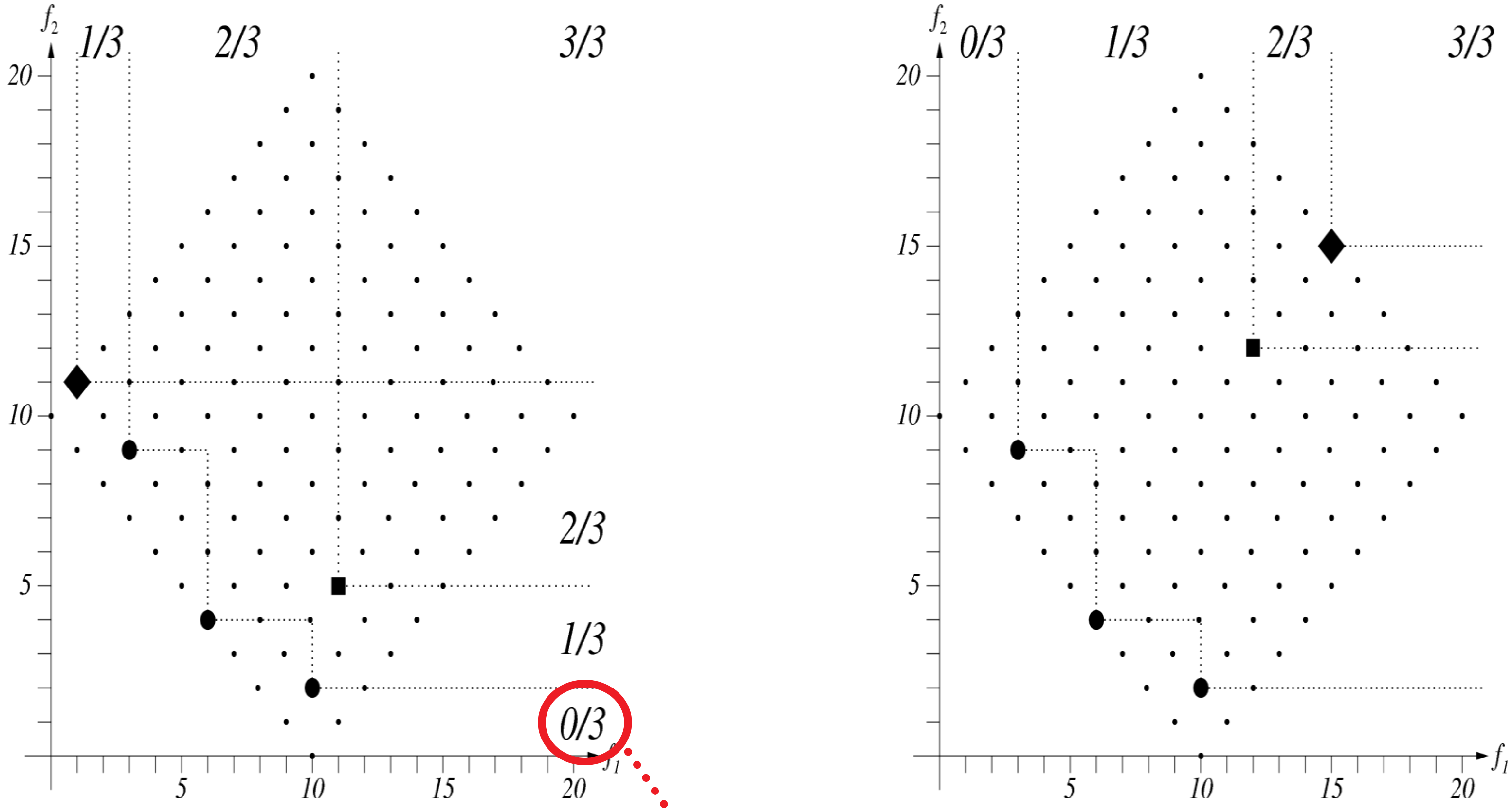


Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions

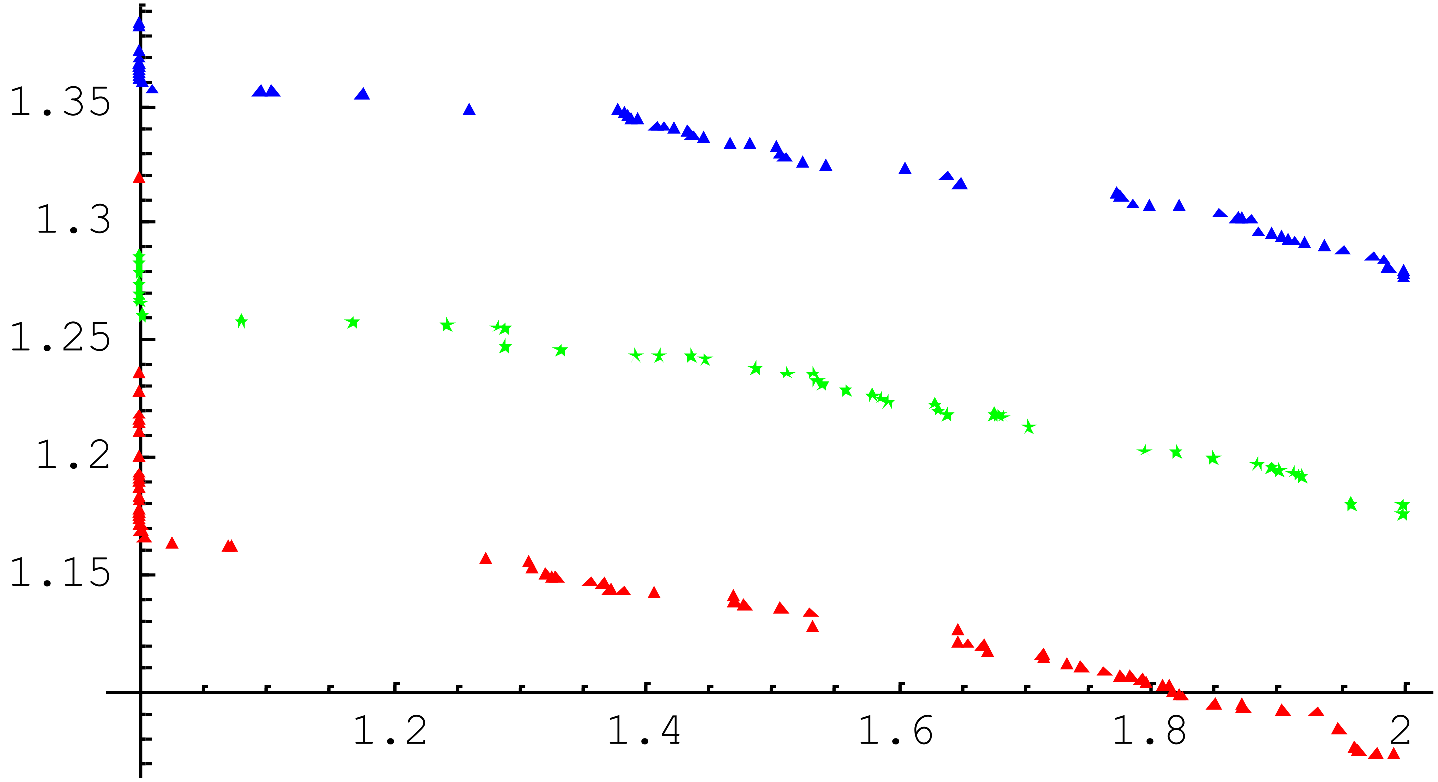
three runs of two multiobjective optimizers



frequency of attaining regions

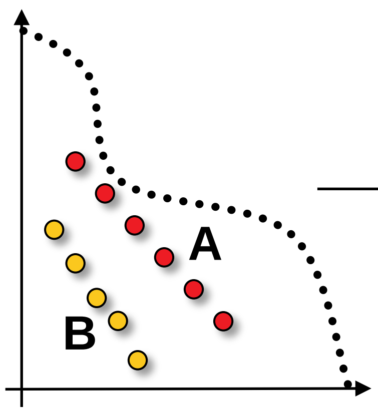
Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
see [Fonseca et al. 2011]

