# GECCO 2018
# Tutorial on
# Evolutionary Multiobjective Optimization

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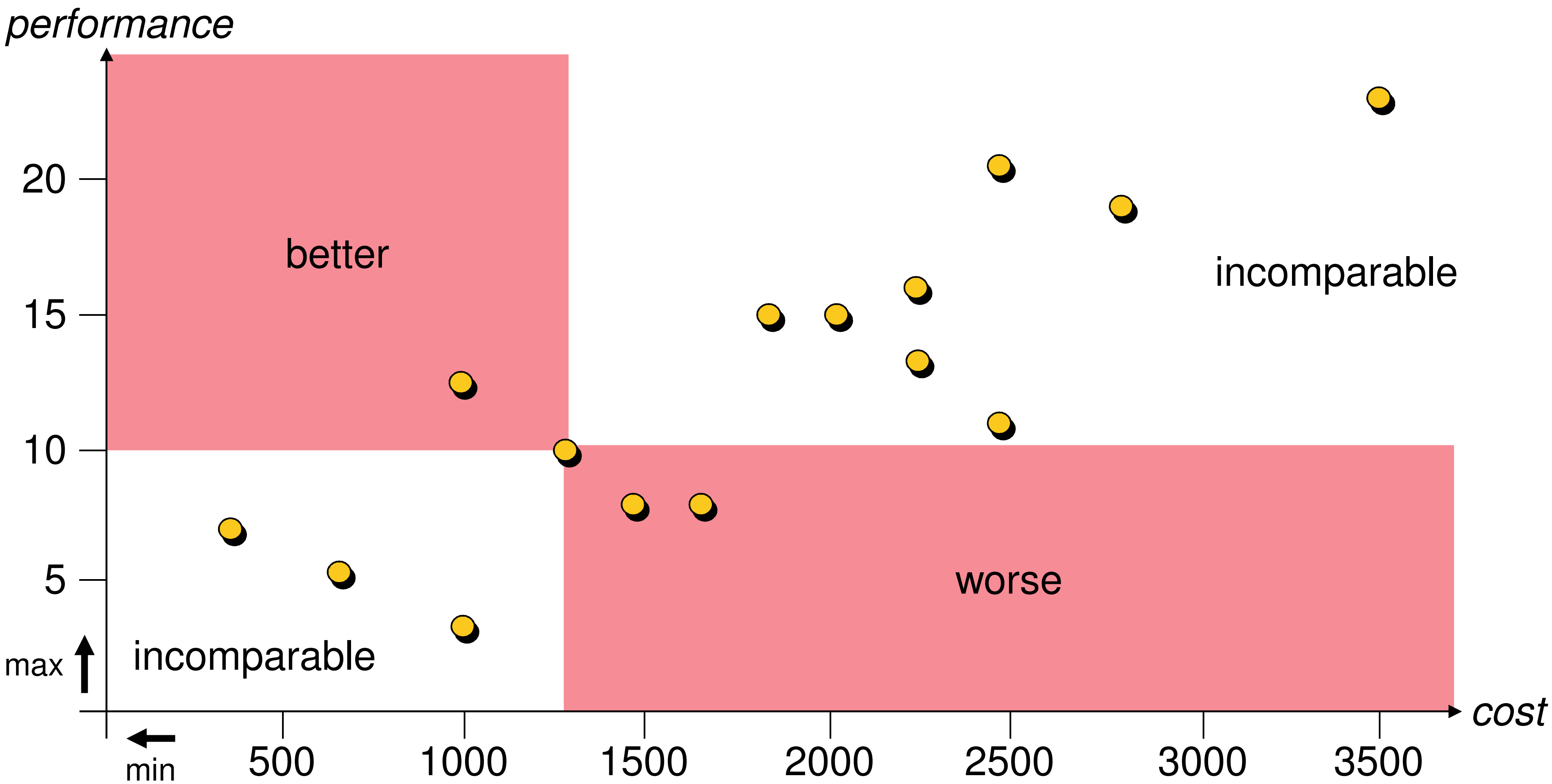
updated slides will be available at
http://www.cmap.polytechnique.fr/~dimo.brockhoff/


ACM ISBN 978-1-4503-5764-7/18/07.
https://doi.org/10.1145/3205651.3207864

# A Brief Introduction to Multiobjective Optimization

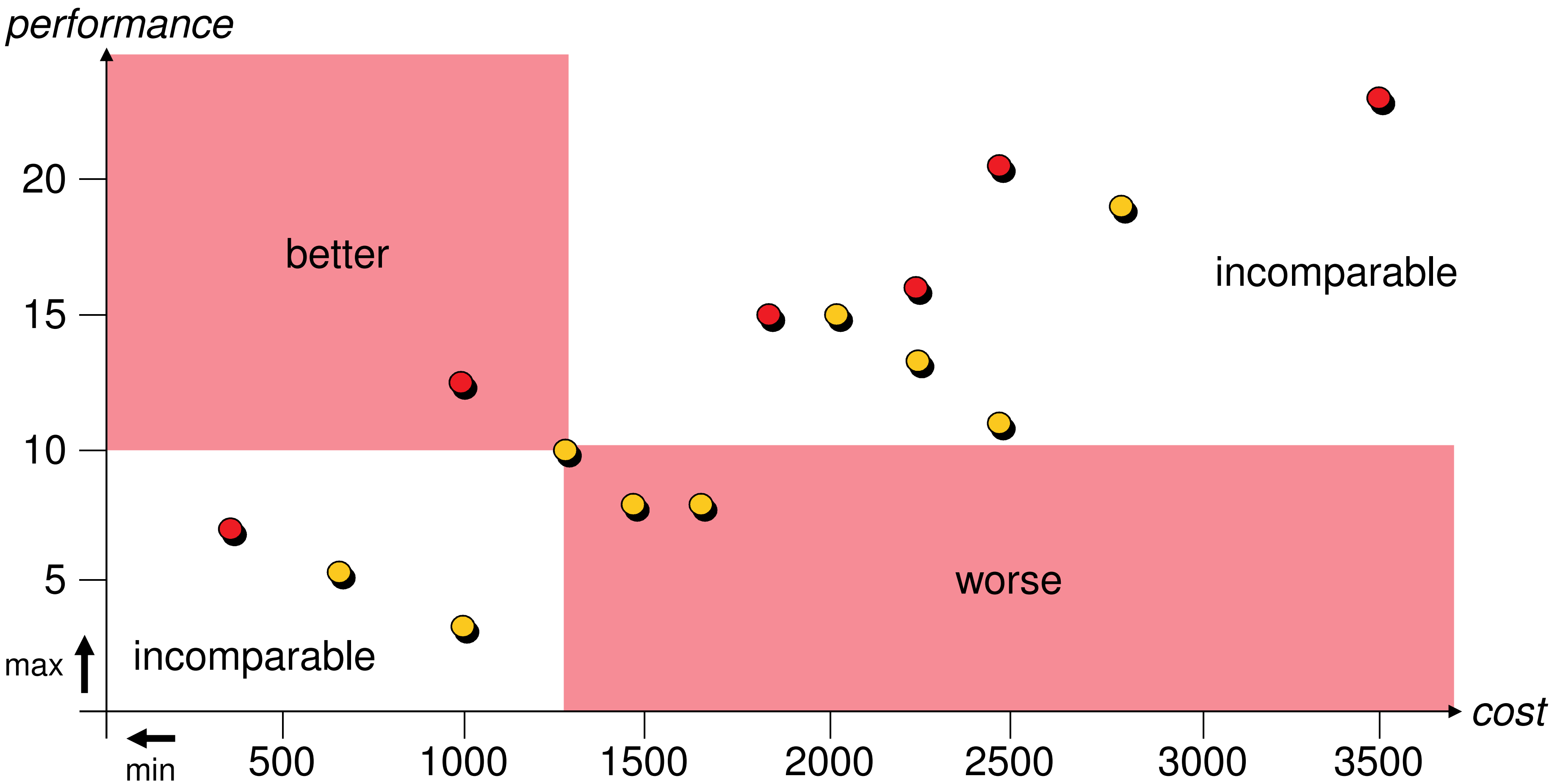
**Multiobjective Optimization**

Multiple objectives that have to be optimized simultaneously



# A Brief Introduction to Multiobjective Optimization

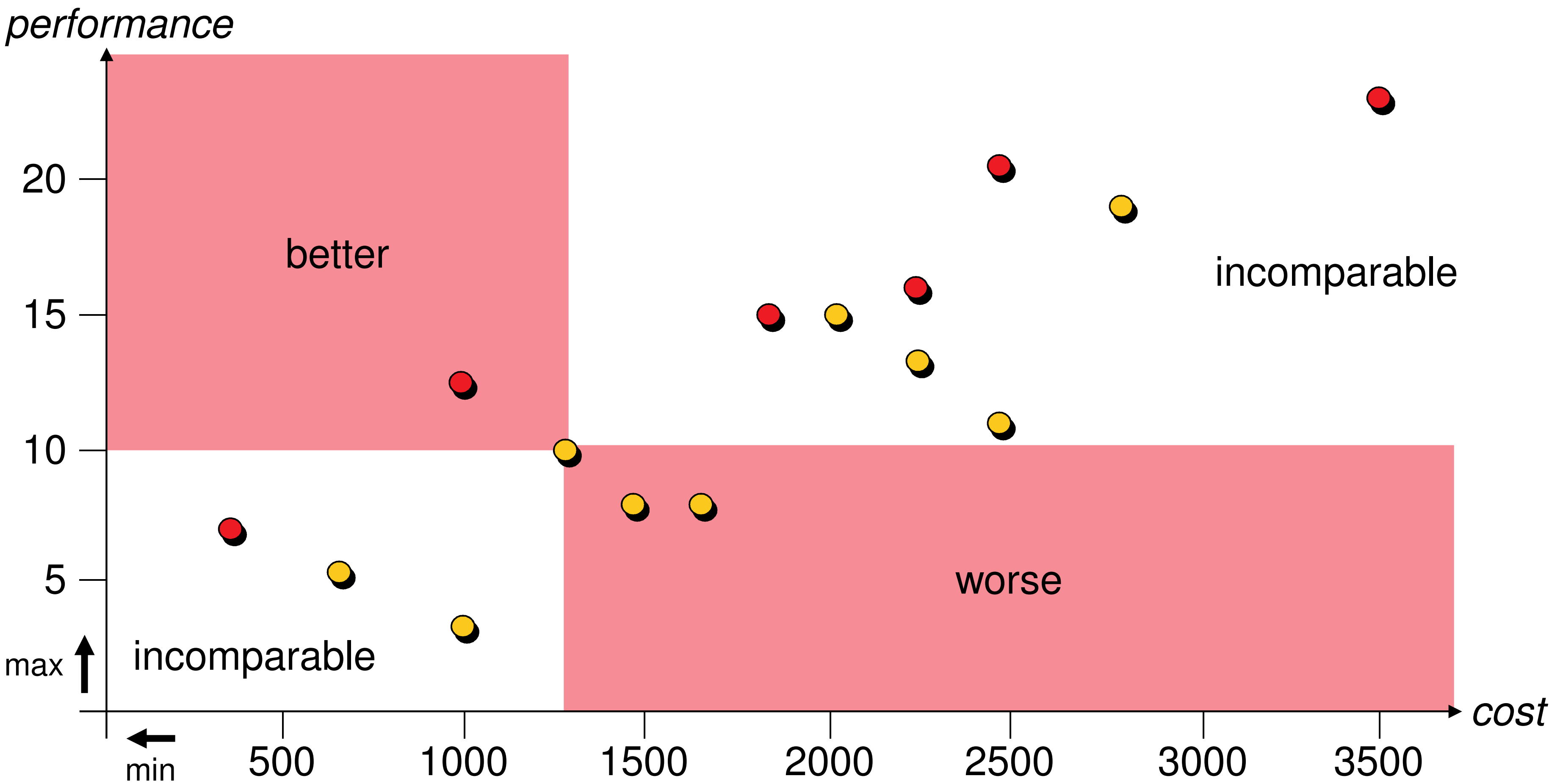
**Observations:** ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



# A Brief Introduction to Multiobjective Optimization

$$u \text{ weakly Pareto dominates } v \ (u \preccurlyeq_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

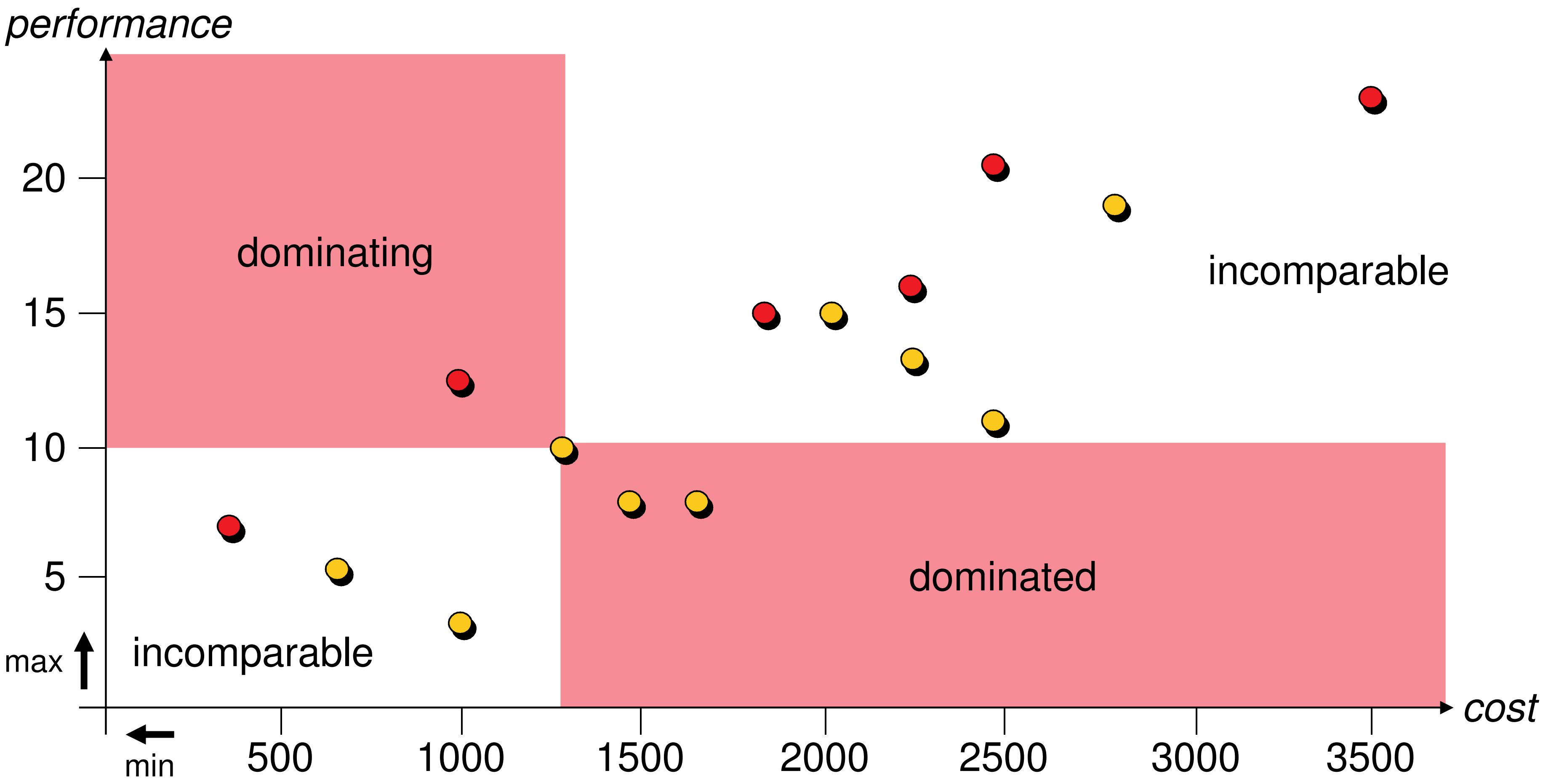
$$u \text{ Pareto dominates } v \ (u \prec_{par} v): \quad u \preccurlyeq_{par} v \ \wedge \ v \not\preccurlyeq_{par} u$$



# A Brief Introduction to Multiobjective Optimization

$$u \textit{ weakly Pareto dominates } v \ (u \leqslant_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

$$u \textit{ Pareto dominates } v \ (u <_{par} v): \quad u \leqslant_{par} v \ \wedge \ v \nleqslant_{par} u$$



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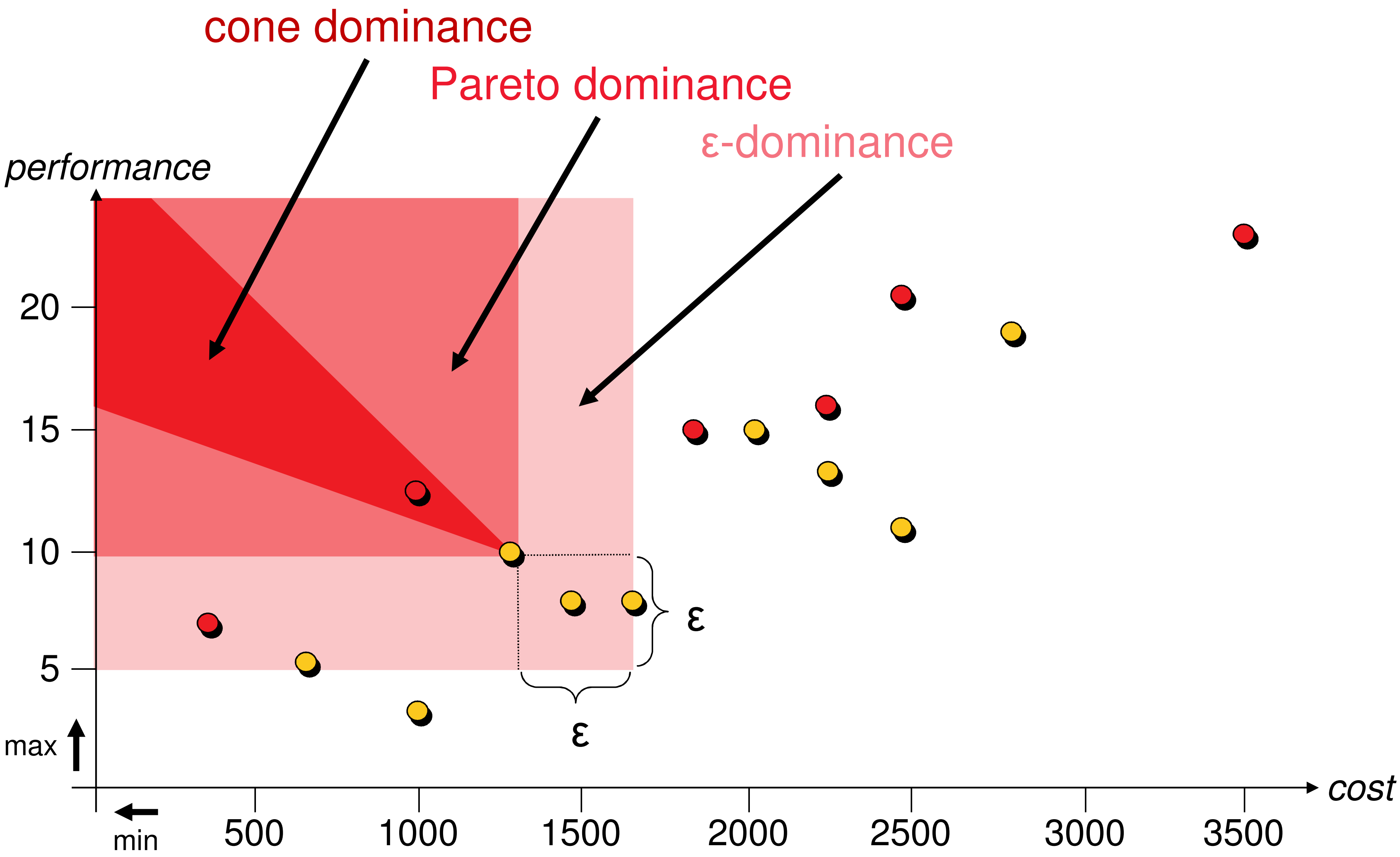
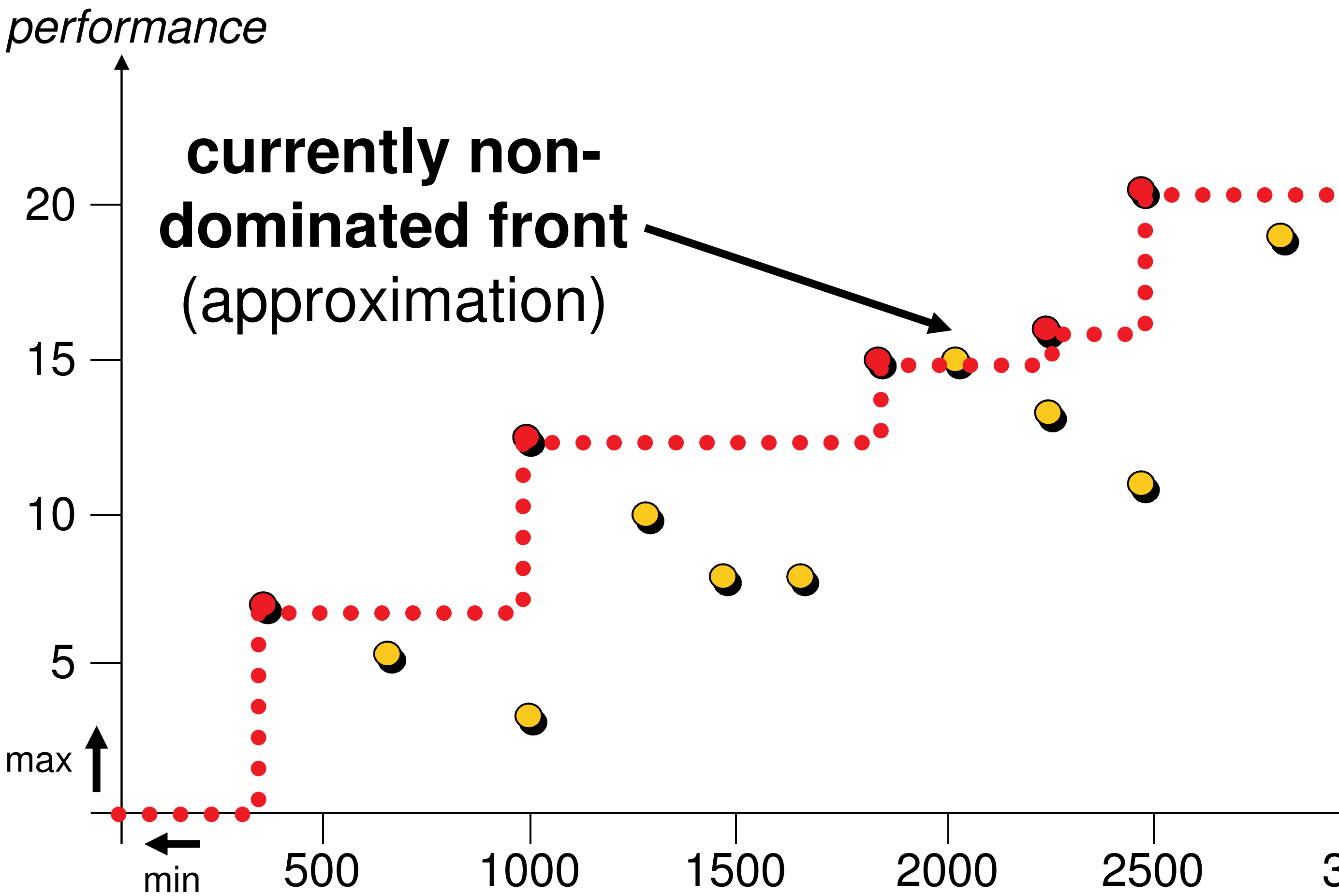
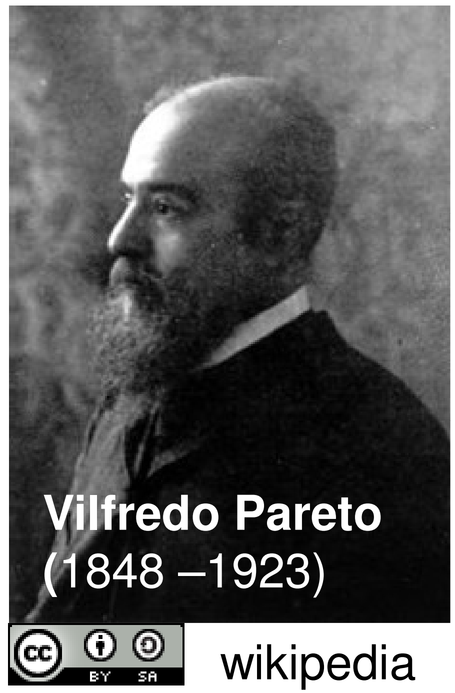


# A Brief Introduction to Multiobjective Optimization

**Pareto set:** set of all non-dominated solutions (decision space)
**Pareto front:** its image in the objective space



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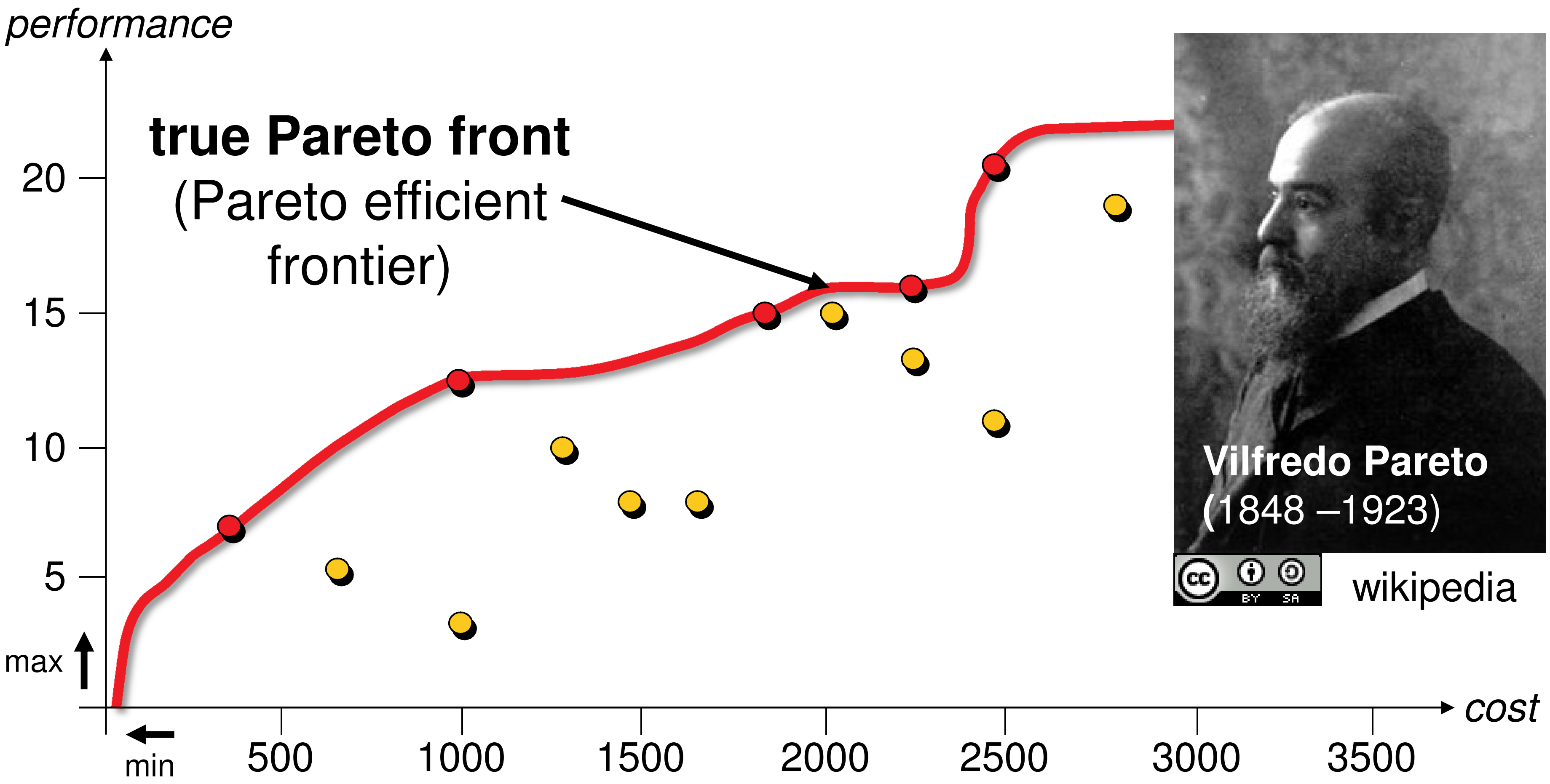

