GECCO 2022 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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The final slides are available at
http://www.cmap.polytechnique.fr/~dimo.brockhoff/

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Overview

Our plan

Discuss history, present and future of multiobjective benchmarking

With respect to different topics

performance assessment / methodology

test functions

Finally, recommendations on good algorithms
This is not an introductory tutorial to multiobjective optimization!

We assume you know basic definitions like

- Objective function
- Pareto dominance/Pareto front/Pareto set
- Ideal/Nadir points
We only consider continuous search spaces
We only consider continuous search spaces

We only consider unconstrained problems
We only consider continuous search spaces

We only consider unconstrained problems

What we present is highly subjective & selective
  • how important do we find each milestone?
  • use version numbering and branches
  • what have we learned from the past?
Overview

1. Performance Assessment
2. Test Problems and Their Visualizations
3. Recommendations from Numerical Results
v0.0.1alpha
... multiobjective EAs were mainly compared visually:

ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II
v0.1beta
<table>
<thead>
<tr>
<th>Problem</th>
<th>MOCSA $\langle d \rangle$</th>
<th>MOCSA S</th>
<th>MOCSA GD</th>
<th>MOCSA ER</th>
<th>NSGA2 $\langle d \rangle$</th>
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<th>NSGA2 GD</th>
<th>NSGA2 ER</th>
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</table>
Table 4: Influence of different $\kappa$ values on the performance of the cone-MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for $\kappa$ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Table 7: Problem-wise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Table 8: Problem-wise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Numbers have their value. But not only tables, please!
v1.0
Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

<table>
<thead>
<tr>
<th>Indicator</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypervolume indicator</td>
<td>6.3431</td>
<td>7.1924</td>
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<tr>
<td>$\epsilon$-indicator</td>
<td>1.2090</td>
<td>0.12722</td>
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<tr>
<td>$R_2$ indicator</td>
<td>0.2434</td>
<td>0.1643</td>
</tr>
<tr>
<td>$R_3$ indicator</td>
<td>0.6454</td>
<td>0.3475</td>
</tr>
</tbody>
</table>

see e.g. [Zitzler et al. 2003]
Empirical Attainment Functions

\[ f_2(x) \]

\[ f_1(x) \]

Run 1

© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]
Empirical Attainment Functions

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Empirical Attainment Functions

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[López-Ibáñez et al. 2010]
Empirical Attainment Functions

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[López-Ibáñez et al. 2010]
The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector $z$ at time $T$:

$$
\alpha_T(z) = \frac{1}{N} \sum_{i=1}^{N} 1\{\mathcal{X}_i \preceq_T z\}
$$

with $\preceq_T$ being the weak dominance relation between a solution set and an objective vector at time $T$. 

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with $\preceq_T$ being the weak dominance relation between a solution set and an objective vector at time $T$.

Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time $T$ in the single-objective case ("fixed budget scenario").
Empirical Attainment Functions in Practice

latest implementation online at http://eden.dei.uc.pt/~cmfonsec/software.html

R package: http://lopez-ibanez.eu/eaftools

see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

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[López-Ibáñez et al. 2010]
Idea:

transfer multiobjective problem into a set problem
define an objective function (“unary quality indicator”) on sets
use the resulting total (pre-)order (on the quality values)
Quality Indicator Approach

Idea:
transfer multiobjective problem into a set problem
define an objective function (“unary quality indicator”) on sets
use the resulting total (pre-)order (on the quality values)

Question:
Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?
⇒ Underlying dominance relation should be reflected!

\[ A \preceq B : \iff \forall_{b \in B} \exists_{a \in A} a \preceq b \]
Monotonicity when quality indicator does not contradict relation

\[ A \preceq B \Rightarrow I(A) \geq I(B) \]
Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation
  \[ A \preceq B \Rightarrow I(A) \geq I(B) \]

- Strict monotonicity: better = higher indicator
  \[ A \preceq B \text{ and } A \neq B \Rightarrow I(A) > I(B) \]
Example: Refinements Using Indicators