Goal: compare two Pareto set approximations A and B



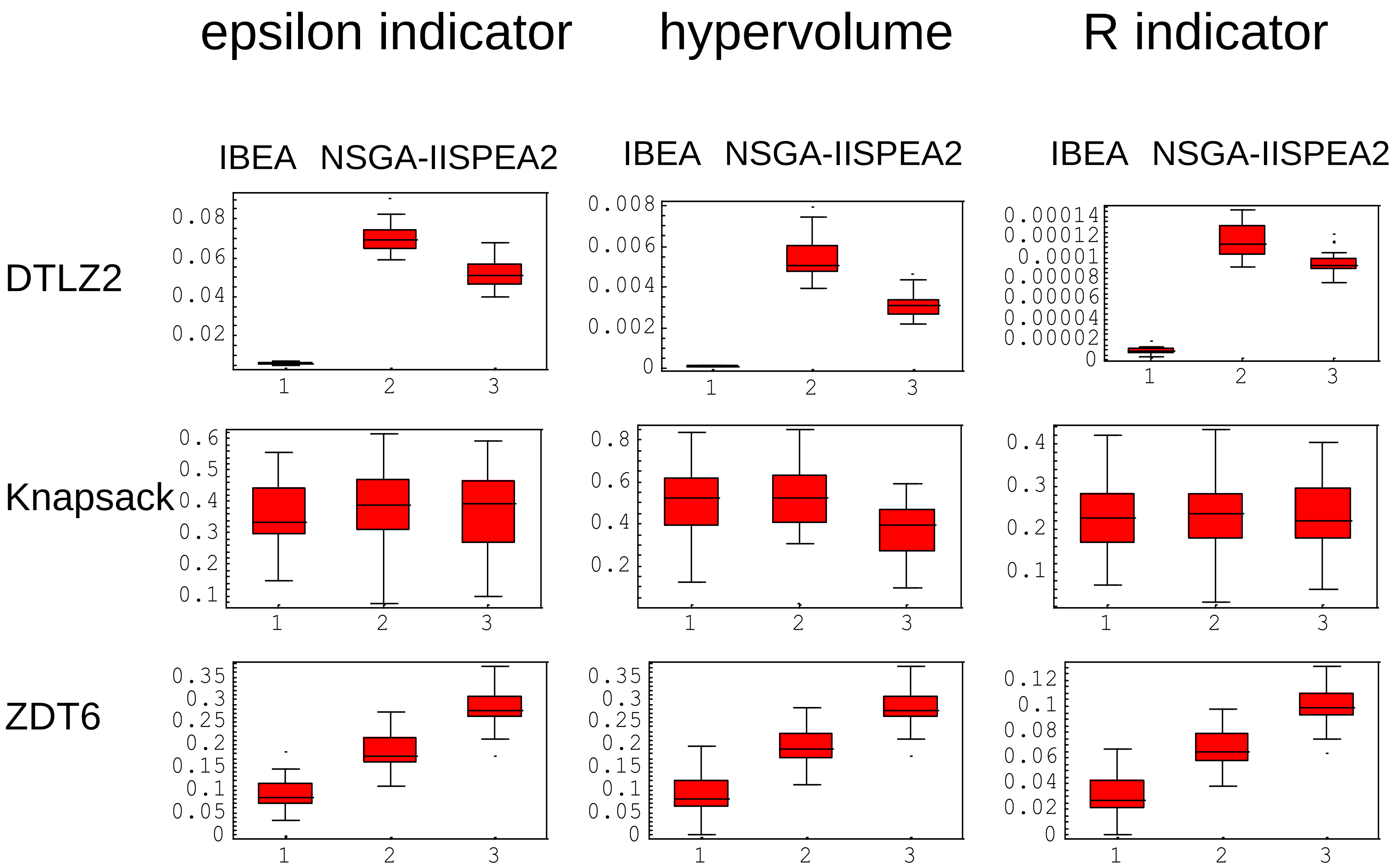
	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow[\textbf{reduction}]{\text{quality measure}} \mathbb{R}^n \xrightarrow[\textbf{interpretation}]{\text{Boolean function}} \text{statement}$$

