GECCO 2023 Tutorial on Benchmarking Multiobjective Optimizers 2.0

Dimo Brockhoff
dimo.brockhoff@inria.fr
Inria – Inventors for the digital world
Institut Polytechnique de Paris

Tea Tušar
tea.tusar@ijs.si
Jožef Stefan Institute

The final slides will be made available at
http://www.cmap.polytechnique.fr/~dimo.brockhoff/

GECCO '23 Companion, July 15–19, 2023, Lisbon, Portugal

ACM ISBN 979-8-4007-0120-7/23/07$15.00
https://doi.org/10.1145/3583133.3595060

Overview

Our plan

- Discuss history, present and future of multiobjective benchmarking

With respect to different topics

- Performance assessment / methodology
- Test functions

Finally, recommendations on good algorithms

Disclaimer

This is not an introductory tutorial to multiobjective optimization!

We assume you know basic definitions like

- Objective function
- Pareto dominance/Pareto front/Pareto set
- Ideal/Nadir points

Disclaimer II

We only consider continuous search spaces

Disclaimer II

We only consider continuous search spaces

We only consider unconstrained problems

Disclaimer II

We only consider continuous search spaces

We only consider unconstrained problems

What we present is highly subjective & selective

- How important do we find each milestone?
- Use version numbering and branches
- What have we learned from the past?

Overview

❶ Performance Assessment

❷ Test Problems and Their Visualizations

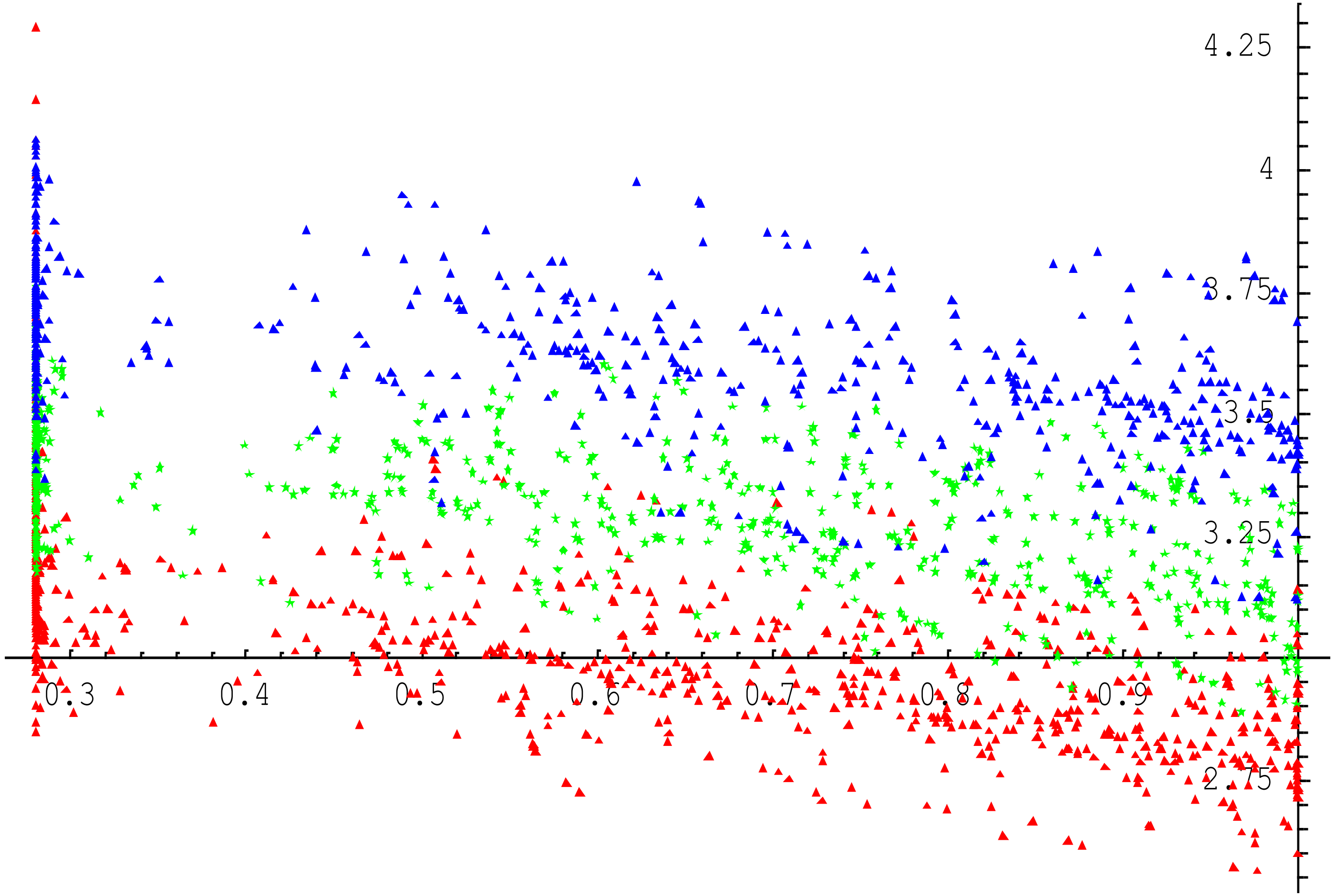
❸ Recommendations from Numerical Results

v0.0.1alpha

Performance Assessment

In the Early Beginnings…

… multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem:

IBEA, SPEA2, NSGA-II

v0.1beta

Performance Assessment

Tables

Problem	MOCSA				NSGA2			
	$\langle d \rangle$	S	GD	ER	$\langle d \rangle$	S	GD	ER
ZDT1	0.0404	0.0055	0.0000	0	0.0270	0.0156	0.0011	0.04
ZDT2	0.0404	0.0082	0.0000	0	0.0292	0.0146	0.0212	0.02
ZDT3	0.0438	0.0148	0.0001	0	0.0329	0.0201	0.0020	0.02
ZDT4	0.0404	0.0097	0.0000	0	0.0328	0.0159	0.0006	0.02
ZDT6	0.0327	0.0150	0.0000	0	0.0216	0.0119	0.0000	0
DTLZ1	0.1114	0.0068	0.0000	0	0.0615	0.0319	0.0000	0
DTLZ2	0.2319	0.0646	0.0021	0.02	0.1361	0.0683	0.0020	0.04
DTLZ3	0.2770	0.0225	0.0000	0	0.1139	0.0739	0.0000	0
DTLZ4	0.2478	0.0424	0.0009	0	0.1630	0.0898	0.0019	0.02
DTLZ5	0.0487	0.0059	0.0000	0	0.0309	0.0176	0.0610	0.06
DTLZ6	0.0484	0.0156	0.0000	0	0.0306	0.0135	0.0000	0
DTLZ7	0.2897	0.0510	0.0011	0.04	0.1880	0.1322	0.0071	0.22

arXiv, 2012

Tables

Table 4: Influence of different κ values on the performance of the coneϵ-MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for κ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		κ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.0001	0.0001
Δ	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
$\lvert\mathcal{H}\rvert$	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	$< 10^{-4}$	$< 10^{-4}$	0.2537	0.3457	0.4842	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.1826	0.1826	0.1826

Metric		κ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
Δ	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
$\lvert\mathcal{H}\rvert$	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		κ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
Δ	M	0.0503	0.6066	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	[illegible]	[illegible]
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	[illegible]	[illegible]
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	[illegible]	[illegible]
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	[illegible]	[illegible]
$\lvert\mathcal{H}\rvert$	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	[illegible]	[illegible]
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	[illegible]	[illegible]

Metric		κ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	[illegible]	[illegible]
γ	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	[illegible]	[illegible]
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0369	0.0281	0.0284	[illegible]	[illegible]
Δ	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	[illegible]	[illegible]
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	[illegible]	[illegible]
$\lvert\mathcal{H}\rvert$	M	14.00	79.50	90.00	92.00	95.00	96.00	95.50	97.00	98.00	[illegible]	[illegible]
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	[illegible]	[illegible]

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ-MOEA	coneϵ-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	γ	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	Δ	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	γ	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	Δ	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	γ	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	Δ	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	γ	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	Δ	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	0.04±2e-3	0.13±3e-3	0.01±6e-4	0.08±3e-3	0.07±3e-3
Zdt3	γ	0.16±2e-2	0.15±1e-2	0.17±10e-3	0.19±1e-2	0.35±3e-2	0.24±2e-2
	Δ	0.67±1e-2	0.85±1e-2	0.57±2e-2	0.49±1e-2	0.33±7e-3	0.50±2e-2
	HV	0.98±2e-3	0.97±3e-3	0.97±2e-3	0.97±2e-3	0.95±4e-3	0.97±3e-3
	CS	0.41±3e-2	0.36±3e-2	0.33±2e-2	0.29±2e-2	0.12±2e-2	0.24±3e-2
Zdt4	γ	0.27±2e-2	0.32±2e-2	0.35±2e-2	0.47±2e-2	0.69±2e-2	0.57±2e-2
	Δ	0.61±9e-3	0.59±1e-2	0.48±1e-2	0.58±1e-2	0.53±1e-2	0.68±1e-2
	HV	0.84±9e-3	0.80±1e-2	0.79±1e-2	0.73±1e-2	0.62±1e-2	0.66±2e-2
	CS	0.73±3e-2	0.58±4e-2	0.57±3e-2	0.41±4e-2	0.15±2e-2	0.21±4e-2
Zdt6	γ	0.06±3e-2	0.04±1e-2	0.04±2e-2	0.04±1e-2	0.01±3e-3	0.03±2e-2
	Δ	0.52±1e-2	0.24±1e-2	0.27±1e-2	0.37±9e-3	0.18±8e-3	0.37±2e-2
	HV	0.96±1e-2	0.98±7e-3	0.98±10e-3	0.97±7e-3	0.99±1e-3	0.98±10e-3
	CS	0.18±2e-2	0.11±2e-2	0.18±2e-2	0.10±1e-2	0.21±2e-2	0.21±2e-2

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ-MOEA	coneϵ-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	γ	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	Δ	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	γ	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	Δ	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.90±7e-4	0.93±4e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	γ	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	Δ	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	γ	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	Δ	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.02±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	γ	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	Δ	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
Dtlz6	γ	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	Δ	0.78±1e-2	0.99±6e-4	0.45±6e-4	0.51±7e-3	0.32±5e-3	0.62±1e-2
	HV	0.99±1e-4	0.99±4e-5	0.99±2e-5	0.99±9e-5	0.99±3e-5	0.99±1e-4
	CS	0.02±6e-4	0.03±8e-4	0.04±8e-4	0.03±8e-4	0.03±8e-4	0.01±7e-4
Dtlz7	γ	0.74±9e-3	0.47±2e-3	0.74±10e-3	0.86±9e-3	0.67±8e-3	0.74±10e-3
	Δ	0.78±1e-2	0.55±9e-3	0.71±9e-3	0.56±1e-2	0.52±8e-3	0.78±8e-3
	HV	0.92±1e-3	0.91±1e-3	0.93±7e-4	0.91±1e-3	0.94±6e-4	0.92±9e-4
	CS	0.06±2e-3	0.08±2e-3	0.06±2e-3	0.02±1e-3	0.09±3e-3	0.07±2e-3
[illegible]	γ	0.45±2e-2	0.03±10e-4	0.24±5e-3	0.52±1e-2	0.83±9e-3	0.58±1e-2
	[illegible]	[illegible]	0.69±9e-3	0.70±9e-3	0.49±8e-3	0.36±8e-3	0.80±1e-2
	[illegible]	[illegible]	0.20±3e-3	0.24±4e-3	0.10±3e-3	0.10±3e-3	0.09±3e-3
[illegible]	[illegible]	[illegible]	0.16±7e-3	0.57±5e-3	0.64±7e-3	0.77±7e-3	0.90±5e-3
	[illegible]	[illegible]	0.71±1e-2	0.52±6e-3	0.49±6e-3	0.45±4e-3	0.61±6e-3
	[illegible]	[illegible]	0.51±1e-2	0.34±7e-3	0.23±8e-3	0.24±8e-3	0.13±7e-3

arXiv, 2020

Numbers have their value.