$I(A) =$ volume of the weakly dominated area in objective space

$I(A) =$ unary hypervolume indicator

$I(A, R) =$ how much needs $A$ to be moved to weakly dominate $R$

$I(A, R) =$ unary epsilon indicator
v1.0.1 – v1.0.100 and counting
Performance Assessment of Multiobjective Optimizers: An Analysis and Review

Eckart Zitzler1, Lothar Thiele1, Marco Laumanns1, Carlos M. Fonseca2, and Viviane Grunert da Fonseca2

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[Zitzler et al. 2003]
Performance indicators in multiobjective optimization

Charles Audet\textsuperscript{a}, Jean Bigeon\textsuperscript{b}, Dominique Cartier\textsuperscript{c}, Sébastien Le Digabel\textsuperscript{a}, Ludovic Salomon\textsuperscript{a,1}

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[Audet et al 2021]

Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

Miqing Li, and Xin Yao\textsuperscript{1}

\textsuperscript{1}CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.
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[Li and Yao 2019]
Focus on indicators which are (strictly) monotone
all hypervolume-based indicators
unary epsilon indicator
R2
IGD+
Focus on indicators which are (strictly) monotone

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

Why is monotonicity important?

- Pareto dominance is the lowest form of preference
- If dominance relation does not hold, we have not defined a true multiobjective problem.
v2.0
With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

We can use our normal tools from single-objective optimization, including
- reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- statistical tests, box plots, …

see for example [Hansen et al. 2021]
convergence graphs is all we have to start with...
convergence graphs is all we have to start with...

Advantage of fixed target view: ratio scale (interpretation of #fevals easier than for f-values)
ECDF:
Empirical Cumulative Distribution Function of the Runtime
Convergence Graph of 15 Runs

quality indicator (to be minimized)

$log_{10}$(function evaluations)
15 Runs ≤ 15 Runtime Data Points

The graph shows the quality indicator (to be minimized) as a function of the logarithm of the number of function evaluations. The target is indicated by the red horizontal line.
Empirical Cumulative Distribution

The ECDF of run lengths to reach the target:

- has for each data point a vertical step of constant size
- displays for each x-value (budget) the count of observations to the left (first hitting times)
Reconstructing A Single Run

quality indicator (to be minimized)

log_{10}(function evaluations)
Reconstructing A Single Run

quality indicator (to be minimized)

50 equally spaced targets

log_{10}(function evaluations)
Reconstructing A Single Run

quality indicator (to be minimized)

$\log_{10}(\text{function evaluations})$
Reconstructing A Single Run

quality indicator (to be minimized)

log_{10}(function evaluations)
Reconstructing A Single Run

The empirical CDF makes a step for each star, is monotonous and displays for each budget the fraction of targets achieved within the budget.
Reconstructing A Single Run

The ECDF recovers the monotonous graph, discretized and flipped.
Reconstructing A Single Run

The ECDF recovers the monotonous graph, discretized and flipped.
Aggregation

quality indicator (to be minimized)

$\log_{10}(\text{function evaluations})$
Aggregation

15 runs
50 targets

quality indicator (to be minimized)

$\log_{10}(\text{function evaluations})$
Aggregation

15 runs
50 targets
Aggregation

15 runs
50 targets
ECDF with 750 steps

quality indicator (to be minimized)

log_{10}(function evaluations)
Aggregation

50 targets from 15 runs integrated in a single graph

quality indicator (to be minimized)

$\log_{10}($function evaluations$)$
area over the ECDF curve = average log runtime (or geometric avg. runtime) over all targets (difficult and easy) and all runs
Worth to Note

ECDF graphs

- should never aggregate over dimension

  dimension is input parameter to algorithm
ECDF graphs

- should never aggregate over dimension
dimension is input parameter to algorithm

- but often over targets and functions

- can show data of more than 1 algorithm at a time
ECDF graphs

- should never aggregate over dimension
  dimension is input parameter to algorithm