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

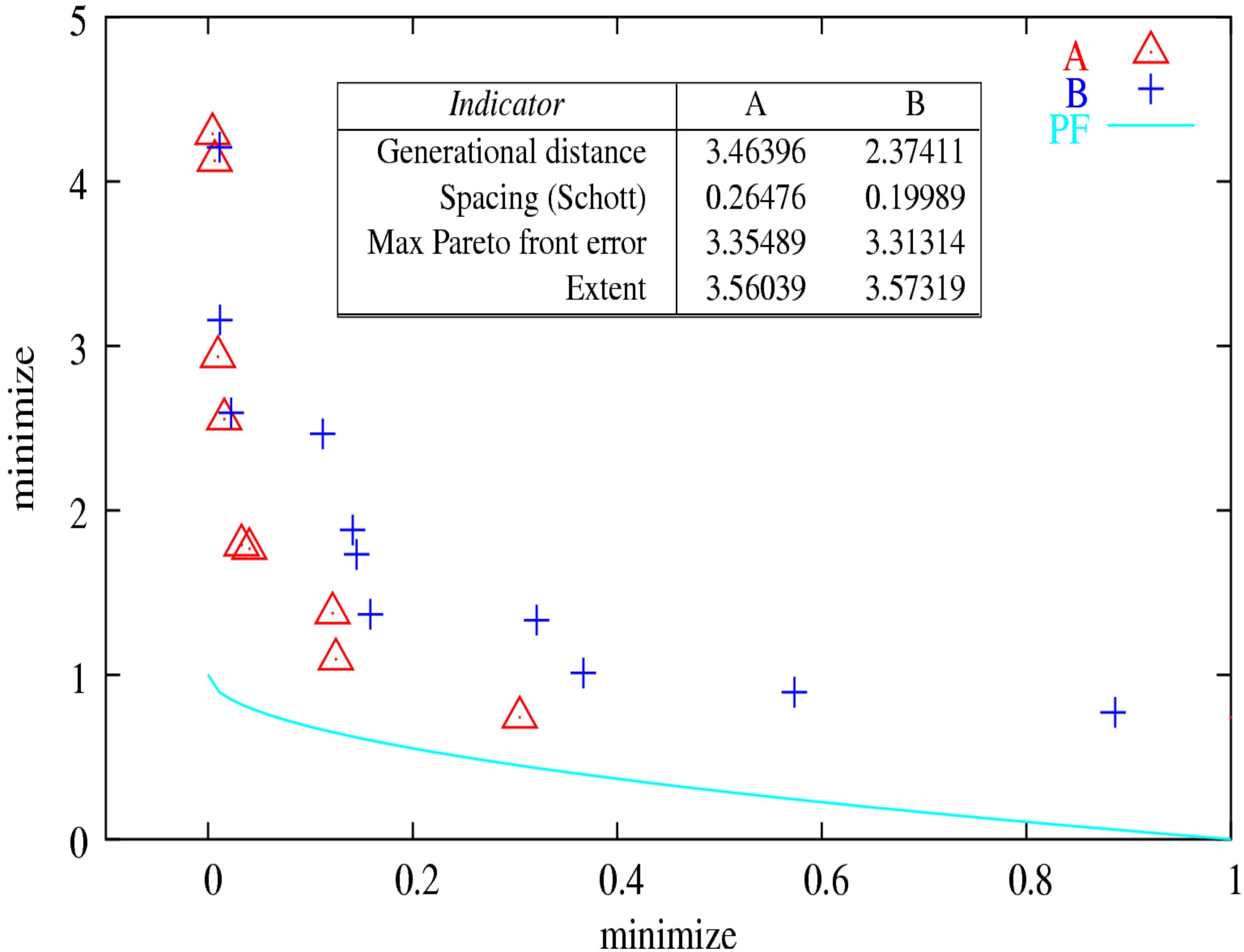
DTLZ2
R

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

Problems With Non-Compliant Indicators



Indicator	A	B
Generational distance	3.46396	2.37411
Spacing (Schott)	0.26476	0.19989
Max Pareto front error	3.35489	3.31314
Extent	3.56039	3.57319

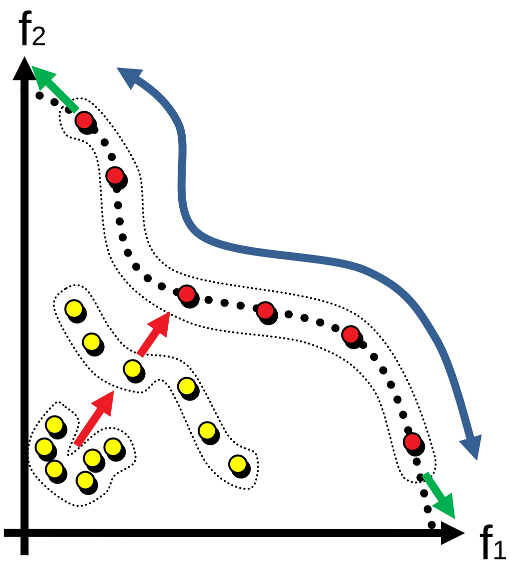
What Are Good Set Quality Measures?

There are **three aspects** [Zitzler et al. 2000]

Comparing different optimization techniques experimentally always involves the notion of performance. In the case of multiobjective optimization, the definition of quality is substantially more complex than for single-objective optimization problems, because the optimization goal itself consists of multiple objectives:

- The distance of the resulting nondominated set to the Pareto-optimal front should be minimized.
- A good (in most cases uniform) distribution of the solutions found is desirable. The assessment of this criterion might be based on a certain distance metric.
- The extent of the obtained nondominated front should be maximized, i.e., for each objective, a wide range of values should be covered by the nondominated solutions.

In the literature, some attempts can be found to formalize the above definition (or parts



Wrong! [Zitzler et al. 2003]

An infinite number of unary set measures is needed to detect in general whether A is better than B

Set Quality Indicators

Open Questions:

- how to design a good benchmark suite?
- are there other unary indicators that are (weak) refinements?
- how to compute indicators efficiently (enough for practice)?
- how to achieve good indicator values?

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Indicator-Based EMO: Optimization Goal

When the goal is to maximize a unary indicator…

- we have a single-objective set problem to solve
- but what is the optimum?
- important: population size μ plays a role!

Multiobjective Problem $\xrightarrow{\text{Indicator}}$ Single-objective Problem

Optimal μ-Distribution:

A set of μ solutions that maximizes a certain unary indicator I among all sets of μ solutions is called optimal μ-distribution for I. [Auger et al. 2009a]

Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal μ-distributions for hypervolume

Optimal μ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with f' the slope of the front

[Friedrich et al. 2011]:

optimal μ-distributions for the hypervolume correspond to ε-approximations of the front

OPT	$1 + \frac{\log(\min\{A/a, B/b\})}{n}$
HYP	$1 + \frac{\sqrt{A/a} + \sqrt{B/b}}{n-4}$
logHYP	$1 + \frac{\sqrt{\log(A/a)\log(B/b)}}{n-2}$

! (probably) does not hold for > 2 objectives

Indicator-Based EMO

Open Questions:

- How do the optimal μ-distributions look like for >2 objectives?
- how to compute certain indicators quickly in practice?
  - several recent improvements for the hypervolume indicator
    [Yildiz and Suri 2012], [Bringmann 2012], [Bringmann 2013]
- how to do indicator-based subset selection quickly?
- what is the best strategy for the subset selection?

further open questions on indicator-based EMO available at
http://simco.gforge.inria.fr/doku.php?id=openproblems

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Articulating User Preferences During Search

What we thought: EMO is preference-less

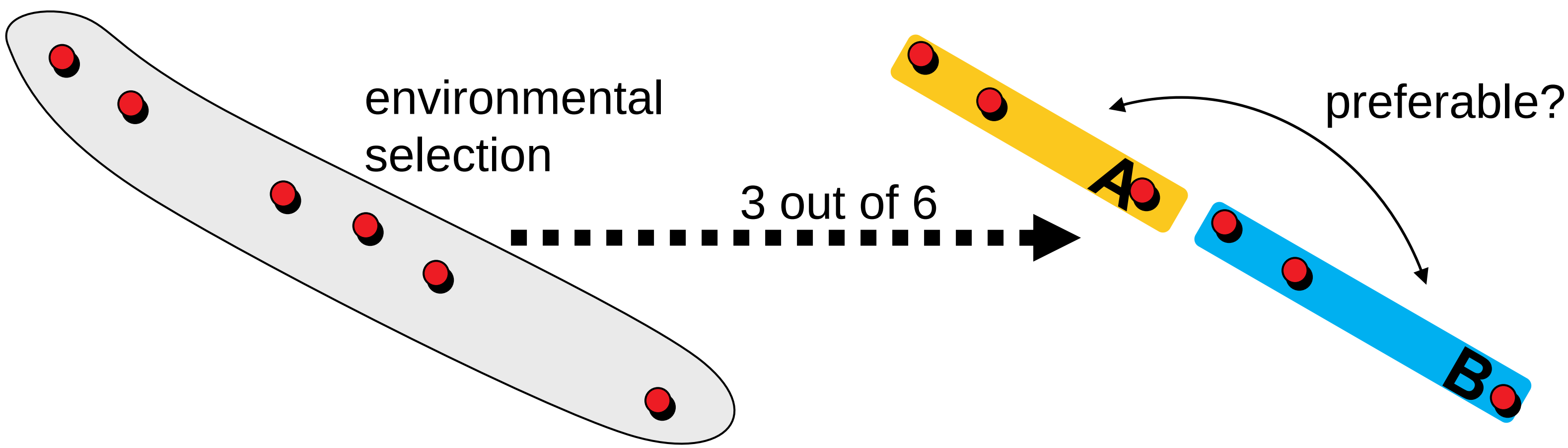
given by the DM.