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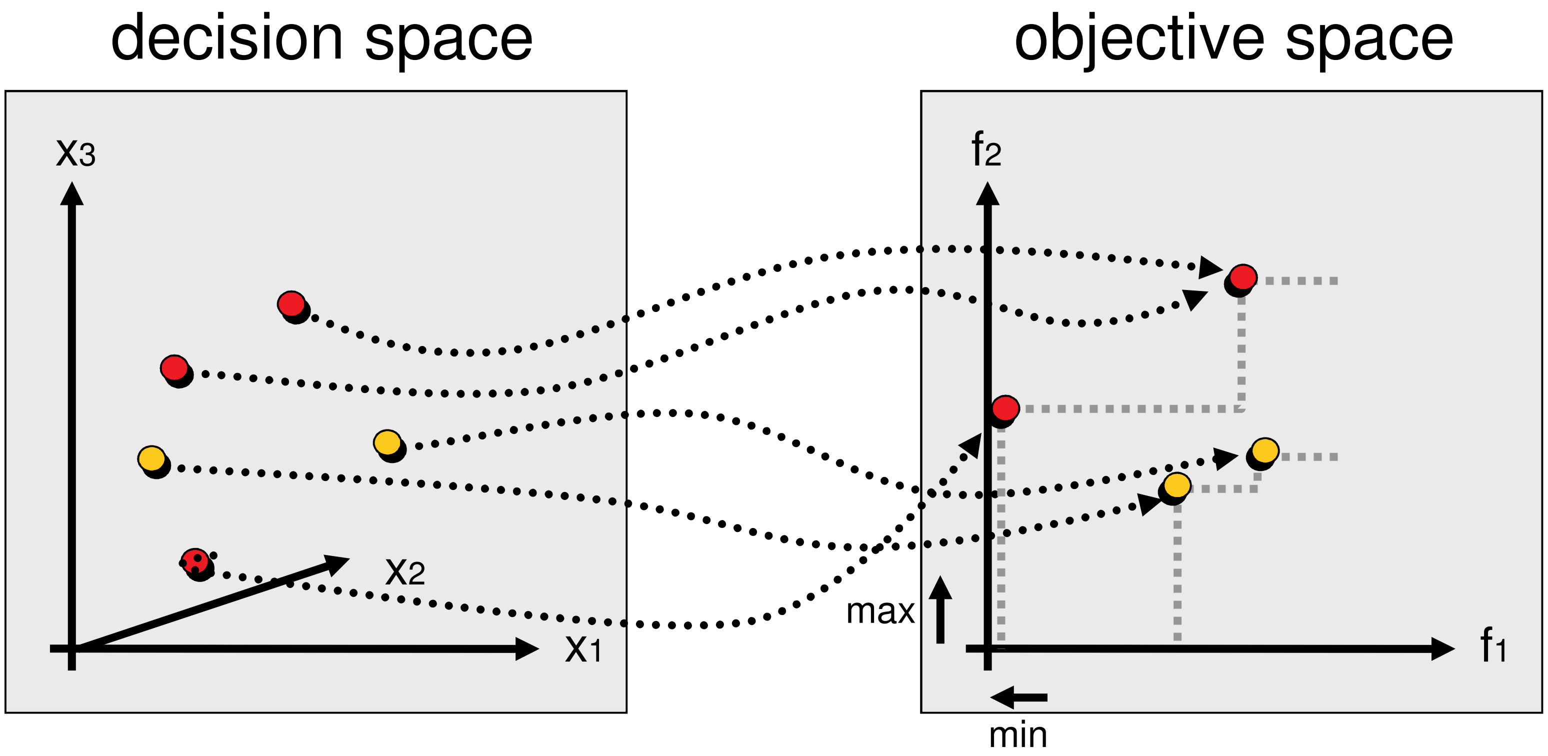
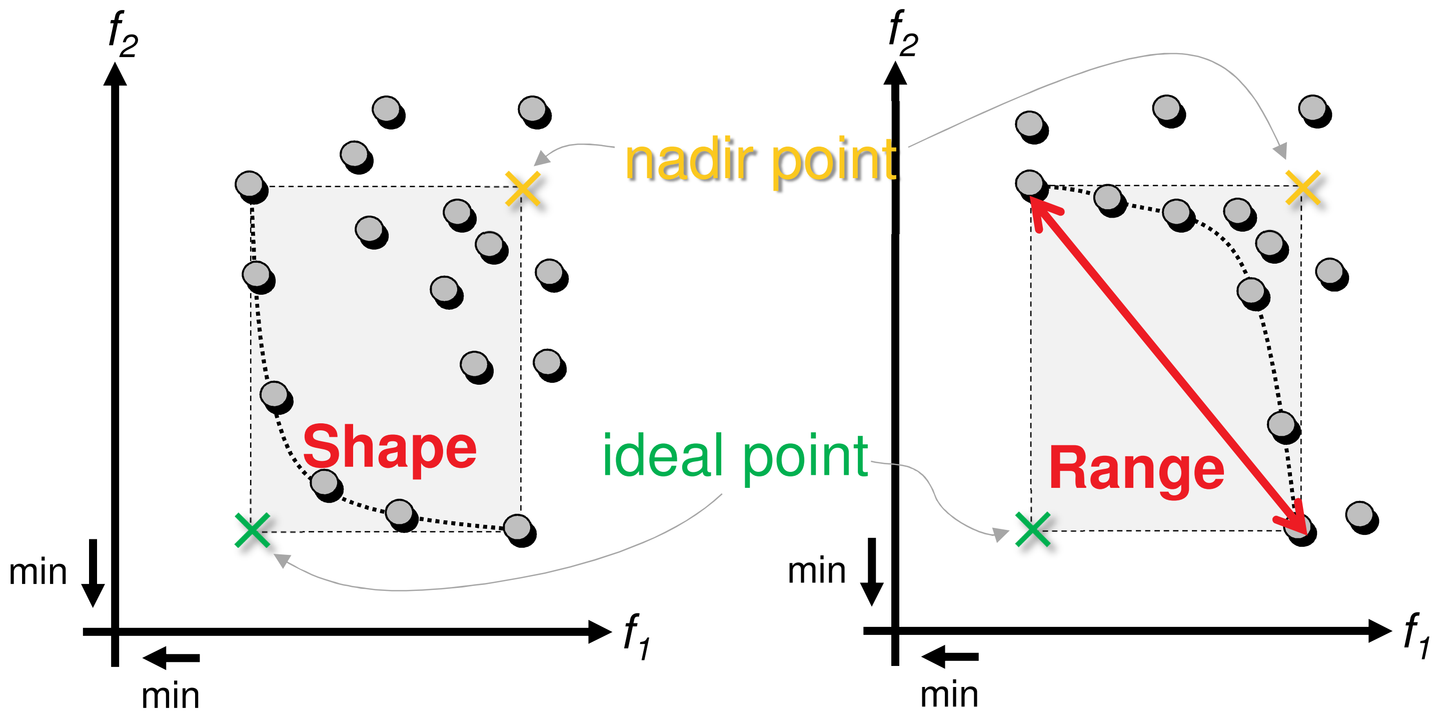



solution of Pareto-optimal set — vector of Pareto-optimal front

non-optimal decision vector — non-optimal objective vector

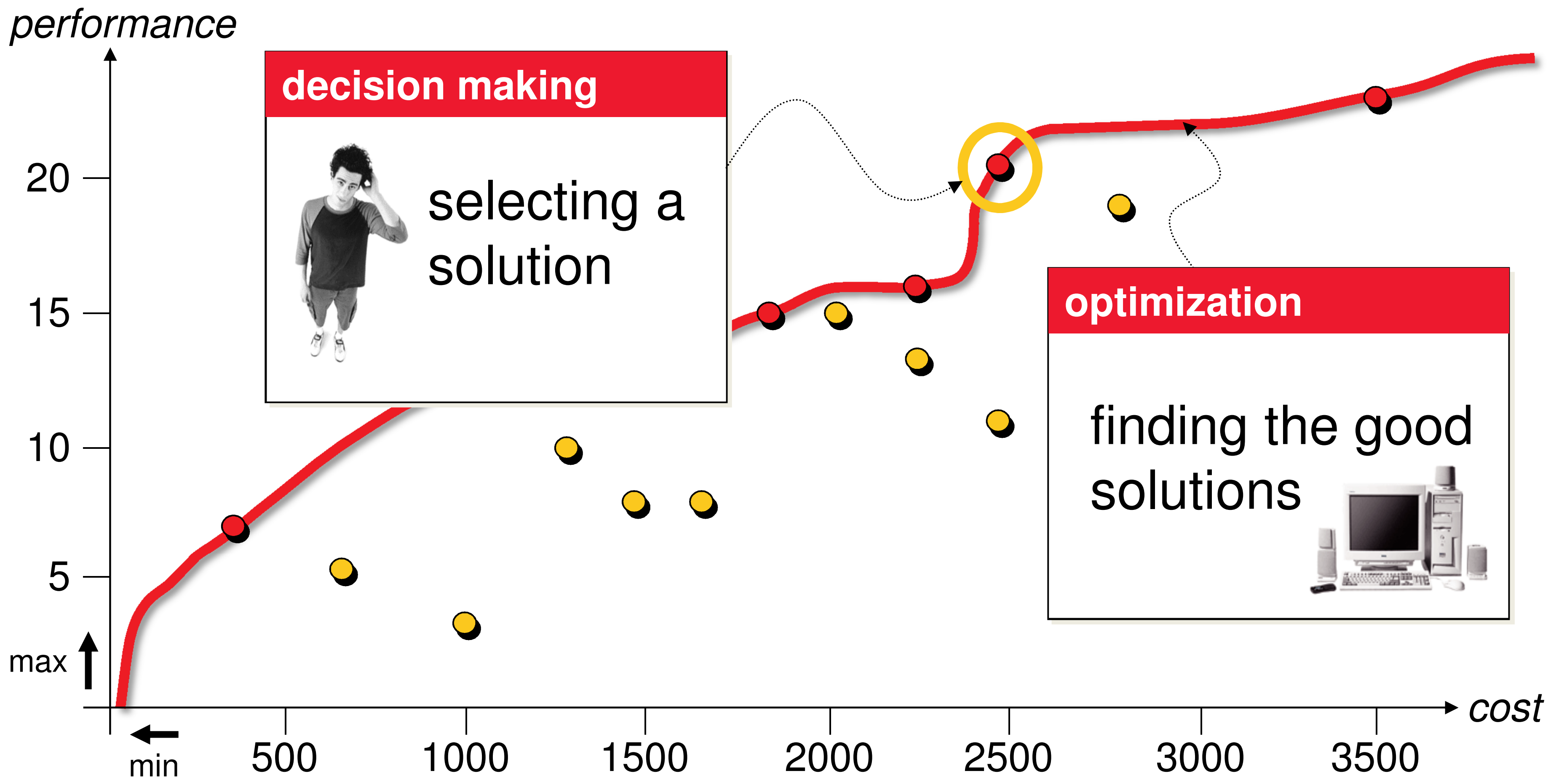
# A Brief Introduction to Multiobjective Optimization




ideal point: best values
nadir point: worst values
} obtained for *Pareto-optimal* points

# Optimization vs. Decision Making

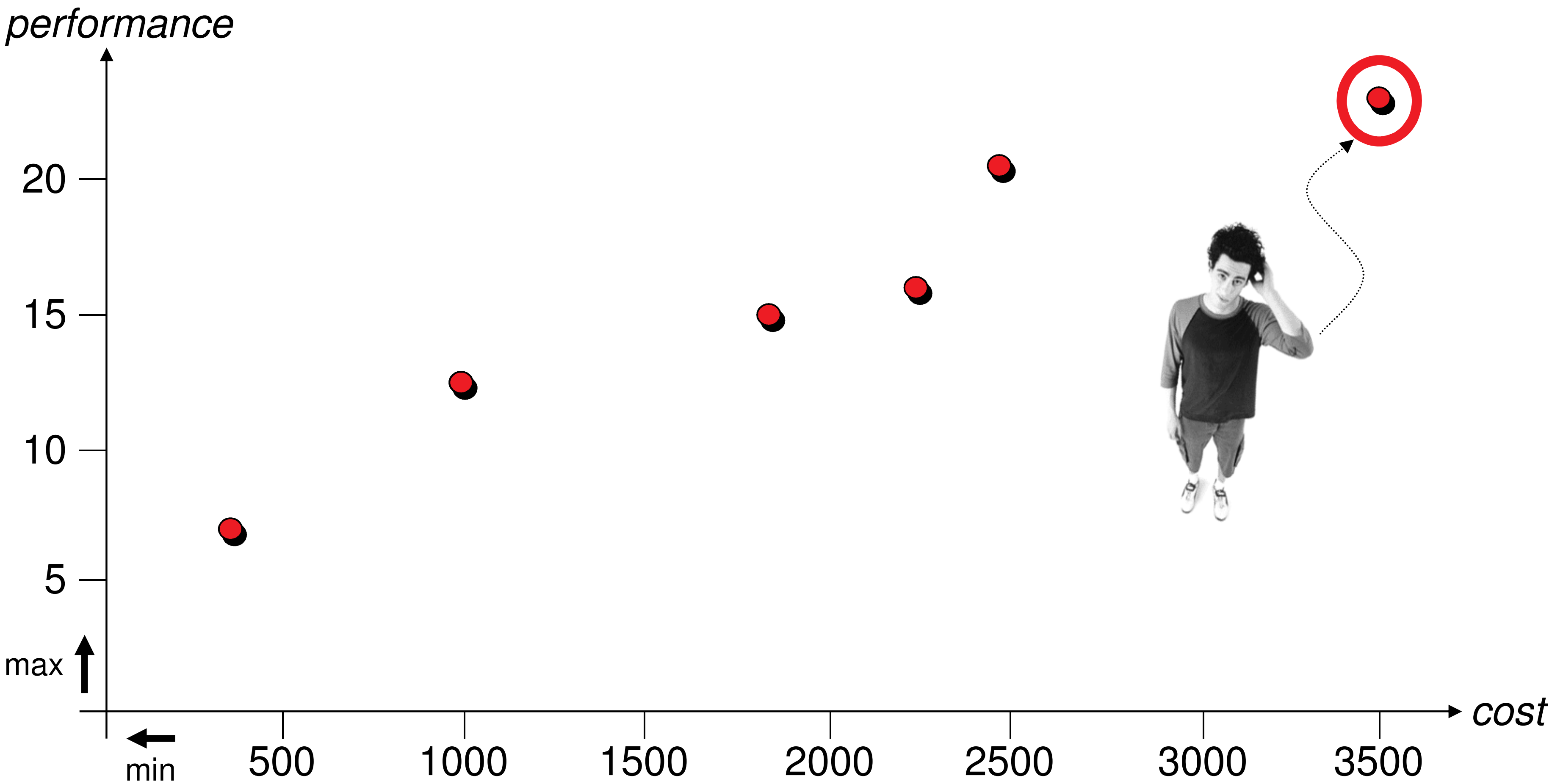
**Multiobjective Optimization**
combination of optimization of a set and a decision for a solution



# Selecting a Solution: Examples

**Possible Approaches:**

❶ **ranking:** performance more important than cost

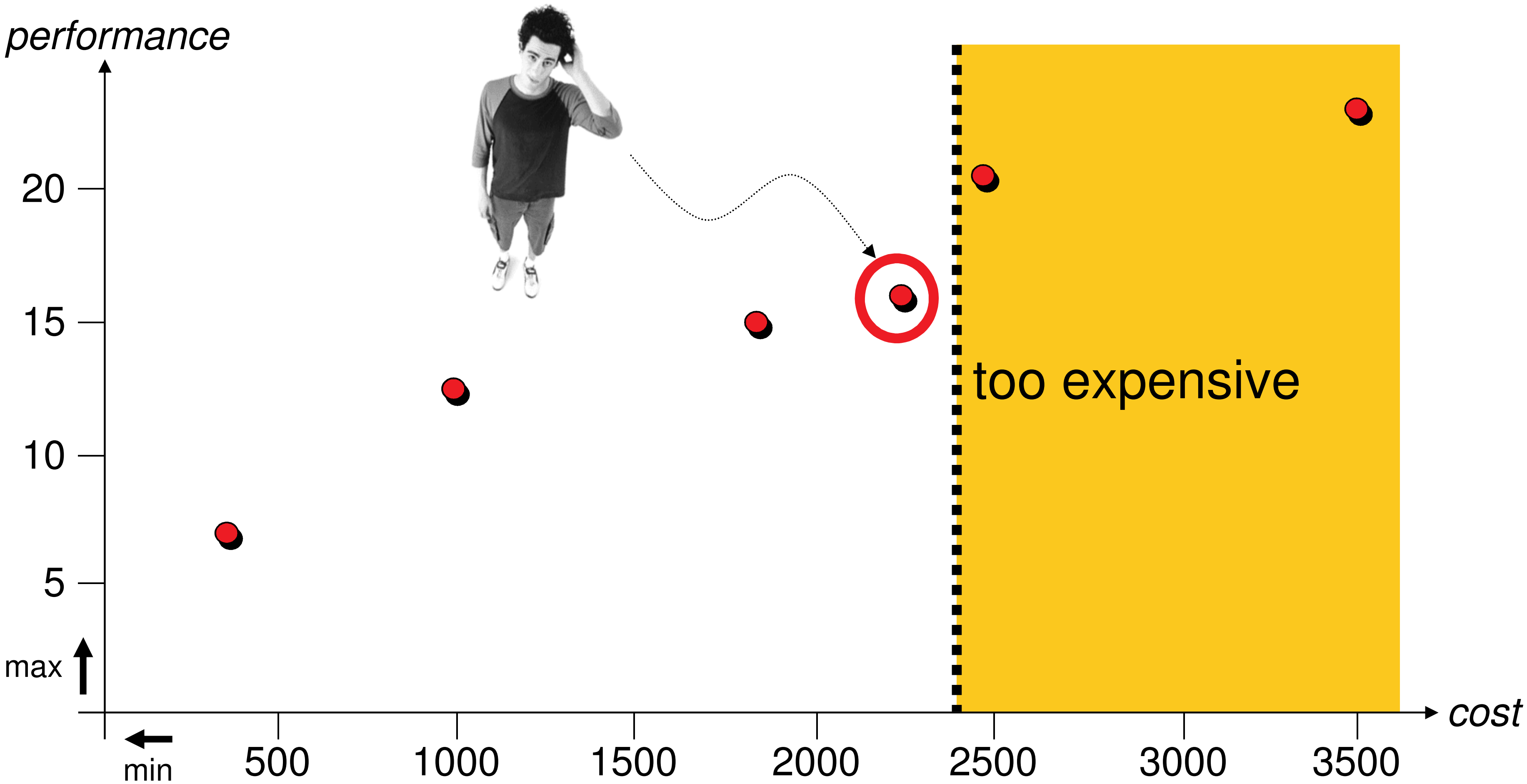


# Selecting a Solution: Examples

**Possible Approaches:**

❶ ranking: performance more important than cost

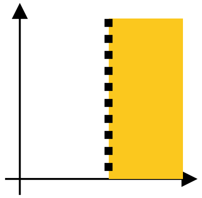
❷ **constraints:** cost must not exceed 2400



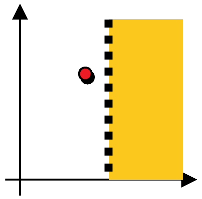
# When to Make the Decision

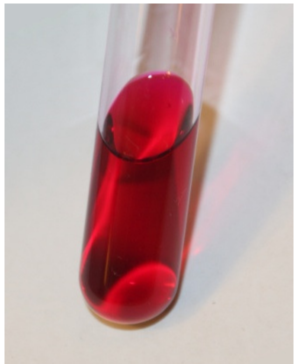
## Before Optimization:

rank objectives,
define constraints,…



search for one
(good) solution





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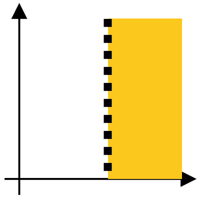

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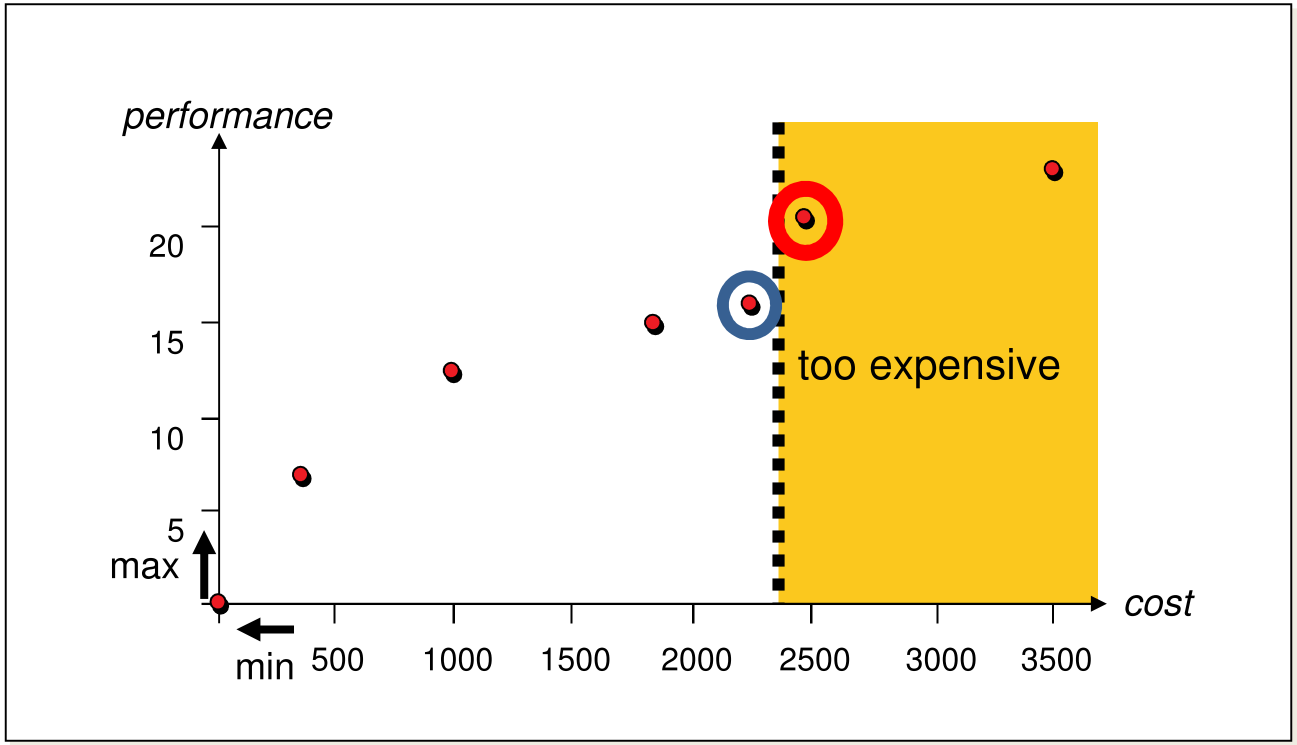

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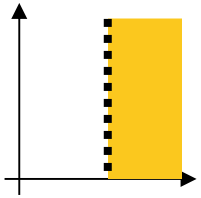

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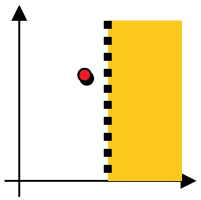
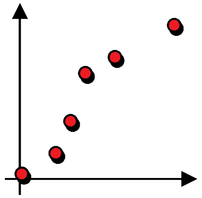
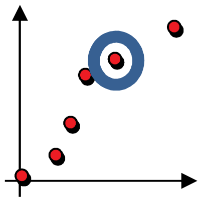


## After Optimization:

search for a set of
(good) solutions



select one solution
considering
constraints, etc.



# When to Make the Decision

## Before Optimization:

rank objectives,
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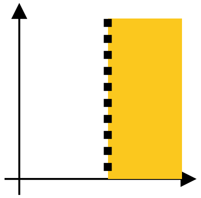

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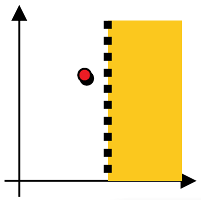

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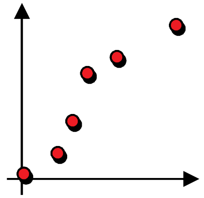

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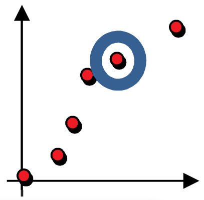


**Focus:** learning about a problem

- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

# Two Communities...

International Society on Multiple Criteria Decision Making

- established field (beginning in 1950s/1960s)
- bi-annual conferences since 1975
- background in economics, math, management and social sciences
- focus on optimization and decision making

- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background in computer science, applied math and engineering
- focus on optimization algorithms

# ...Slowly Merge Into One

International Society on
Multiple Criteria Decision Making

- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

## Blackbox optimization

$$x \in X \longrightarrow \boxed{f} \longrightarrow (f_1(x), \ldots, f_k(x))$$

only mild assumptions

→ EMO therefore well-suited for real-world engineering problems

**objectives**

non-linear

noisy

uncertain

non-differentiable

expensive (integrated simulations, real experiments)

**problem**

many objectives

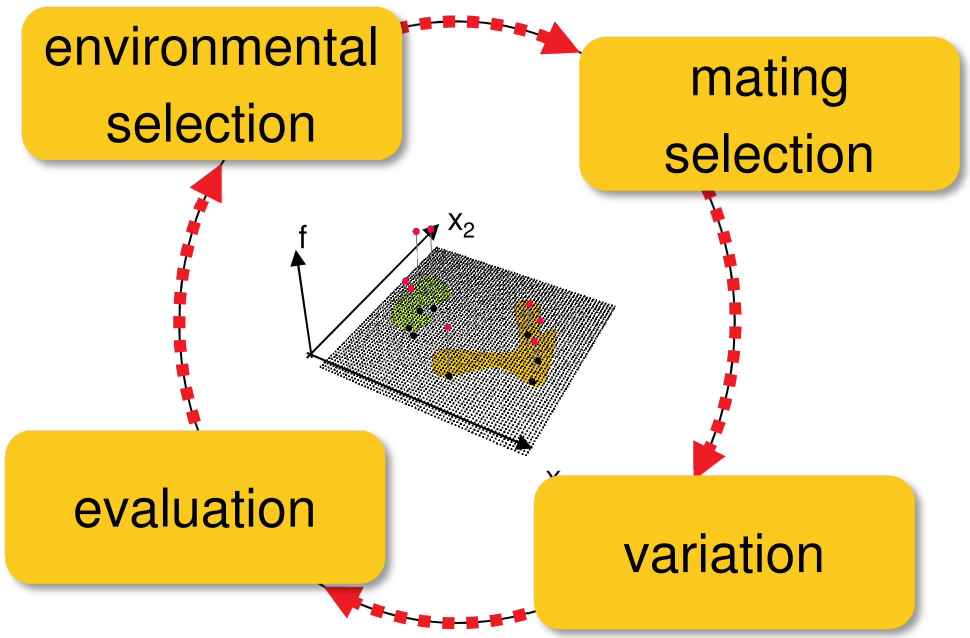
huge search spaces

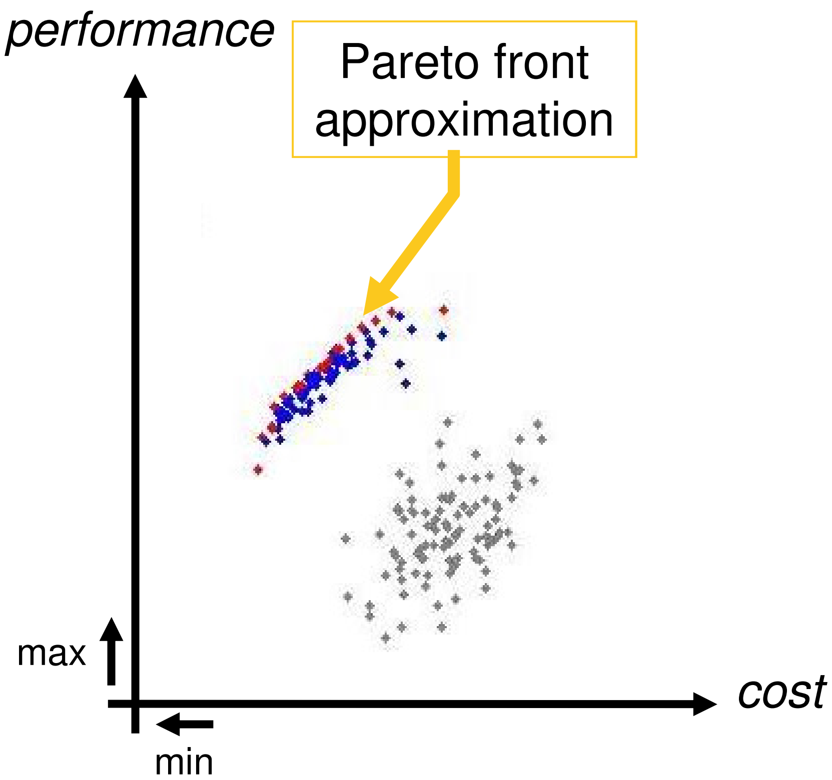
many constraints

# The Other Main Difference

## Evolutionary Multiobjective Optimization

- set-based algorithms
- therefore possible to approximate the Pareto front in one run

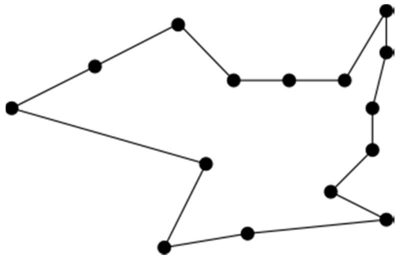


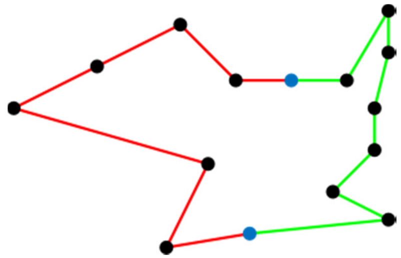
# Multiobjectivization

Some problems are easier to solve in a multiobjective scenario

example: TSP
[Knowles et al. 2001]



$\pi \in S_n \to f(\pi)$



$\pi \in S_n \to (f_1(\pi, a, b), f_2(\pi, a, b))$

**Multiobjectivization**

by addition of new "helper objectives" [Jensen 2004]

job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], VRP [Watanabe and Sakakibara 2007], ...

by decomposition of the single objective

TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], ...