- but often over targets and functions

- can show data of more than 1 algorithm at a time

- are an extension of data profiles

introduced by Moré and Wild [Moré and Wild 2009]

but for multiple and absolute targets
ECDF graphs

- should never aggregate over dimension
  - dimension is input parameter to algorithm
- but often over targets and functions
- can show data of more than 1 algorithm at a time
- are an extension of data profiles
  - introduced by Moré and Wild [Moré and Wild 2009]
  - but for multiple and absolute targets
- are COCO’s main performance visualization tool
Example ECDF (later more)

- bbof-biobj f1-f55, 5-D
- 58 targets: 1.-1.0e-4
- 10 instances

Fraction of function target pairs

log10(# f-evals / dimension)
In single-objective optimization: scaling behavior mandatory to investigate
In single-objective optimization: scaling behavior mandatory to investigate

In multiobjective optimization:
  actually two dimensions: search and objective space
In single-objective optimization: scaling behavior mandatory to investigate

In multiobjective optimization:
- actually two dimensions: search and objective space
- but former almost never looked at right now 😞

~10 papers from EMO’21 and PPSN/GECCO/CEC’21 change dimension
but 50+ papers have a “fixed” dimension
Mostly Overlooked: Scaling with Dimension

- In single-objective optimization: scaling behavior mandatory to investigate
- In multiobjective optimization:
  actually two dimensions: search and objective space
  but former almost never looked at right now 😞

~10 papers from EMO’21 and PPSN/GECCO/CEC’20 change dimension
but 50+ papers have a “fixed” dimension

but in practice search space scalability almost more important
number of objectives often fixed
A Few General Recommendations

- always display everything you have
- look at single runs
- do each experiment at least twice

 (= look at the variance of your results)
A Few General Recommendations

- always display everything you have
- look at single runs
- do each experiment at least twice
  (= look at the variance of your results)
- as quality indicators, use hypervolume, R2, or epsilon indicator
  or any indicator which is at least monotone
A Few General Recommendations

- always **display everything** you have
- look at **single runs**
- do each experiment **at least twice**
  
  (= look at the *variance* of your results)
- as quality indicators, use hypervolume, R2, or epsilon indicator or any indicator which is at least monotone
- see also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

Recommended Experimental Setup (w/ or w/o COCO)

1. Benchmarking Experiment
2. Choosing Algorithms for Comparison
   - see https://numbbo.github.io/data-archive/
3. Postprocessing
   - python -m cocopp resultfolder/ ALG2 ALG3
4. Displaying and Discussing Summary Results
5. Investigating and Discussing Complementary Results
6. Processed Data Sharing
   - provide html output somewhere
7. Raw Data Sharing
   - easy with COCO archive module & through issue tracker
Overview

1. Performance Assessment
2. Test Problems and Their Visualizations
3. Recommendations from Numerical Results
Introduction

Test Problems (1)

Artificial problems (continuous and unconstrained)

- **v0.1**: Individual problems
- **v0.2**: MOP suite (unscalable problems)
- **v0.5**: ZDT suite (scalable number of variables)
- **v1.0**: DTLZ suite (scalable number of variables and objectives)
- **v1.2**: WFG suite
- **v1.3**: Other suites with a bottom-up construction
- **v1.5**: Suites of distance-based problems
- **v2.0**: The bbo-biobj(-ext) suite
Visualization of multiobjective landscapes

Low-dimensional search spaces
  Dominance ratio
  Local dominance
  Gradient path length

  PLOT

Any-dimensional search spaces
  Line cuts
  Optima network
Test Problems (2)

Artificial problems (other)
Constrained problems
Mixed-integer problems

Real-world problems
v0.1: Individual problems
v0.2: Suites of unscalable problems
v0.5: Suites of scalable problems (number of variables)

Conclusions
Introduction

Why use test problems?
Introduction

Why use test problems?