But not *only* tables, please!

v1.0

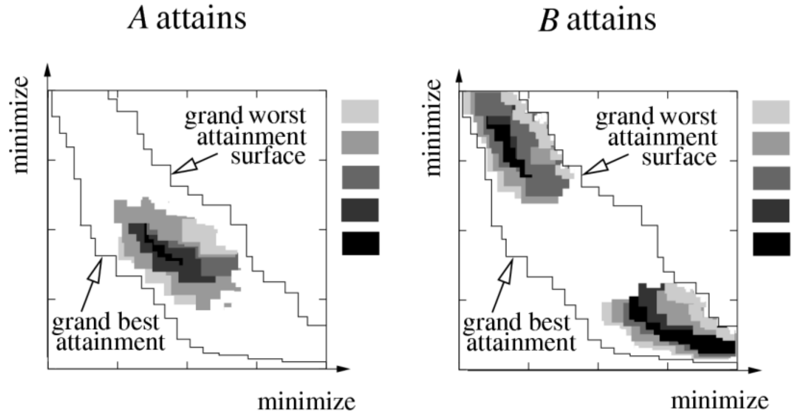
Performance Assessment

v1.0: Two Approaches for Empirical Studies

Attainment function approach

[Fonseca and Fleming 1996]

- Applies statistical tests directly to the approximation set
- Detailed information about how and where performance differences occur



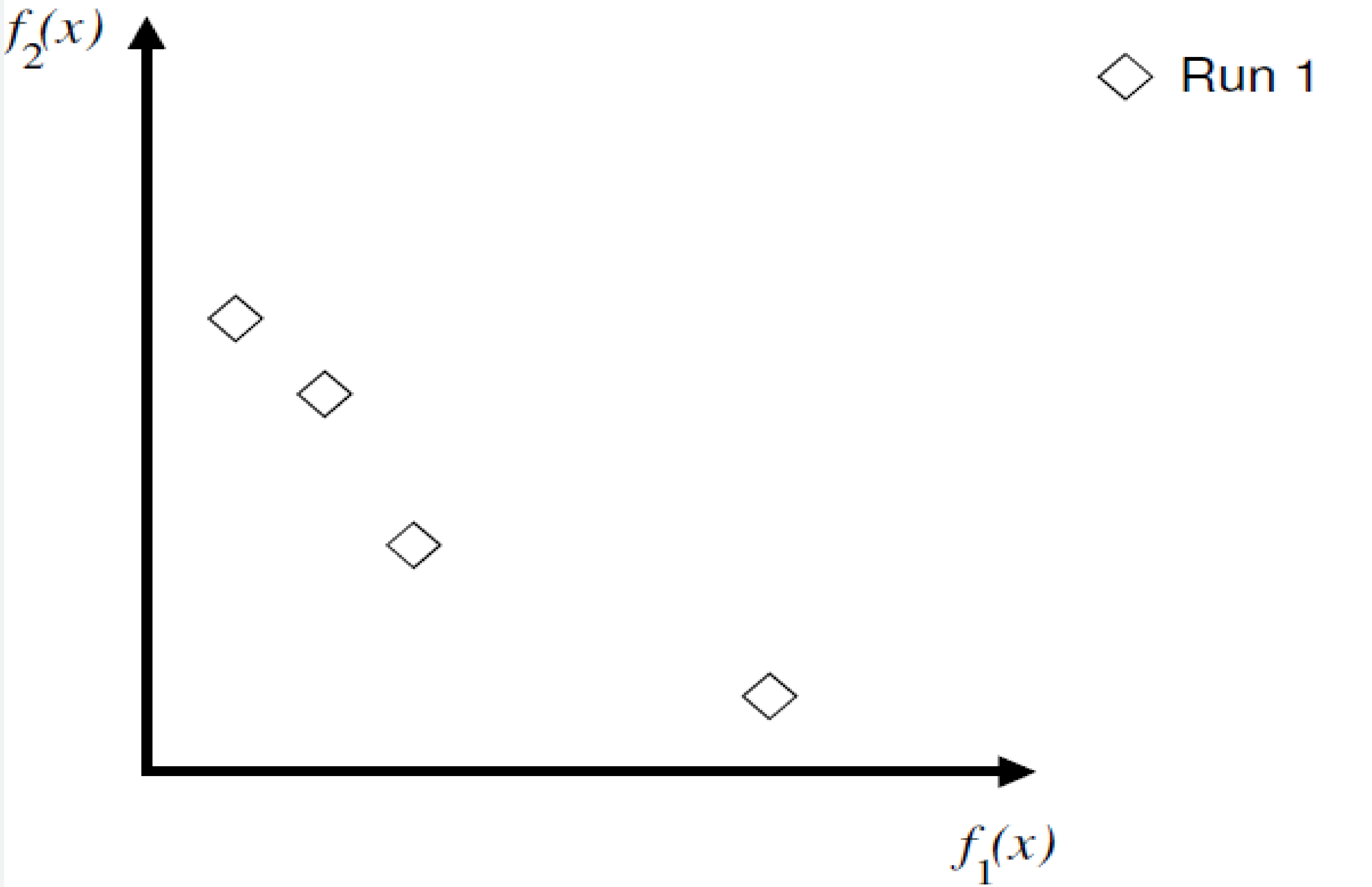
Quality indicator approach

- Reduces each approximation set to a single quality value
- Applies statistical tests to the quality values

Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

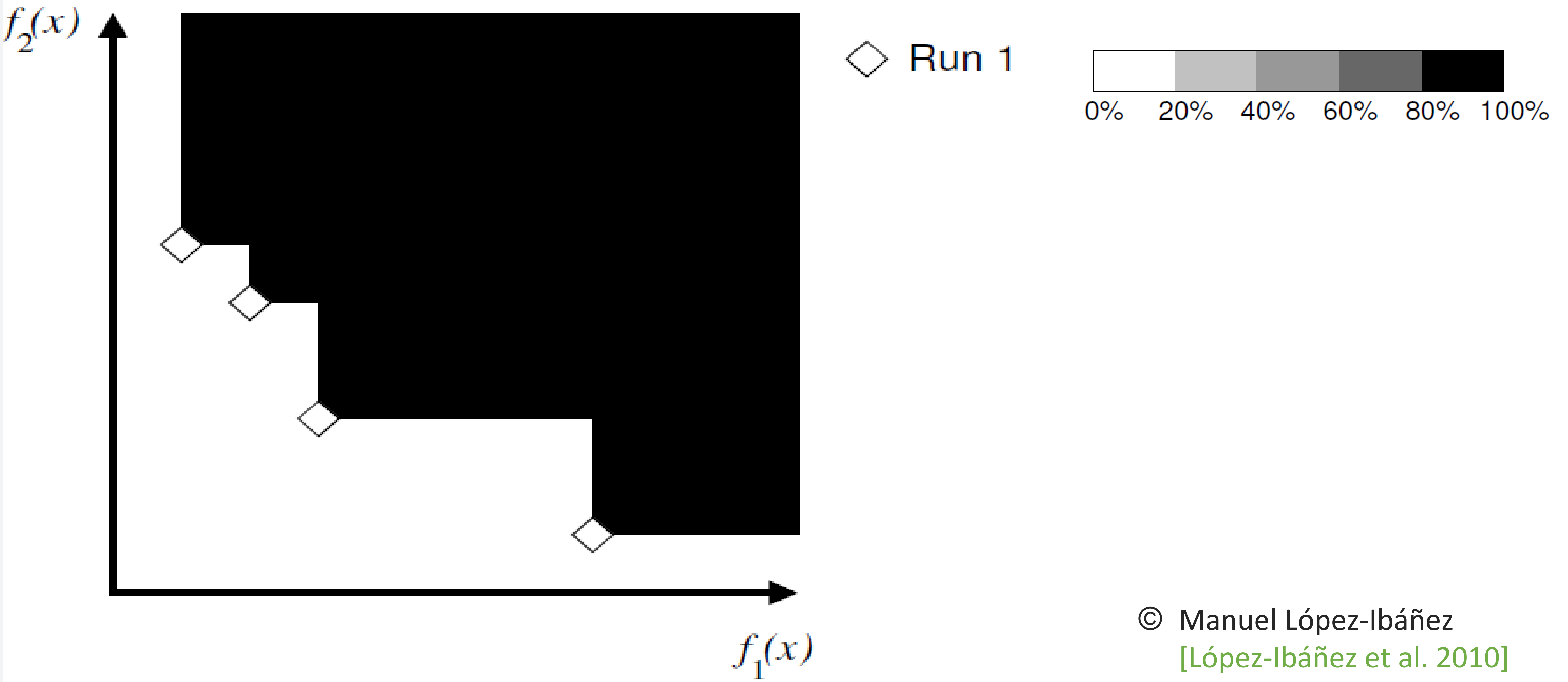
see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions



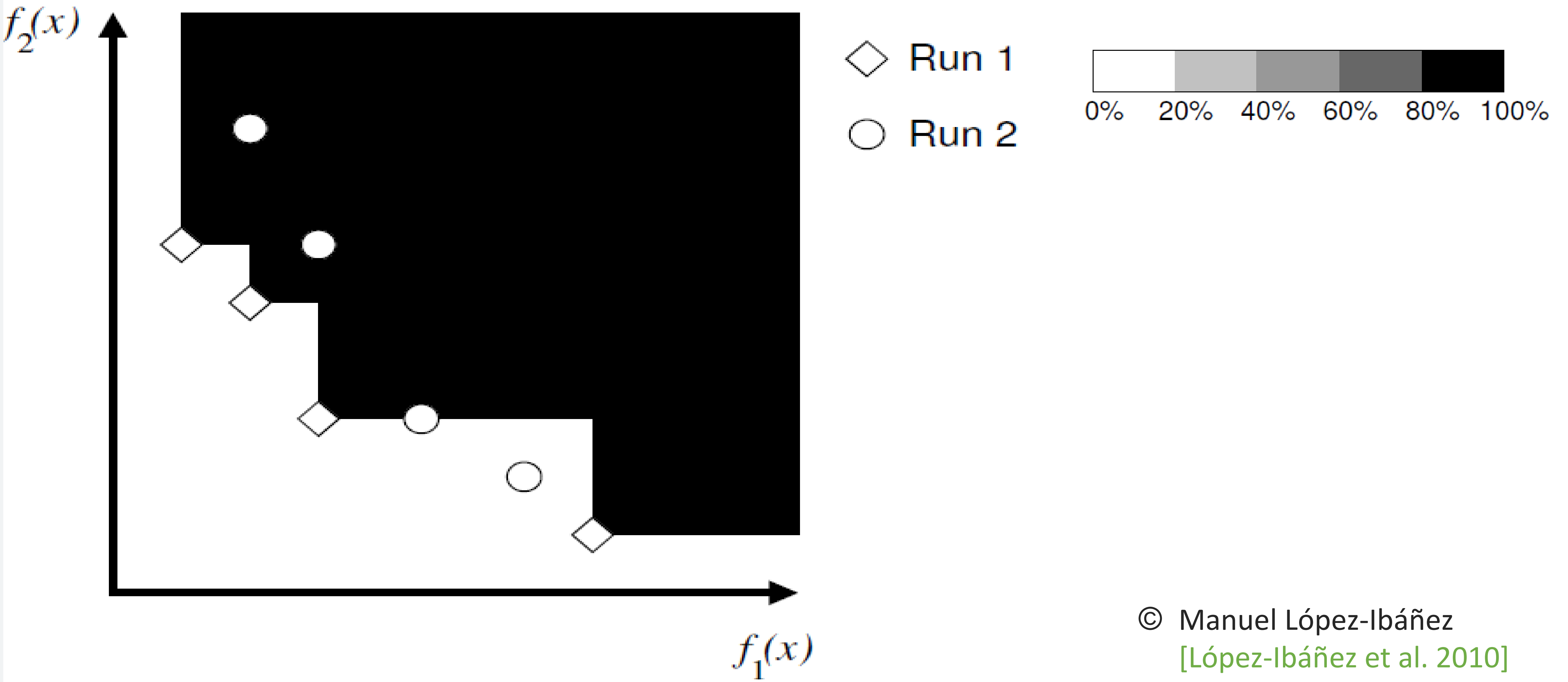
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



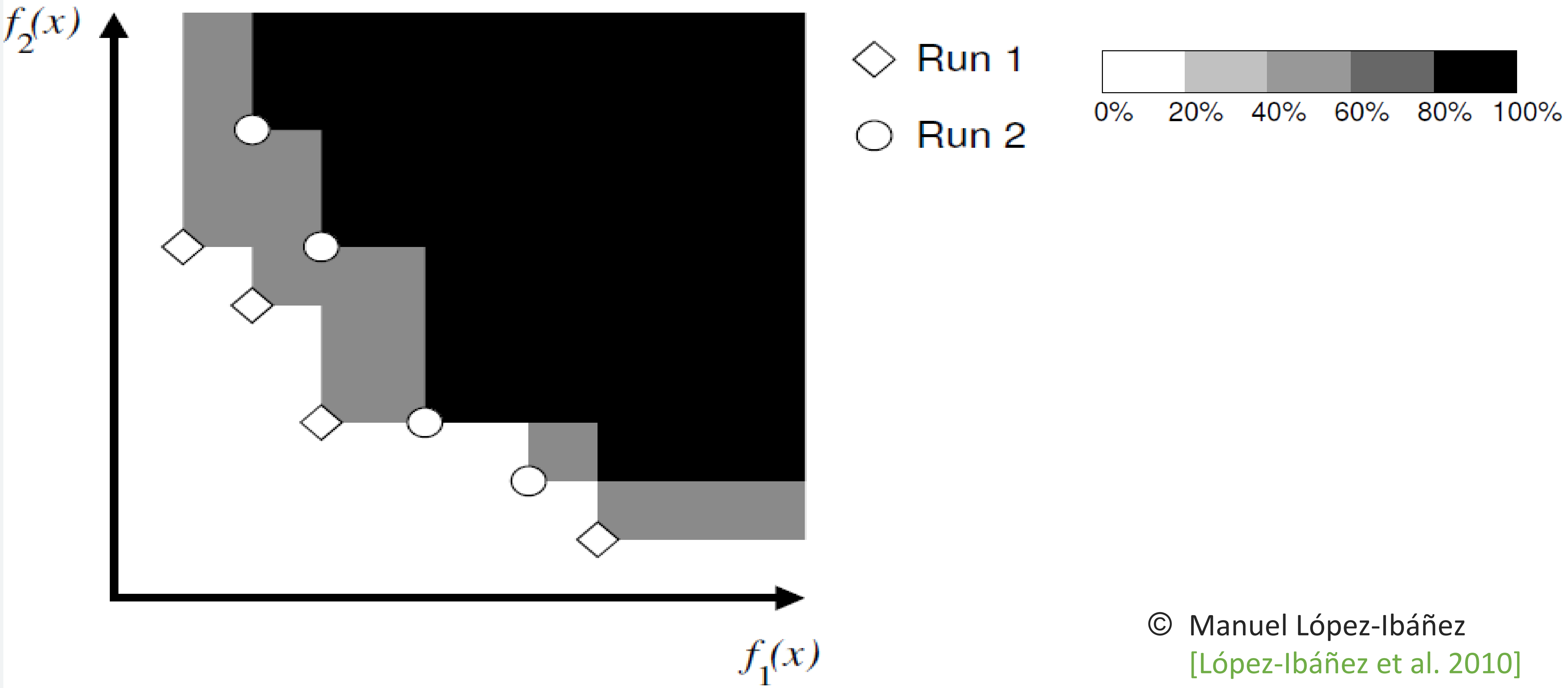
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