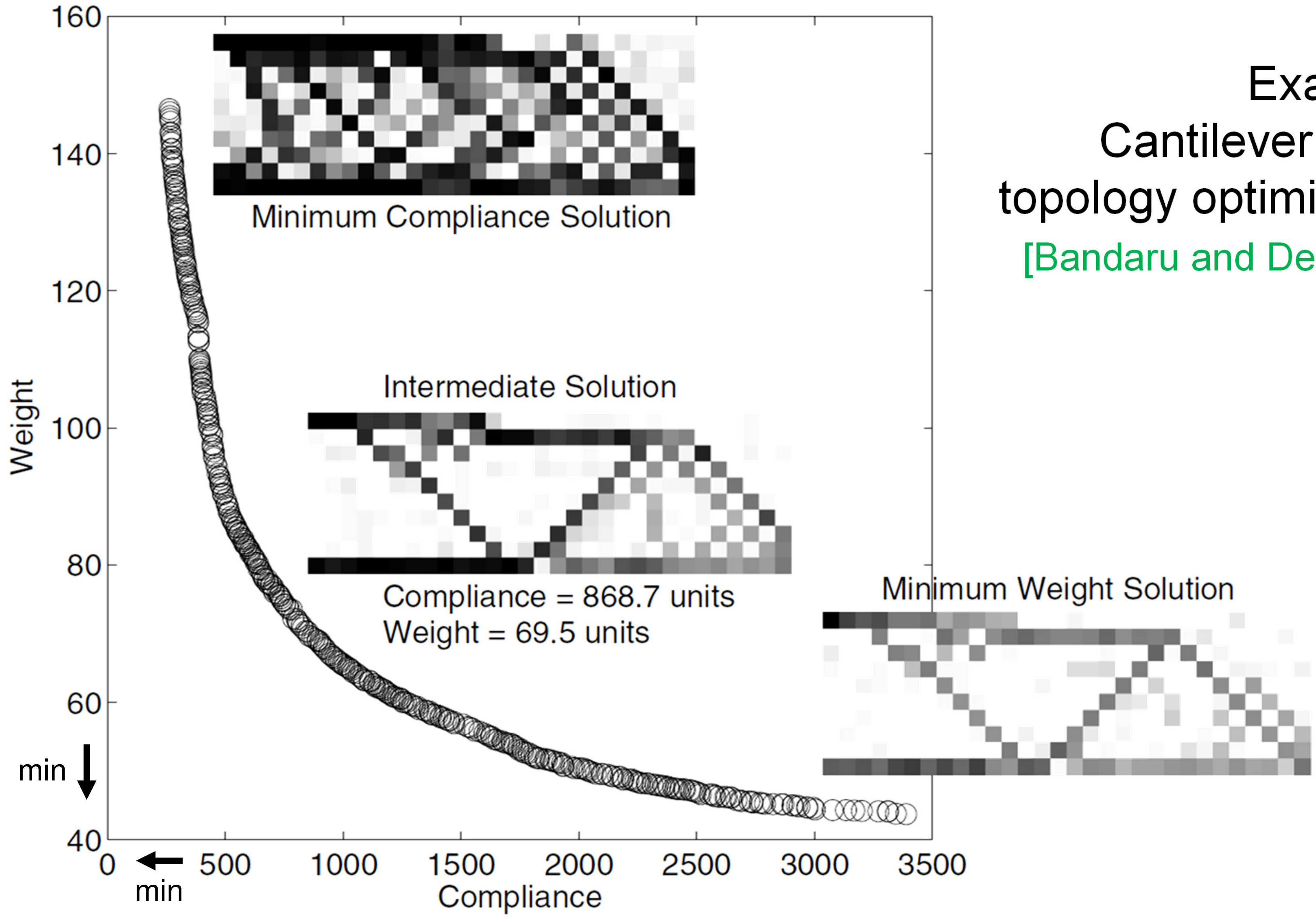
also backed up by theory e.g. [Brockhoff et al. 2009, Handl et al. 2008b]

related to *constrained* and *multimodal* single-objective optimization

see also this recent overview: [Segura et al. 2013]

# Innovization

Often innovative design principles among solutions are found




Example:
Cantilever beam
topology optimization
[Bandaru and Deb 2015]

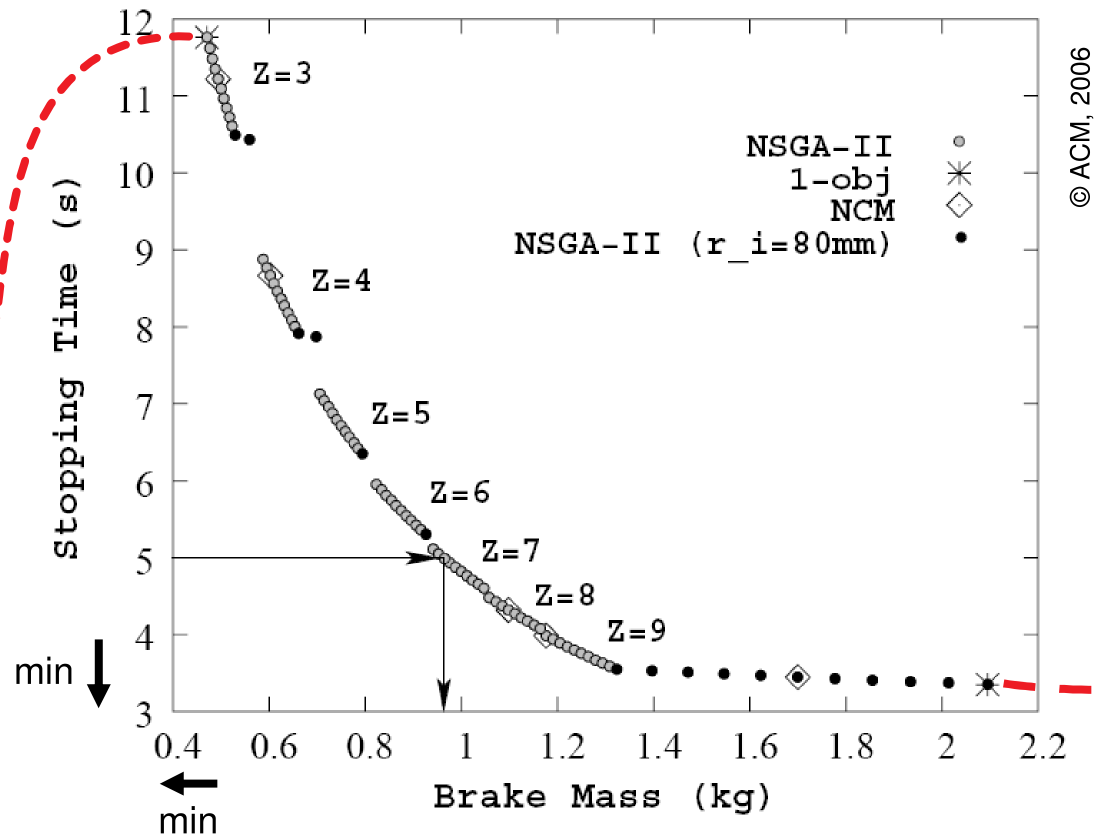
# Innovization

Often innovative design principles among solutions are found

Example:

Clutch brake design

[Deb and Srinivasan 2006]




| Solution | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $f_1$ | $f_2$ |
|---|---|---|---|---|---|---|---|
| Min. $f_1$ | 70 | 90 | 1.5 | 1000 | 3 | 0.4704 | 11.7617 |
| Min. $f_2$ | 80 | 110 | 1.5 | 1000 | 9 | 2.0948 | 3.3505 |

# Innovization

Often innovative design principles among solutions are found

**Innovization** [Deb and Srinivasan 2006]

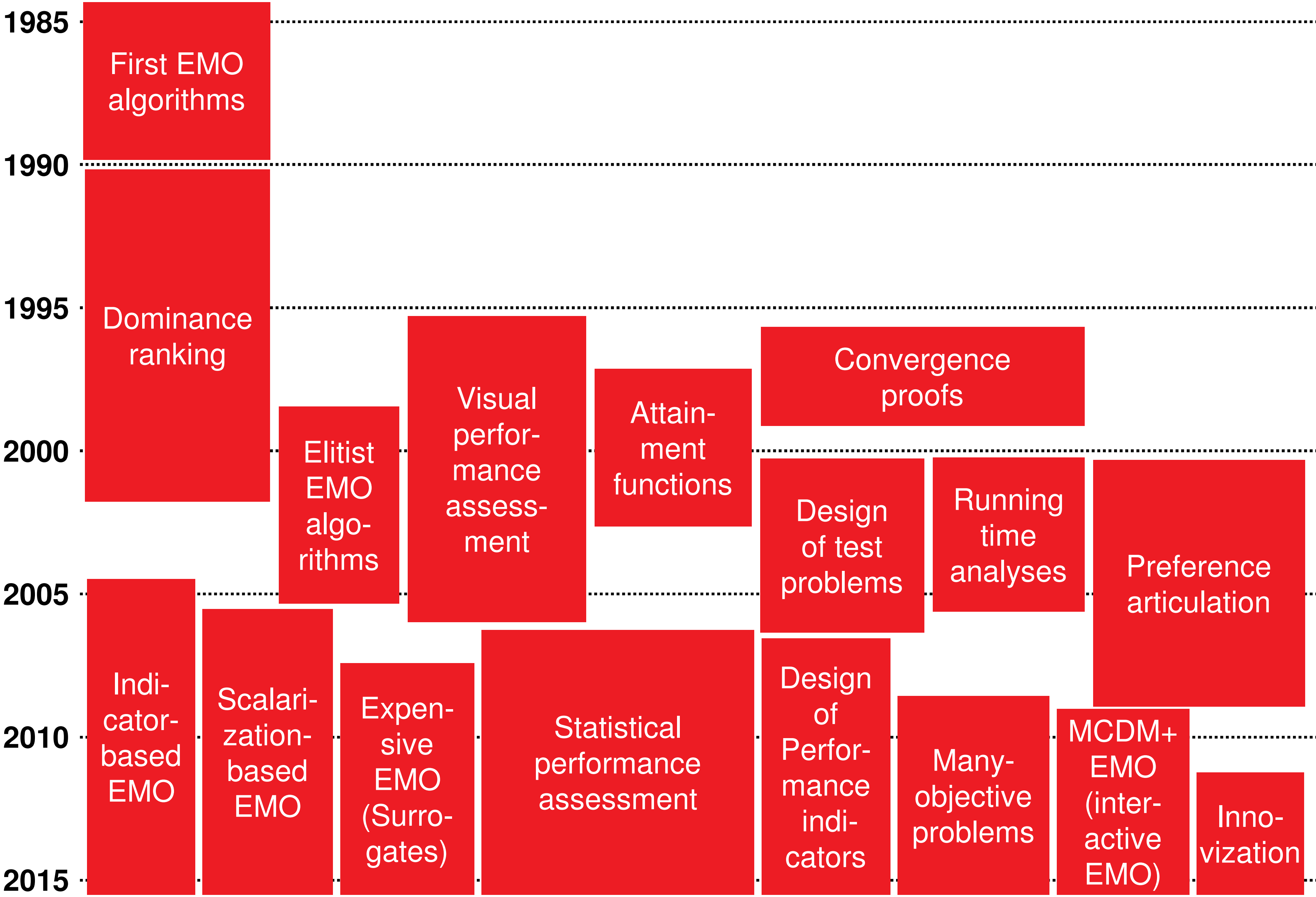
= using machine learning techniques to find new and innovative design principles among solution sets

= learning from/about a multi-objective optimization problem

**Other examples:**

- SOM for supersonic wing design [Obayashi and Sasaki 2003]
- Biclustering for processor design and knapsack [Ulrich et al. 2007]
- Successful case studies in engineering (noise barrier design, polymer extrusion, friction stir welding) [Deb et al. 2014]

# The History of EMO At A Glance



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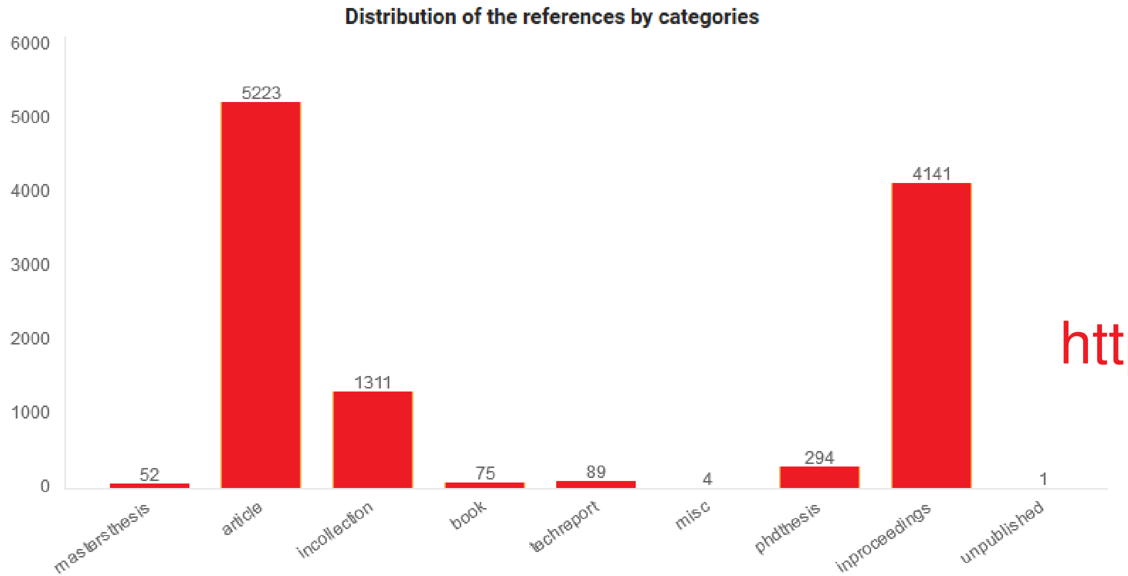
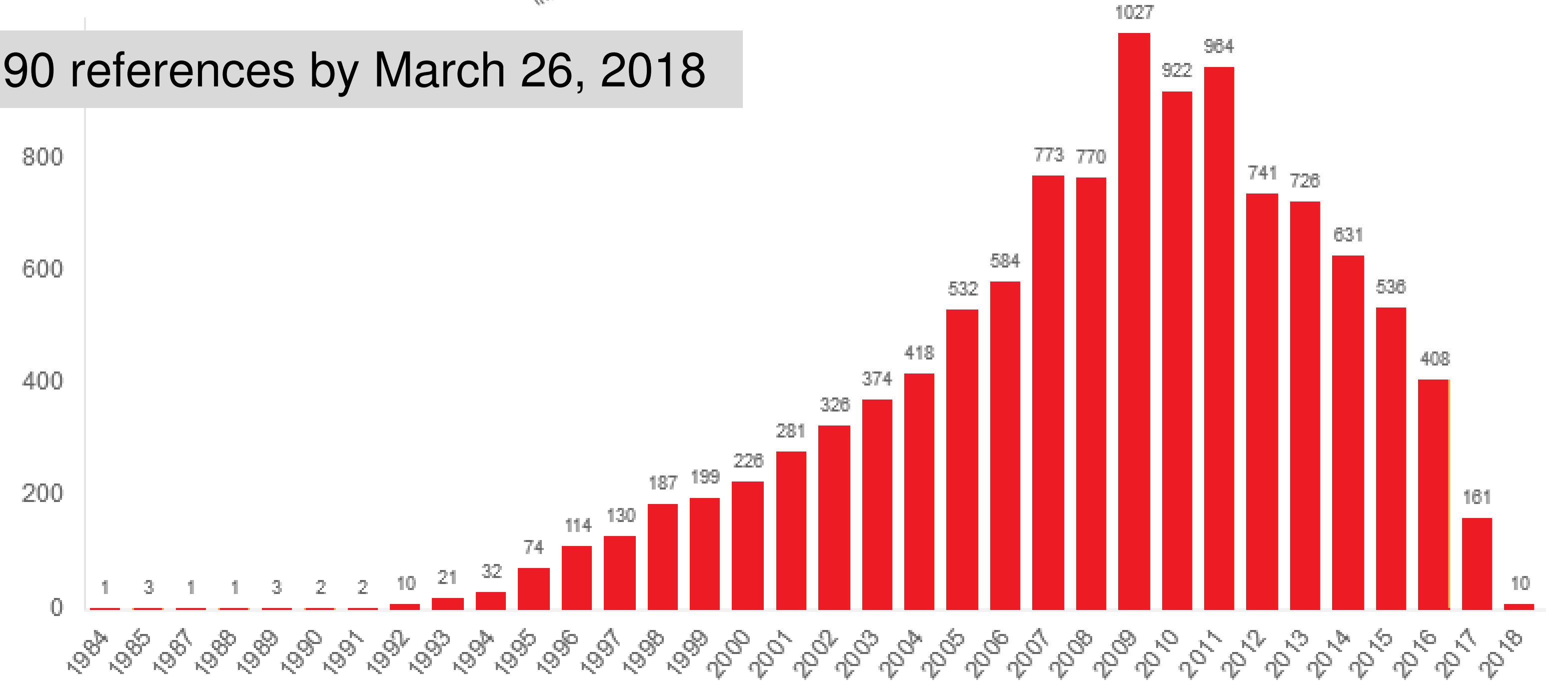



https://emoo.cs.cinvestav.mx/

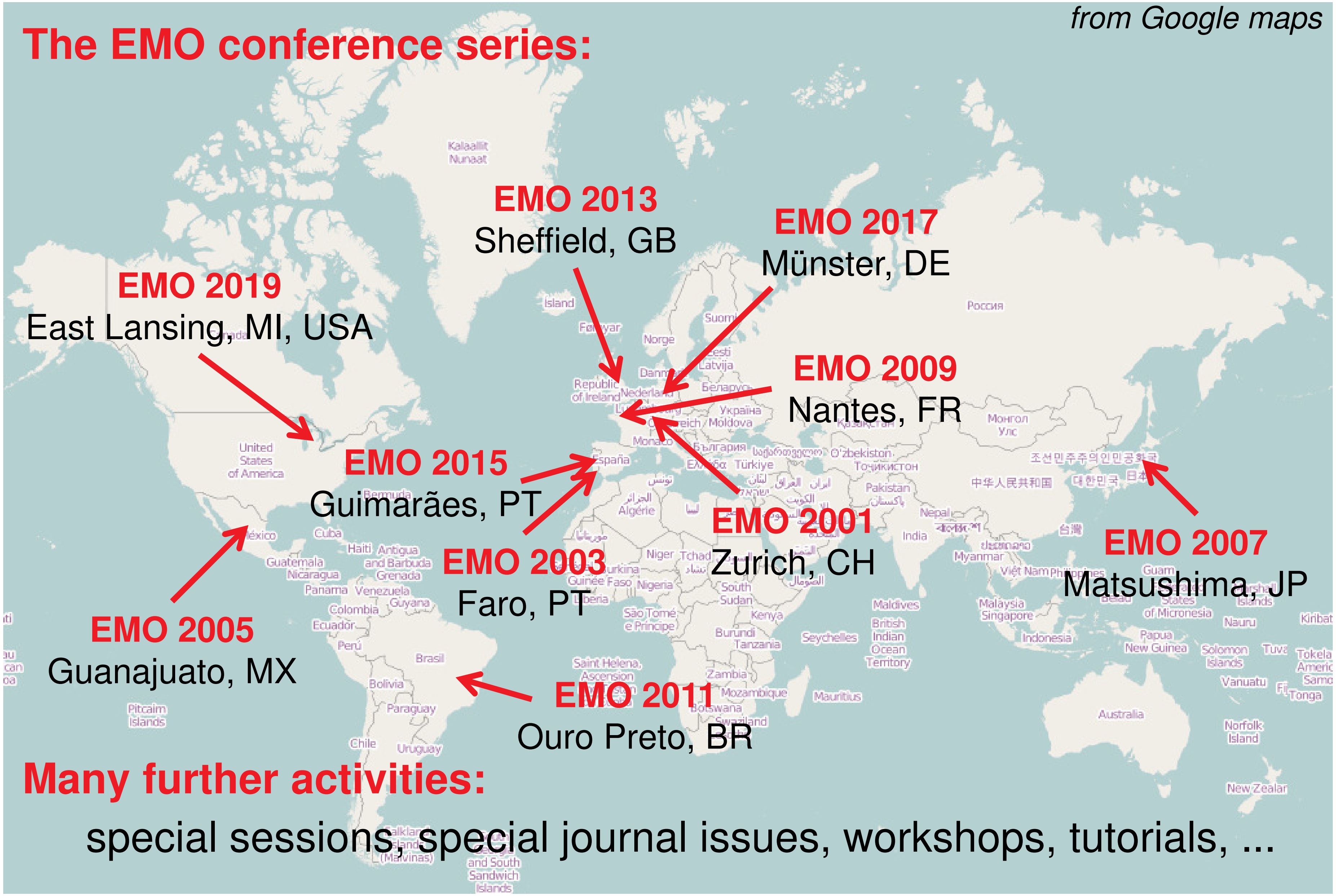
Overall: 11190 references by March 26, 2018



# The EMO Community

*from Google maps*

## The EMO conference series:




## Many further activities:

special sessions, special journal issues, workshops, tutorials, ...

# Overview

The Big Picture

Basic Algorithm Design Principles and Concepts

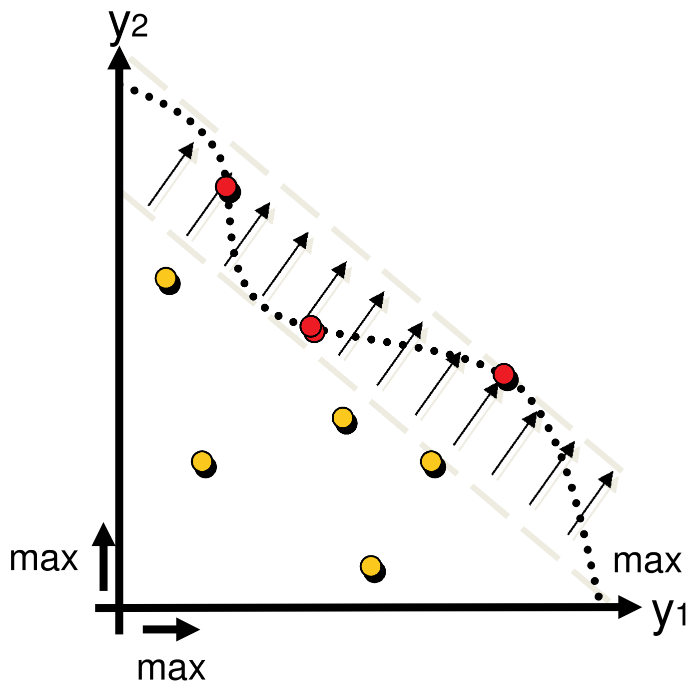
Performance Assessment and Benchmarking

Preference Articulation

# Fitness Assignment: Principal Approaches

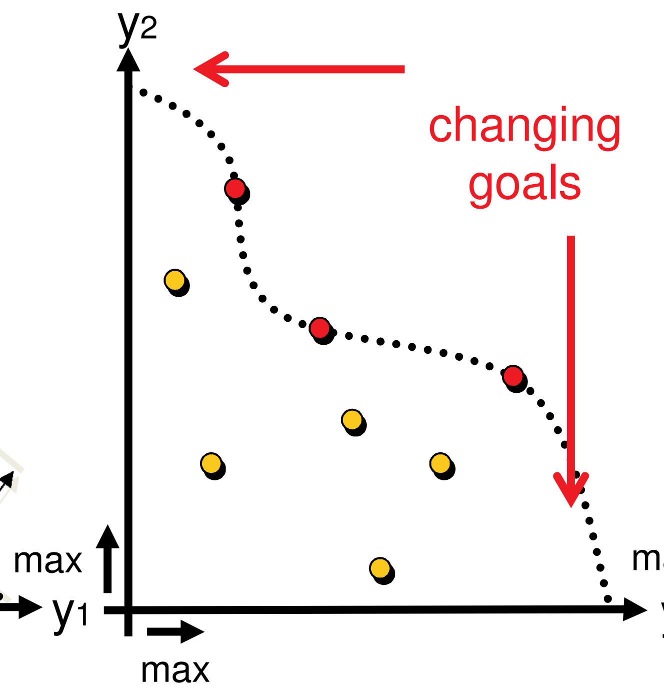
**aggregation-based**

*problem decomposition (multiple single-objective optimization problems)*



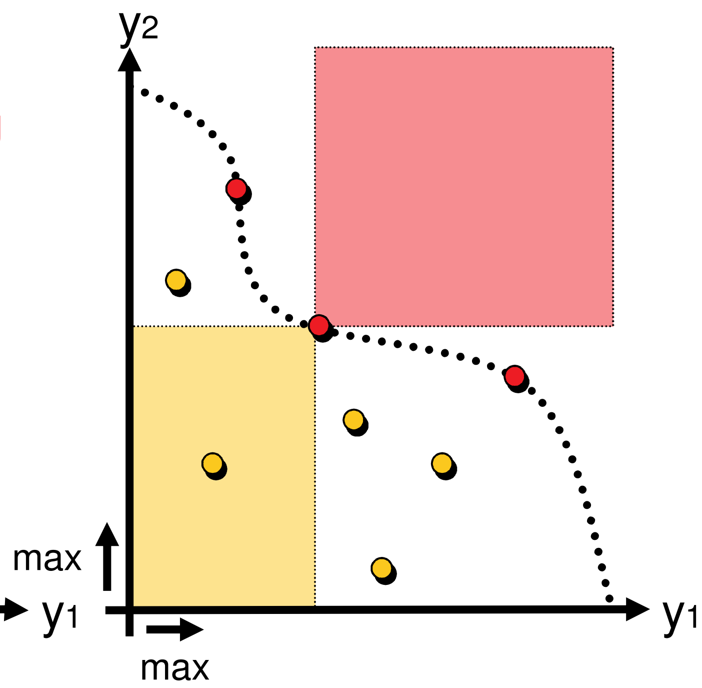

**criterion-based**

*VEGA*




**dominance-based**

*SPEA2, NSGA-II*
*“modern” EMOA*




solution-oriented
scaling-dependent

set-oriented
scaling-independent

# Solution-Oriented Problem Transformations

parameters

multiple objectives $(f_1(x), f_2(x), \ldots, f_k(x)) \longrightarrow$ **transformation** $\longrightarrow$ single objective $s(x)$