Expert knowledge
Landscape analysis
Desirable Characteristics of a Problem Set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]
Artificial problems (continuous and unconstrained)
Individual problems

Minimize = \[
\begin{align*}
& f_1(x) = x^2 \\
& f_2(x) = (x - 2)^2
\end{align*}
\]

[Schaffer 1985]

Minimize = \[
\begin{align*}
& f_1(x) = \sum_{i=1}^{2} \left[ -10 \exp \left( -0.2 \sqrt{x_i^2 + x_{i+1}^2} \right) \right] \\
& f_2(x) = \sum_{i=1}^{3} \left[ |x_i|^{0.8} + 5 \sin(x_i^3) \right]
\end{align*}
\]

[Kursawe 1991]
Individual problems

Minimize = \[
\begin{align*}
    f_1 (x) &= 1 - \exp \left[ - \sum_{i=1}^{n} \left( x_i - \frac{1}{\sqrt{n}} \right)^2 \right] \\
    f_2 (x) &= 1 - \exp \left[ - \sum_{i=1}^{n} \left( x_i + \frac{1}{\sqrt{n}} \right)^2 \right]
\end{align*}
\]

[Fonseca and Fleming 1995]

Minimize = \[
\begin{align*}
    f_1 (x, y) &= \left[ 1 + (A_1 - B_1 (x, y))^2 + (A_2 - B_2 (x, y))^2 \right] \\
    f_2 (x, y) &= (x + 3)^2 + (y + 1)^2
\end{align*}
\]

where = \[
\begin{align*}
    A_1 &= 0.5 \sin(1) - 2 \cos(1) + \sin(2) - 1.5 \cos(2) \\
    A_2 &= 1.5 \sin(1) - \cos(1) + 2 \sin(2) - 0.5 \cos(2) \\
    B_1 (x, y) &= 0.5 \sin(x) - 2 \cos(x) + \sin(y) - 1.5 \cos(y) \\
    B_2 (x, y) &= 1.5 \sin(x) - \cos(x) + 2 \sin(y) - 0.5 \cos(y)
\end{align*}
\]

[Poloni et al. 1996]
Individual problems

Minimize = \[
\begin{align*}
  f_1(x, y) &= 0.5 \left( x^2 + y^2 \right) + \sin\left( x^2 + y^2 \right) \\
  f_2(x, y) &= \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\
  f_3(x, y) &= \frac{1}{x^2+y^2+1} - 1.1 \exp\left( - \left( x^2 + y^2 \right) \right)
\end{align*}
\]

[Viennet et al. 1996]
MOP Suite

MOP = Multi-Objective Problem
[Van Veldhuizen 1999]

Properties

- A collection of 7 test problems from the literature (including the 5 shown before)
- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- Many problems have optimal solutions on the boundary or middle of the search space
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries
- The Pareto set is hard to compute for some problems
ZDT = Zitzler, Deb, Thiele

[Zitzler et al. 2000]

The same construction for all problems (following Deb’s toolkit [Deb 1999])

Given

\[
x = \{x_1, \ldots, x_n\}
\]

Minimise

\[
f_1(y)
\]

Distance f. Front shape

\[
f_2(y, z) = g(z) h(f_1(y), g(z))
\]

where

\[
y = \{x_1, \ldots, x_j\}
\]

\[
z = \{x_{j+1}, \ldots, x_n\}
\]

The separation of variables was done to simplify problem construction.
ZDT Suite

Properties

- 6 test problems, but ZDT5 is regularly omitted, because it has a binary encoding
- Scalable in the number of (distance) variables
- All problems have 2 objectives
- 4 problems have optimal solutions on the boundary and 1 in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known
v1.0
DTLZ = Deb, Thiele, Laumanns, Zitzler
[Deb et al. 2005]

**Desired Features of Test Problems**

1. Have controllable difficulty to converge to the Pareto front, a widely-distributed set of Pareto-optimal solutions
2. Scalable number of variables
3. Scalable number of objectives
4. Simple to construct
5. Pareto front easy to comprehend, both the Pareto set and front known
6. Similar difficulties to those present in real-world problems
Problem Design Approaches

1. Multiple single-objective functions approach
2. Bottom-up approach
   1. Choose a Pareto front
   2. Build the objective space
   3. Construct the search space (add difficulties using the function $g$)
3. Constraint surface approach (for constrained problems)
Properties

- Originally 9 problems, but then 2 were dropped
- Scalable number of distance variables, $M - 1$ position variables
- Scalable number of objectives
- Objectives separable in practice (optimizing one variable at a time will yield at least one global optimum)
- Linear, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known
- Most problems have the Pareto set in the middle of the search space