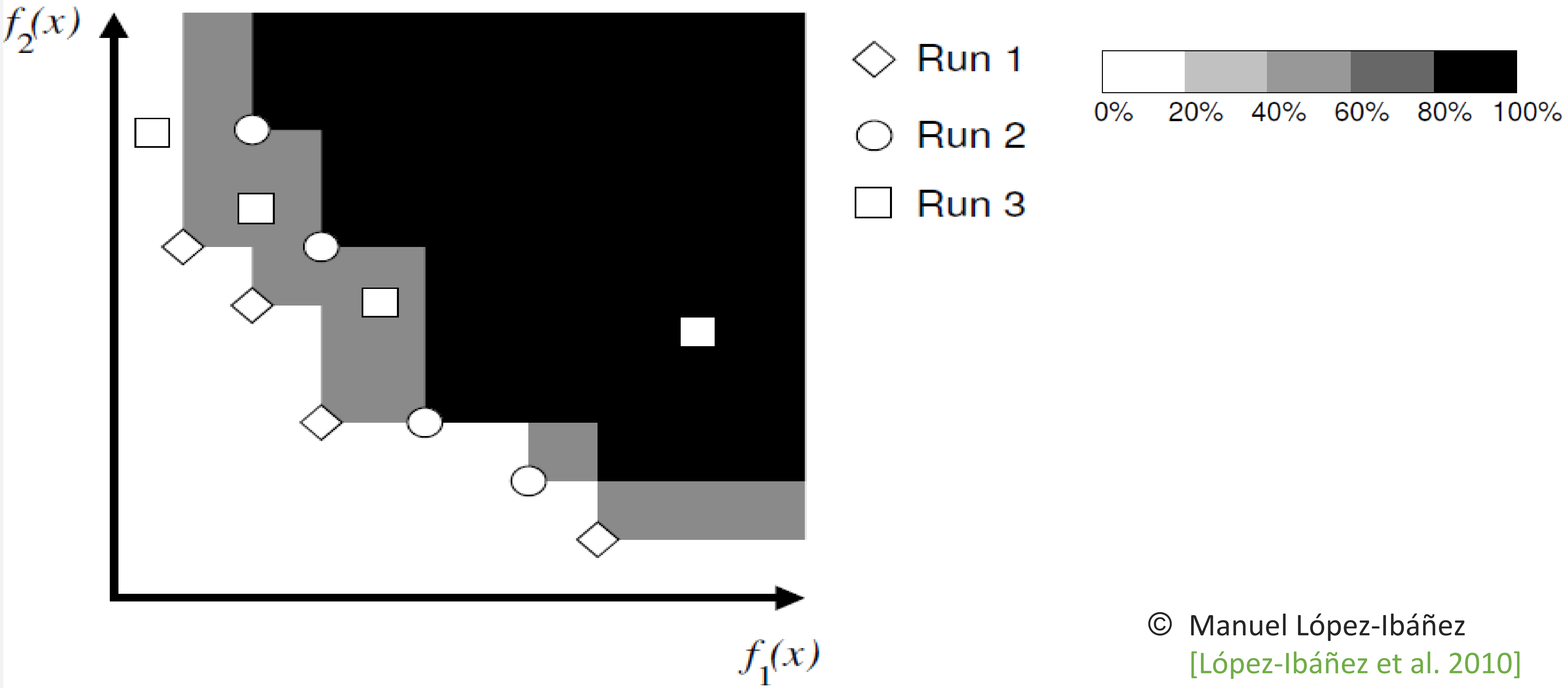
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



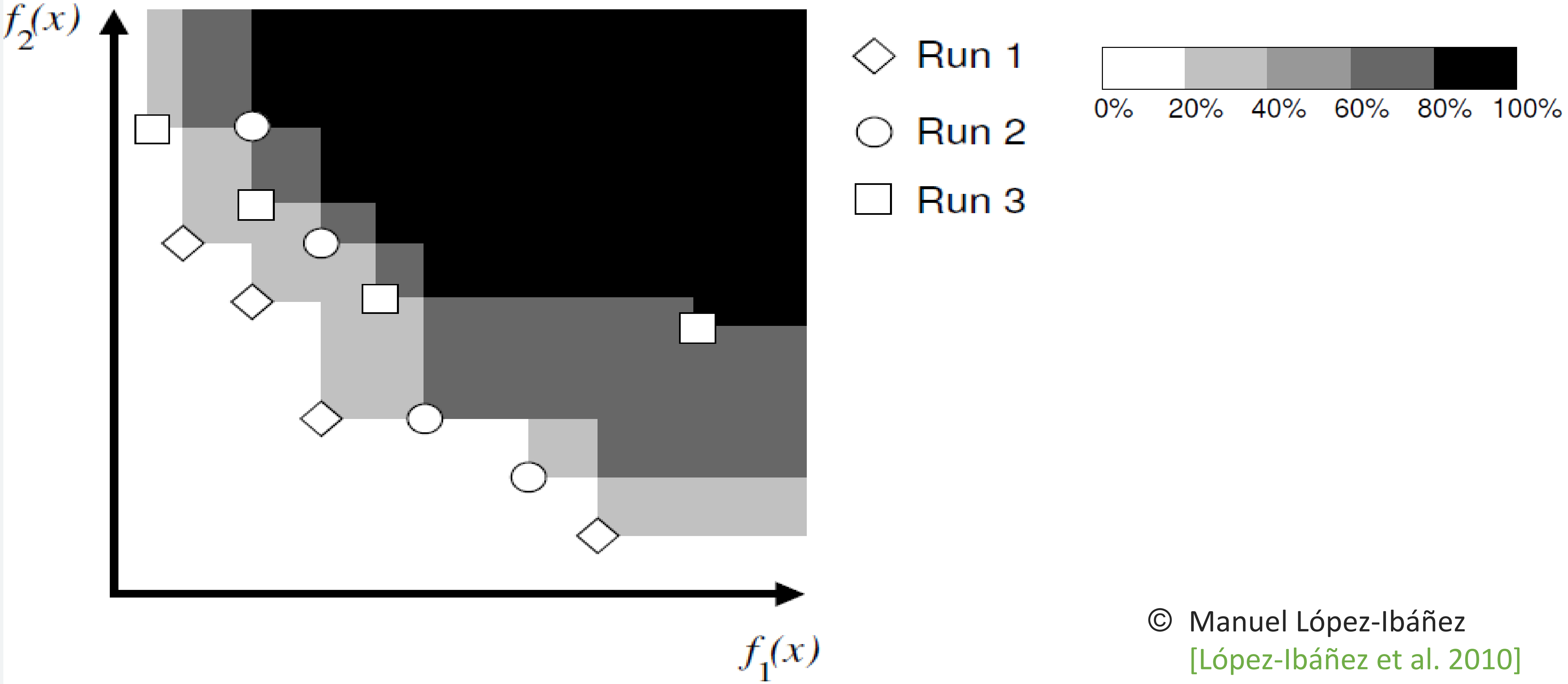
© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Definition

The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector z at time T:

$$\alpha_T(z) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time T.

Empirical Attainment Functions: Definition

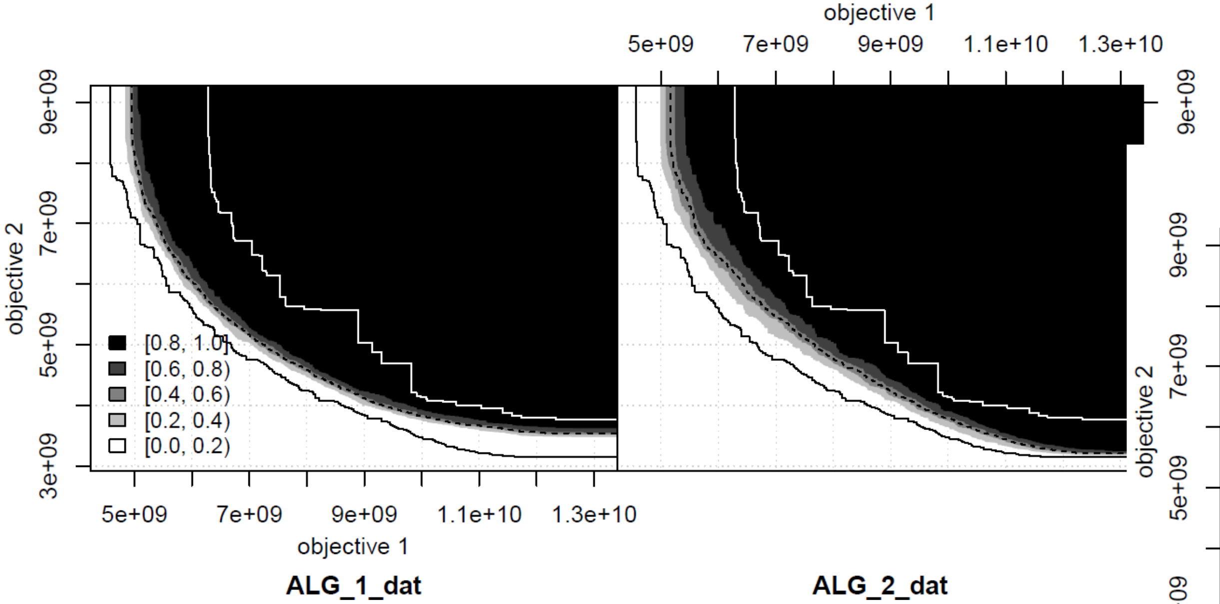
The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector z at time T:

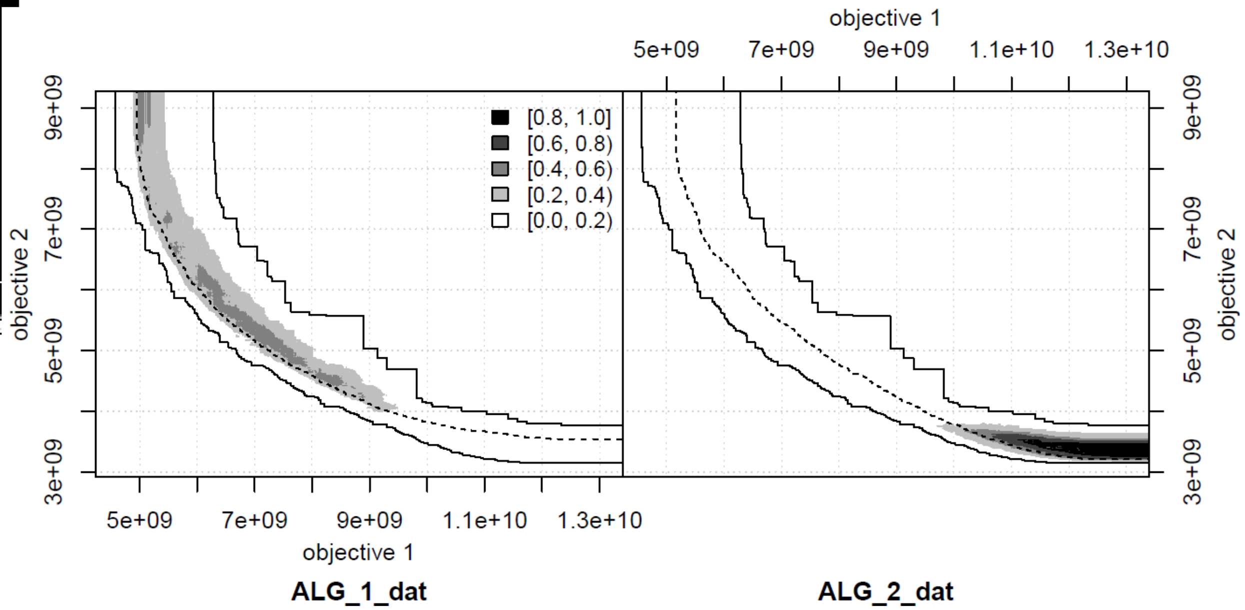
$$\alpha_T(z) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time T.

Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time T in the single-objective case ("fixed budget scenario").