Search before decision making: Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

Decision making during search: The DM can articulate preferences during

[Zitzler 1999]

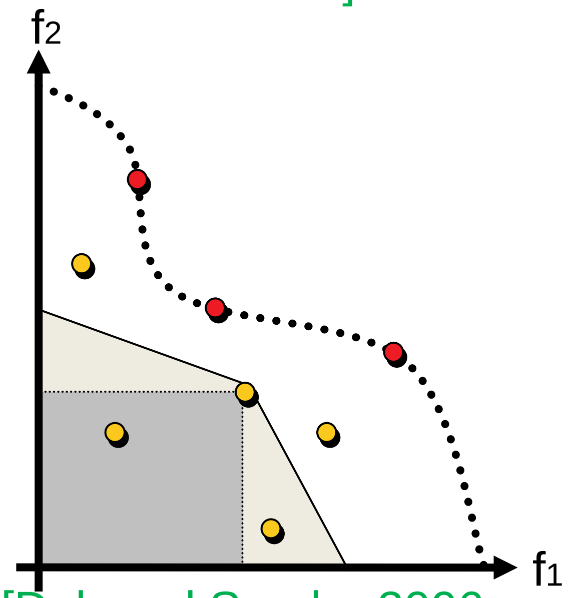
What we learnt: EMO just uses weaker preference information



Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is too large

[Branke 2008], [Rachmawati and Srinivasan 2006], [Coello Coello 2000]



❶ Refine/modify dominance relation, e.g.:

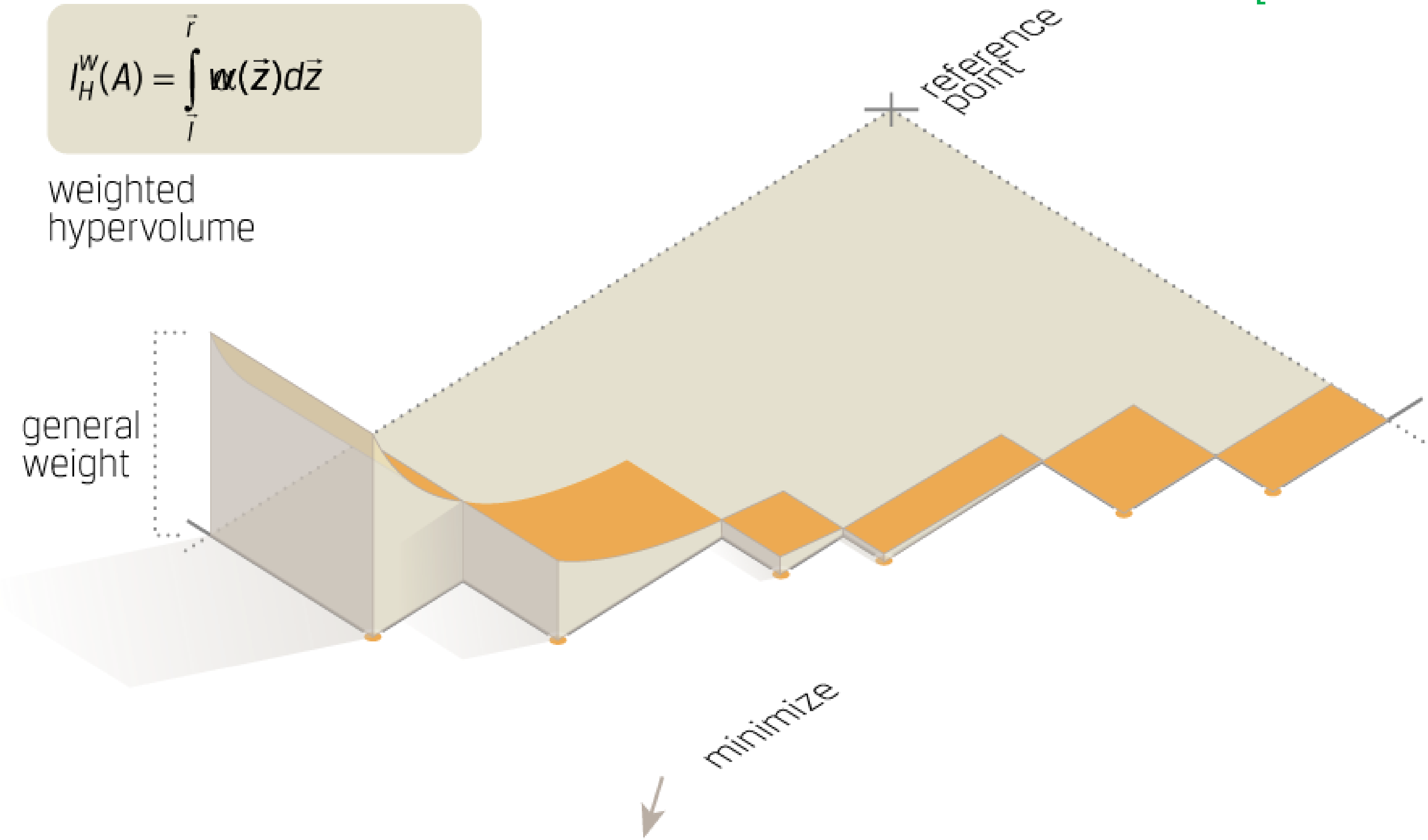
- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of cones
  [Branke and Deb 2004]

❷ Use quality indicators, e.g.:

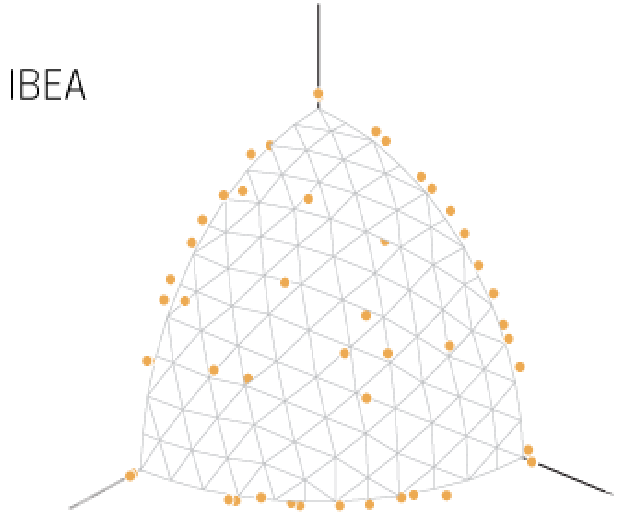
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on binary quality indicators [Zitzler and Künzli 2004]
- based on the hypervolume indicator (now) [Zitzler et al. 2007]

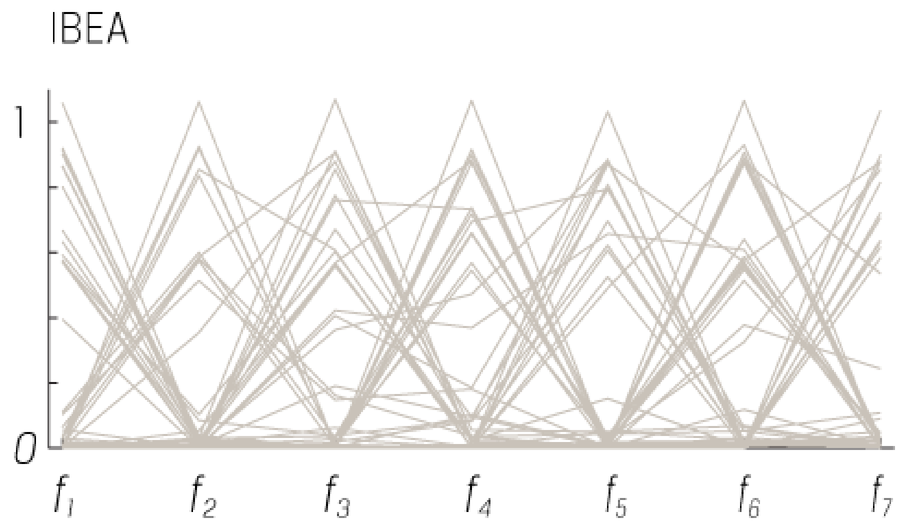
Example: Weighted Hypervolume Indicator

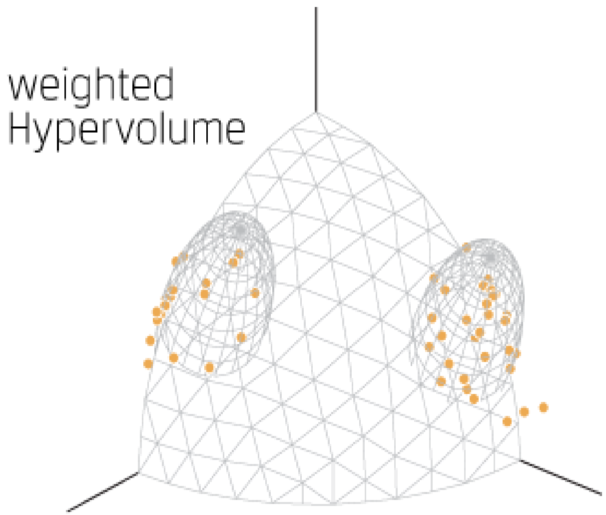
[Zitzler et al. 2007]