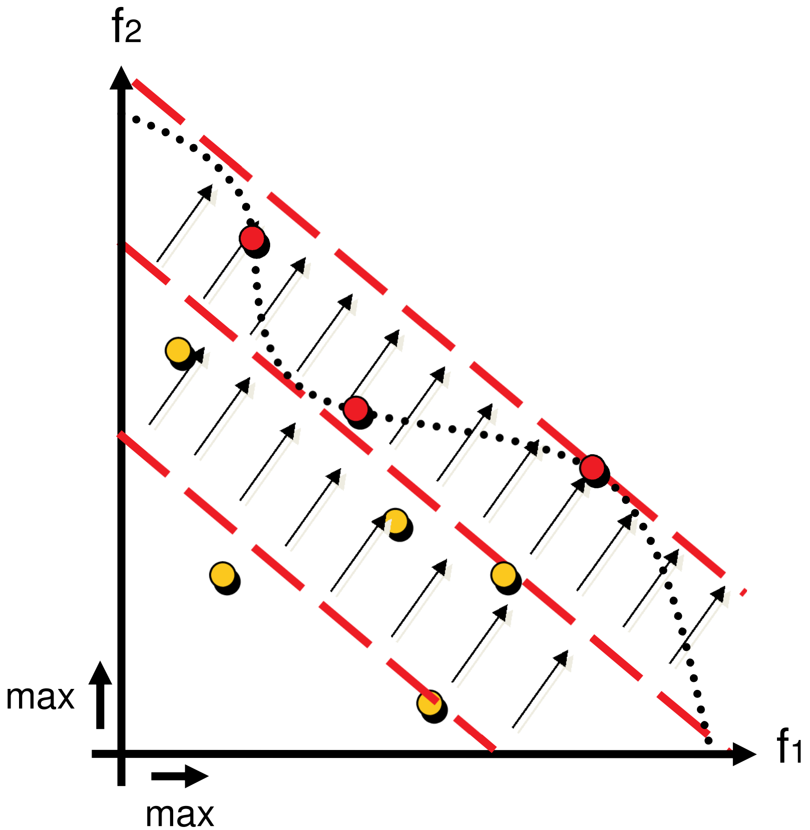
---

A scalarizing function $s$ is a function $s : Z \to \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

# Solution-Oriented Problem Transformations

multiple objectives

$(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** (parameters ↓) → single objective $s(x)$



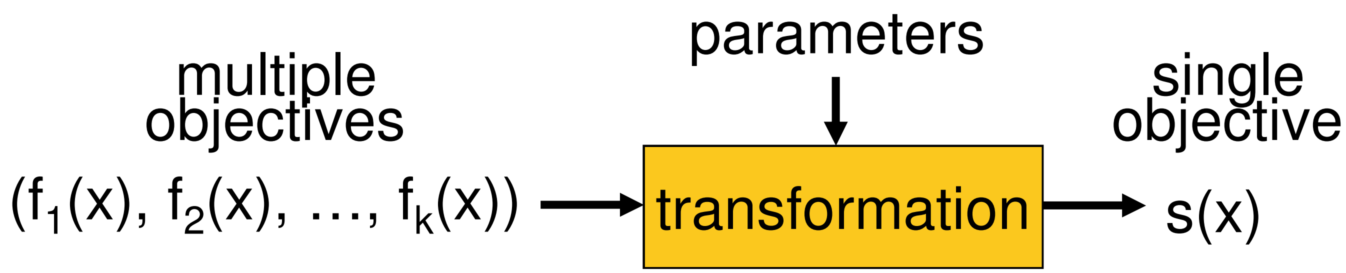

**Example 1:** weighted sum approach

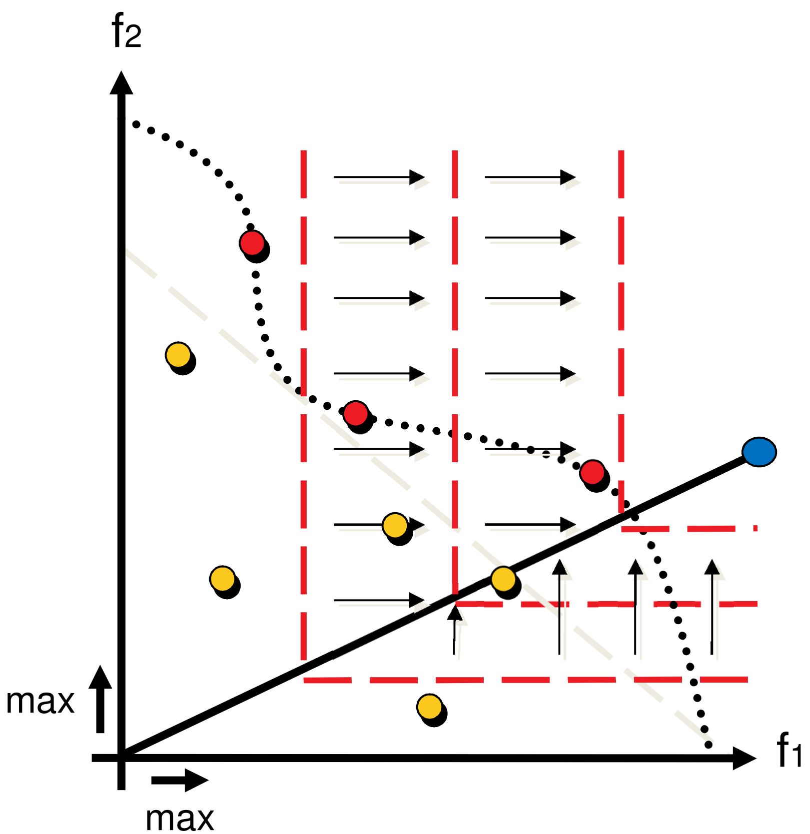
$(w_1, w_2, \ldots, w_k)$ ↓

→ $y = w_1 y_1 + \ldots + w_k y_k$ →

Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex

# Solution-Oriented Problem Transformations



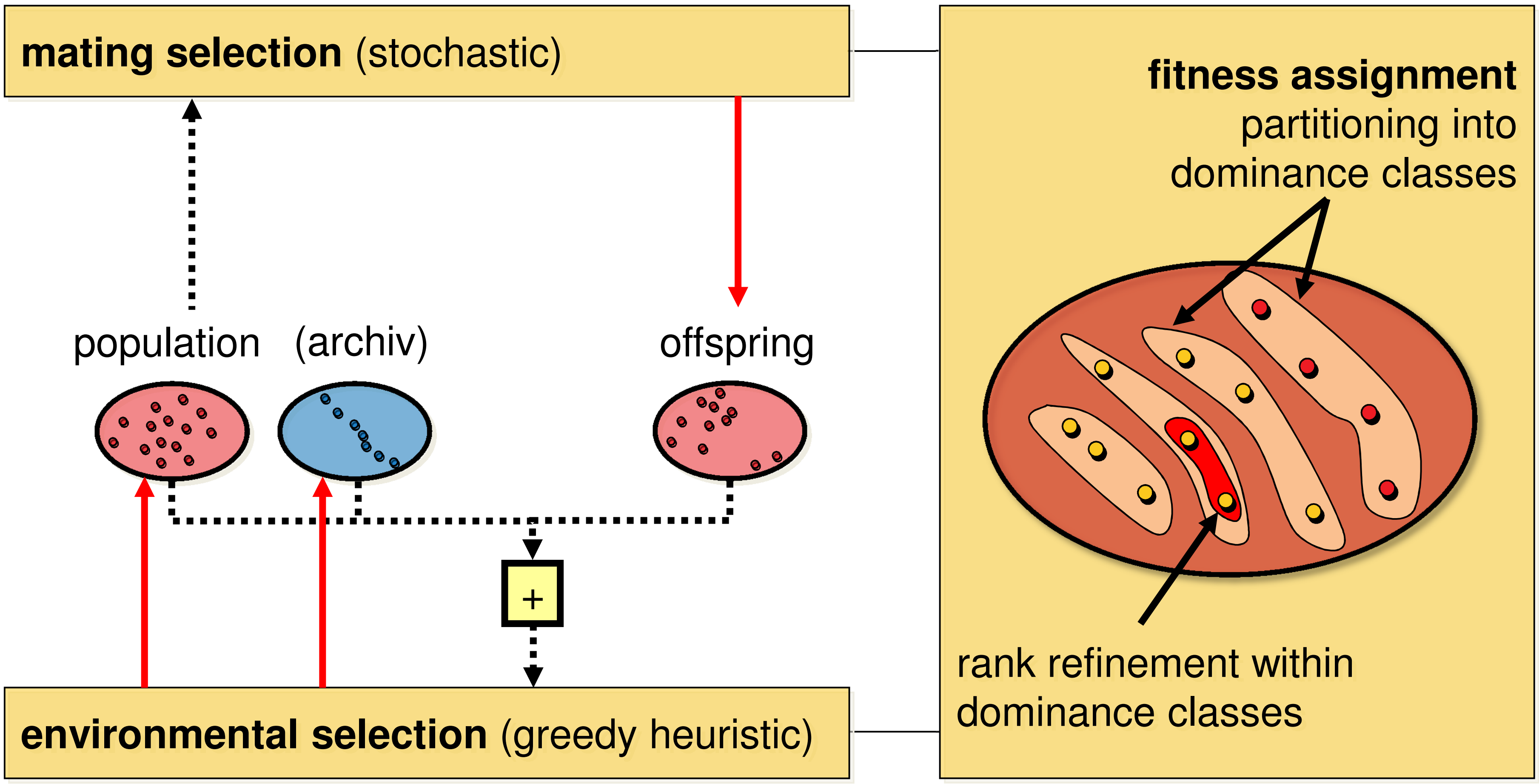

**Example 2:** weighted Tchebycheff

$$(\lambda_1, \lambda_2, \ldots, \lambda_k)$$

$$y = \max_i | \lambda_i(z_i - y_i) |$$

Several other scalarizing functions are known, see e.g. [Miettinen 1999]

# General Scheme of Most Set-Oriented EMO

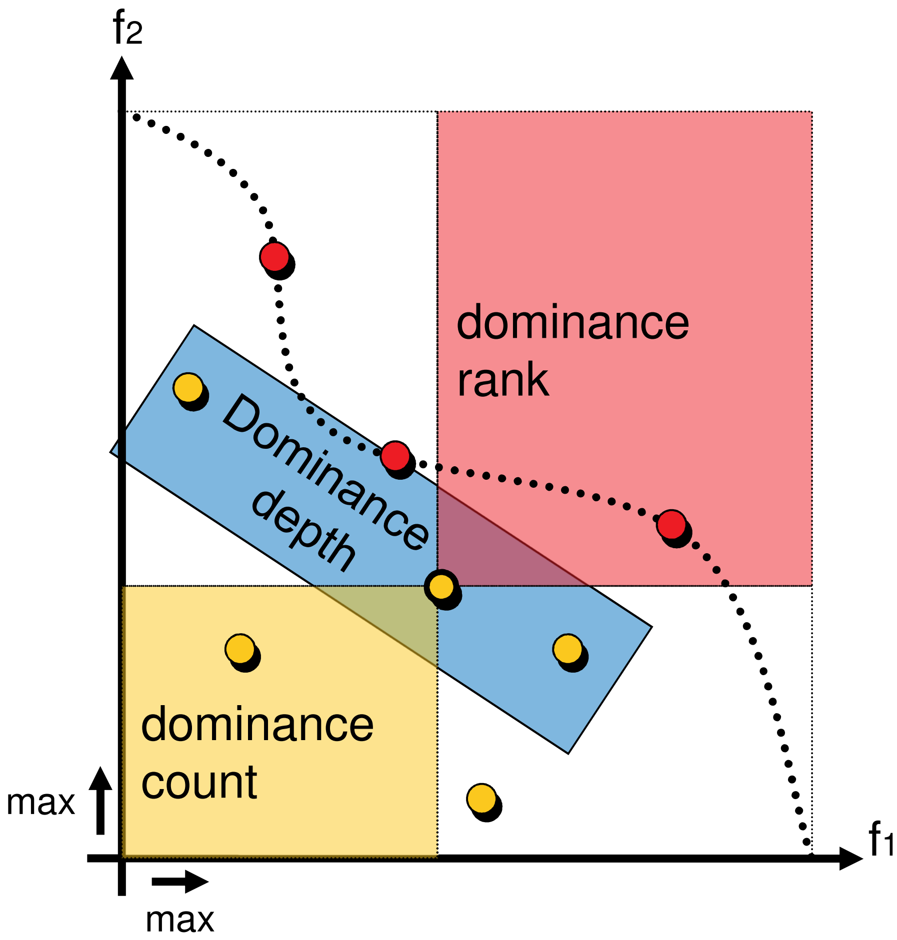


# Ranking of the Population Using Dominance

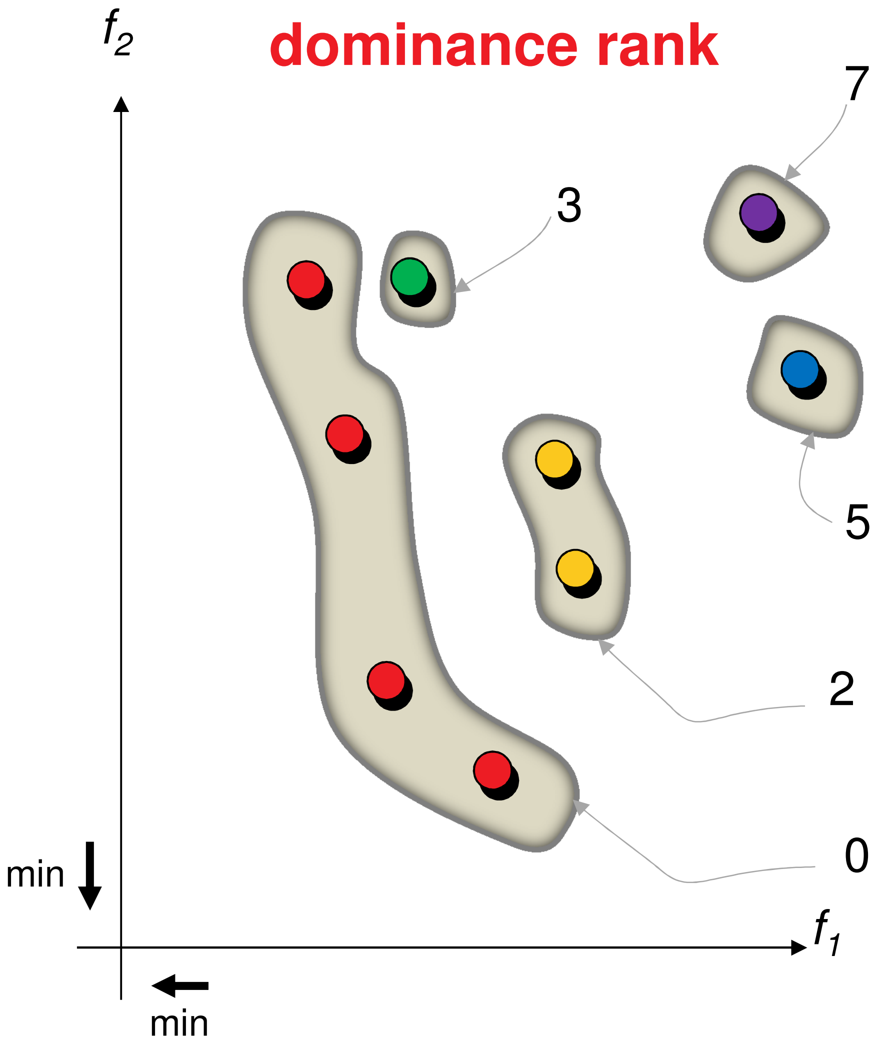
... goes back to a proposal by David Goldberg in 1989.

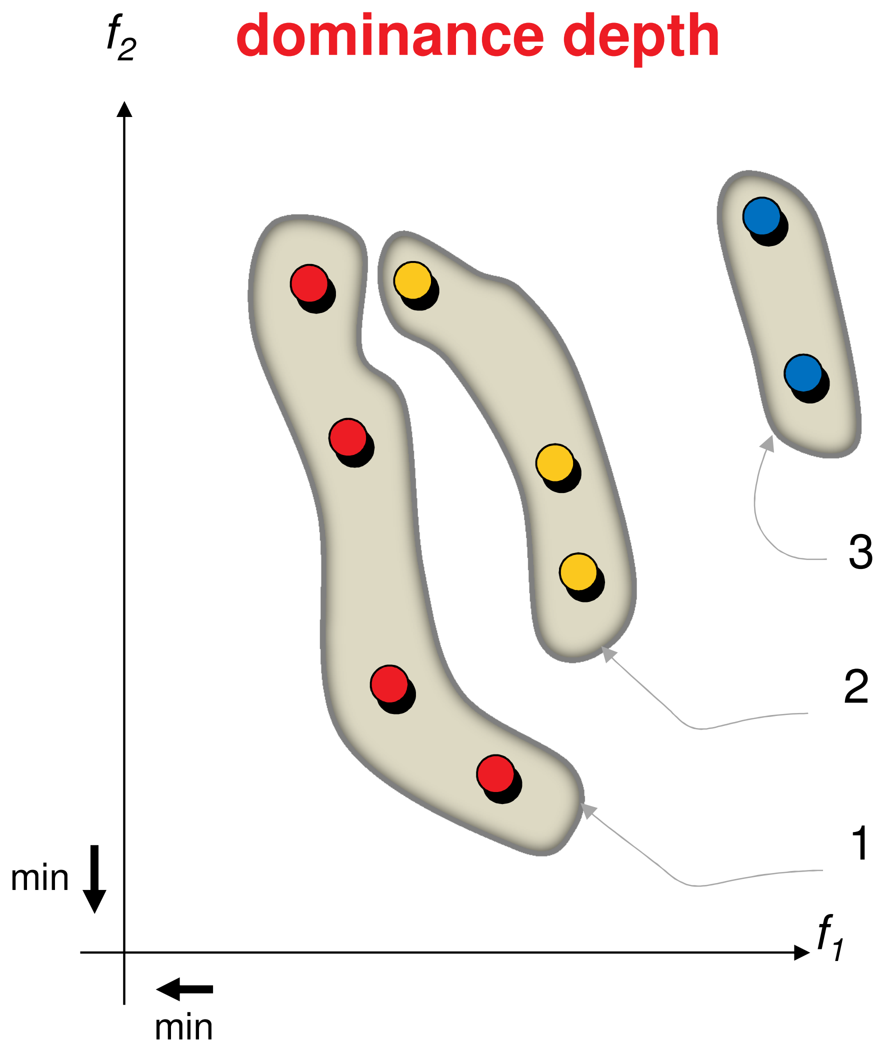
... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  *MOGA, NPGA*
- **dominance count:** how many individuals does an individual dominate?
  *SPEA, SPEA2*
- **dominance depth:** at which front is an individual located?
  *NSGA, NSGA-II, most of the recently proposed algorithms*



# Illustration of Dominance-Based Partitioning



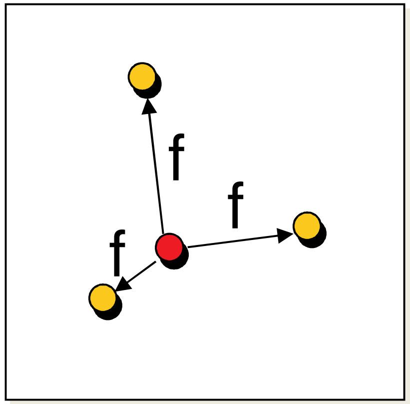



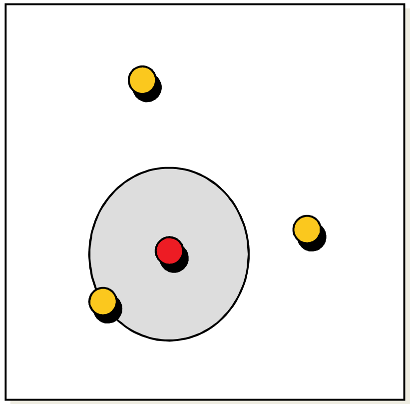
# Refinement of Dominance Rankings

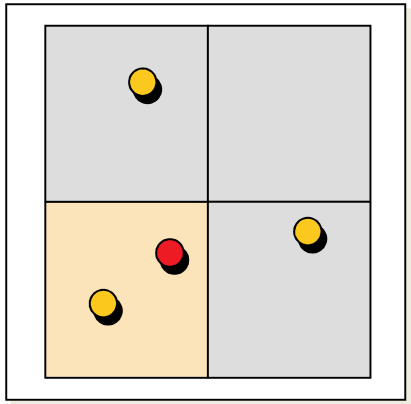
**Goal:** rank incomparable solutions within a dominance class

❶ Diversity information

| Kernel method | k-th nearest neighbor | Histogram method |
|---|---|---|
| diversity = function of the distances | diversity = function of distance to k-th nearest neighbor | diversity = number of elements within box(es) |



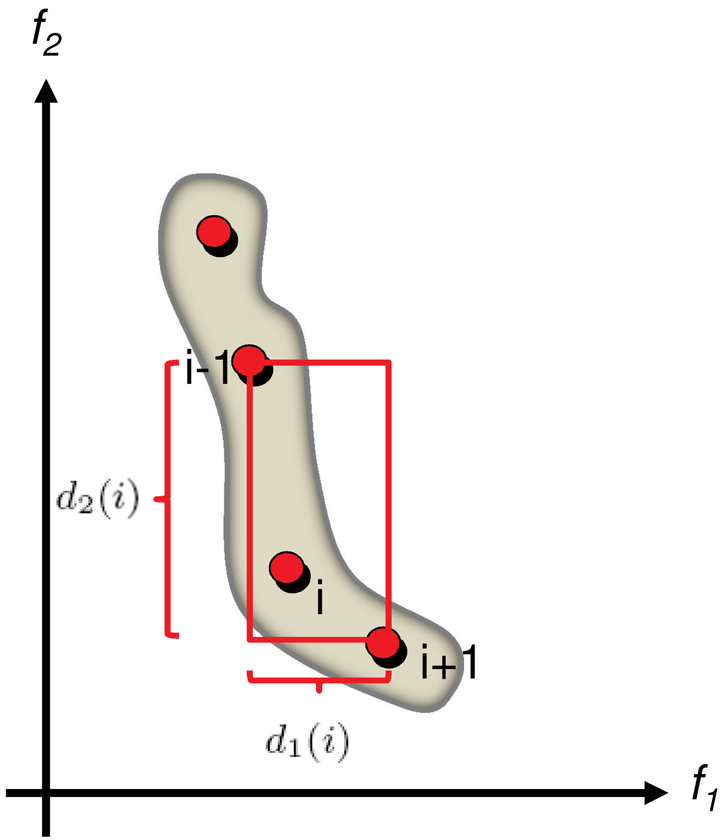


❷ (Contribution to a) quality indicator

# Example: NSGA-II Diversity Preservation

## Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective

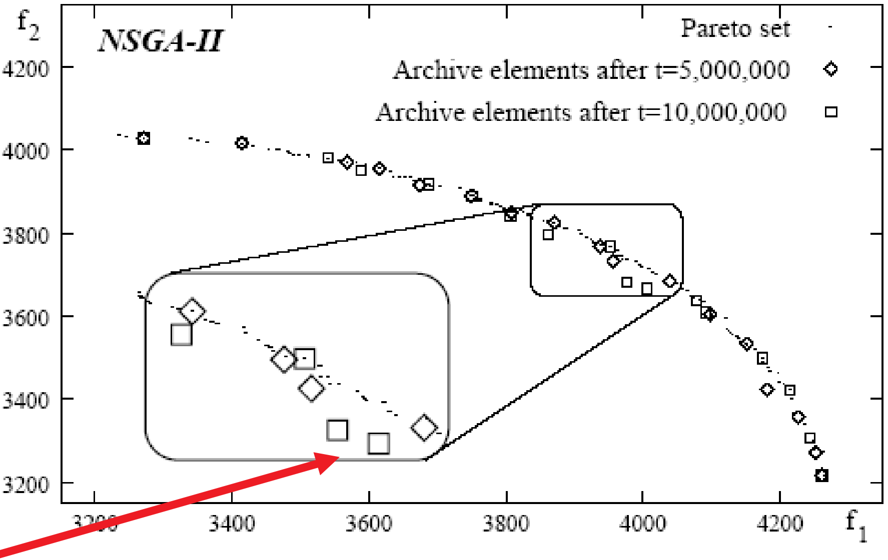


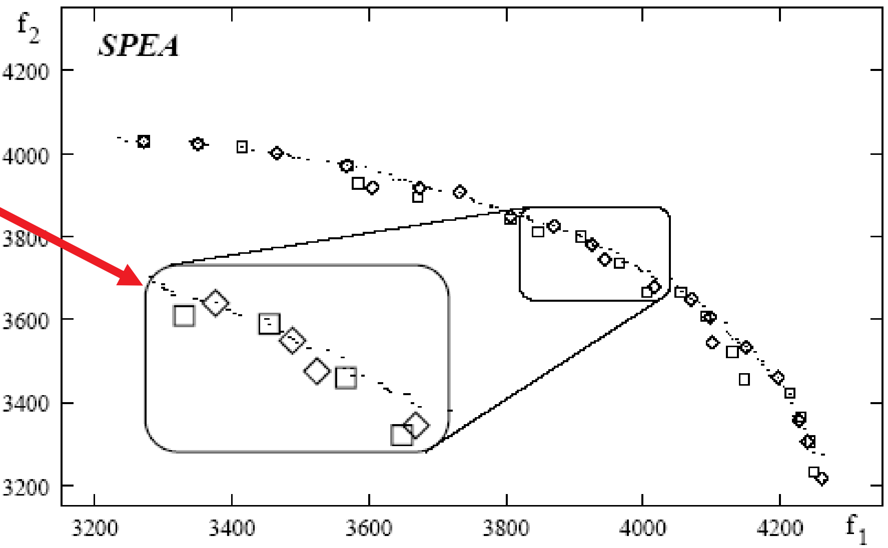

$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

# SPEA2 and NSGA-II: Deteriorative Cycles

Selection in SPEA2 and NSGA-II can result in *deteriorative* cycles

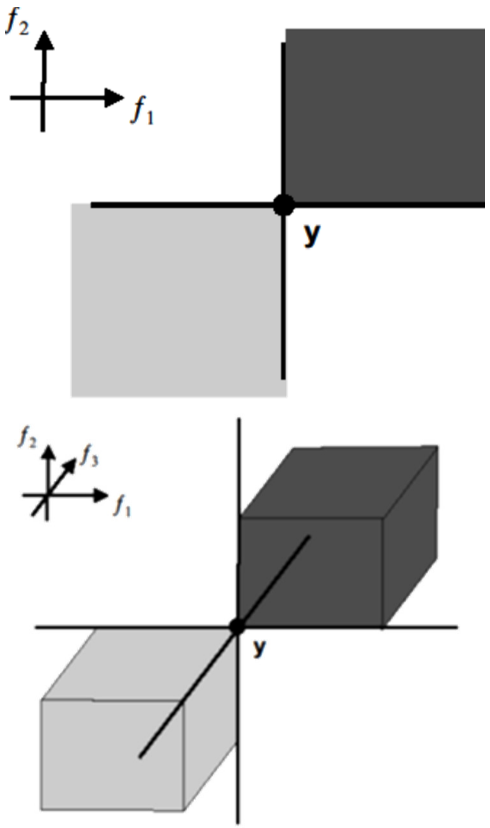
non-dominated solutions already found can be lost