Empirical Attainment Functions in Practice





© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

latest implementation online at
http://eden.dei.uc.pt/~cmfonsec/software.html
R package: **http://lopez-ibanez.eu/eaftools**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

Quality Indicator Approach

Idea

- Transfer multiobjective problem into a set problem
- Define an objective function (“unary quality indicator”) on sets
- Use the resulting total (pre-)order (on the quality values)

Quality Indicator Approach

Idea

- Transfer multiobjective problem into a set problem
- Define an objective function ("unary quality indicator") on sets
- Use the resulting total (pre-)order (on the quality values)

Underlying dominance relation should be reflected!

$$A \preccurlyeq B :\Leftrightarrow \forall_{b \in B} \exists_{a \in A}\ a \preccurlyeq b$$

Monotonicity and Strict Monotonicity

Monotonicity when quality indicator does not contradict relation

$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

Monotonicity and Strict Monotonicity

Monotonicity when quality indicator does not contradict relation

$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

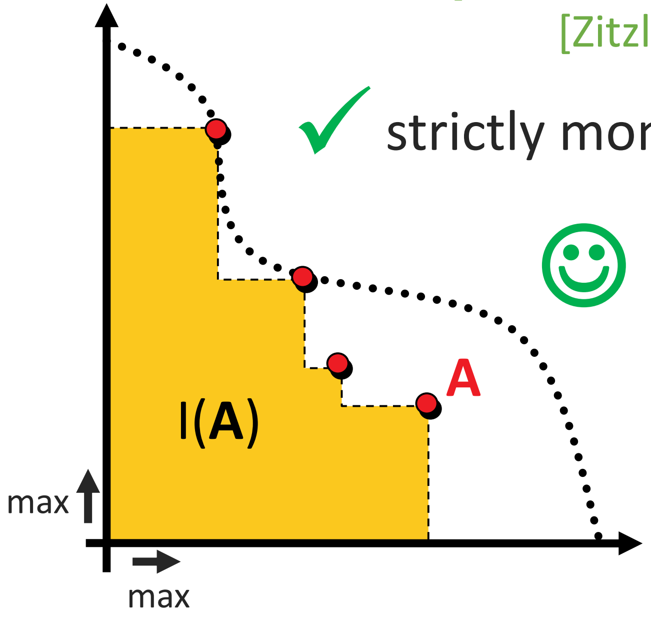
Strict monotonicity: better = higher indicator

$$A \preccurlyeq B \text{ and } A \neq B \Rightarrow I(A) > I(B)$$

Example: Refinements Using Indicators

I(A) = volume of the weakly dominated area in objective space

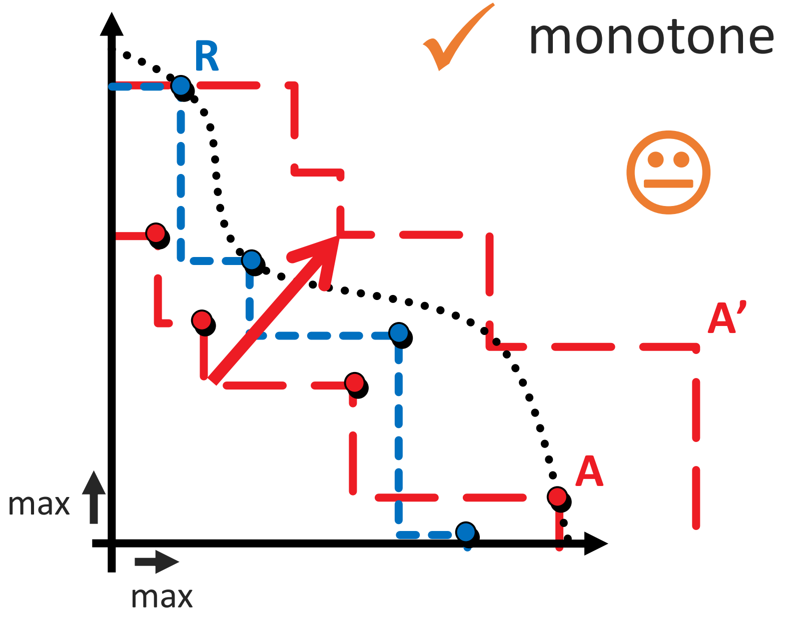
[Zitzler and Thiele 1989]
[Zitzler et al. 2007]



Unary hypervolume indicator

I(A,R) = how much A needs to be moved to weakly dominate R

[Zitzler et al. 2003]



Unary epsilon indicator

v1.0.1 – v1.0.100 and counting

Performance Assessment

Many Indicators Available

Performance Assessment of Multiobjective Optimizers: An Analysis and Review

Eckart Zitzler[1], Lothar Thiele[1], Marco Laumanns[1],
Carlos M. Fonseca[2], and Viviane Grunert da Fonseca[2]

[1] Computer Engineering and Networks Laboratory (TIK)
Department of Information Technology and Electrical Engineering
Swiss Federal Institute of Technology (ETH) Zurich, Switzerland
Email: {zitzler, thiele, laumanns}@tik.ee.ethz.ch

[2]ADEEC and ISR (Coimbra)
Faculty of Sciences and Technology
University of Algarve, Portugal
Email: cmfonsec@ualg.pt, vgrunert@csi.fct.ualg.pt

[Zitzler et al. 2003]

22 indicators

Even More Indicators…

Performance indicators in multiobjective optimization

Charles Audet[a], Jean Bigeon[b], Dominique Cartier[c], Sébastien Le Digabel[a], Ludovic Salomon[a,1]

[a]GERAD and Département de mathématiques et génie industriel, École Polytechnique de Montréal, C.P. 6079, Succ. Centre-ville, Montréal, Québec, H3C 3A7, Canada.
[b]Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France.
[c]Collège de Maisonneuve, 3800 Rue Sherbrooke E, Montréal, Québec, H1X 2A2, Canada.

[Audet et al 2021]

63 indicators

Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

Miqing Li, and Xin Yao[1]

[1]CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.
*Email: limitsing@gmail.com, x.yao@cs.bham.ac.uk

[Li and Yao 2019]

100 indicators

Many Indicators: What Do We Do?

Focus on indicators which are (strictly) monotone

- All hypervolume-based indicators [Zitzler et al. 2007]
- Unary epsilon indicator [Zitzler et al. 2003]
- R2 [Hansen and Jaszkiewicz 1998]
- IGD+ [Ishibuchi et al. 2015]

v2.0

Performance Assessment

Benchmarking Multiobjective Optimizers 2.0

With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

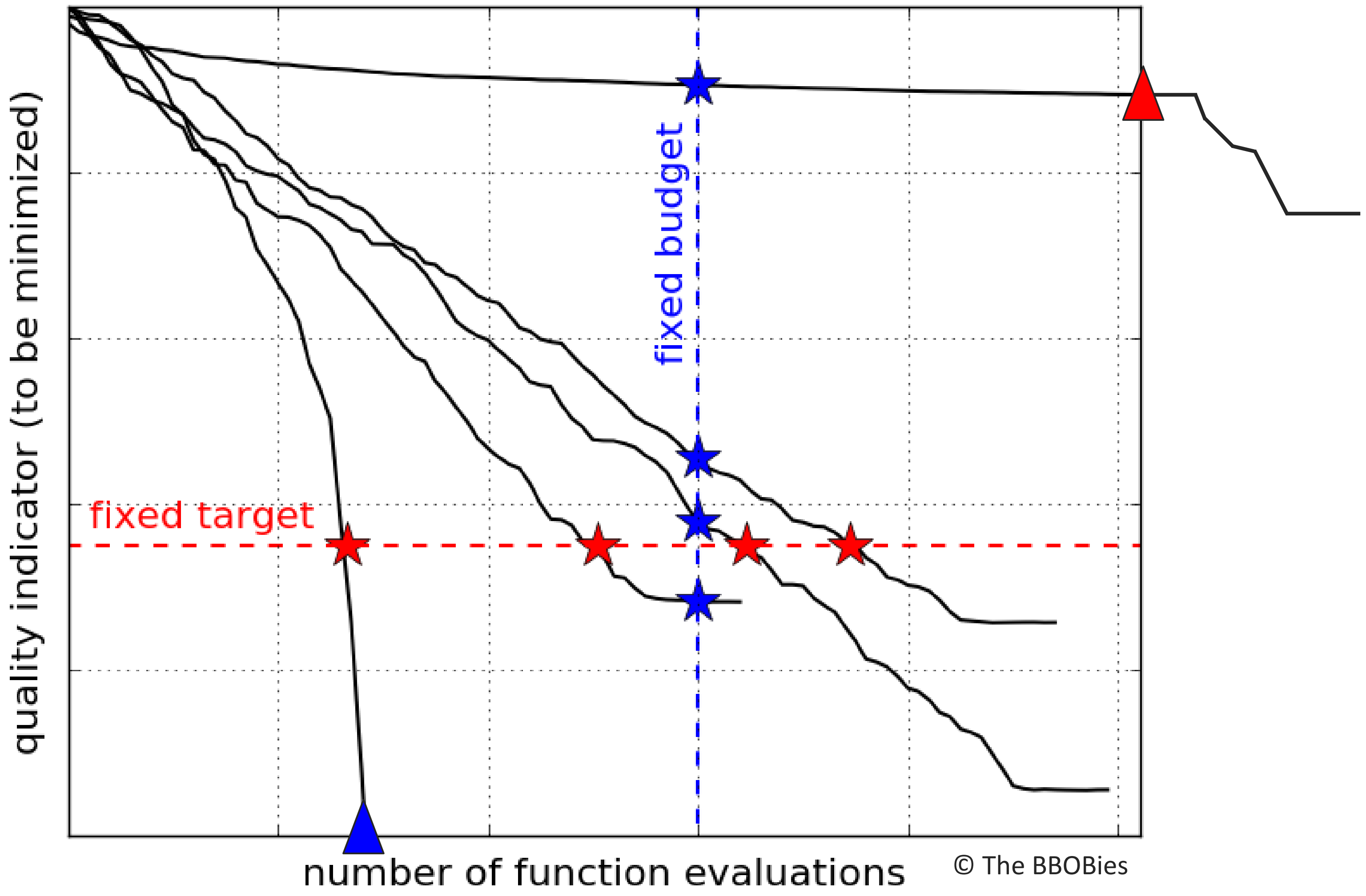
We can use our normal tools from single-objective optimization, including

- Reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- Statistical tests, box plots, …

see for example [Hansen et al. 2021]

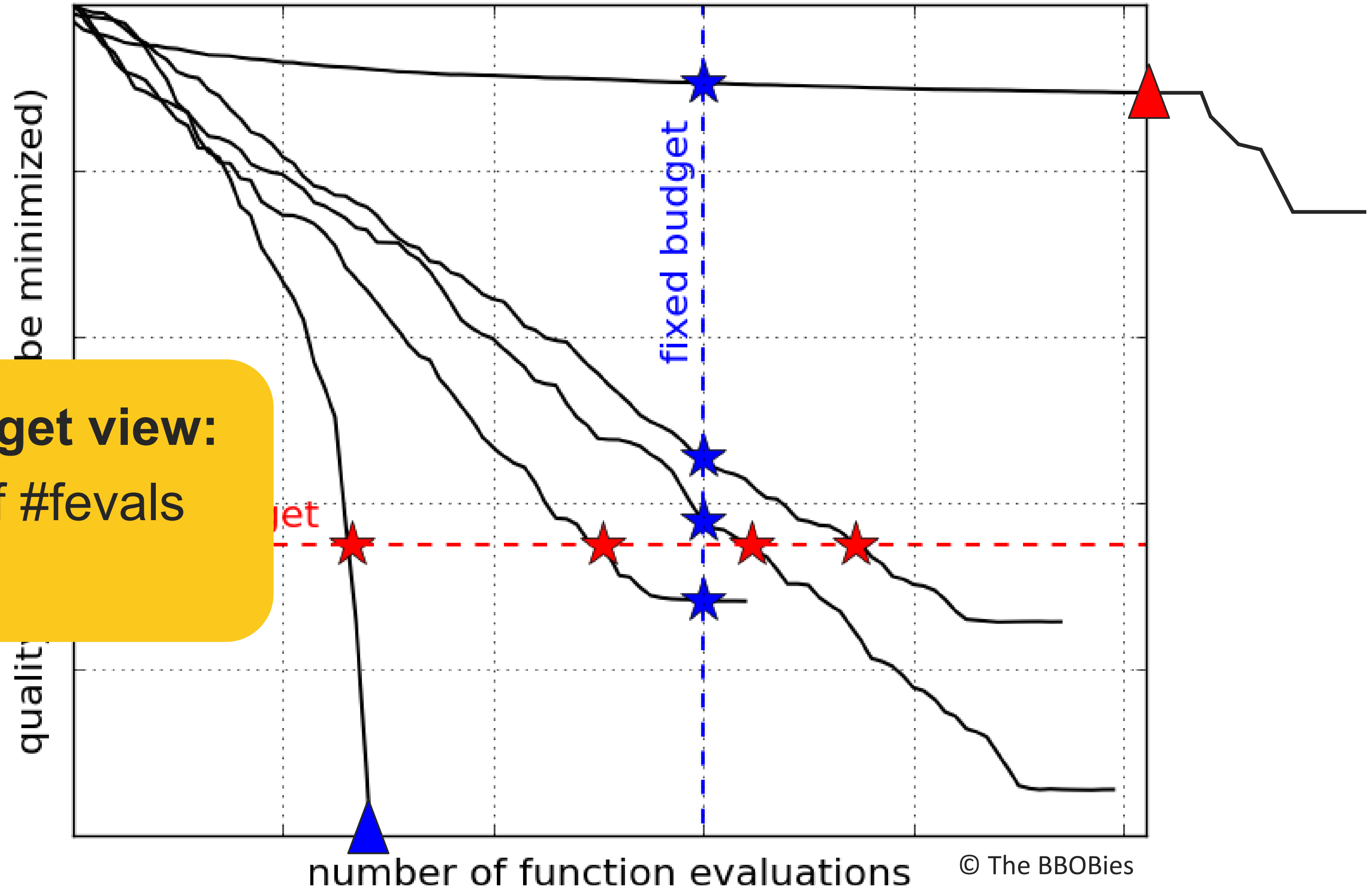
Measuring Performance Empirically

Convergence graphs is all we have to start with...