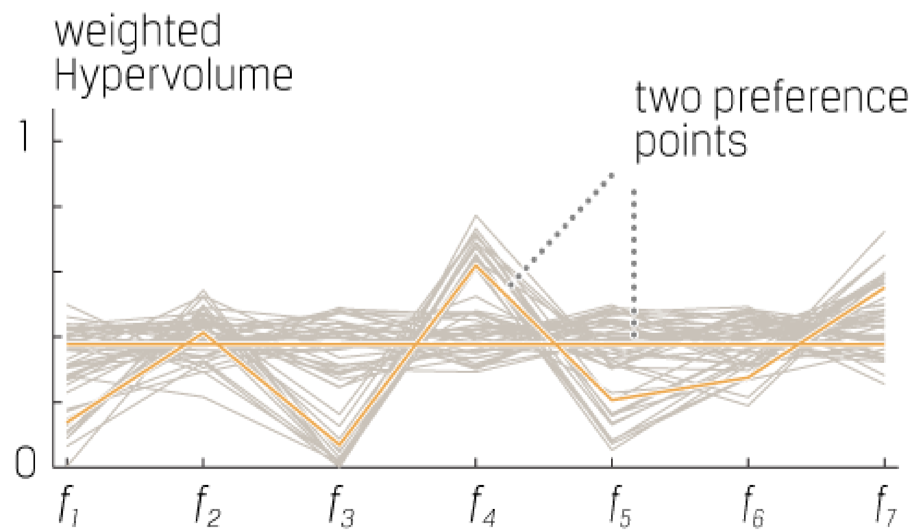


Weighted Hypervolume in Practice









[Auger et al. 2009b]

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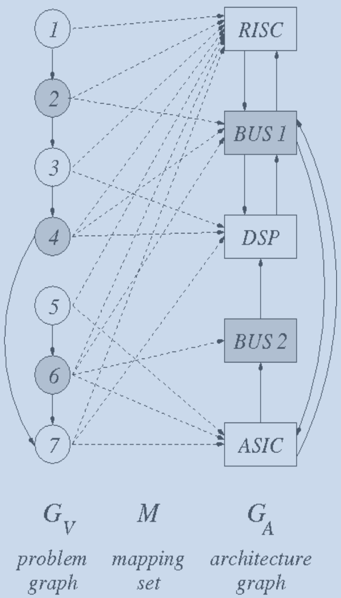
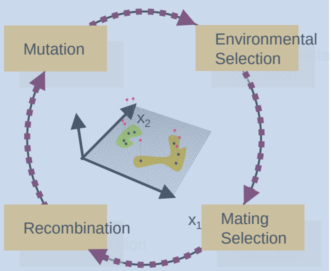
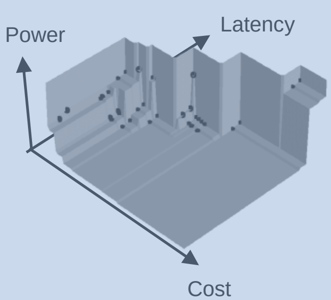
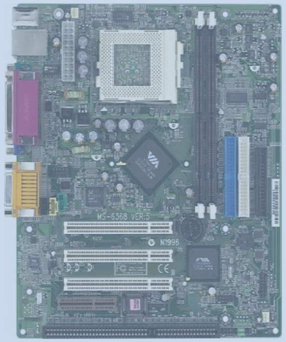
Application: Design Space Exploration