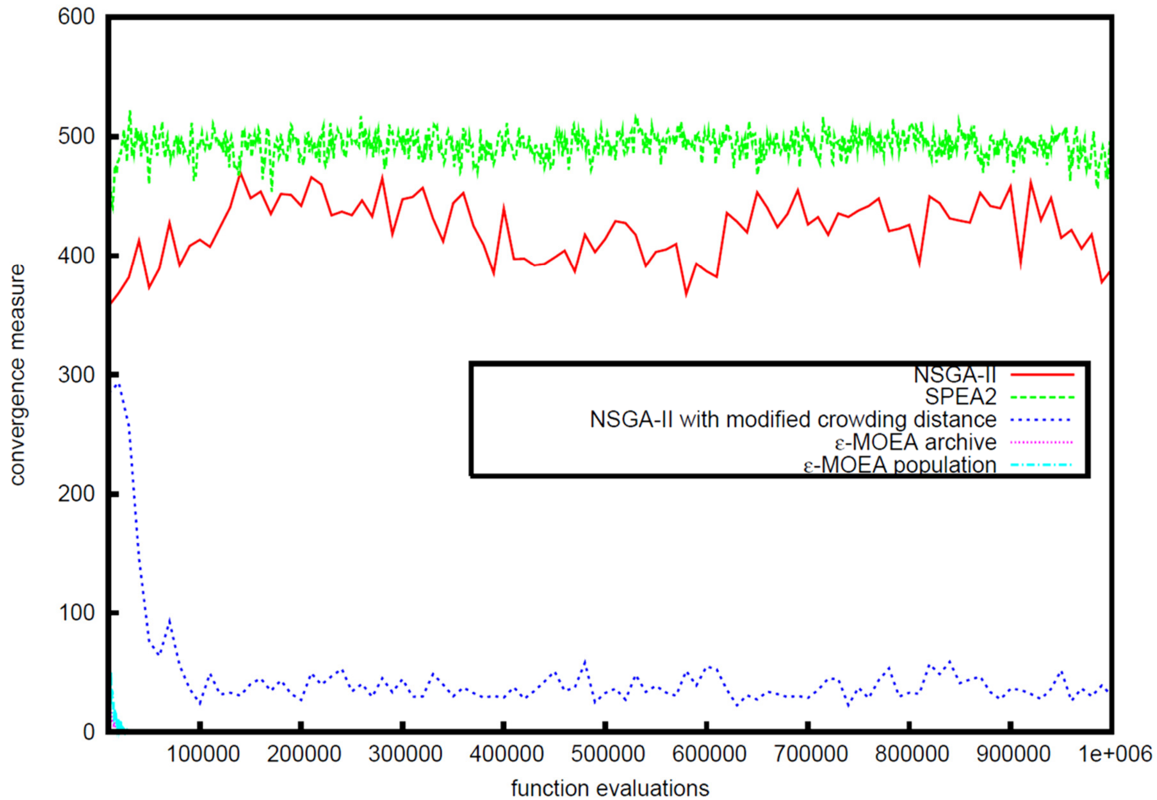


# Remark: Many-Objective Optimization

- high number of objectives
  → percentage of non-dominated solutions within a random sample quickly approaches 100 %
  → optimization is mainly guided by diversity criterion
  → apply secondary criterion compliant with dominance relation



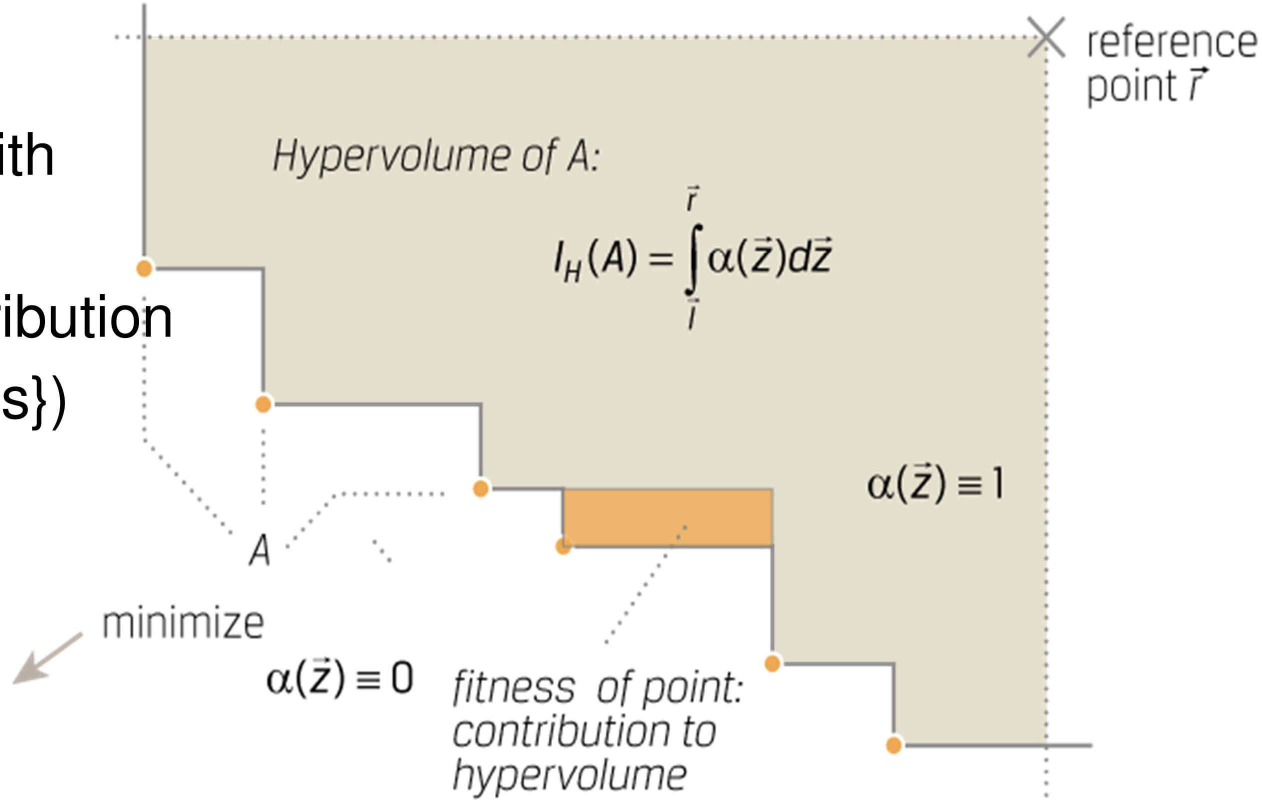
# Hypervolume-Based Selection

**Latest Approach** (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

**Main idea**

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P)-I_H(P / \{s\})$ iteratively




**But:**

- can also result in cycles if reference point is not constant [Judt et al. 2011]
- expensive to compute exactly [Bringmann and Friedrich 2009]
- less and less practically important [Guerreiro and Fonseca 2017]

# Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator $I$ is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

Multiobjective Problem $\xrightarrow{\text{Indicator}}$ Single-objective Problem

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]

- Generalizable also to contribution to larger sets

  **HypE** [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

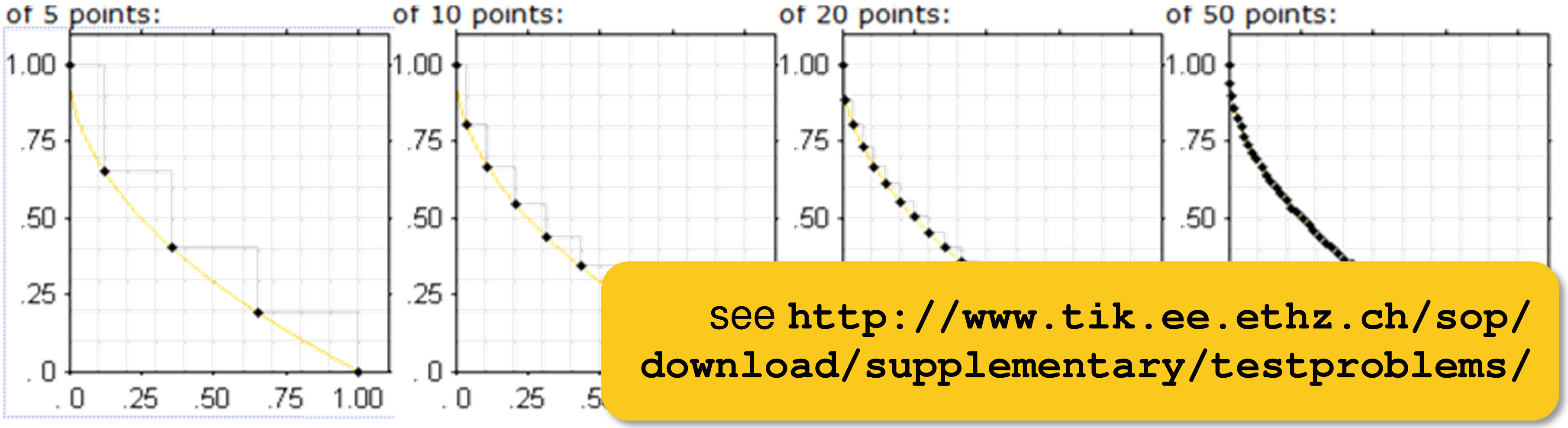
# The Optimization Goal in Indicator-Based EMO

## When the goal is to maximize a unary indicator…

- we have a single-objective problem on sets
- but what is the optimum?
- important: population size µ plays a role!

## Optimal µ-Distribution:

A set of µ solutions that maximizes a certain unary indicator I among all sets of µ solutions is called optimal µ-distribution for I. [Auger et al. 2009a]




see `http://www.tik.ee.ethz.ch/sop/download/supplementary/testproblems/`

# Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal μ-distributions for hypervolume

## Optimal μ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with $f'$ the slope of the front

[Friedrich et al. 2011]:

optimal μ-distributions for the hypervolume correspond to ε-approximations of the front

| | |
|---|---|
| **OPT** | $1+\frac{\log(\min\{A/a, B/b\})}{n}$ |
| **HYP** | $1+\frac{\sqrt{A/a}+\sqrt{B/b}}{n-4}$ |
| **logHYP** | $1+\frac{\sqrt{\log(A/a)\,\log(B/b)}}{n-2}$ |

! (probably) does not hold for > 2 objectives

# Indicator-Based EMO

## Open Questions:

- How do the optimal μ-distributions look like for >2 objectives?
- how to compute certain indicators quickly in practice?
  - several recent improvements for the hypervolume indicator
    [Yildiz and Suri 2012], [Bringmann 2012], [Bringmann 2013]
    [Guerreiro and Fonseca 2017]
- how to do indicator-based subset selection quickly?
  - also here several recent improvements
    [Kuhn et al. 2014], [Bringmann et al. 2014], [Guerreiro et al. 2015]
- what is the best strategy for the subset selection?

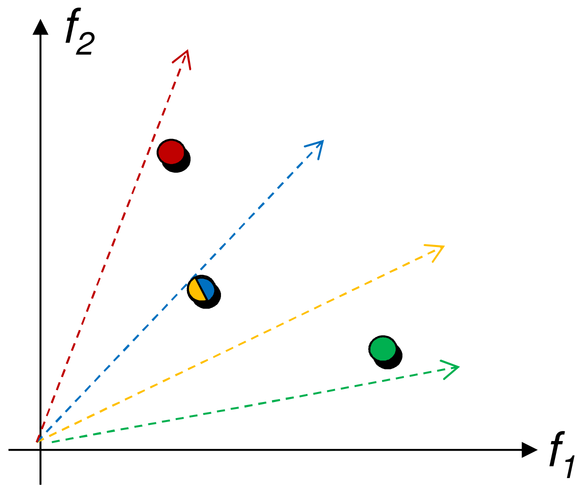
further open questions on indicator-based EMO available at
**http://simco.gforge.inria.fr/doku.php?id=openproblems**

# Decomposition-Based Selection: MOEA/D

**MOEA/D:** Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

**Ideas:**

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



# Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

## Open Question:

- how to achieve fast convergence to a *set*?

## Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007, Voß et al. 2010, Krause et al. 2016]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015, Hadka and Reed 2013]

# Overview

The Big Picture

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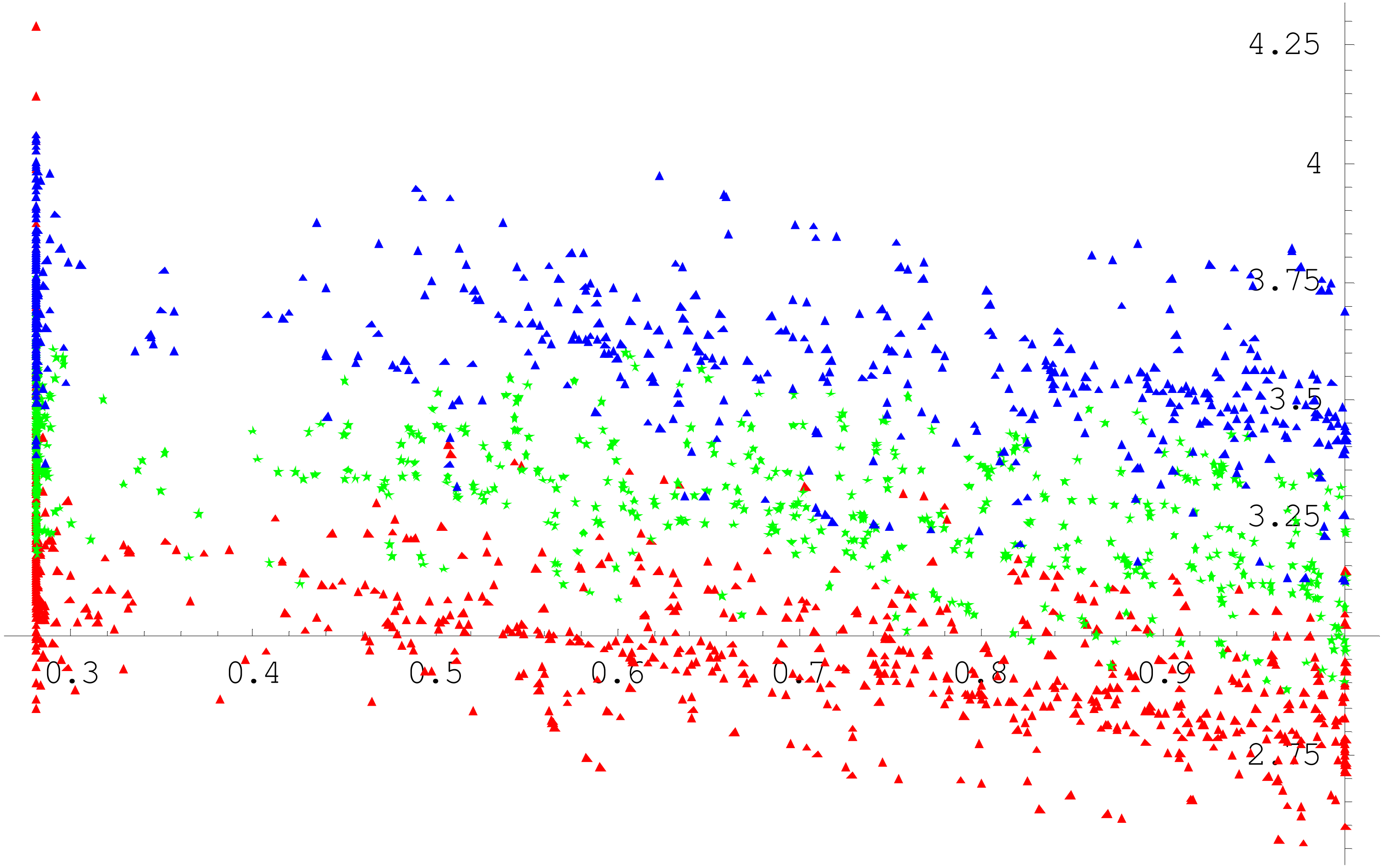
Preference Articulation

# Once Upon a Time...

... multiobjective EAs were mainly compared visually:




ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

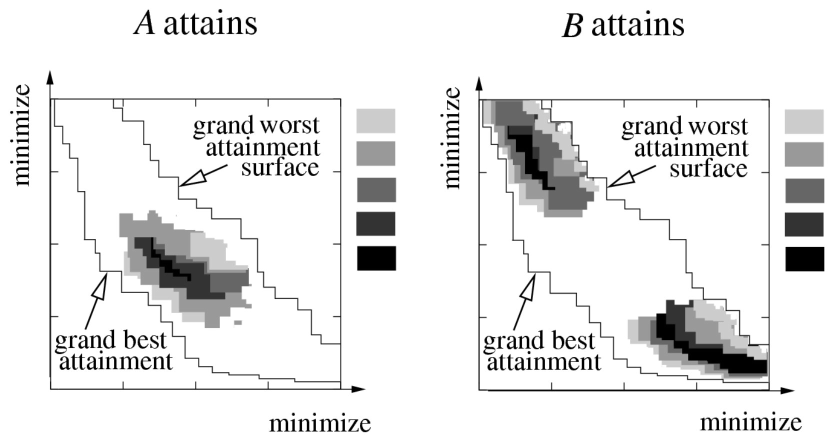
# Two Approaches for Empirical Studies

## Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

## Quality indicator approach

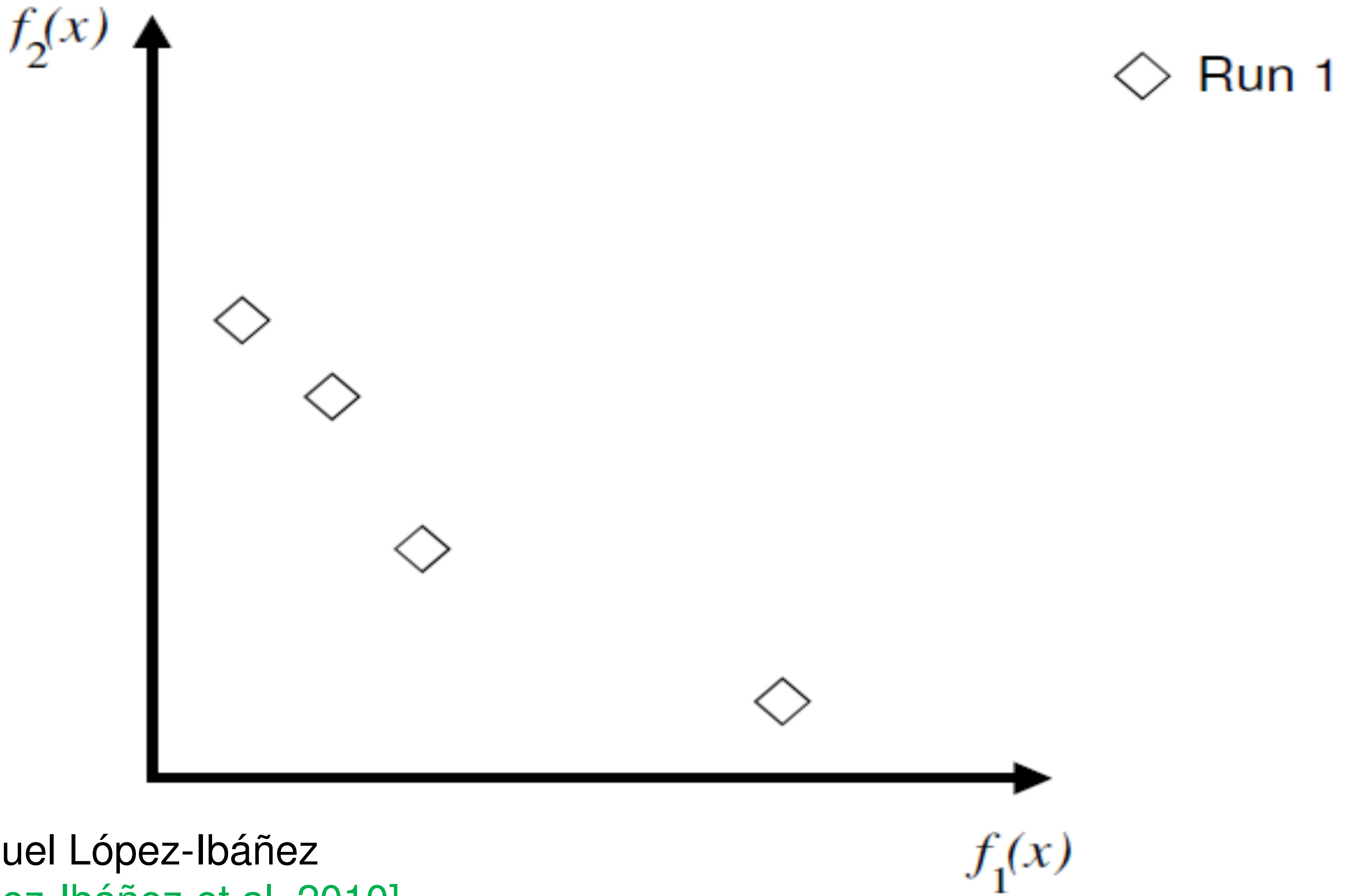
- reduces each approximation set to a single quality value
- applies statistical tests to the quality values




| *Indicator* | A | B |
|---|---|---|
| Hypervolume indicator | 6.3431 | 7.1924 |
| $\epsilon$-indicator | 1.2090 | 0.12722 |
| $R_2$ indicator | 0.2434 | 0.1643 |
| $R_3$ indicator | 0.6454 | 0.3475 |

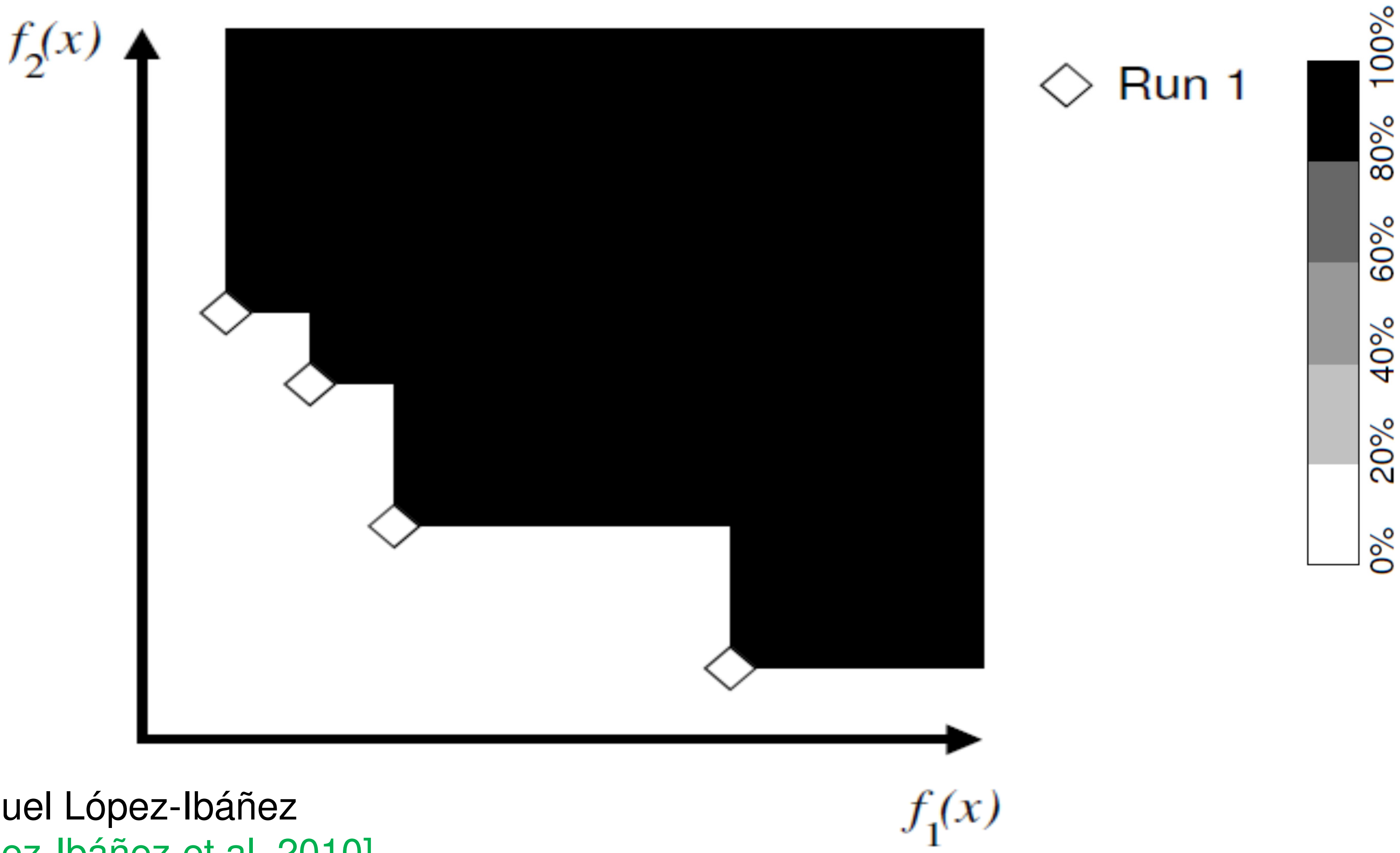
see e.g. [Zitzler et al. 2003]

# Empirical Attainment Functions




© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

# Empirical Attainment Functions

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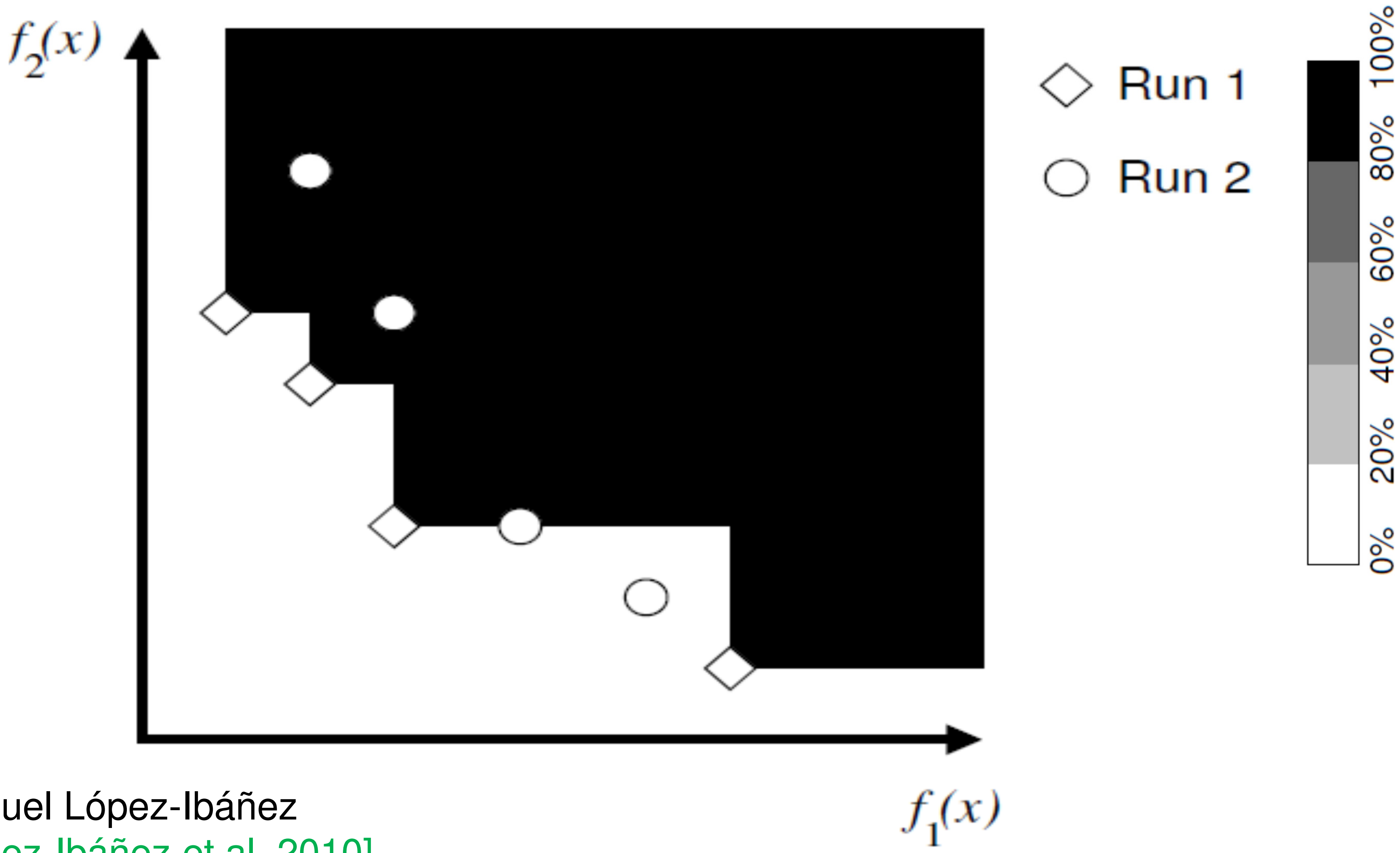
[López-Ibáñez et al. 2010]