Measuring Performance Empirically

Convergence graphs is all we have to start with...

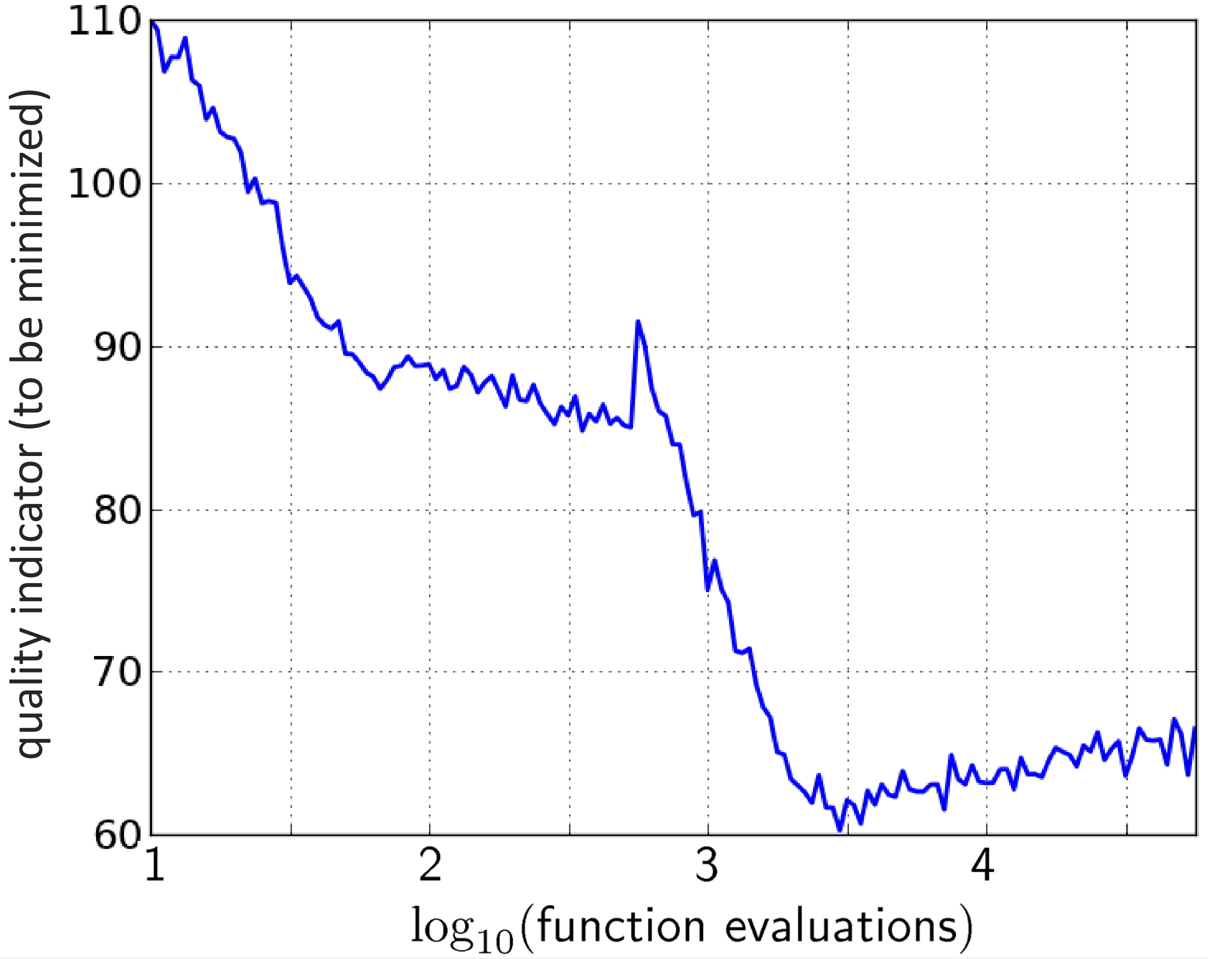


Advantage of the fixed target view:
Ratio scale (interpretation of #fevals easier than for f-values)