Specification → Optimization → Evaluation → Implementation



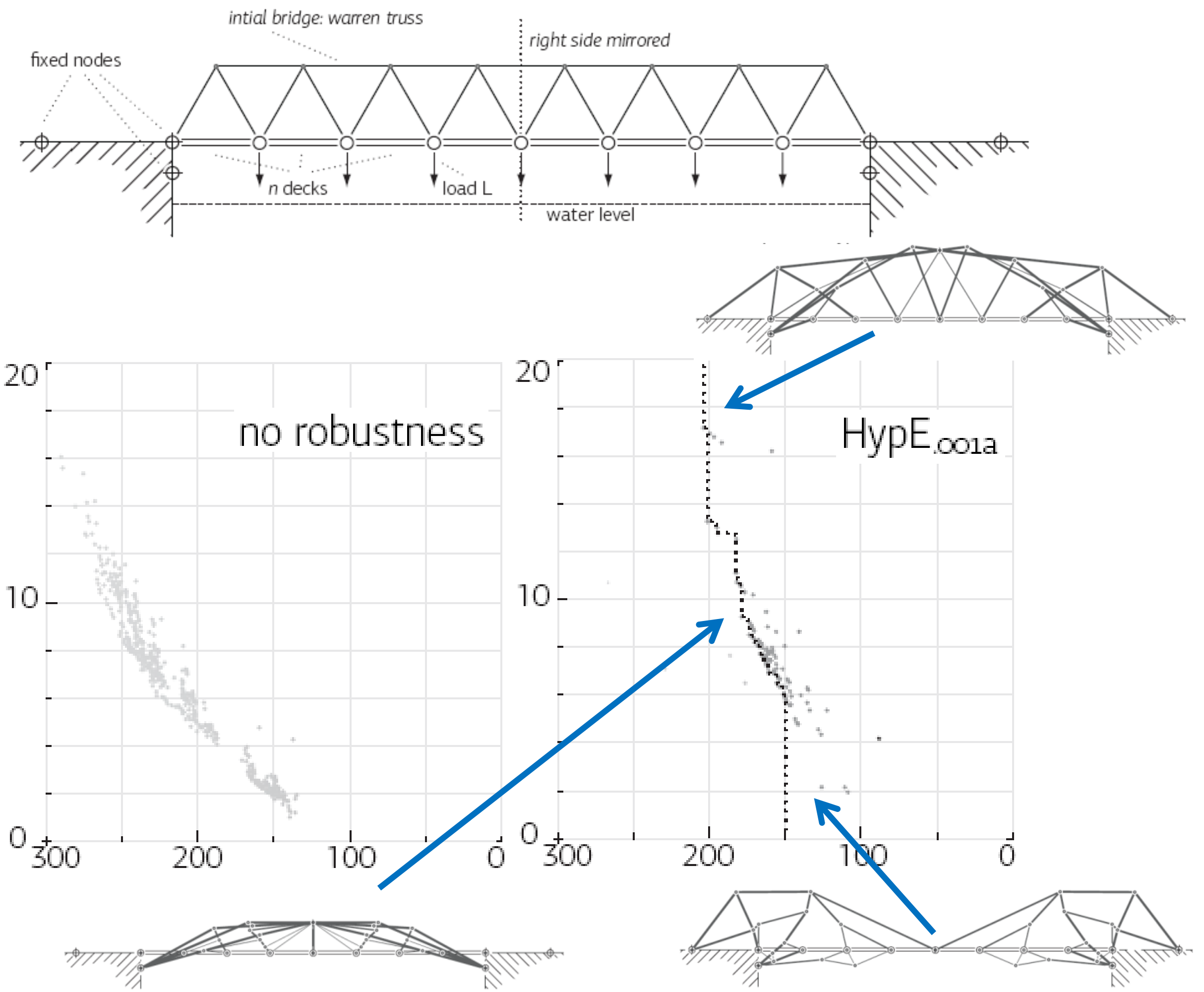


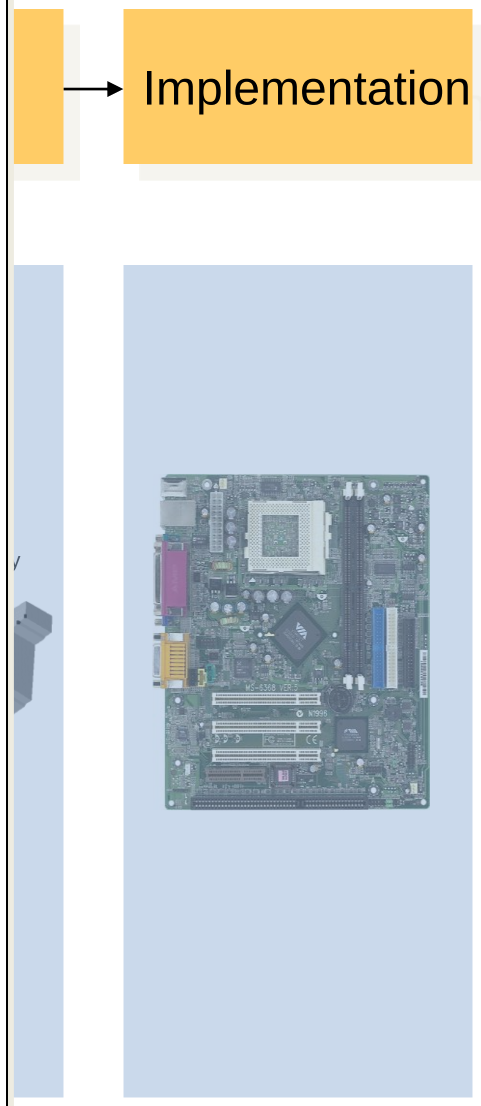




Truss Bridge Design

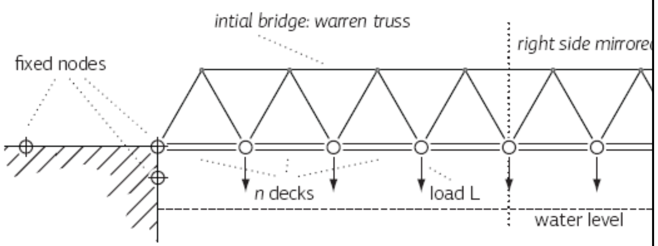
[Bader 2010]

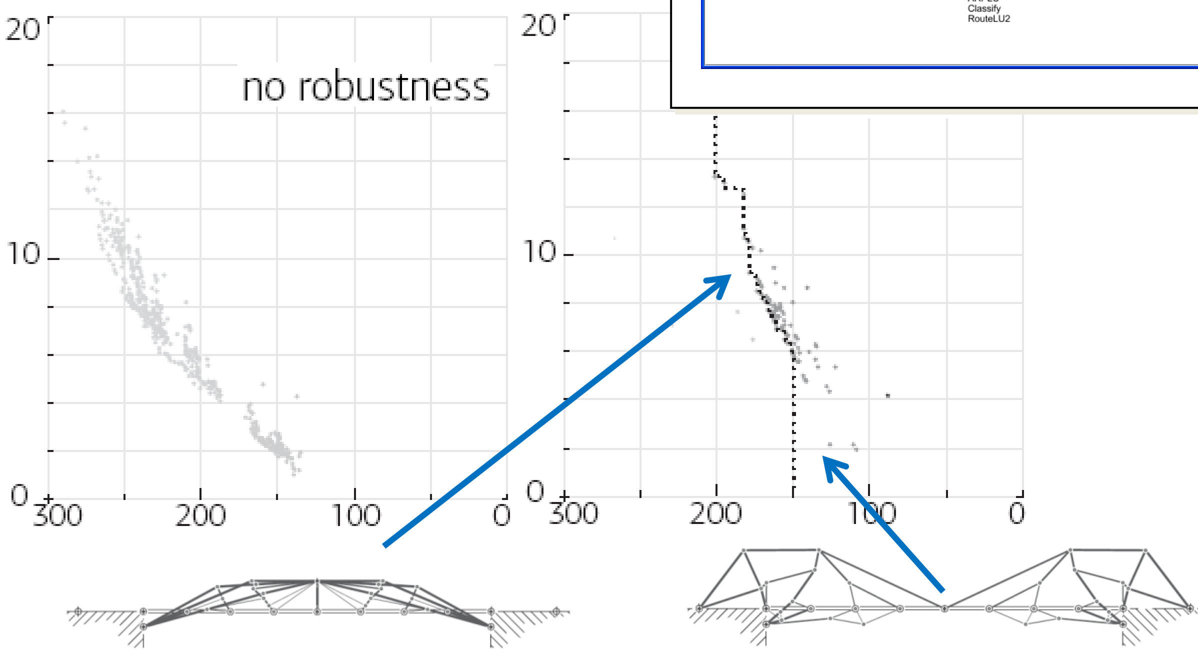




Truss Bridge Design

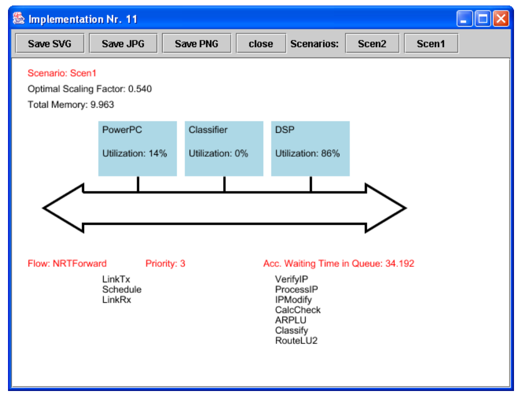
[Bader 2010]

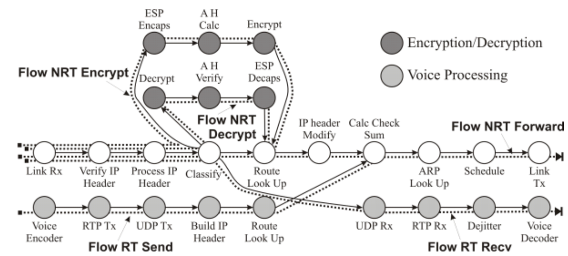


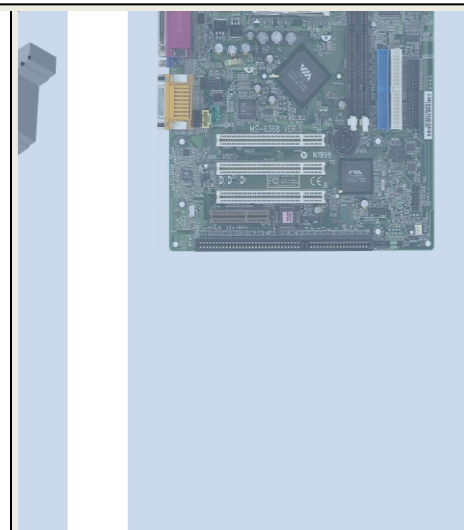


Network Processor Design

[Thiele et al. 2002]