# Empirical Attainment Functions

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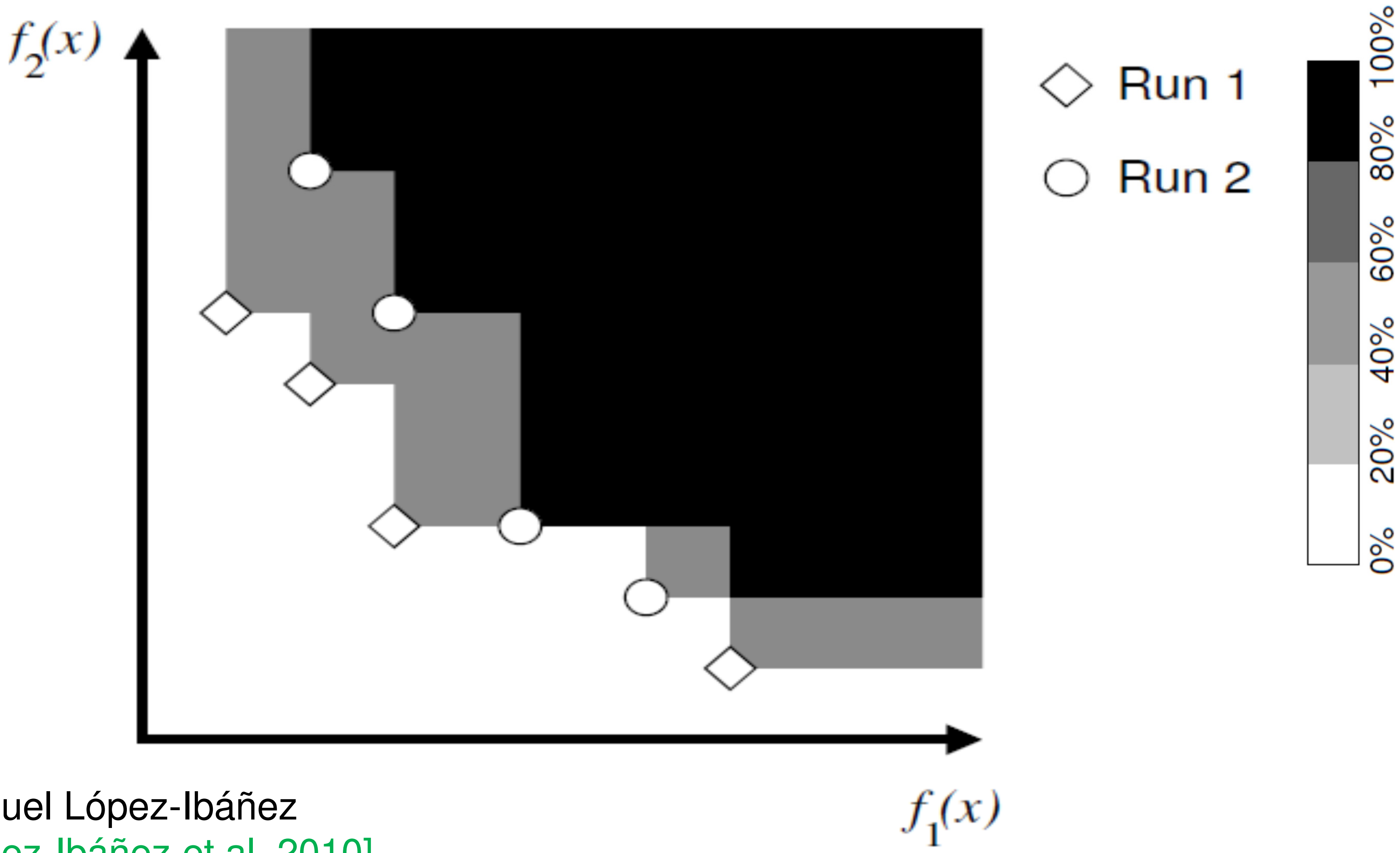
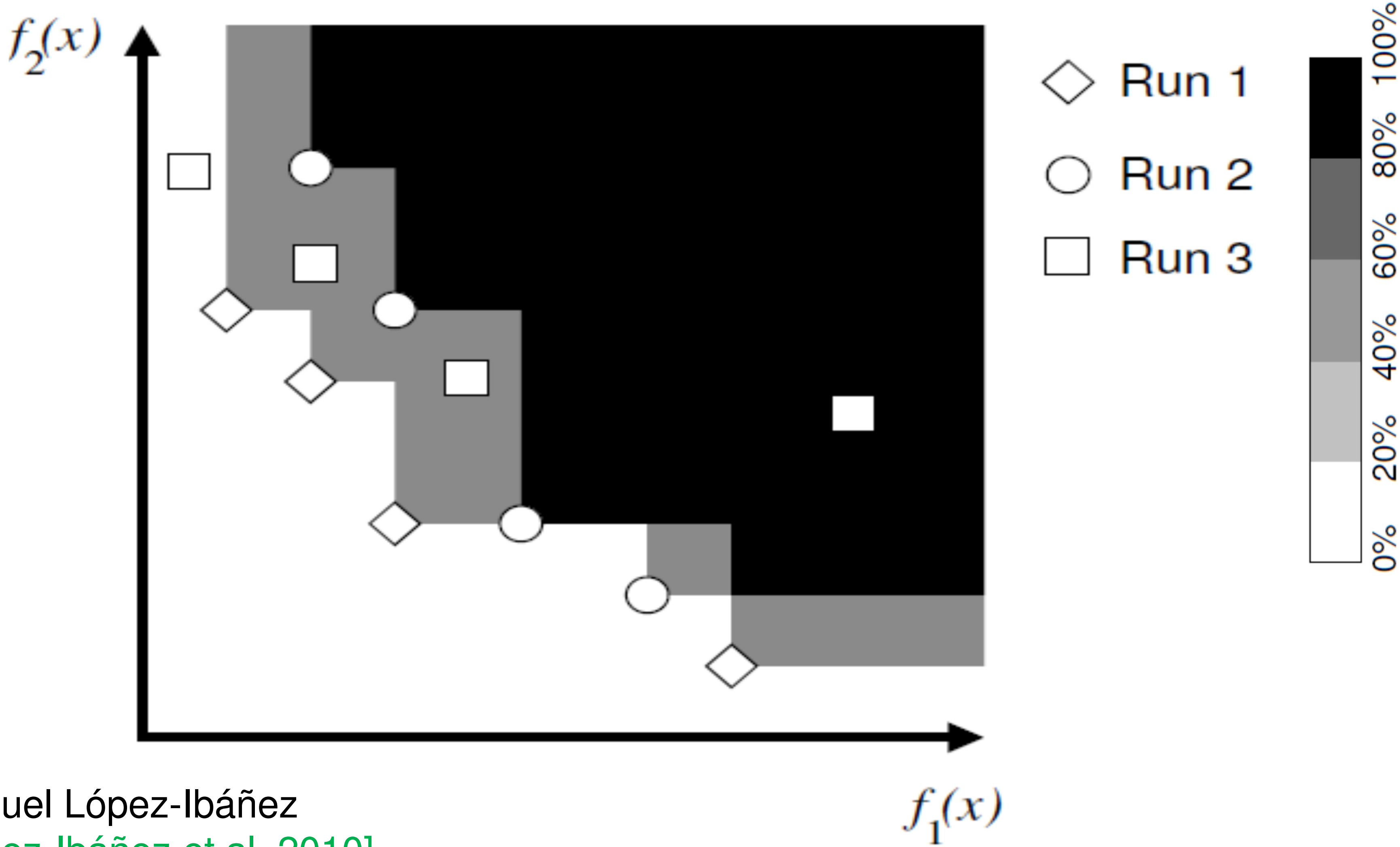
[López-Ibáñez et al. 2010]





[López-Ibáñez et al. 2010]

# Empirical Attainment Functions

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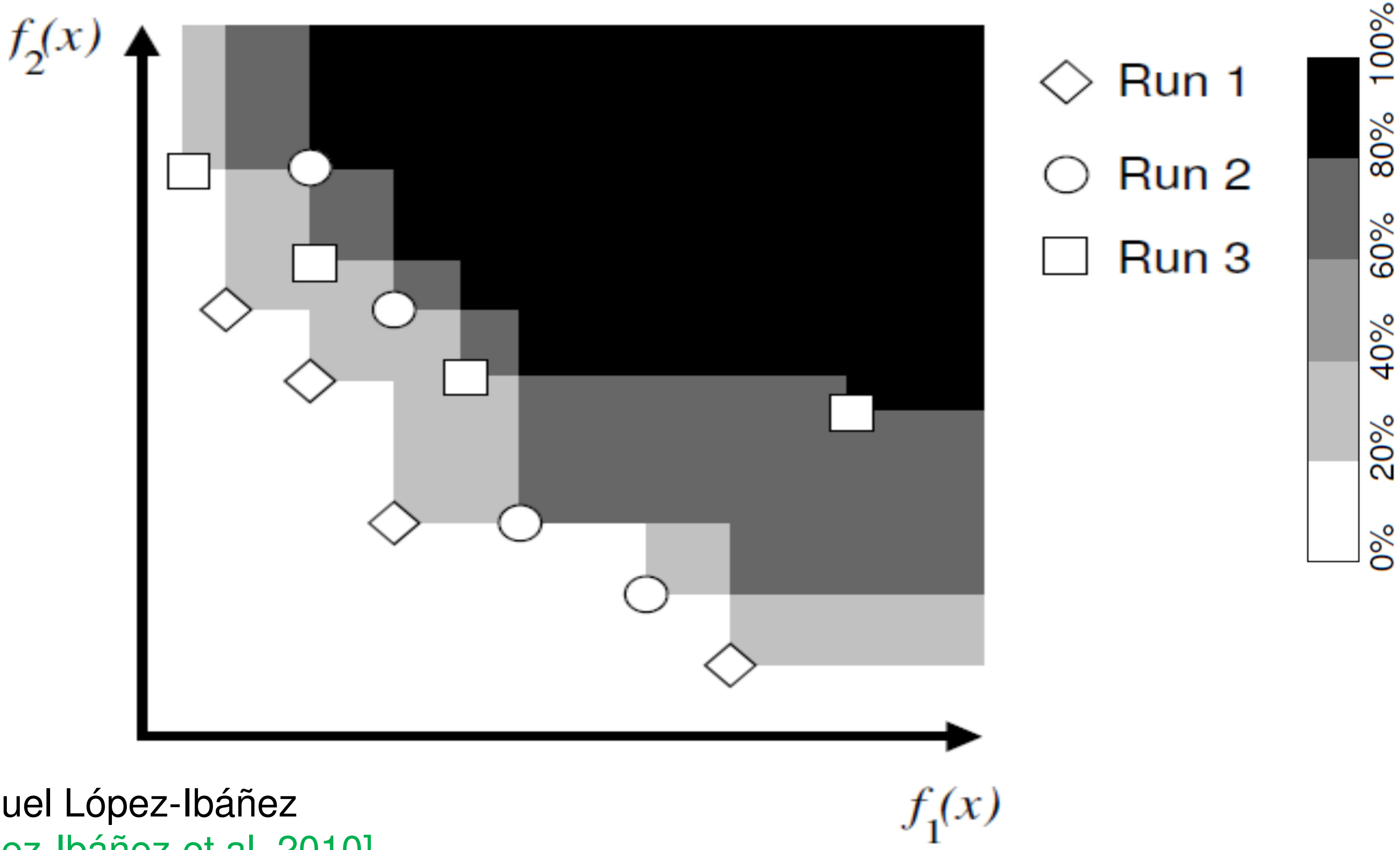
[López-Ibáñez et al. 2010]

# Empirical Attainment Functions






[López-Ibáñez et al. 2010]

# Empirical Attainment Functions: Definition

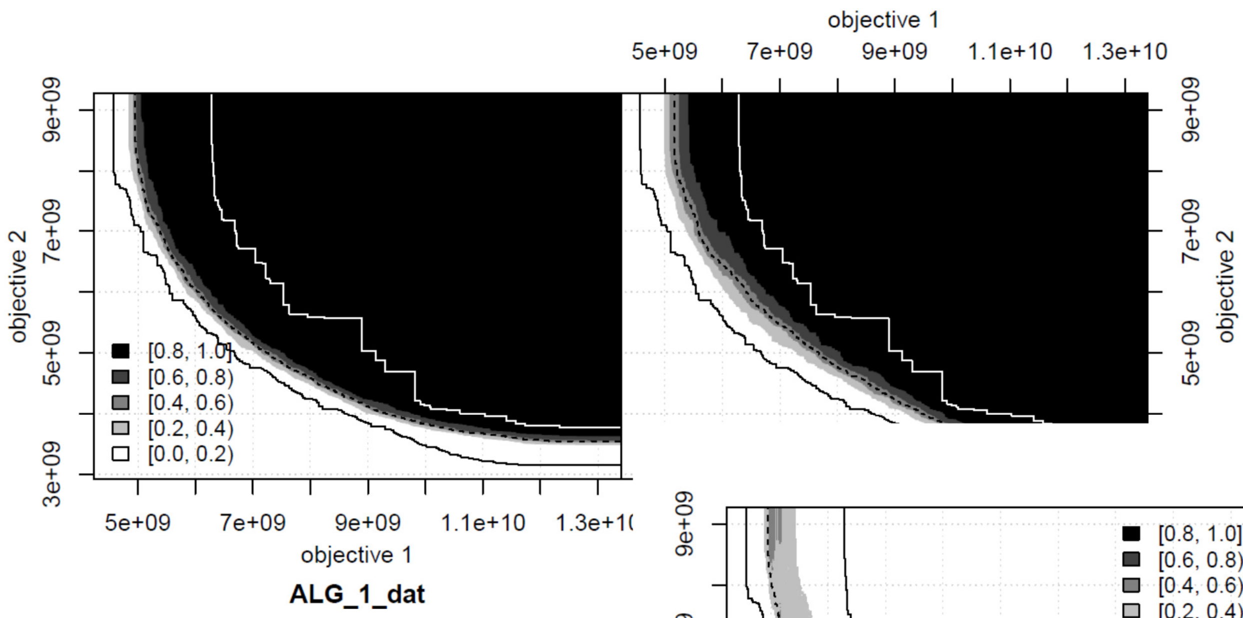
The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector $z$ at time $T$:

$$\alpha_T(z) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time $T$.

Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time T in the single-objective case ("fixed budget scenario").

# Empirical Attainment Functions in Practice

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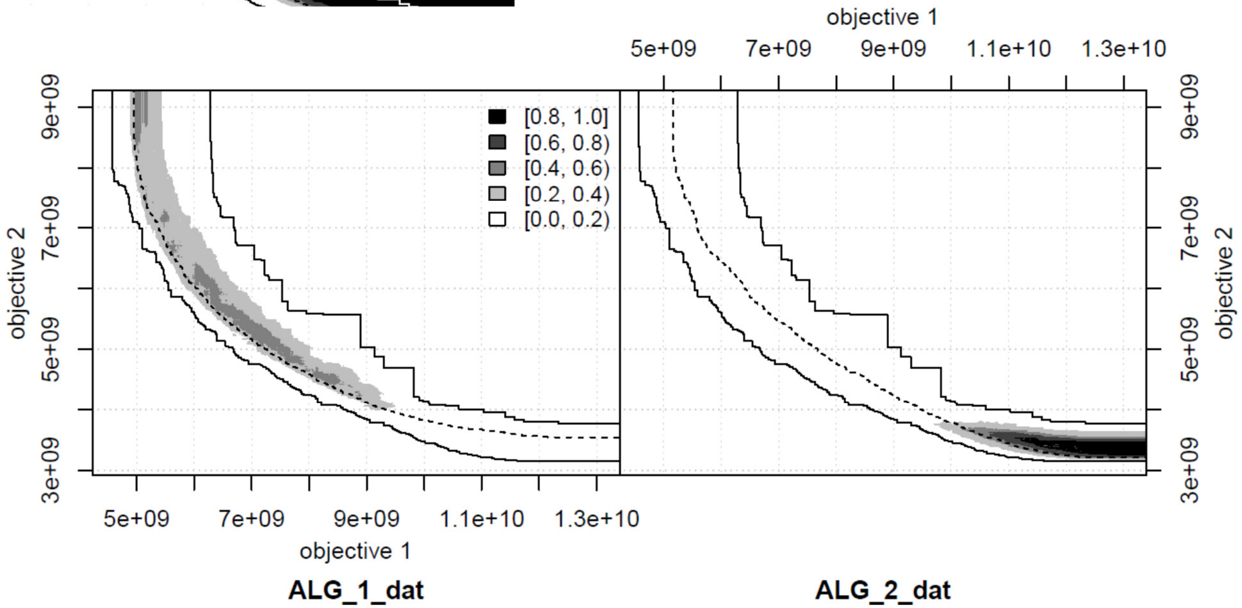
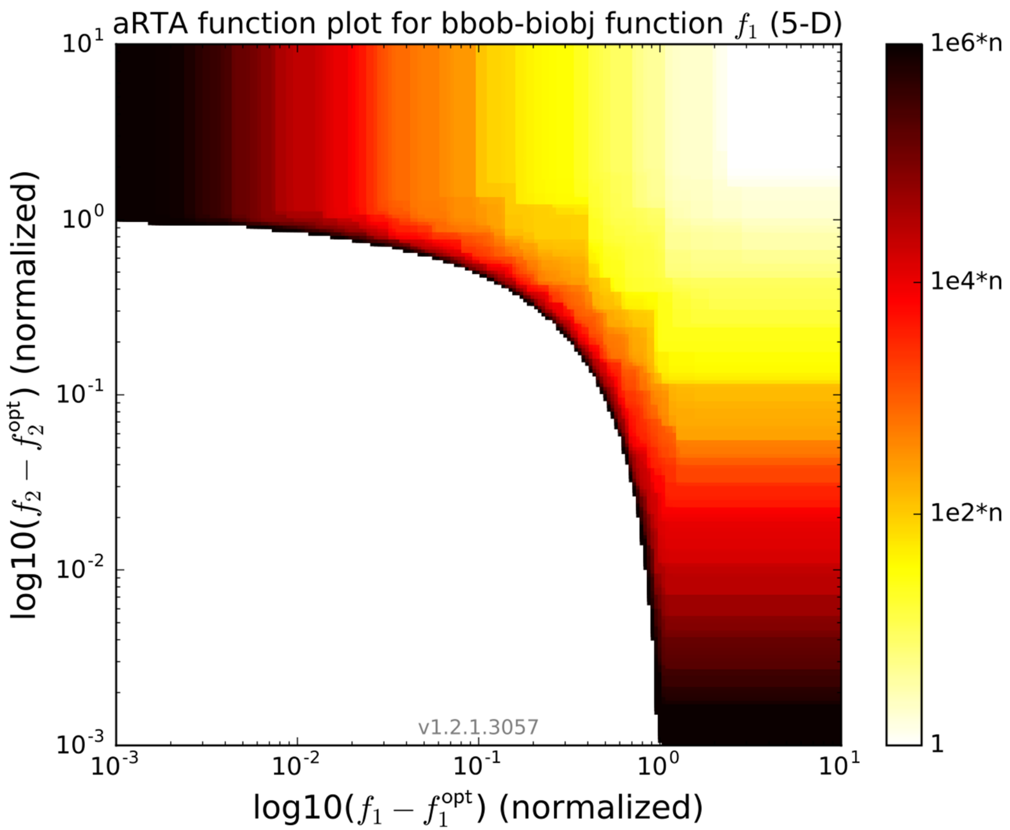
[López-Ibáñez et al. 2010]




latest implementation online at
**`http://eden.dei.uc.pt/~cmfonsec/software.html`**
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

# Plotting Average Runtimes

**Note:** success probability can be naturally replaced by the average runtime of an artificially restarted algorithm (aRT):




code available at **`http://github.com/numbbo/coco/`**

see also [Brockhoff et al. 2017]

# Quality Indicator Approach

## Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

## Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \leq_{par} y$$

# Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

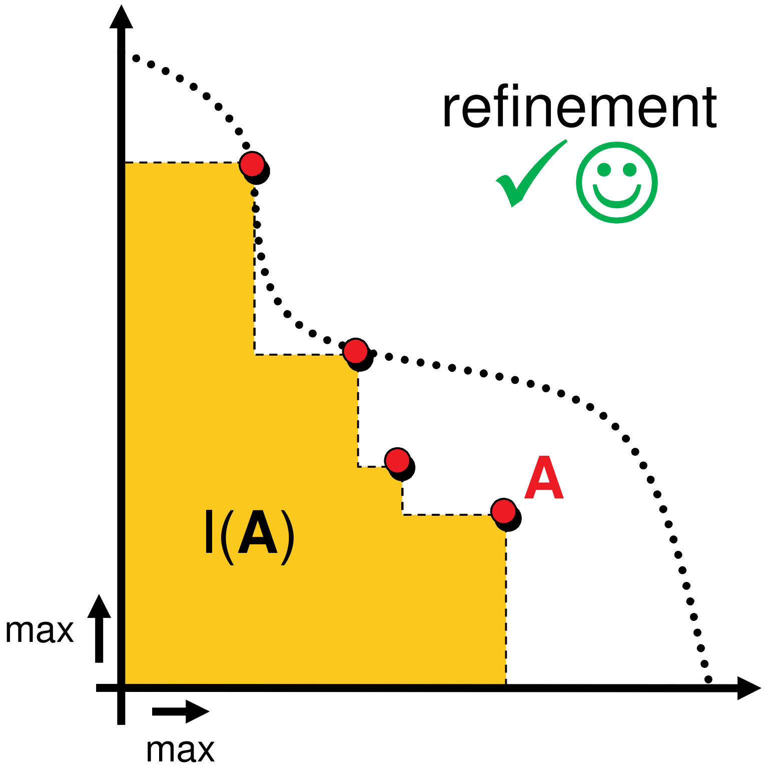
# Example: Refinements Using Indicators

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

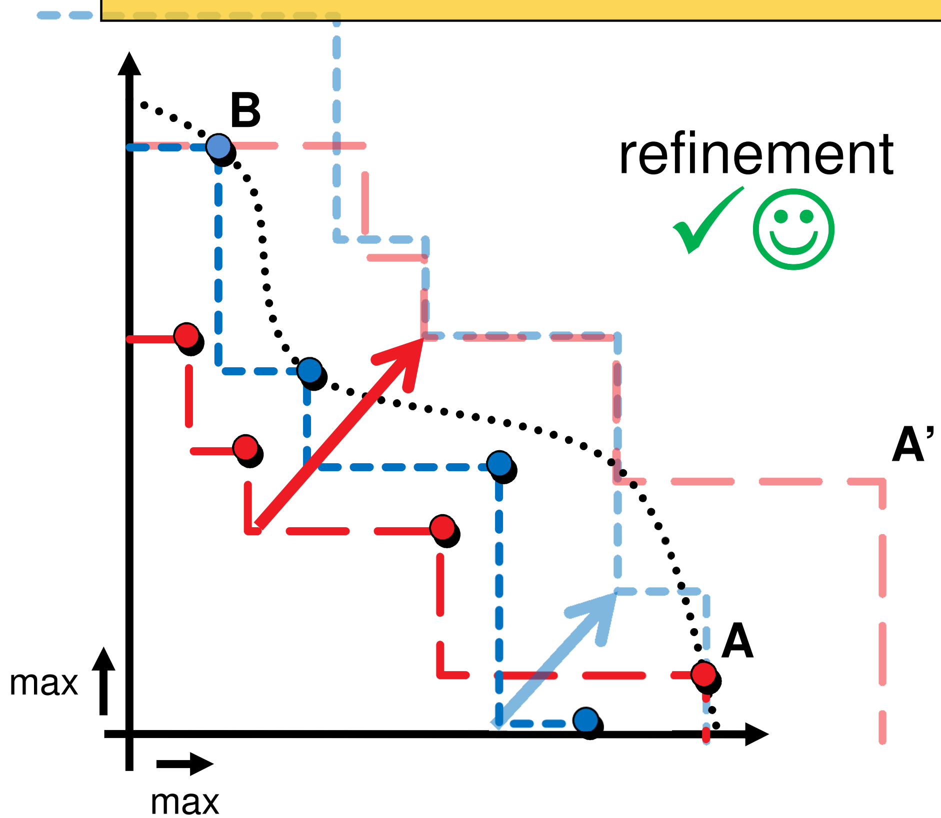
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B




unary hypervolume indicator




binary epsilon indicator

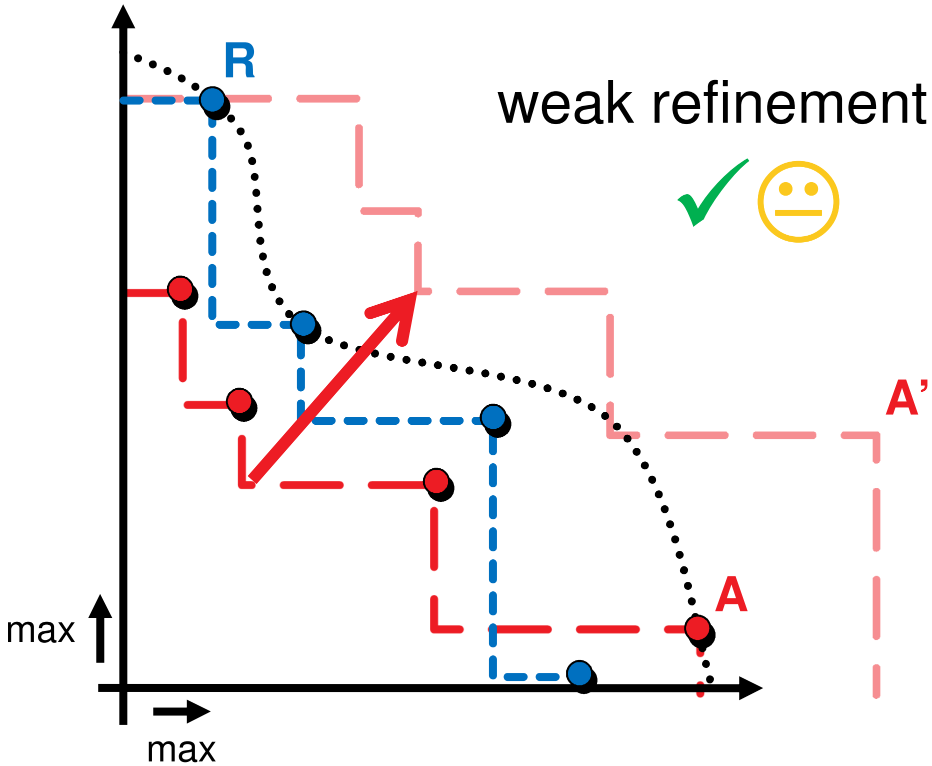
# Example: Weak Refinement / No Refinement

$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,R) \leq I(B,R)$

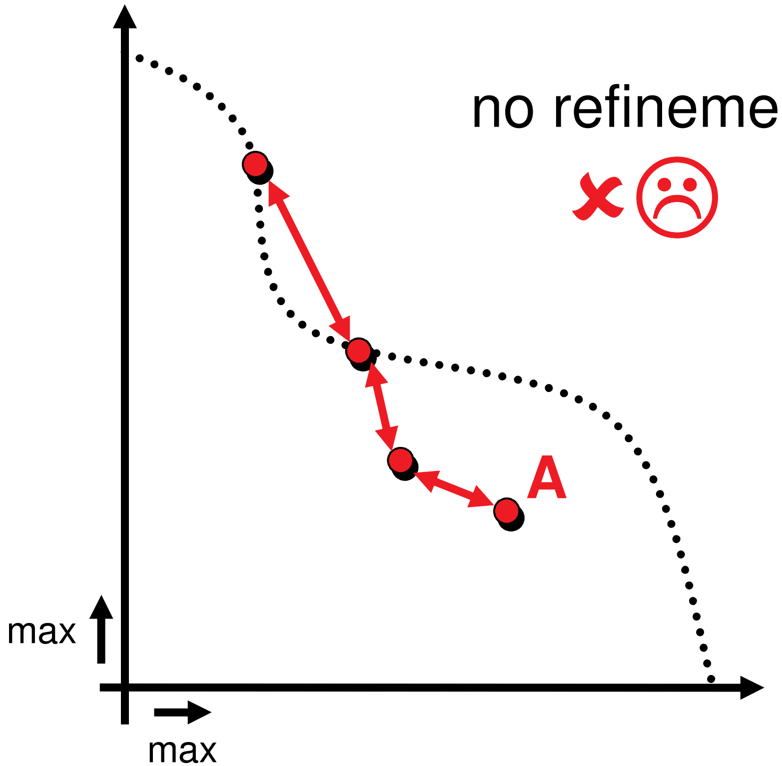
$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \leq I(B)$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances



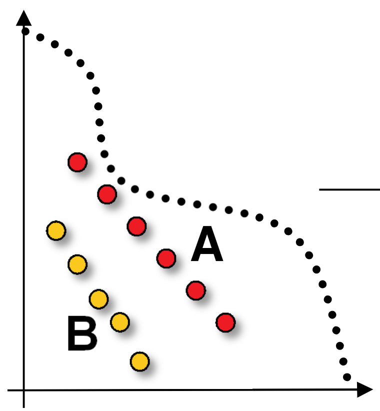

unary epsilon indicator




unary diversity indicator

# Quality Indicator Approach

**Goal:** compare two Pareto set approximations A and B




| | A | B |
|---|---|---|
| hypervolume | *432.34* | *420.13* |
| distance | *0.3308* | *0.4532* |
| diversity | *0.3637* | *0.3463* |
| spread | *0.3622* | *0.3601* |
| cardinality | *6* | *5* |