Note that although the suite is scalable in the number of variables, this is rarely used in benchmark studies.
v1.2
WFG = Walking Fish Group
[Huband et al. 2006]

Recommendations for multiobjective test problems

1. No extremal variables
2. No medial variables
3. Scalable number of variables
4. Scalable number of objectives
5. Dissimilar variable domains
6. Dissimilar objective ranges
7. Pareto set and front known
Recommendations for multiobjective test suites

[Huband et al. 2006]

1. A few unimodal test problems to test convergence velocity relative to different Pareto optimal geometries and bias conditions
2. Cover the three core types of geometries: degenerate Pareto fronts, disconnected Pareto fronts, and disconnected Pareto sets
3. The majority of problems should be multimodal with a few deceptive problems
4. The majority of problems should be nonseparable
5. Contain problems that are both nonseparable and multimodal to be representative of real-world problems
Properties

- 9 problems constructed from a combination of shape functions and several transformations
- Scalable number of variables (2 variables are not supported for some of the problems)
- Scalable number of objectives
- Includes also nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts
- The Pareto sets and fronts are known
- Optimal solutions do not lie on the boundary or the middle of the search space, but the Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables
Problems constructed with the bottom-up approach

[Zapotecas et al. 2019]

- IHR test suite of 5 rotated ZDT problems [Igel et al. 2007]
- LZ test suite of 9 problems with complicated Pareto sets [Li and Zhang 2009]
- SZDT test suite of 7 scalable problems with complicated Pareto sets [Saxena et al. 2011]
- Convex DTLZ problem [Deb and Jain 2014]
- Inverted DTLZ problem [Jain and Deb 2014]
- MNI test suite of 2 test problems with diverse shapes of the Pareto front [Masuda et al. 2016]
- LSMOP test suite of 9 test problems for large-scale optimization with variable dependencies [Cheng et al. 2017b]
- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]
CEC Competition Suites

Information about all CEC competitions:
https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm

- 13 test problems for CEC 2007 [Huang et al. 2007]
  OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
  4 shifted ZDT, 1 rotated ZDT
  2 shifted DTLZ, 1 rotated DTLZ
  3 WFG

- 13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]
  10 with complicated Pareto sets (4 from the LZ suite)
  2 extended rotated DTLZ
  1 WFG
CEC Competition Suites

- 15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]
  7 modified DTLZ problems
  2 distance minimization problems
  3 WFG problems
  1 SZDT problem
  2 LSMOP problem
- 22 test problems for CEC 2019 [Liang et al. 2019]
  2 SYM-PART
  Omni-test [Deb and Tiwari 2008]
  19 MMF problems
- 24 test problems for CEC 2020 [Liang et al. 2020]
  24 MMF problems
64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)

- CEC 2020
- GECCO 2020
- PPSN 2020
- EMO 2021
General idea
[Ishibuchi et al. 2010]

Minimize \( f(x) = (f_1(x), f_2(x), ..., f_k(x)) \)

\( f_i(x) = \min\{\text{dis}(x, a_{i1}), \text{dis}(x, a_{i2}), ..., \text{dis}(x, a_{im})\} \)

- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions
- Unlikely to be relevant for real-world problems
- Based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]
Distance-Based Problems

Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]
v2.0
Motivation
[Brockhoff et al. 2016]

- Most other suites are constructed based on the desired Pareto front properties
- Consequently, problems have artificial properties not likely to exist in real-world problems
  - Distance and position parameters (DTLZ-like problems)
  - Linear objectives (distance-based problems)
- In real-world problems each objective is a separate function
- Go back to basics – use single-objective functions for each objective
Construction

- Use the functions from the **bbob** suite
  - Well-understood
  - Scalable in the number of variables and *parametrized*
  - 24 functions categorized in 5 groups based on their properties
    - Separable
    - Low or moderate conditioning
    - High conditioning and unimodal
    - Multimodal with global structure
    - Multimodal with weak global structure