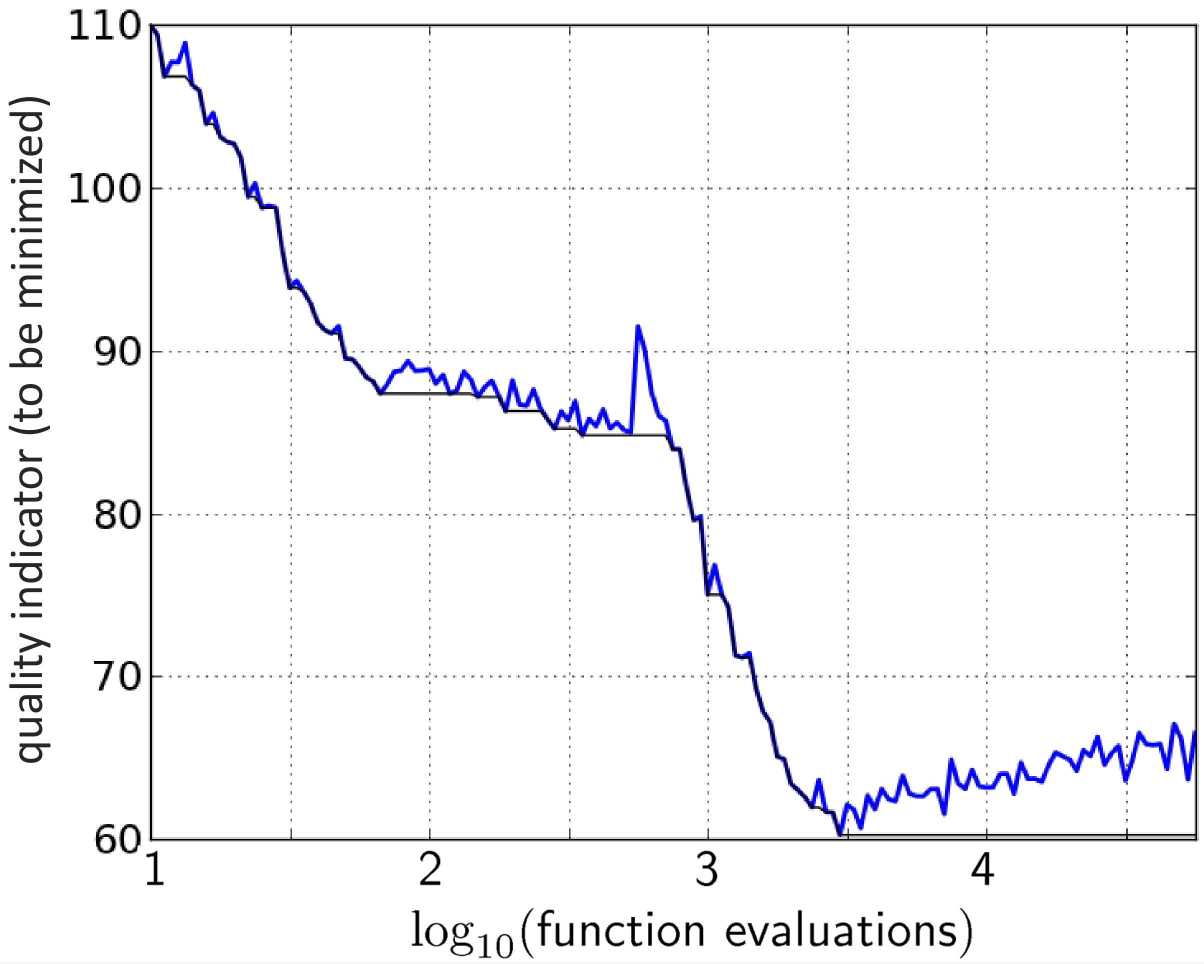
ECDF

Empirical Cumulative Distribution Function of the Runtime

A Convergence Graph

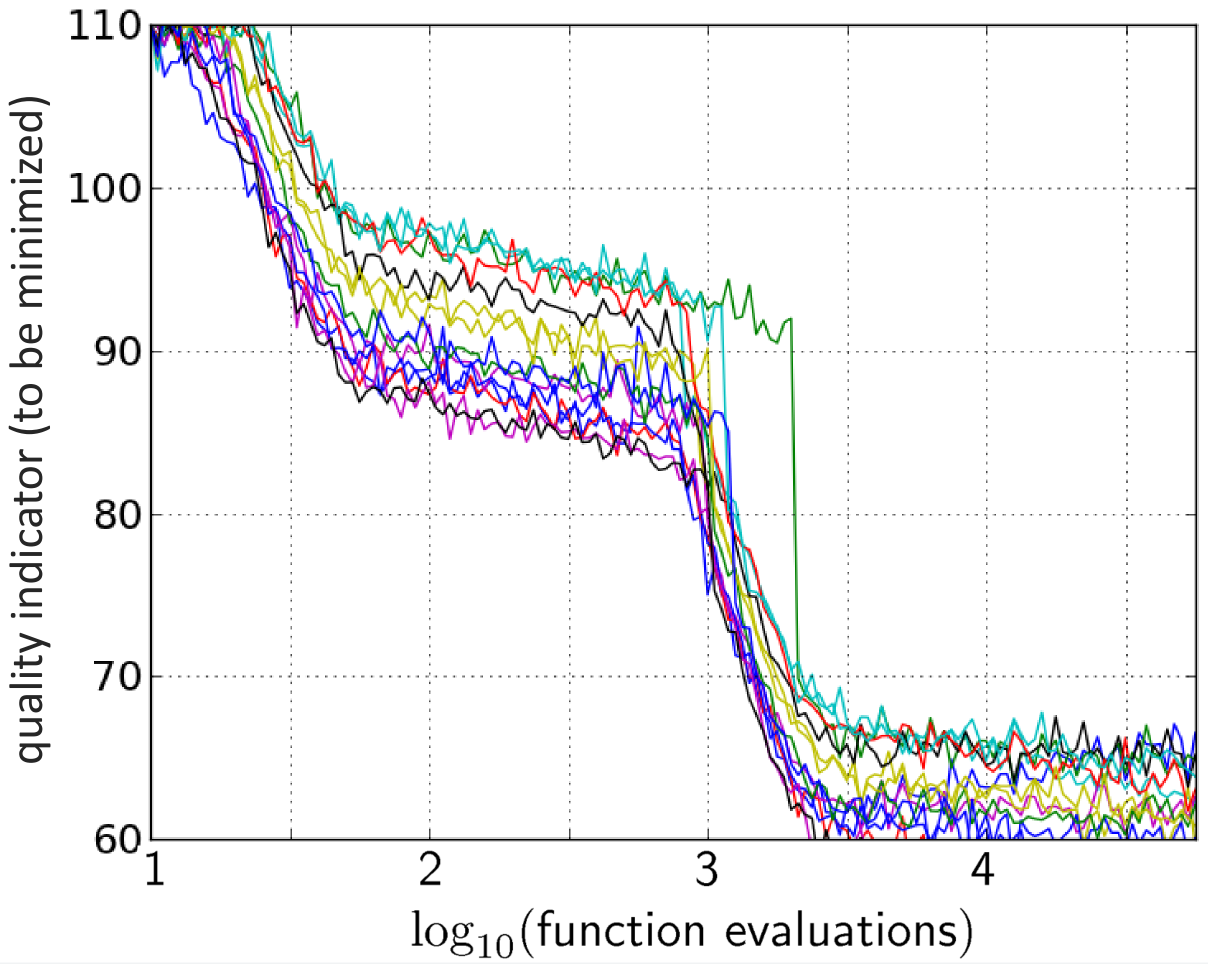


First Hitting Time is Monotonous



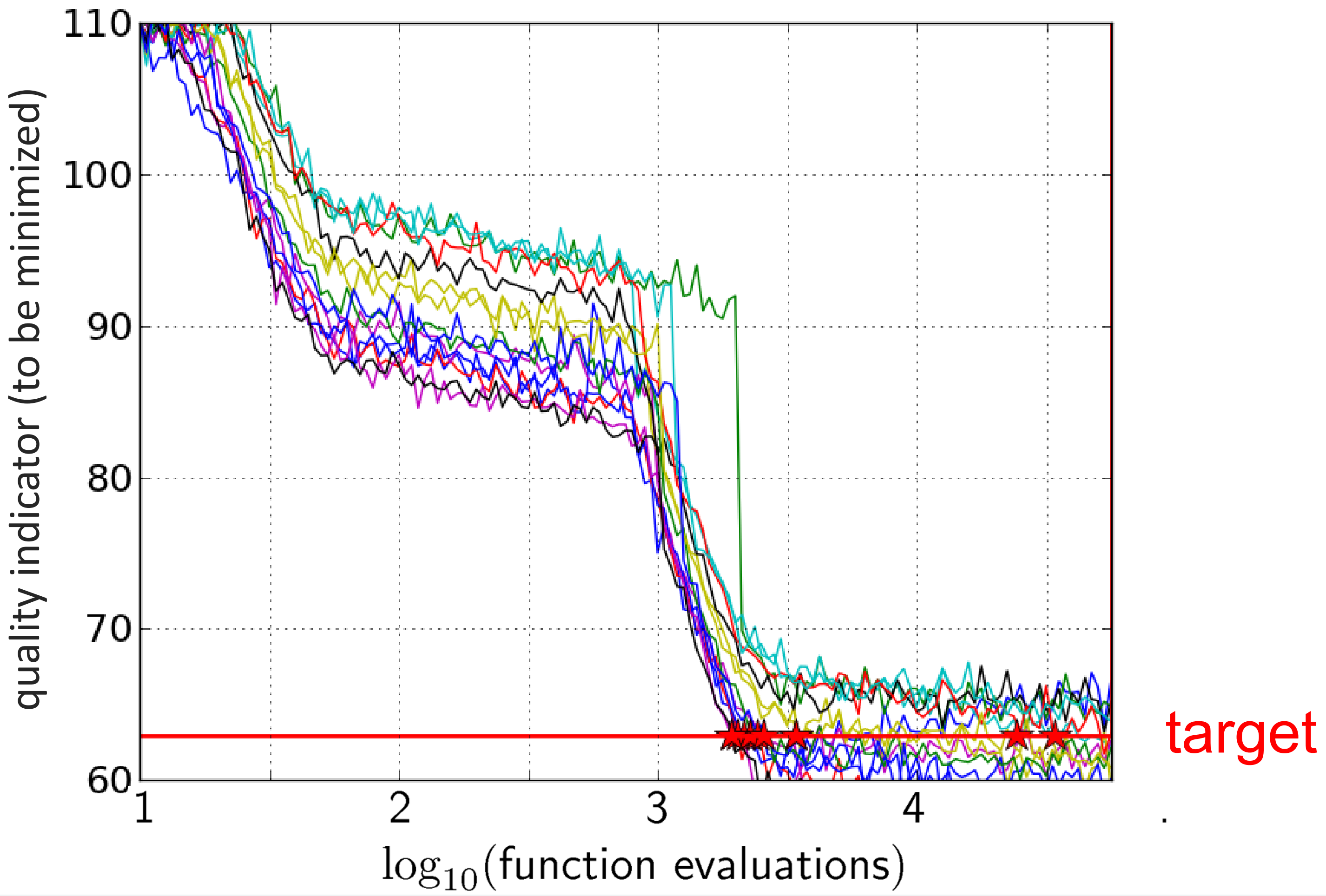
.

15 Runs

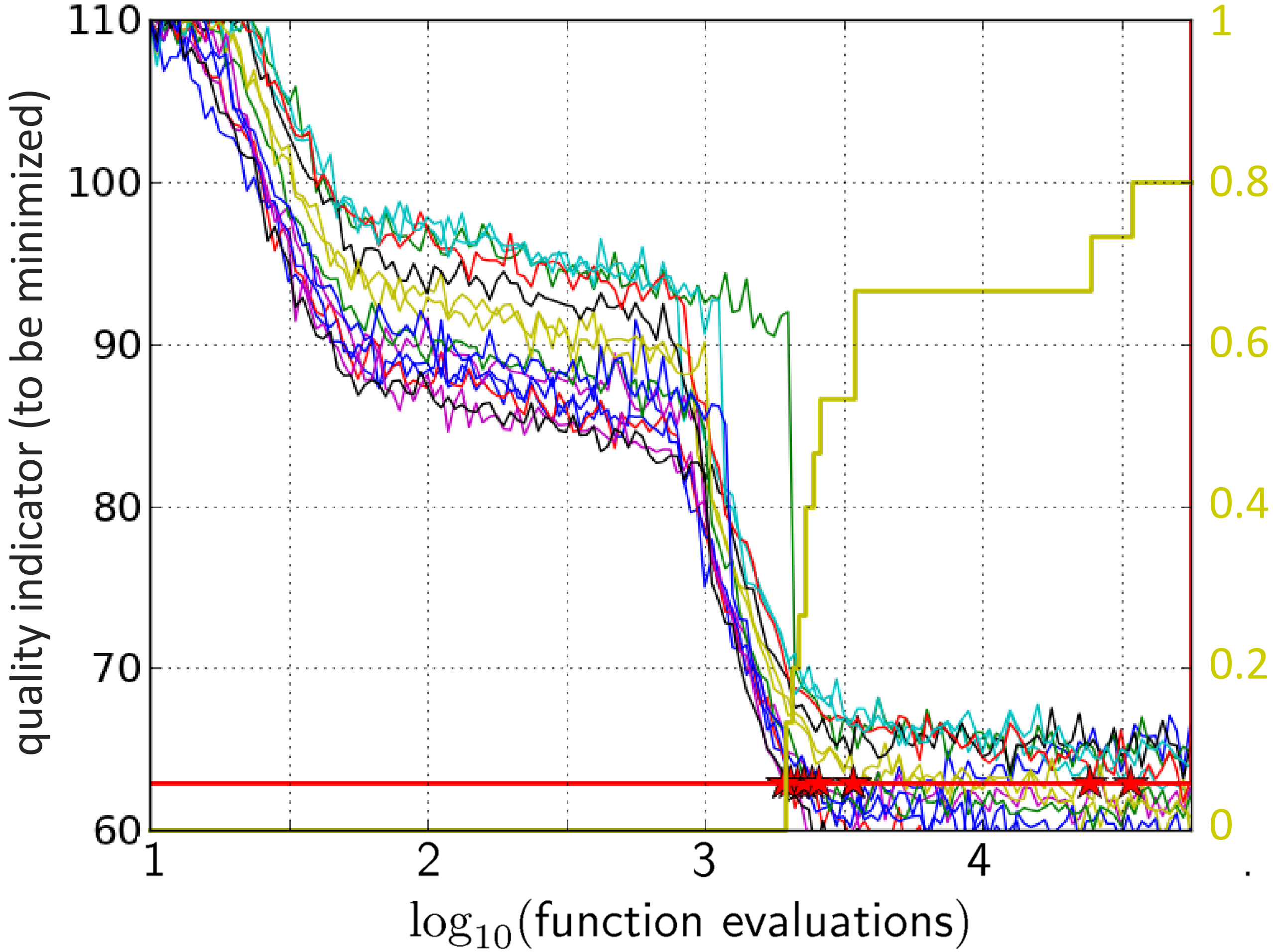


quality indicator (to be minimized)
110
100
90
80
70
60
1
2
3
4
log10(function evaluations)

15 Runs ≤ 15 Runtime Data Points



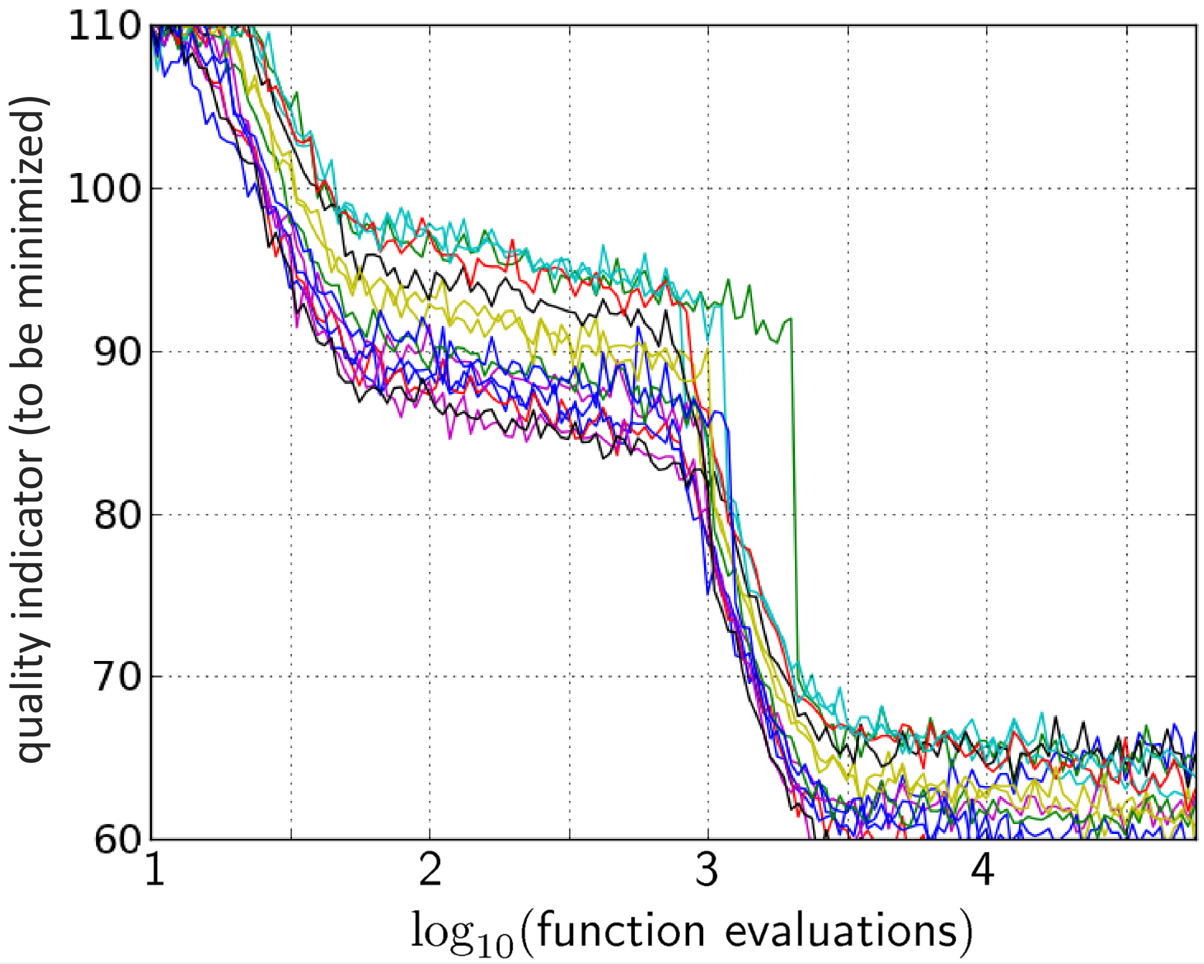
Empirical Cumulative Distribution



The ECDF of run lengths to reach the target

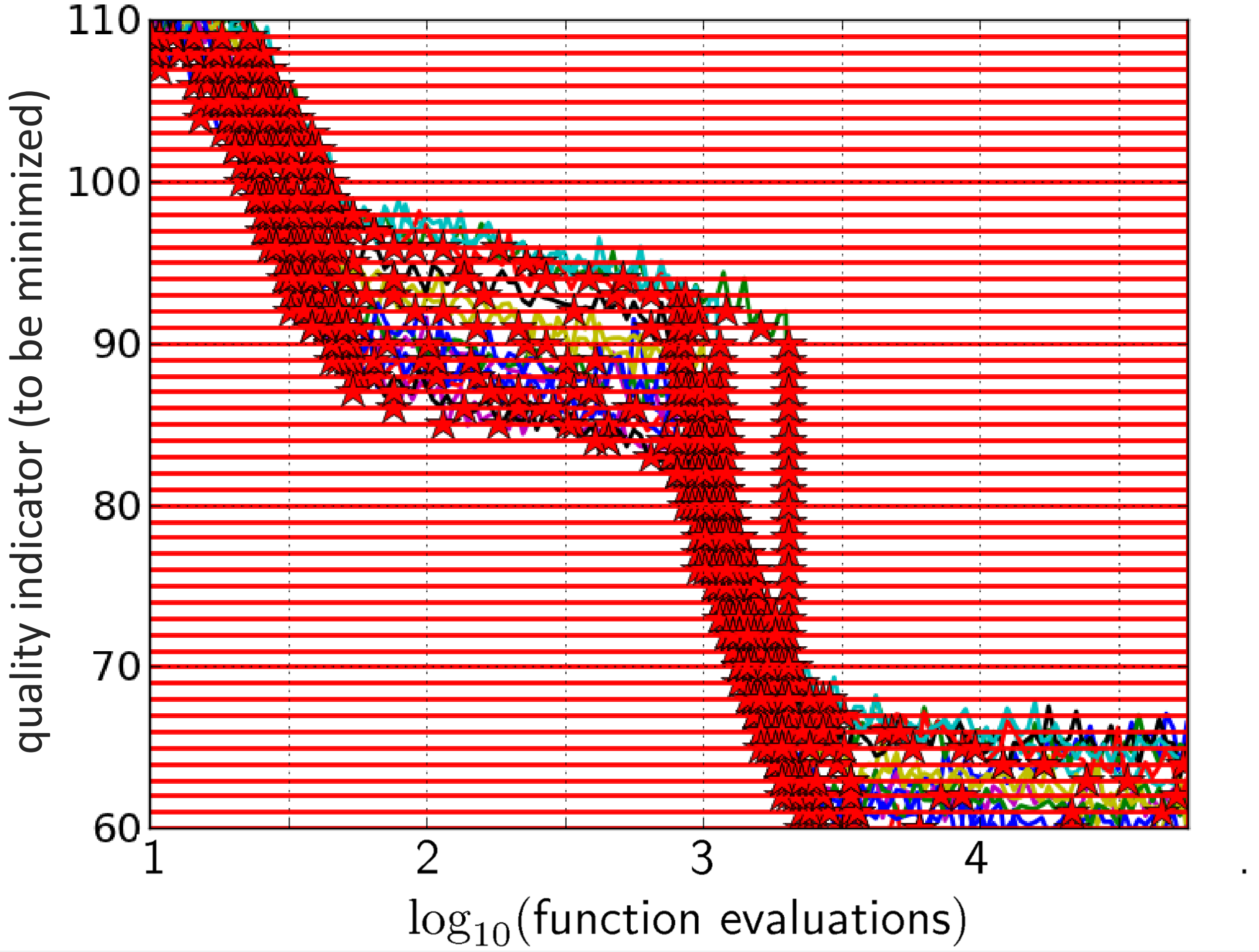
- Has for each data point a vertical step of constant size
- Displays for each x-value (budget) the count of observations to the left (first hitting times)