→ "**A** better"

**Comparison method** C = quality measure(s) + Boolean function

$$A, B \xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$$

**reduction** **interpretation**

# Example: Box Plots

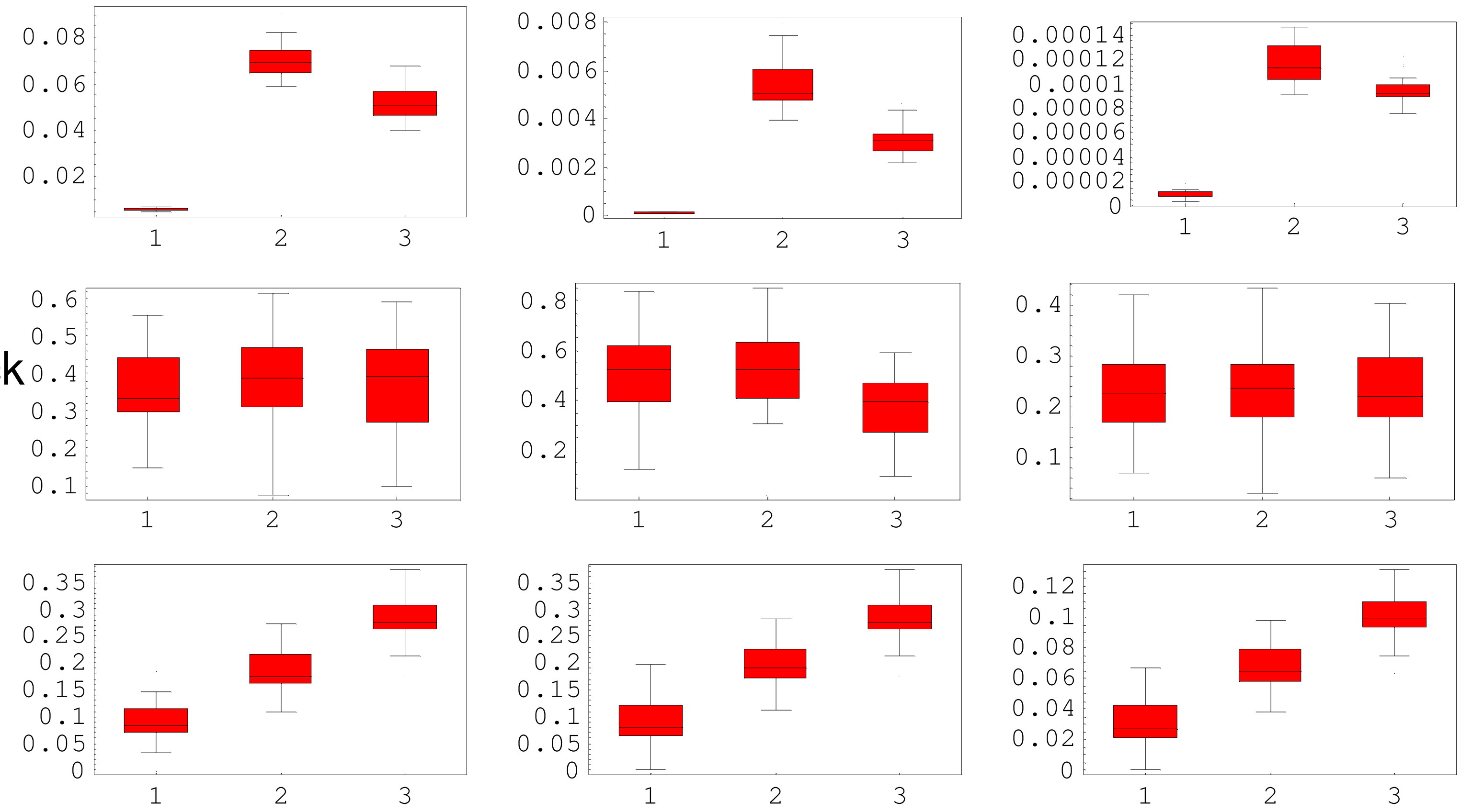
epsilon indicator | hypervolume | R indicator

IBEA NSGA-IISPEA2 | IBEA NSGA-IISPEA2 | IBEA NSGA-IISPEA2

DTLZ2

Knapsack

ZDT6



# Statistical Assessment (Kruskal Test)

**ZDT6**
Epsilon

| is better than | IBEA | NSGA2 | SPEA2 |
|---|---|---|---|
| IBEA | | ~0 ☺ | ~0 ☺ |
| NSGA2 | 1 | | ~0 ☺ |
| SPEA2 | 1 | 1 | |

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

**DTLZ2**
R

| is better than | IBEA | NSGA2 | SPEA2 |
|---|---|---|---|
| IBEA | | ~0 ☺ | ~0 ☺ |
| NSGA2 | 1 | | 1 |
| SPEA2 | 1 | ~0 ☺ | |

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

**Knapsack**/Hypervolume: $H_0$ = No significance of any differences

# Automated Benchmarking

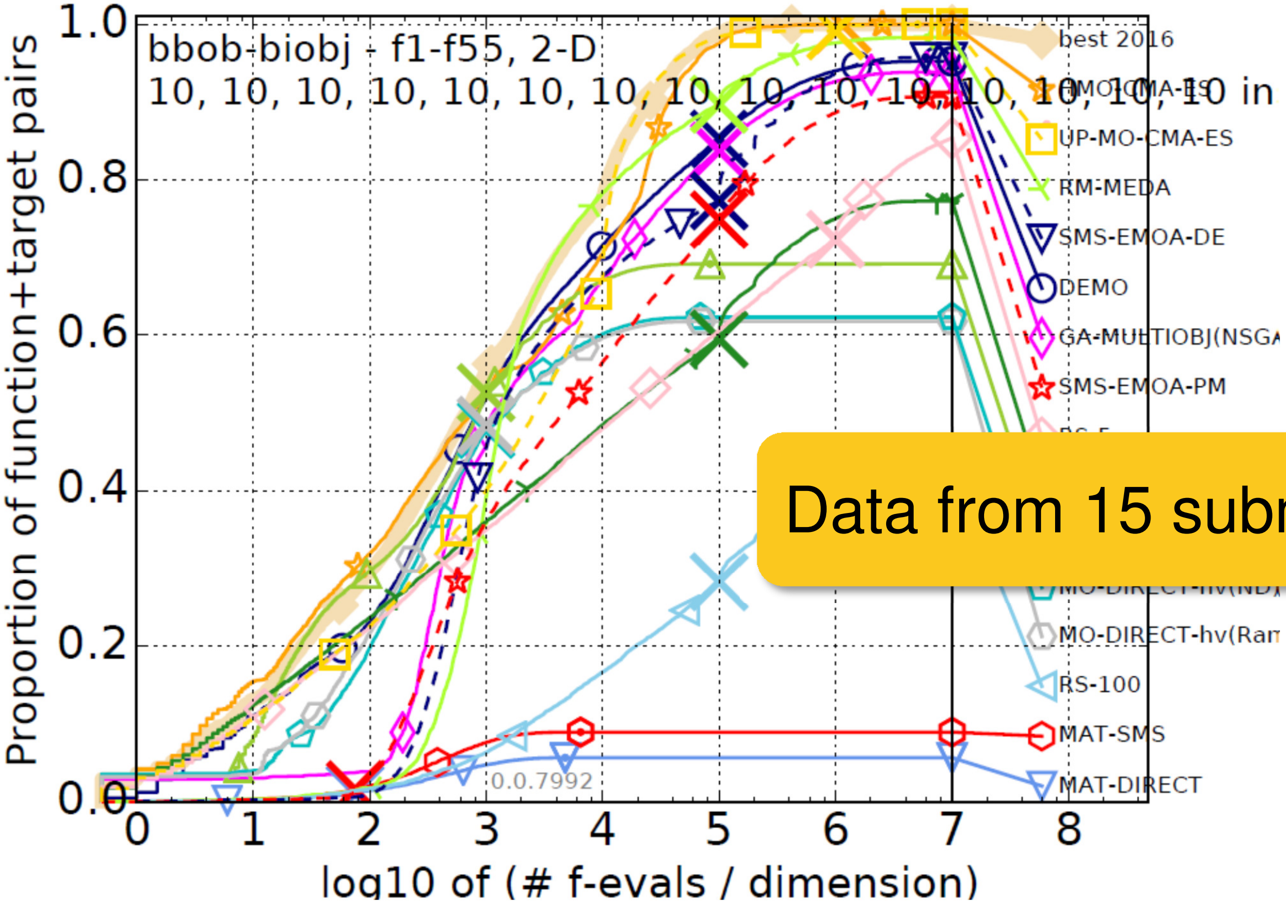
- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform

  `https://github.com/numbbo/coco`

- Release of a bi-objective test suite at BBOB-2016 workshop

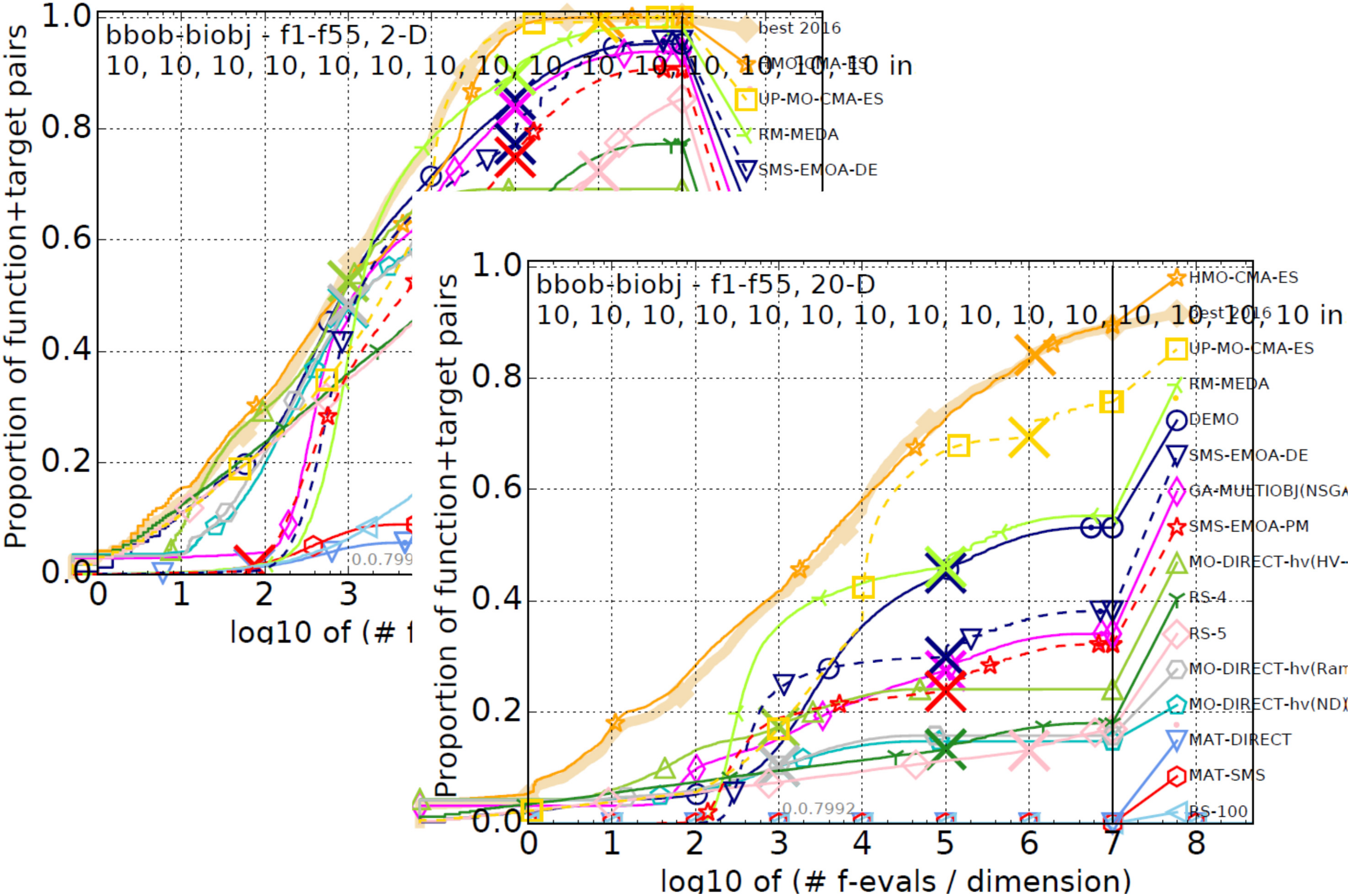
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports average runtimes (aRT) to reach target precision
- COCO provides data profiles, scaling plots, scatter plots, tables, statistical tests, etc. automatically

# Exemplary BBOB-2016 Results




Data from 15 submitted algorithms

# Exemplary BBOB-2016 Results



# A Few Recommendations

- always display *everything* you have
- look at *single runs*
- do each experiment at least twice

  (= look at the *variance* of your results)
- as quality indicators, use hypervolume, R2, or epsilon indicator
- see also the tutorial by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

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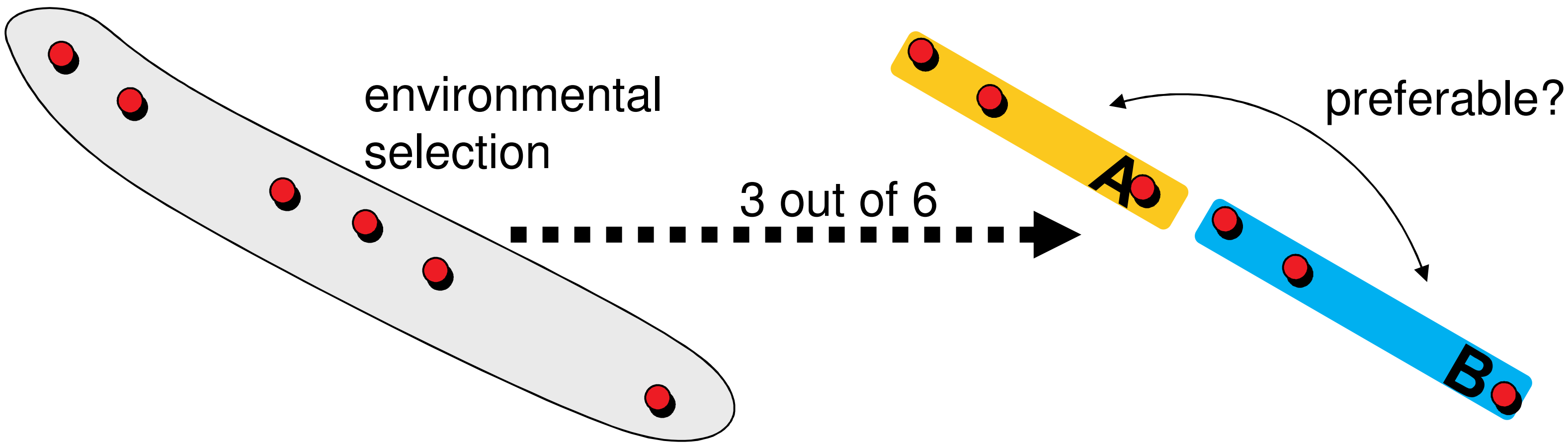
# Articulating User Preferences During Search

**What we thought:** EMO is preference-less

> **Search before decision making:** Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

**What we learnt:** EMO just uses weaker preference information



# Incorporation of Preferences During Search

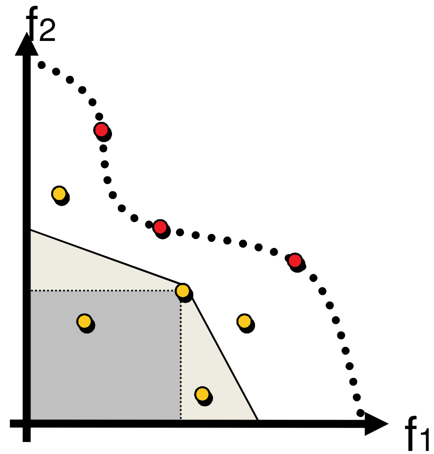
**Nevertheless...**

- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]

❶ **Refine/modify dominance relation, e.g.:**

- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of dominance cones
  [Branke and Deb 2004]

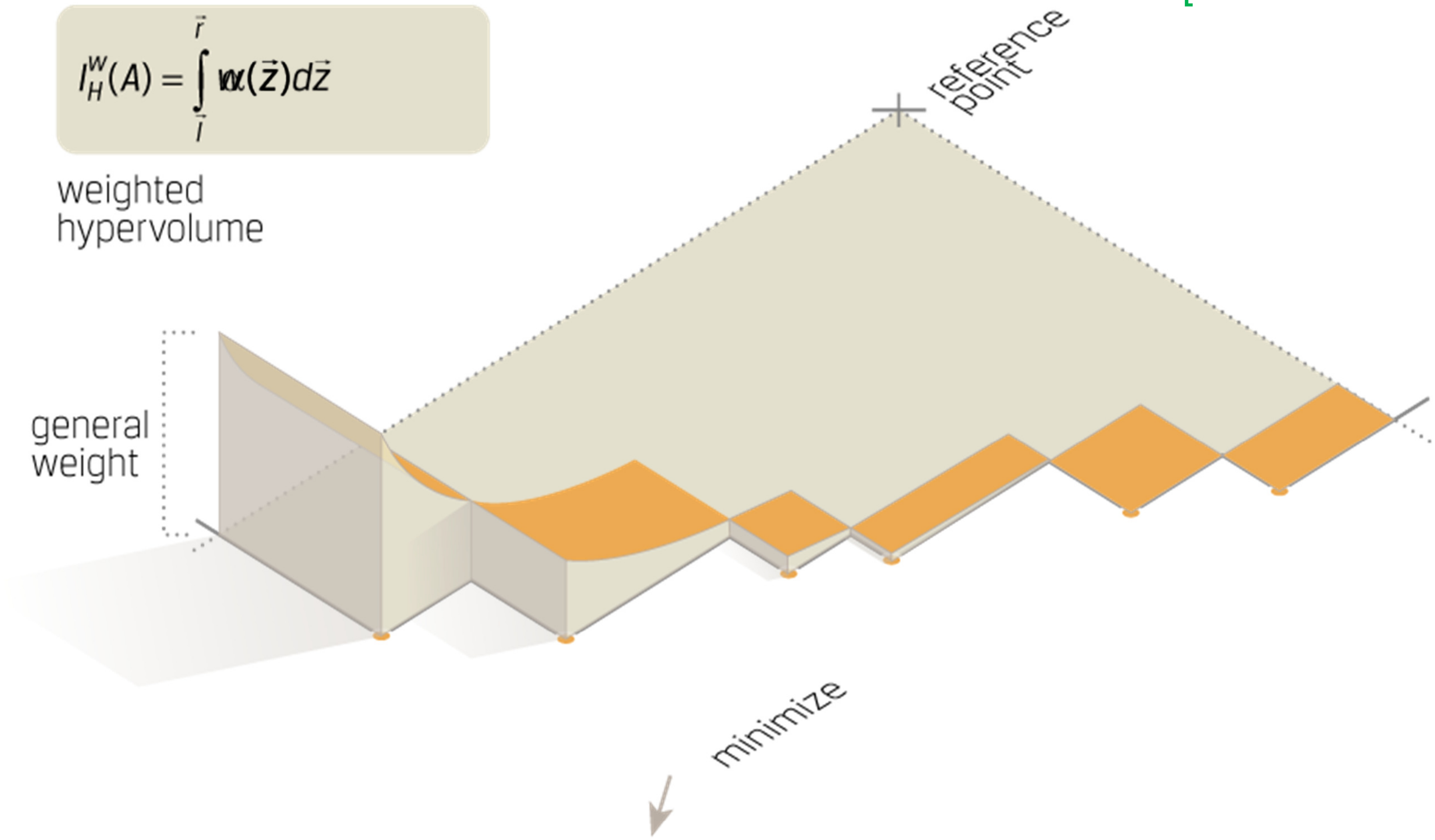



❷ **Use quality indicators, e.g.:**

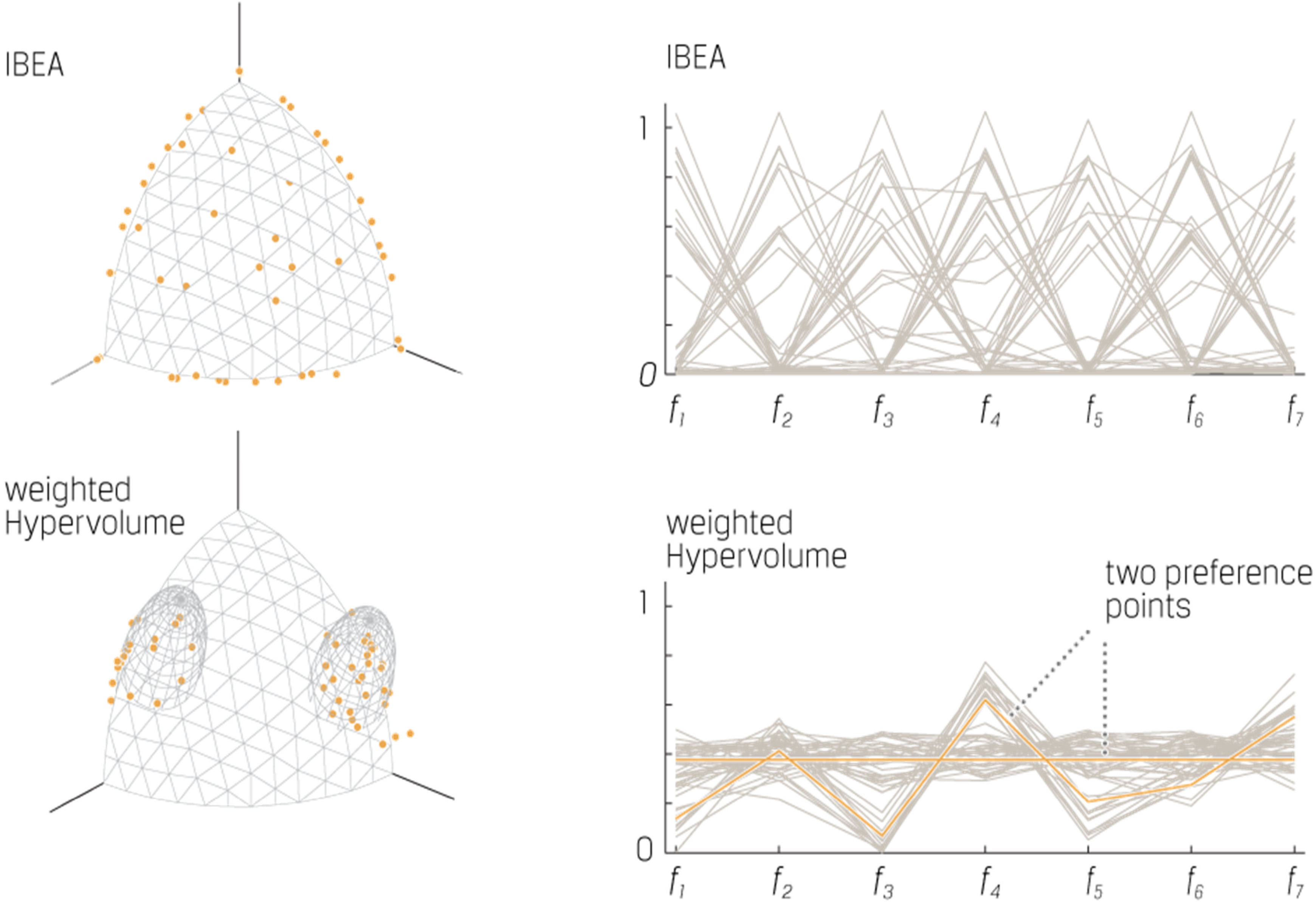
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator
  [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

# Example: Weighted Hypervolume Indicator

[Brockhoff et al. 2013]



# Weighted Hypervolume in Practice

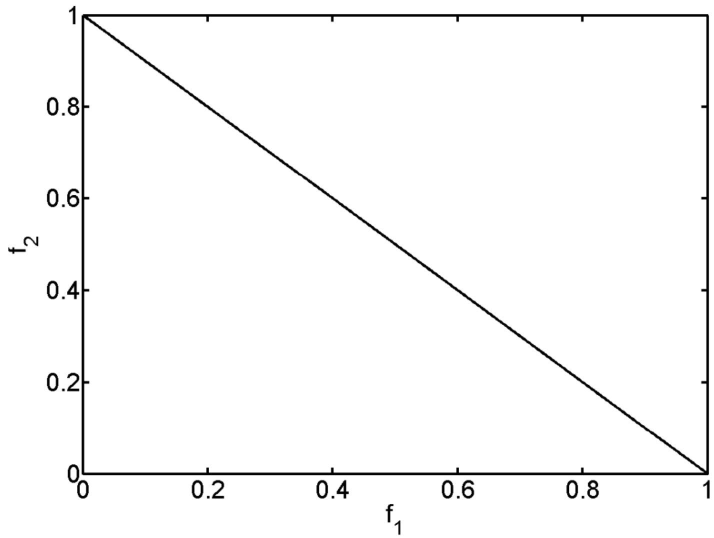


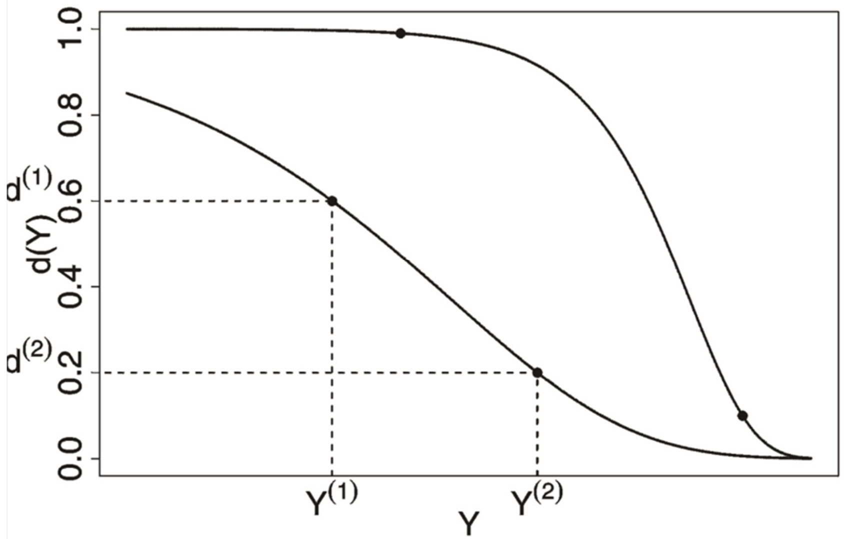

[Auger et al. 2009b]

# Example: Desirability Function (DF)-SMS-EMOA

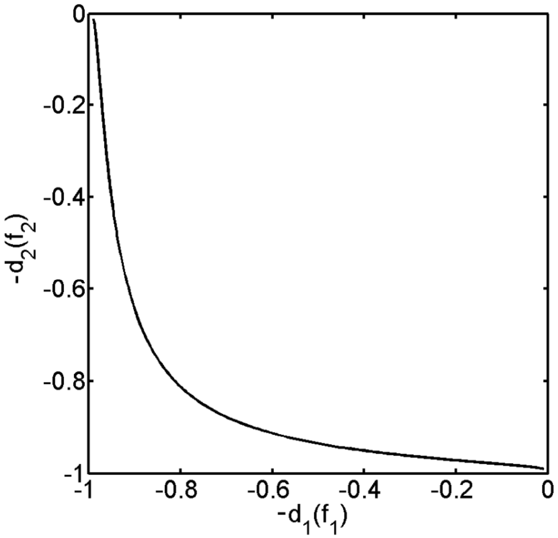
[Wagner and Trautmann 2010]