- How to avoid an explosion in the number of problems?
bbob-biobj Suite

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- \( f_1 \): Sphere
- \( f_2 \): Ellipsoid separable
- \( f_6 \): Attractive sector
- \( f_8 \): Rosenbrock original
- \( f_{13} \): Sharp ridge
- \( f_{14} \): Sum of different powers
- \( f_{15} \): Rastrigin
- \( f_{17} \): Schaffer F7 (condition 10)
- \( f_{20} \): Schwefel \( x \times \sin(x) \)
- \( f_{21} \): Gallagher 101 peaks
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- $f_1$: Sphere
- $f_2$: Ellipsoid separable
- $f_3$: Rastrigin separable
- $f_4$: Skew Rastrigin-Bueche
- $f_5$: Linear slope
- $f_6$: Attractive sector
- $f_7$: Step-ellipsoid
- $f_8$: Rosenbrock original
- $f_9$: Rosenbrock rotated
- $f_{10}$: Ellipsoid
- $f_{11}$: Discus
- $f_{12}$: Bent cigar
- $f_{13}$: Sharp ridge
- $f_{14}$: Sum of different powers
- $f_{15}$: Rastrigin
- $f_{16}$: Weierstrass
- $f_{17}$: Schaffer F7 (condition 10)
- $f_{18}$: Schaffer F7 (condition 1000)
- $f_{19}$: Griewank-Rosenbrock F8F2
- $f_{20}$: Schwefel x*sin(x)
- $f_{21}$: Gallagher 101 peaks
- $f_{22}$: Gallagher 21 peaks
- $f_{23}$: Katsuuras
- $f_{24}$: Lunacek bi-Rastrigin
Properties

- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Problem instances more diverse than for the single-objective suite
- Currently limited to 2 objectives
- Unknown Pareto set and front (but known single-objective optima)
- Available approximations of the Pareto fronts (and sets for lower-dimensional problems)
Visualization of multiobjective landscapes
Low-dimensional search spaces

- Dominance ratio [Fonseca 1995]
- Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])
- Local dominance [Fieldsend et al. 2019]
- PLOT [Shaepereimeier et al. 2020]

Any-dimensional search spaces

- Optima network [Liefooghe et al. 2018, Fieldsend and Alyahya 2019]

Various visualizations of bobb-biobj-ext problems:
https://numbbo.github.io/bbob-biobj/

Visualizations of bobb-biobj and other multi-objective suites using PLOT:
https://schaepermeier.shinyapps.io/moPLOT/
Problems for demonstration

- Double sphere problem `bbob-biobj F_1 = (f_1, f_1)`, instance 1
- Sphere-Gallagher problem `bbob-biobj F_{10} = (f_1, f_{21})`, instance 1
- Double Gallagher problem `bbob-biobj F_{55} = (f_{21}, f_{21})`, instance 1

Gallagher = Gallagher’s Gaussian
101-me Peaks Function
Discretized search space (501 x 501 grid)

- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale

[References: Fonseca 1995]
Adjusted from [Kerschke and Grimme 2017]

- Compute the **bi-objective gradient** for all grid points

$$\nu = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum
Local Dominance

- **Green**: Dominance-neutral local optima regions
  Points that are mutually nondominated with all their 8 neighbors (not equal to Pareto sets)

- **Pink**: Basins of attraction
  Points that are dominated by at least one neighbor and whose dominating paths lead to the same green region

- **White**: Boundary regions
  Points whose dominating paths lead to different green regions

[Fieldsend et al. 2019]
Comparison of Problem Landscapes

ZDT4

Two problems where both objectives are separable, first is unimodal and second is multimodal

bbob-biobj-ext $F_{56}$

$f_1$ Sphere function

$f_3$ Rastrigin function
Comparison of Problem Landscapes

DTLZ1

Two problems where both objectives are separable and multimodal

bbob-biobj-ext $F_{62}$

$f_3$ Rastrigin function

$f_4$ Skew Rastrigin-Bueche
Comparison of Problem Landscapes

WFG9

Two problems where both objectives are nonseparable and multimodal

$b\text{bob-biobj-ext } F_{82}$

$f_{17}$ Schaffer F7

$f_{19}$ Griewank-Rosenbrock
Other artificial problems
Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019a]
- DAS-CMOP [Fan et al. 2019b]
- MW [Ma and Wang 2019]
- DOC [Liu and Wang 2019]
- Eq-DLTZ and Eq-IDTLZ [Cuate et al. 2020]
- CLSMOP [He et al. 2021]