Aggregation



15 runs

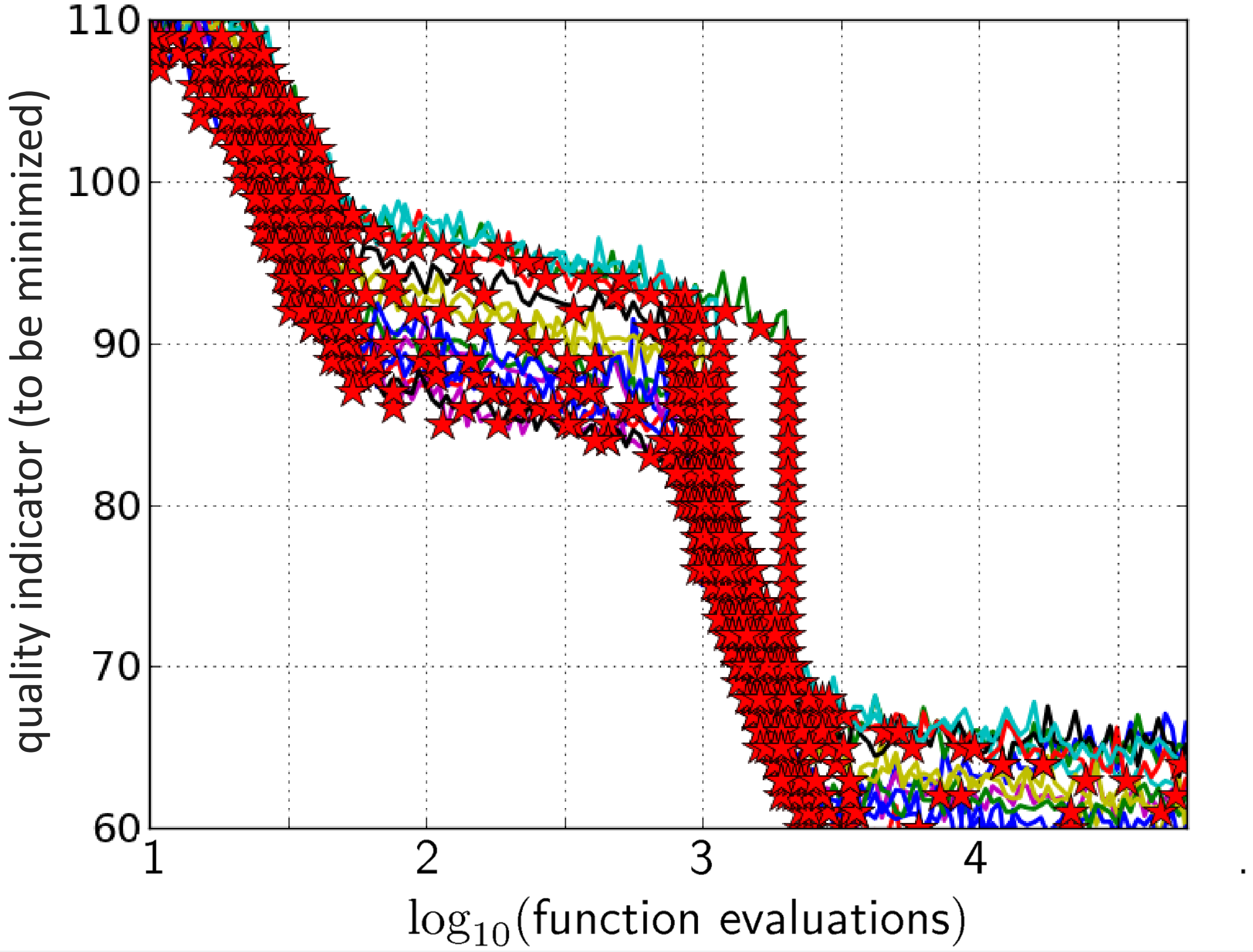
Aggregation



15 runs

50 targets

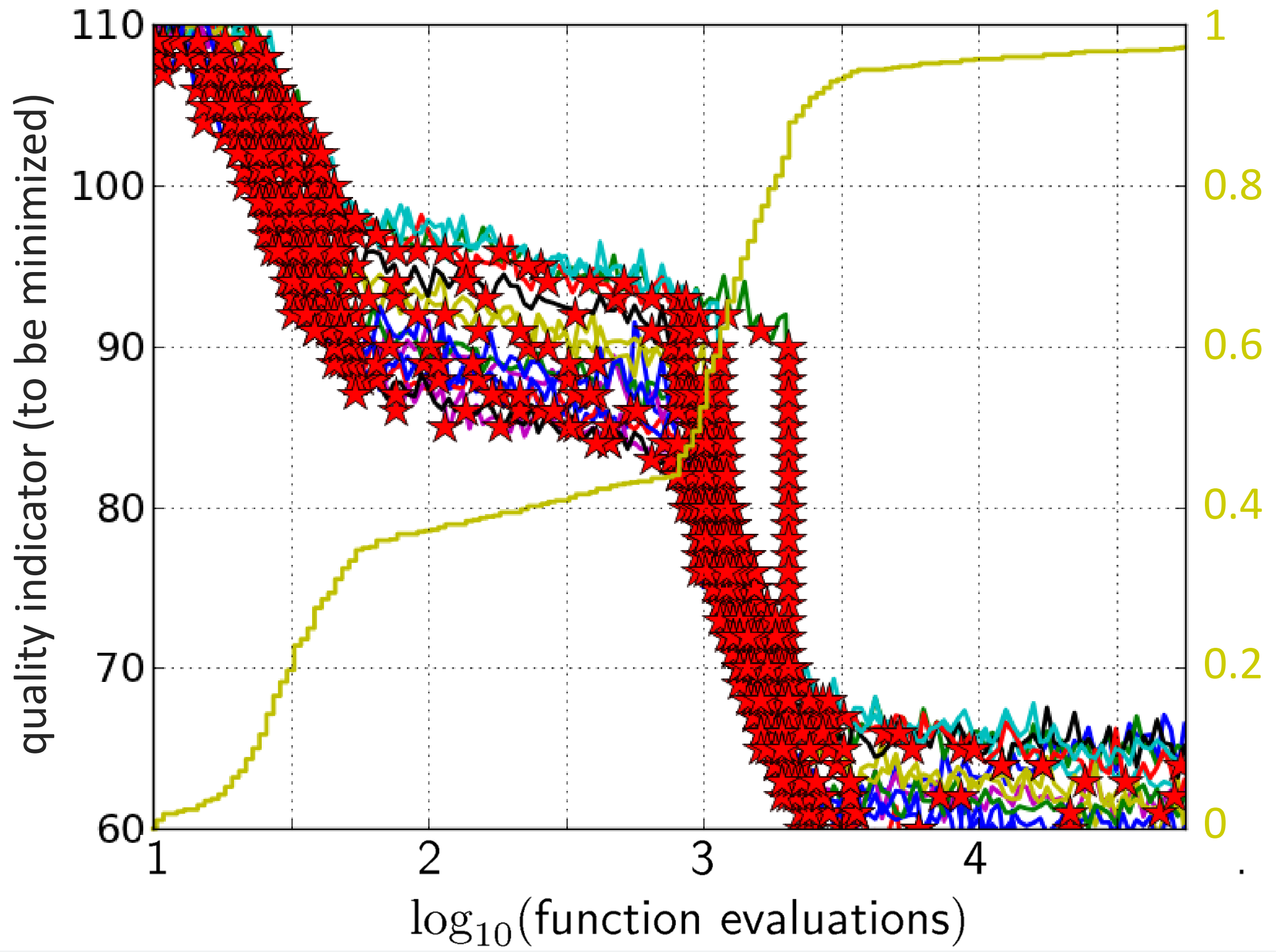
Aggregation



15 runs

50 targets

Aggregation

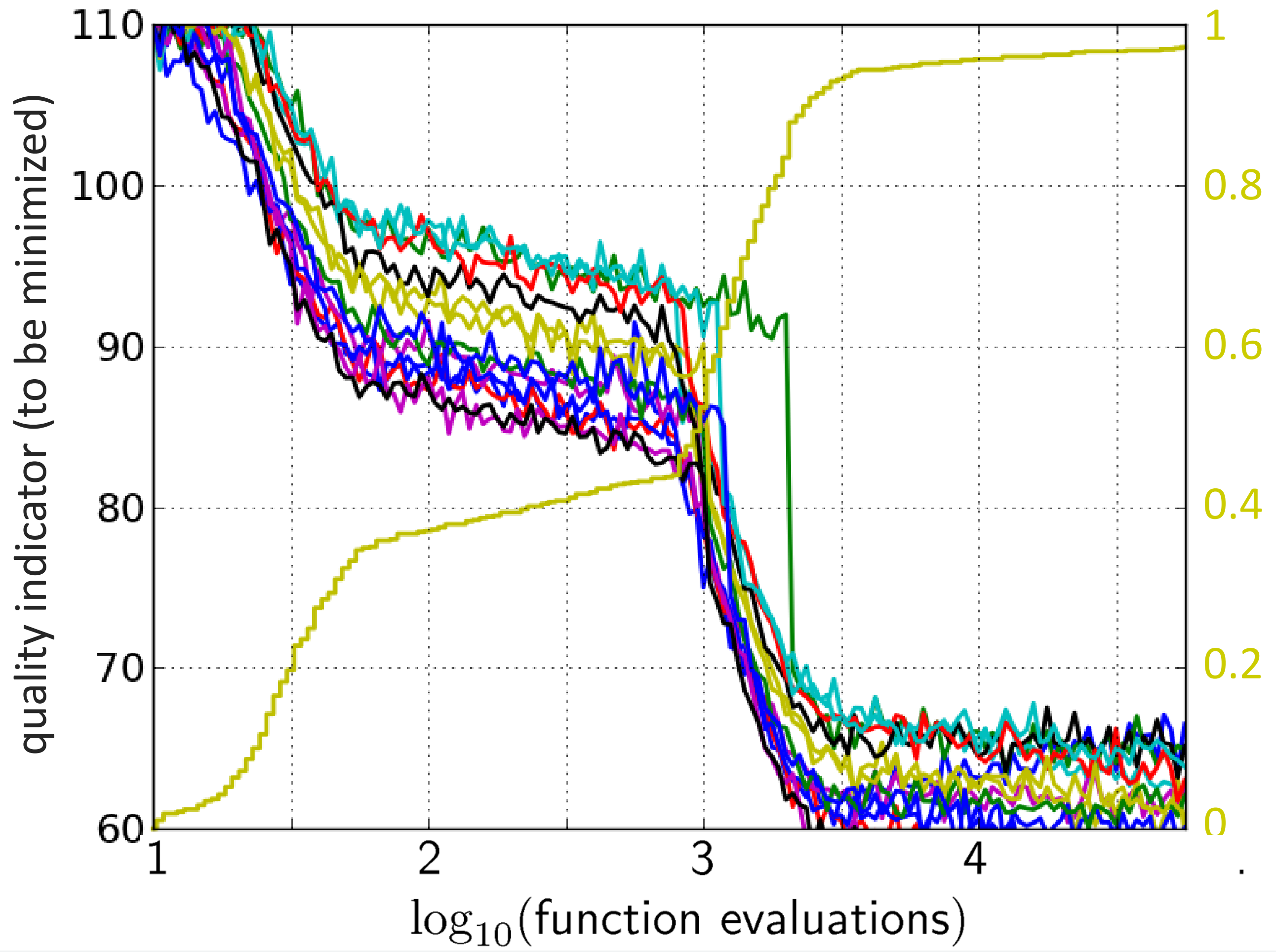


15 runs

50 targets

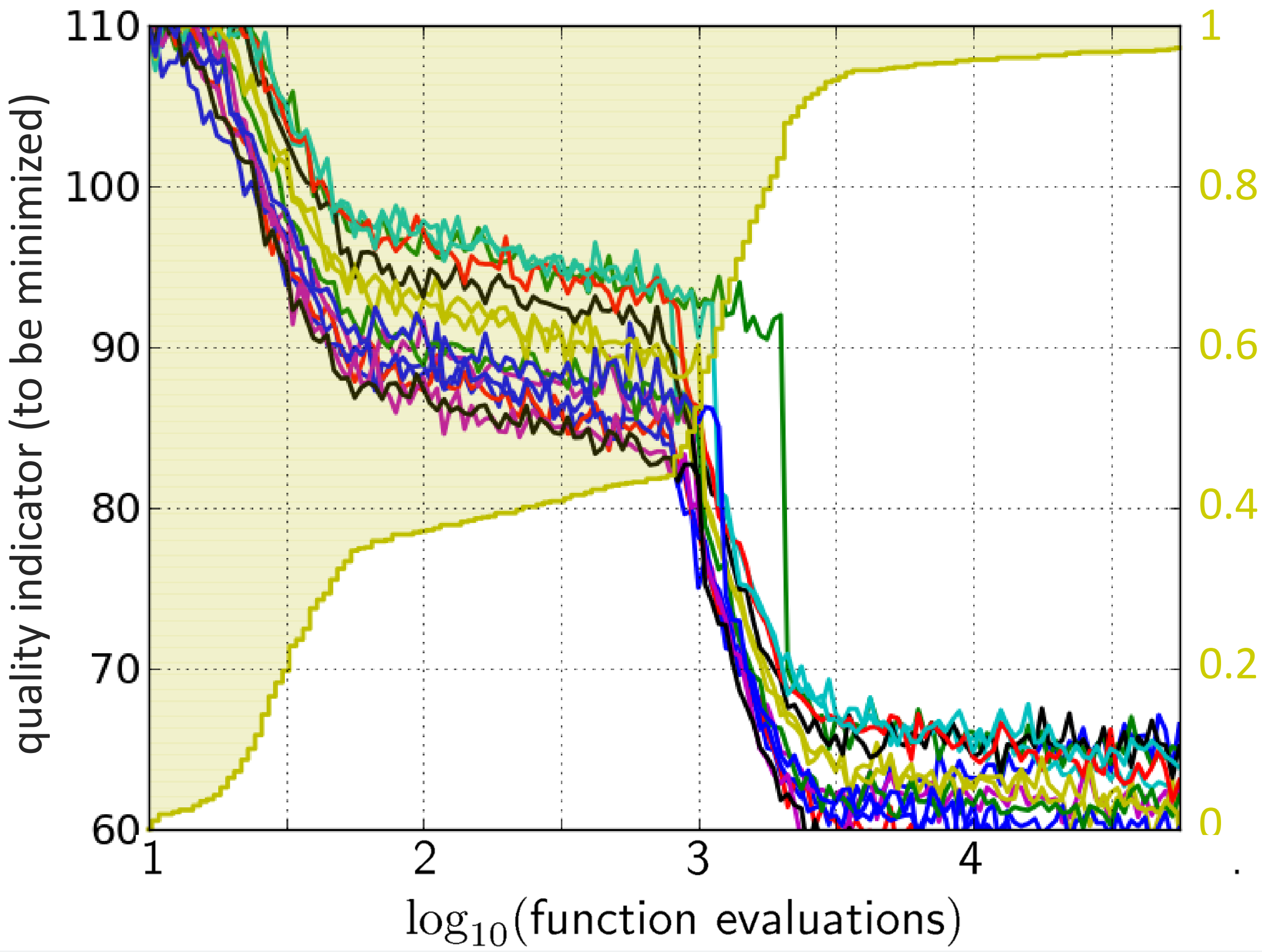
ECDF with 750 steps

Aggregation



50 targets from 15 runs integrated in a single graph

Interpretation



50 targets from 15 runs integrated in a single graph

area over the ECDF curve

=

average log runtime

(or geometric avg. runtime) over all targets (difficult and easy) and all runs

Worth to Note

ECDF graphs

- Should never aggregate over dimension

Dimension is input parameter to algorithm

Worth to Note

ECDF graphs

- Should never aggregate over dimension

Dimension is input parameter to algorithm

- But often over targets and functions
- Can show data of more than 1 algorithm at a time

Worth to Note

ECDF graphs

- Should never aggregate over dimension

Dimension is input parameter to algorithm

- But often over targets and functions
- Can show data of more than 1 algorithm at a time
- Are an extension of data profiles
  - Introduced by Moré and Wild for single and relative targets [Moré and Wild 2009]
  - But here for multiple and absolute targets

Worth to Note

ECDF graphs

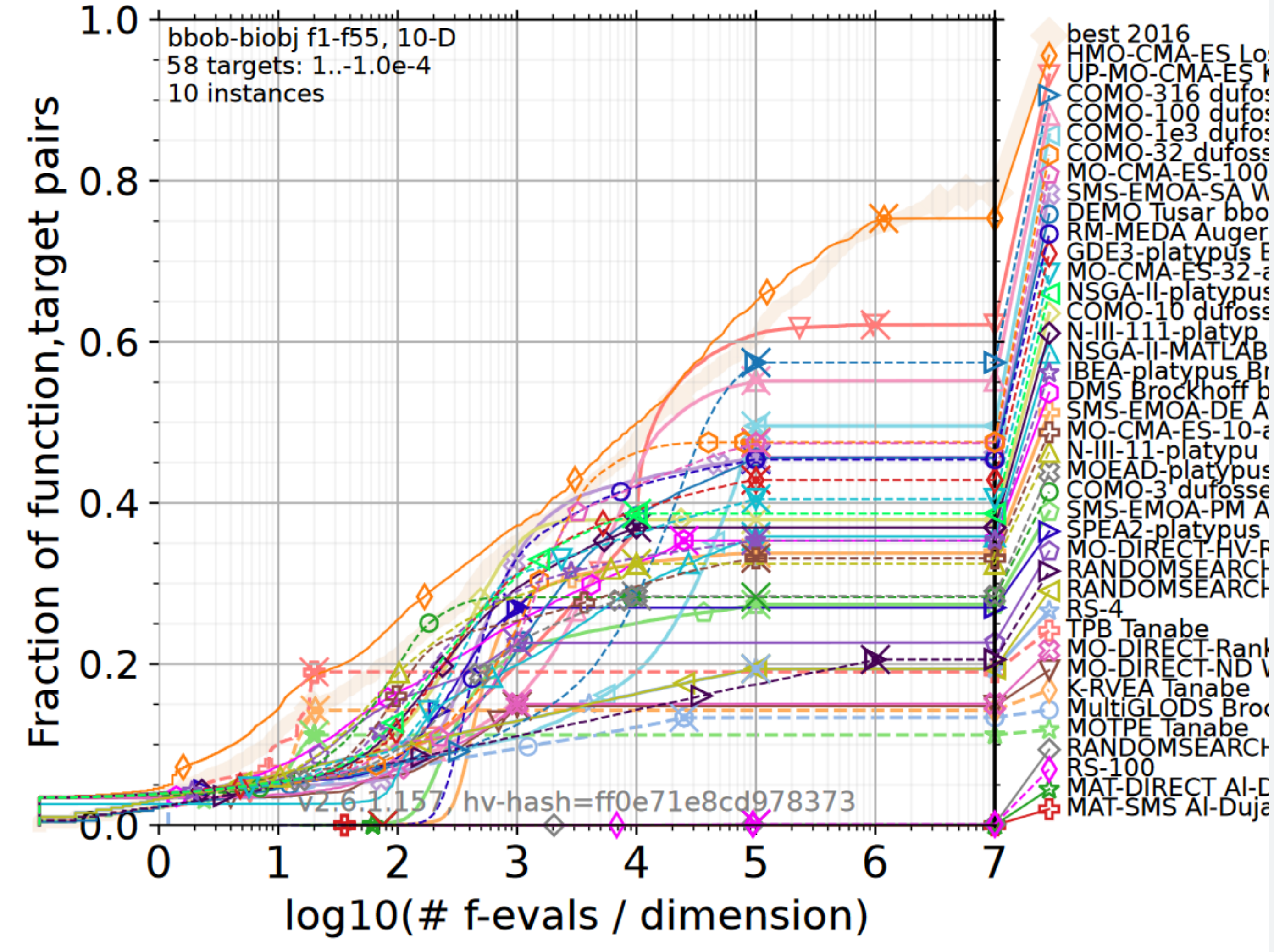
- Should never aggregate over dimension

Dimension is input parameter to algorithm

- But often over targets and functions
- Can show data of more than 1 algorithm at a time
- Are an extension of data profiles
  - Introduced by Moré and Wild for single and relative targets [Moré and Wild 2009]
  - But here for multiple and absolute targets
- Are COCO's main performance visualization tool

[Hansen et al. 2021] – `https://github.com/numbbo/coco`

Example ECDF (later more)



Mostly Overlooked: Scaling with Dimension

In single-objective optimization:

- Scaling behavior mandatory to investigate

Mostly Overlooked: Scaling with Dimension

In single-objective optimization:

- Scaling behavior mandatory to investigate

In multiobjective optimization:

- Actually two dimensions: search and objective space

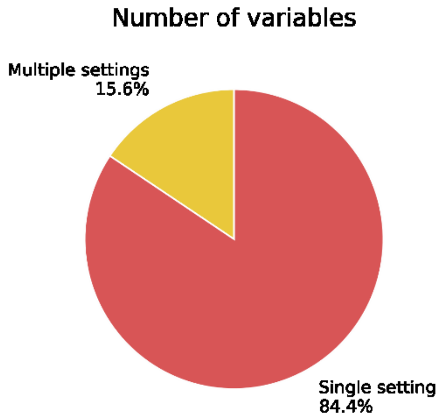
Mostly Overlooked: Scaling with Dimension

In single-objective optimization:

- Scaling behavior mandatory to investigate

In multiobjective optimization:

- Actually two dimensions: search and objective space
- But former almost never looked at right now ☹



~10 papers from EMO’21 and PPSN/GECCO/CEC’20 change dimension

but 50+ papers have a “fixed” dimension