Shape of the untransformed Pareto front

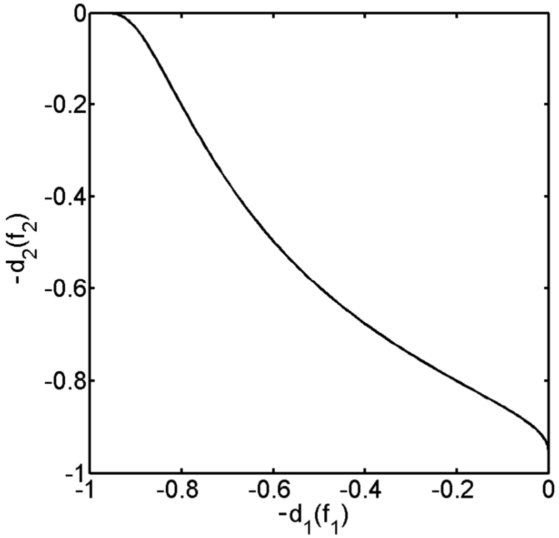


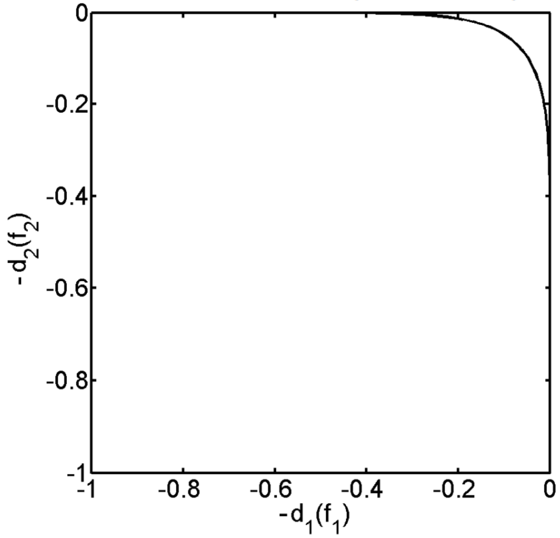

Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 1 & 0.01 \end{pmatrix}$



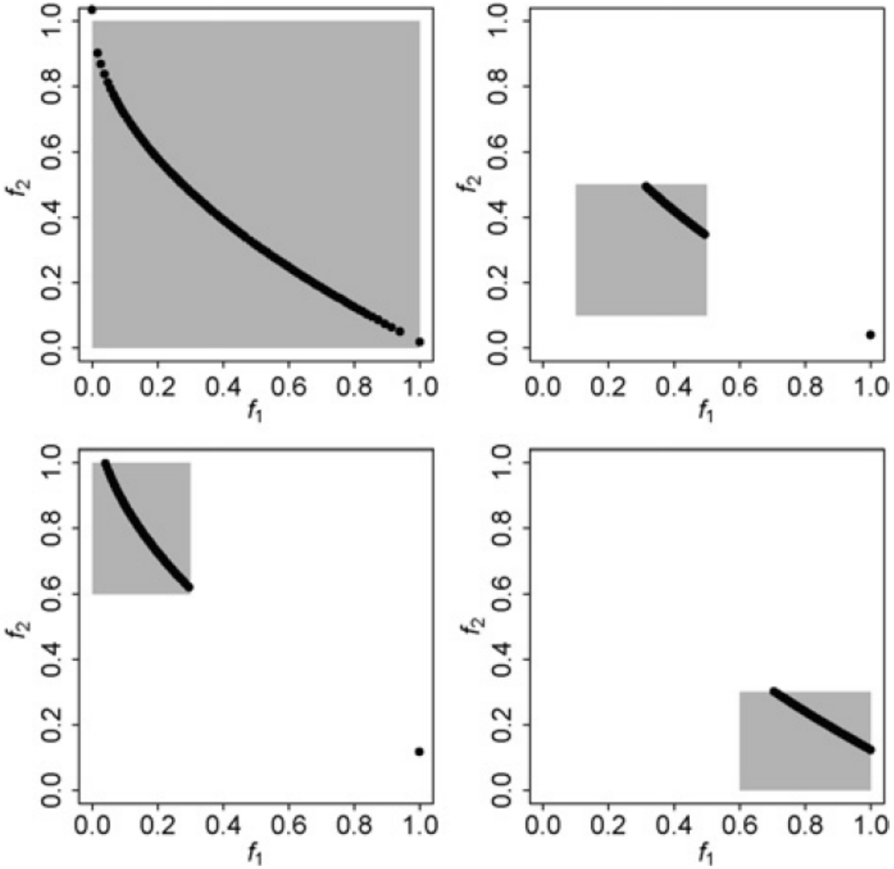

Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.75 & 0.01 \end{pmatrix}$

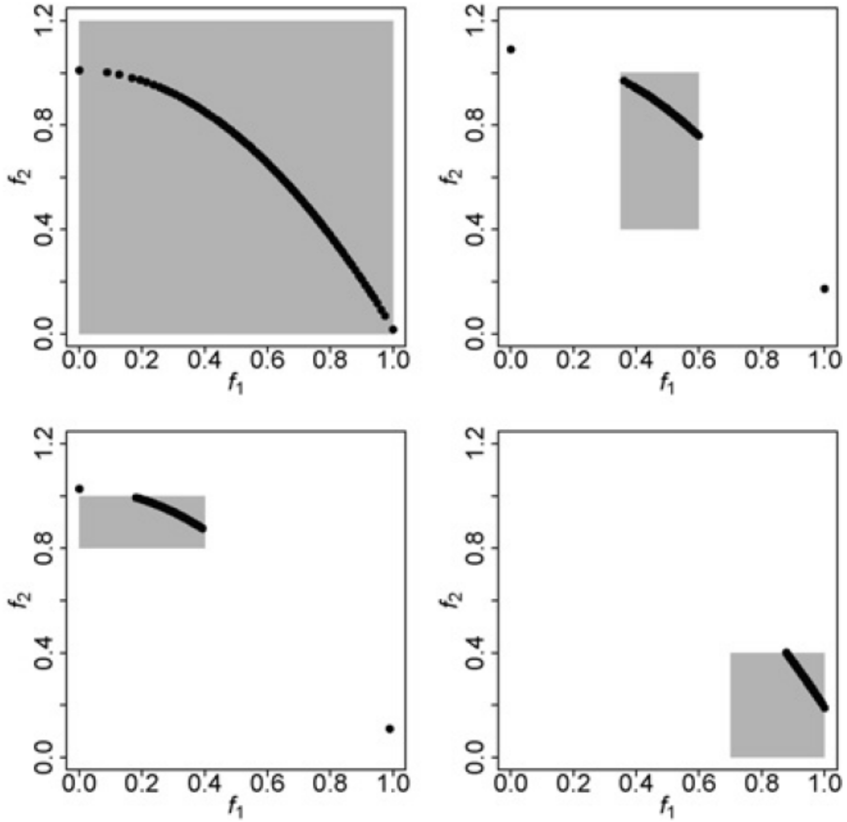



Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.55 & 0.01 \end{pmatrix}$



# DF-SMS-EMOA in Practice



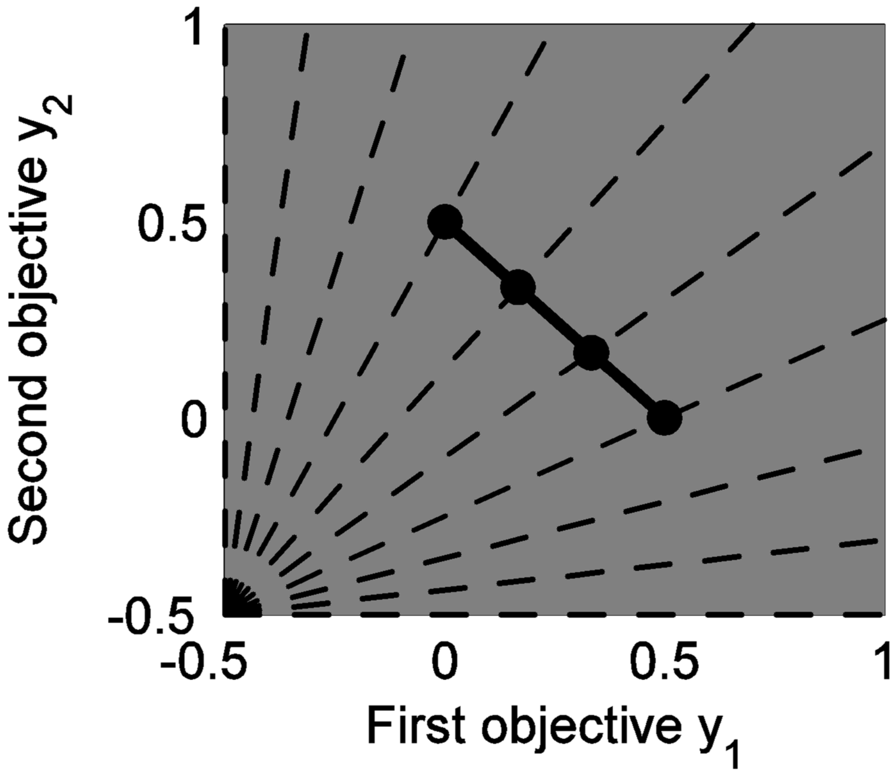



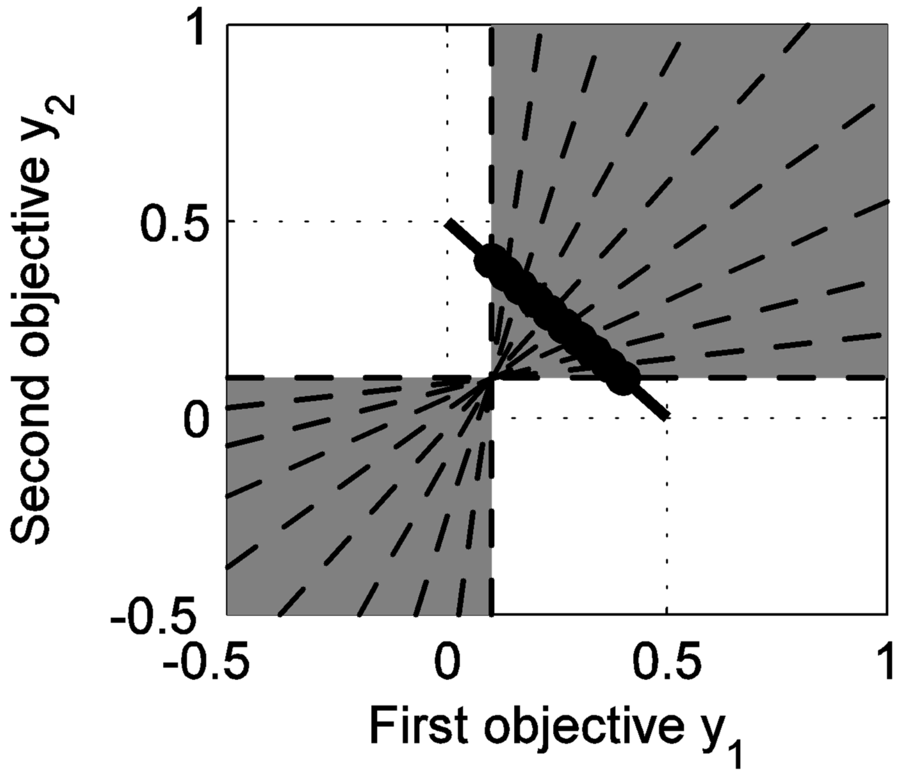
# Example: R2-EMOA

## Concept

Integration of preferences by varying the scalarizing functions

## Position of ideal point



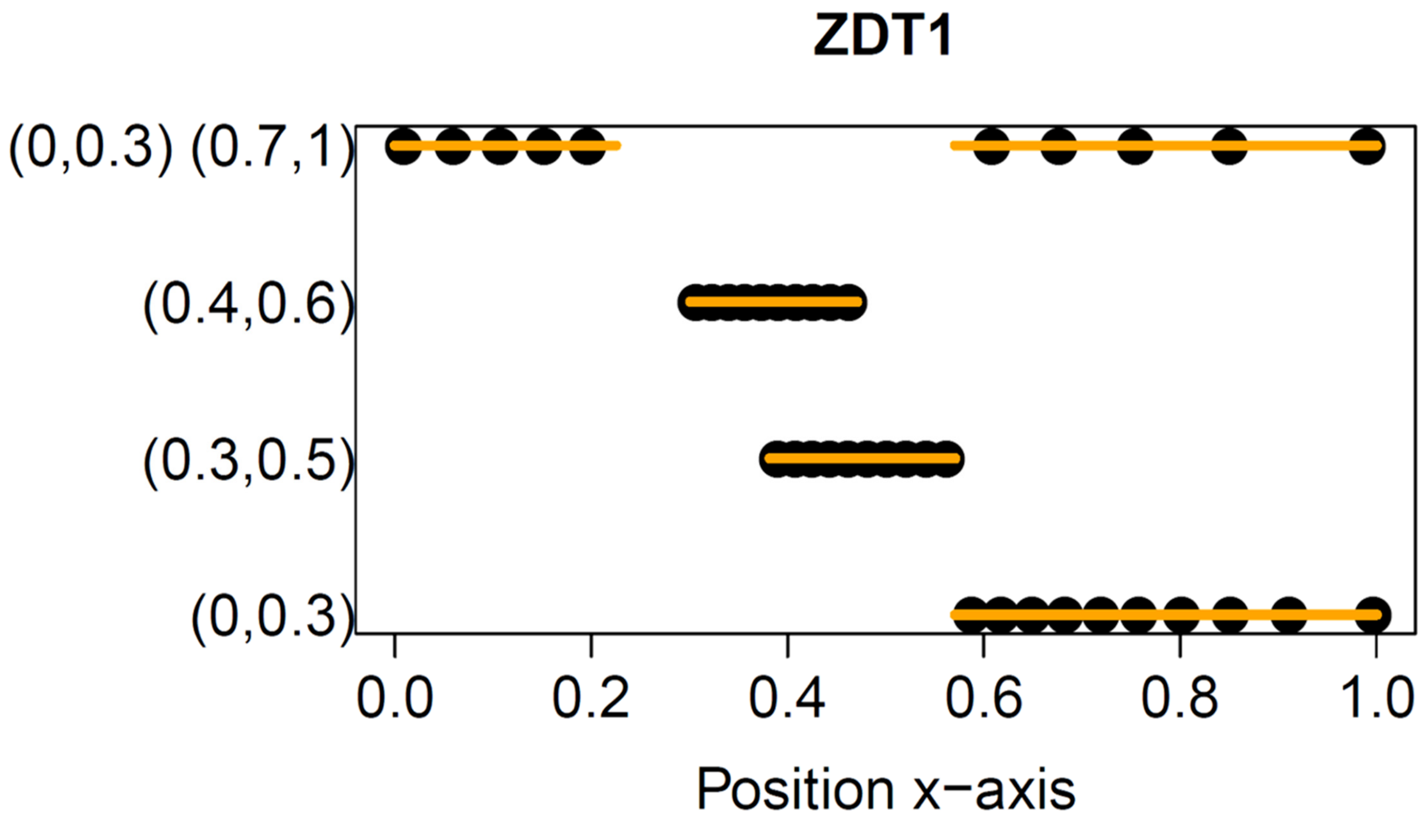



# Example: R2-EMOA

## Concept

Integration of preferences by varying the scalarizing functions

## Restriction of the weight space



# Interactive Approaches

## Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

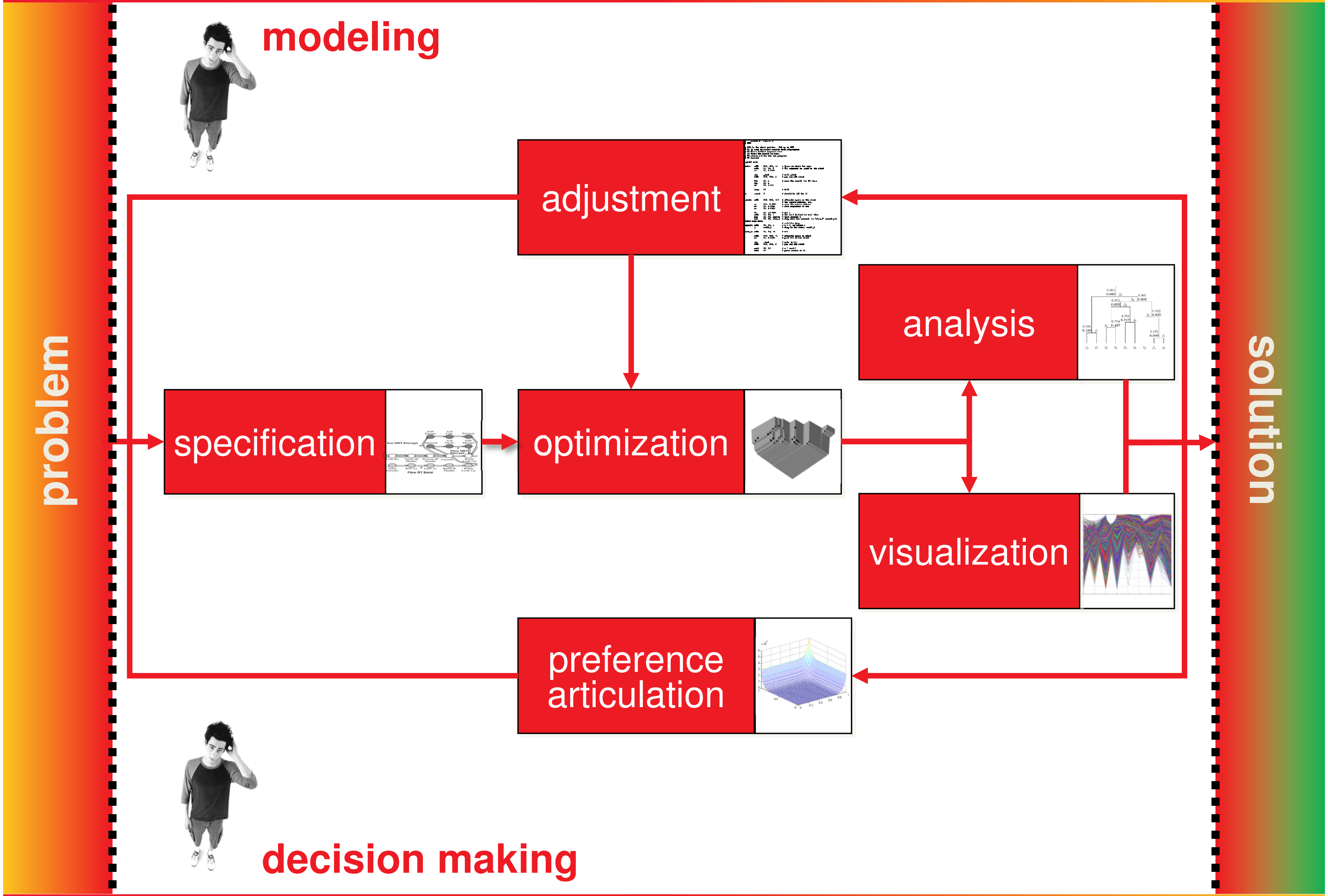
## Examples

- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

## Issues/Open Questions

- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

# Conclusions: EMO as Interactive Decision Support



# The EMO Community

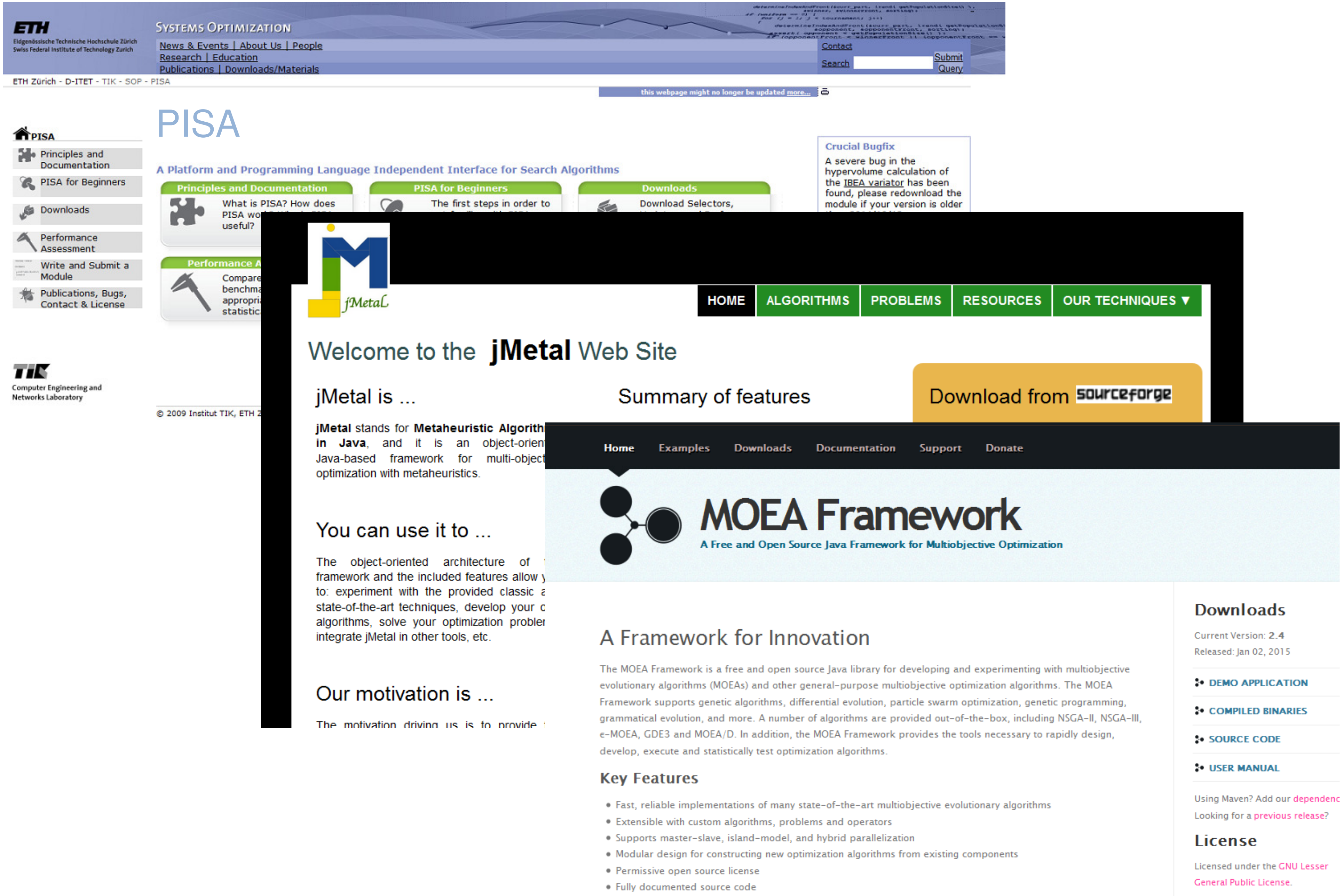
## Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

## Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

# Software



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A Framework for Innovation
The MOEA Framework is a free and open source Java library for developing and experimenting with multiobjective evolutionary algorithms (MOEAs) and other general-purpose multiobjective optimization algorithms. The MOEA Framework supports genetic algorithms, differential evolution, particle swarm optimization, genetic programming, grammatical evolution, and more. A number of algorithms are provided out-of-the-box, including NSGA-II, NSGA-III, ε-MOEA, GDE3 and MOEA/D. In addition, the MOEA Framework provides the tools necessary to rapidly design, develop, execute and statistically test optimization algorithms.
Key Features
• Fast, reliable implementations of many state-of-the-art multiobjective evolutionary algorithms
• Extensible with custom algorithms, problems and operators
• Supports master-slave, island-model, and hybrid parallelization
• Modular design for constructing new optimization algorithms from existing components
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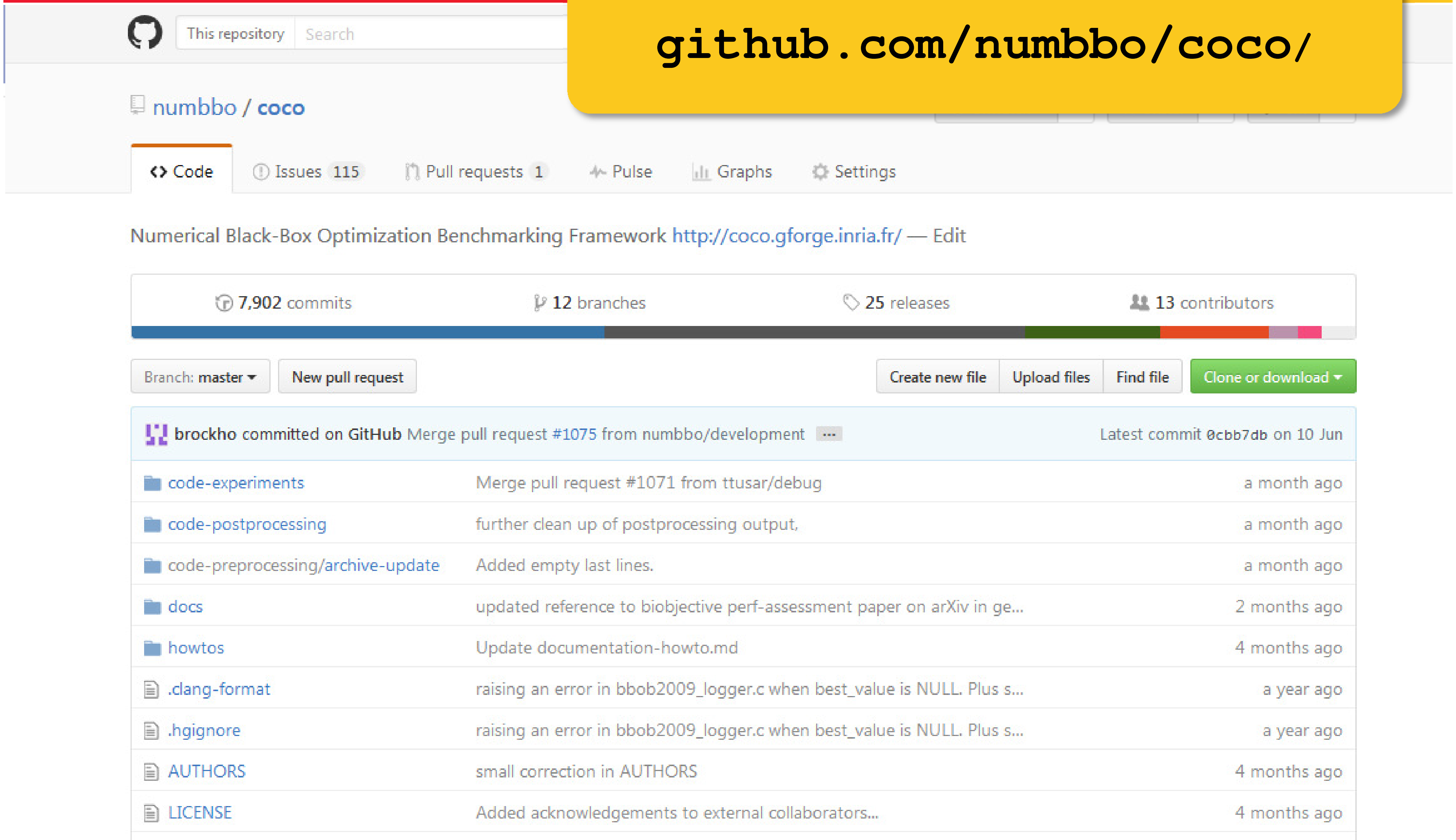
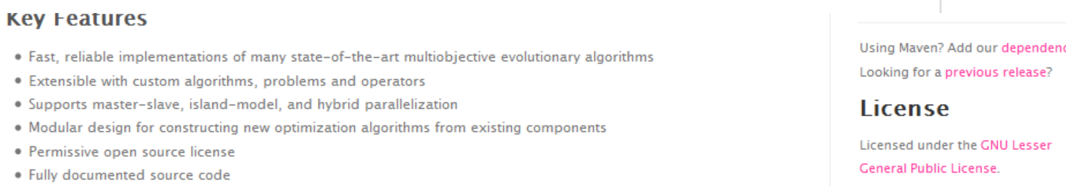
# Software

**github.com/numbbo/coco/**



# Perspectives

## Challenging Open (Research) Directions

- from algorithms to toolkits
  - libraries of modules for each task (selection, variation, etc.)
  - problem-specific algorithm configuration/ parameter tuning
- benchmarking
  - comparison with classical approaches
  - design/selection of practically relevant problems
  - Algorithm/toolkit recommendations for practice
- integration of EMO and MCDM into one field
- interactive preference articulation and learning
- interactive problem design
- integration of problem-specific knowledge

**Questions?**

# Additional Slides

# Instructor Biography: Dimo Brockhoff

**Dimo Brockhoff**

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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at Inria Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011, Dimo has been a permanent researcher at Inria: from 2011 till 2016 with the Inria Lille - Nord Europe research center and since October 2016 with the Saclay - Ile-de-France research center, co-located with CMAP, Ecole Polytechnique. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

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# Acknowledgements

I would like to thank in particular Eckart Zitzler and Tobias Wagner who contributed significantly to the content of these slides over the years.

Many thanks go also to Carlos Fonseca and Manuel López-Ibáñez for pointing out some mistakes during my PPSN 2016 presentation on the same topic that should be corrected in these slides.