Analysis and visualization of multiobjective problems with constraints: 
[https://vodopijaaljosa.github.io/cmop-web/](https://vodopijaaljosa.github.io/cmop-web/)

Tutorial on Multiobjective optimization in the presence of constraints: 
[https://dis.ijs.si/filipic/cec2021tutorial/](https://dis.ijs.si/filipic/cec2021tutorial/)
Suites of multiobjective mixed-integer problems

- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbo-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function.
Real-world problems
v0.1
Real-World Problems

Individual problems

- Radar waveform design problem with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the HBV rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]
- Lunar lander landing site selection problem with 2 variables, 3 objectives and 2 constraints [JSEC and JAXA 2018]
- Wind turbine design problem with 32 variables, 5 objectives and 22 constraints [JSEC 2019]
v0.2
Suites of unscalable problems

- DDMOP suite of 7 test problems with a different number of variables (5–17) and objectives (2–10) [He et al. 2020]
- Two suites of previously published problems [Tanabe and Ishibuchi 2020]
  - RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
    - 11 continuous, 1 integer, 4 mixed-integer
  - CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
    - 6 continuous, 1 integer, 1 mixed-integer
- RCM suite of 50 problems with a different number of variables (2–34), objectives (2–5) and constraints (1–29) [Kumar et al. 2021]
v0.5
Suites of scalable problems

- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]
- Framework with scalable pathfinding problems (5 different objectives) [Weise and Mostaghim 2022]
Conclusions

We should think about the usefulness of our research

Results of a questionnaire on the properties of real-world problems has shown their diversity [van der Blom et al. 2020]

[https://sites.google.com/view/macoda-rwp/home](https://sites.google.com/view/macoda-rwp/home)

Most research is done on continuous unconstrained problems

Although the test problems are scalable, most studies use a fixed number of variables
Problem suites constructed with the bottom-up approach have unrealistic properties.

Algorithms are overfitting to these problems (especially the overused DTLZ and WFG) [Ishibuchi et al. 2017]

Using separate functions for the objectives looks like a step in the right direction.
Overview

1. Performance Assessment
2. Test Problems and Their Visualizations
3. Recommendations from Numerical Results
python -m cocopp bbo-biobj*
Aggregated Results Over All 55 Functions

58 targets: 1..1.0e-4
10 instances

Fraction of function-target pairs

log10(# f-evals / dimension)
Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions
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- many-objective problems
  - problems/suites
  - indicators
  - efficient implementations
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  simulation crashes
  parallelism
  dynamic changes
  interactive decision making
  …
Multiobjective Benchmarking 3.0?

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- real-world benchmarking?
  - simulation crashes
  - parallelism
  - dynamic changes
  - interactive decision making
- ...
- benchmarking results from more classical approaches
Three “New Year” Resolutions
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1. Show convergence graphs/ECDF
   anything else than tables for fixed budget
Three “New Year” Resolutions

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2. Use “most realistic” problems
Three “New Year” Resolutions

1. Show convergence graphs/ECDF anything else than tables for fixed budget
2. Use “most realistic” problems
3. Show scaling with (search & objective space) dimension
Three “New Year” Resolutions

1. Show convergence graphs/ECDF anything else than tables for fixed budget

2. Use “most realistic” problems

3. Show scaling with (search & objective space) dimension

Thank you!
Supplementary Material
Bibliography


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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. After postdoctoral research positions at Inria Saclay Ile-de-France in Orsay and at Ecole Polytechnique in Palaiseau, both in France, Dimo has been a permanent researcher at Inria: from 2011 till 2016 with the Inria Lille - Nord Europe research center and since October 2016 with the Saclay - Ile-de-France research center, co-located with CMAP, Ecole Polytechnique, IP Paris. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.
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