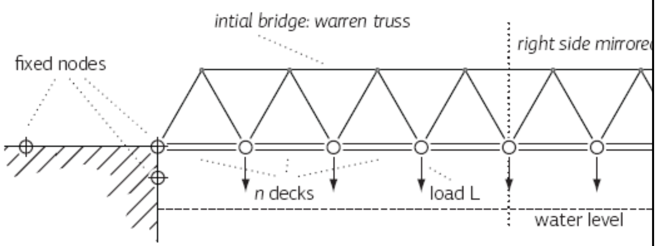


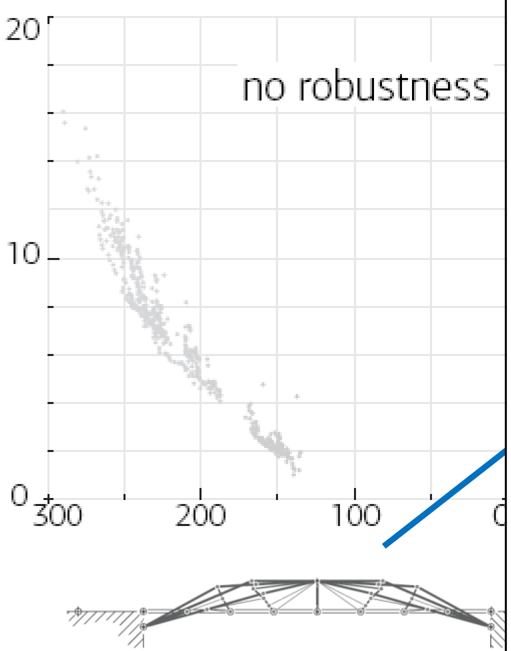


Application: Design Space Exploration

Truss Bridge Design

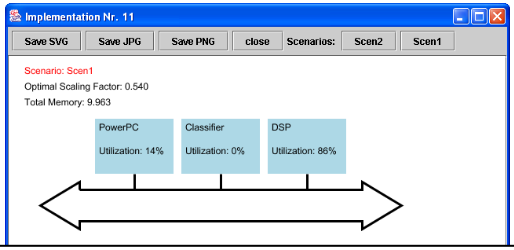
[Bader 2010]

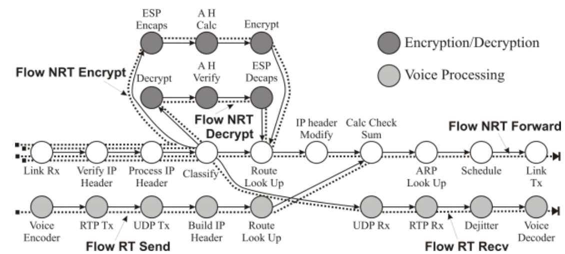




Network Processor Design

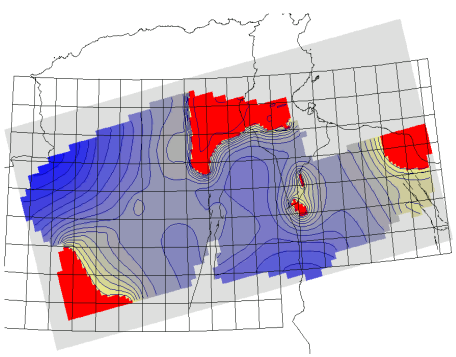
[Thiele et al. 2002]

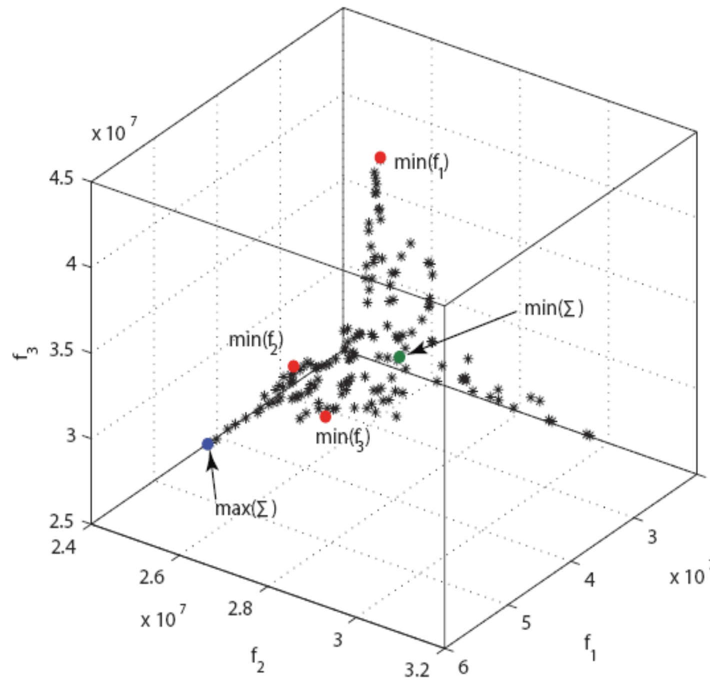




Water resource management

[Siegfried et al. 2009]



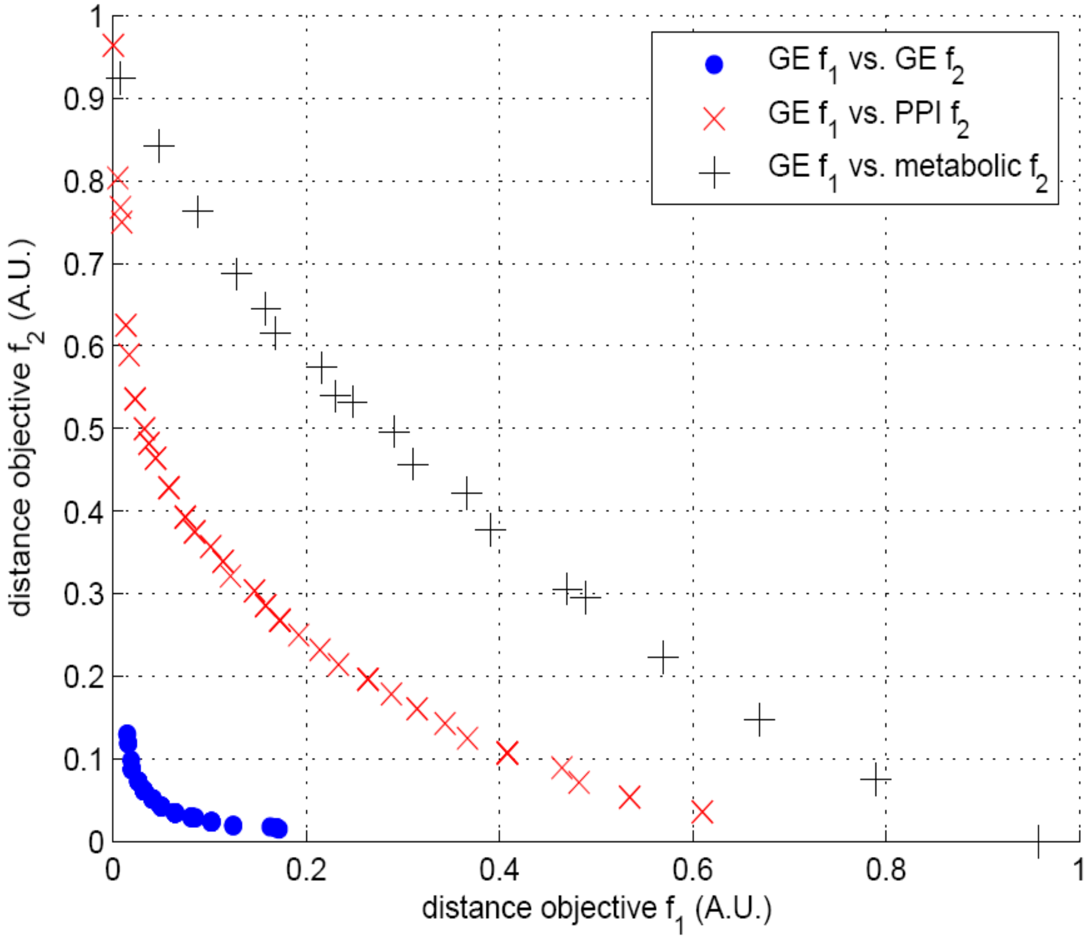


Application: Trade-Off Analysis

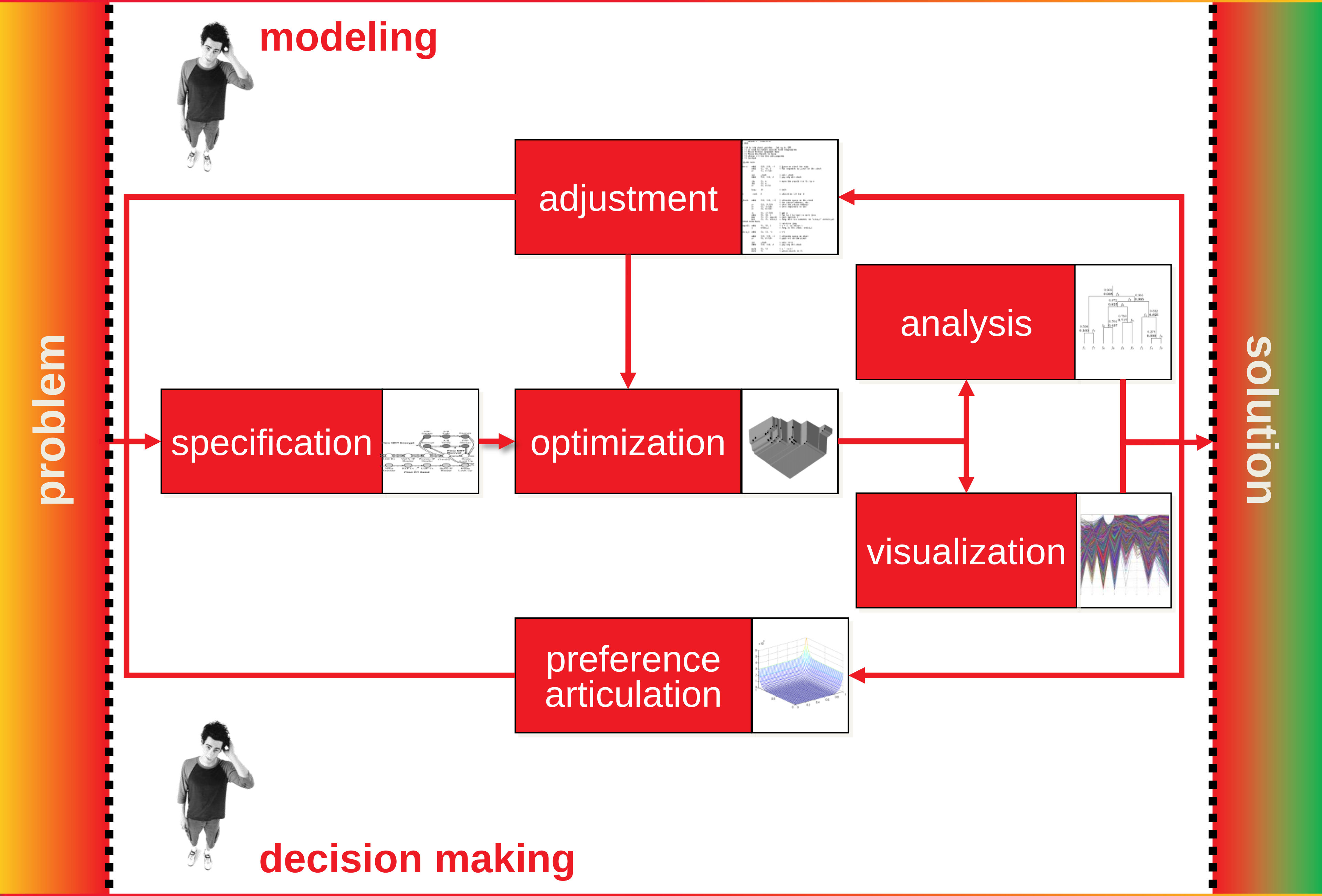
Module identification from biological data [Calonder et al. 2006]

Find group of genes wrt different data types:

- similarity of gene expression profiles
- overlap of protein interaction partners
- metabolic pathway map distances



Conclusions: EMO as Interactive Decision Support



The EMO Community

Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…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this webpage might no longer be updated more...

PISA
- Principles and Documentation
- PISA for Beginners
- Downloads
- Performance Assessment
- Write and Submit a Module
- Publications, Bugs, Contact & License

Download of Selectors, Variators and Performance Assessment

This page contains the currently available variators and selector (see also Principles of PISA) as well as performance assessment tools (see also Performance Assessment). The variators are mainly test and benchmark problems that can be used to assess the performance of different optimizers. EXPO is a complex application form the are of computer design that can be used as a benchmark problem too. The selectors are state-of-the-art evolutionary multi-objective optimization methods. If you want to write or submit a module, please look at Write and Submit a Module. Links to documentation on the PISA specification can be found at Documentation.

Jaroslav Hajek pointed out a severe bug in the WFG selector, please redownload the module if your version is older than 2010/02/03.

Optimization Problems (variator)

GWLAB - Multi-Objective Groundwater Management
Package: in Matlab
more...

LOTZ - Demonstration Program
Source: in C
Binaries: Solaris, Windows, Linux
more...

LOTZ2 - Leading Ones Trailing Zeros
Source: in C
Binaries: Solaris, Windows, Linux
more...

LOTZ2 - Java Example Variator
Source: in Java
Binaries: Windows, Linux
more...

Knapsack Problem
Source: in C
Binaries: Solaris, Windows, Linux
more...

EXPO - Network Processor Design Problem

Optimization Algorithms (selector)

SPAM - Set Preference Algorithm for Multiobjective Optimization
Source: in C
Binaries: Windows, Linux 32bit, Linux 64bit
more...

SHV - Sampling-based HyperVolume-oriented algorithm
Source: in C
Binaries: Windows, Linux 32bit, Linux 64bit
more...

SIBEA - Simple Indicator Based Evolutionary Algorithm

and many more:
jmetal, Shark,
MOEA Framework,
...

Perspectives

Challenging Open (Research) Directions

- Benchmarking
  - comparison with classical approaches
  - where are real strengths of EMO (how much better?)
  - algorithm recommendations for practice
- Many-objective Optimization
- growing EMO and MCDM to one field

Questions?

Additional Slides

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

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DOLPHIN team
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40, avenue Halley - Bât B - Park Plaza
59650 Villeneuve d'Ascq
France

After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at INRIA Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011 he has been a junior researcher (now CR1) at INRIA Lille - Nord Europe in Villeneuve d'Ascq, France. His research interests are focused on evolutionary multiobjective optimization (EMO), in particular on many-objective optimization, benchmarking, and theoretical aspects of indicator-based search.

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