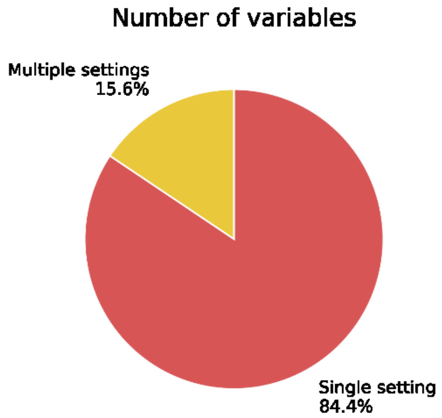
Mostly Overlooked: Scaling with Dimension

In single-objective optimization:

- Scaling behavior mandatory to investigate

In multiobjective optimization:

- Actually two dimensions: search and objective space
- But former almost never looked at right now ☹



~10 papers from EMO'21 and PPSN/GECCO/CEC'20 change dimension

but 50+ papers have a "fixed" dimension

- But in practice search space scalability almost more important

Number of objectives often fixed

A Few General Recommendations

- Always display everything you have
- Look at single runs
- Do each experiment at least twice

(= look at the *variance* of your results)

A Few General Recommendations

- Always display everything you have
- Look at single runs
- Do each experiment at least twice

  (= look at the *variance* of your results)
- As quality indicators, use hypervolume, R2, or epsilon indicator

  or any indicator which is at least monotone

A Few General Recommendations

- Always display everything you have
- Look at single runs
- Do each experiment at least twice

  (= look at the *variance* of your results)
- As quality indicators, use hypervolume, R2, or epsilon indicator

  or any indicator which is at least monotone
- See also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

  http://www.cmap.polytechnique.fr/~nikolaus.hansen/gecco2018-experimentation-guide-slides.pdf

Recommended Experimental Setup (w/ or w/o COCO)

❶ **Benchmarking Experiment**

❷ **Choosing Algorithms for Comparison**

See `https://numbbo.github.io/data-archive/`

❸ **Postprocessing**

```
python -m cocopp resultfolder/ ALG2 ALG3
```

❹ **Displaying and Discussing Summary Results**

❺ **Investigating and Discussing Complementary Results**

❻ **Processed Data Sharing**

Provide html output somewhere

❼ **Raw Data Sharing**

Easy with COCO archive module & through issue tracker

Overview

❶ Performance Assessment

❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

Test Problems and Their Visualizations

Introduction

Test Problems (1)

Artificial problems (continuous and unconstrained)

- **v0.1:** Individual problems
- **v0.2:** MOP suite (unscalable problems)
- **v0.5:** ZDT suite (scalable number of variables)
- **v1.0:** DTLZ suite (scalable number of variables and objectives)
- **v1.2:** WFG suite
- **v1.3:** Other suites with a bottom-up construction
- **v1.5:** Suites of distance-based problems
- **v2.0:** The bbob-biobj(-ext) suite

Test Problems and Their Visualizations

Visualization of multiobjective landscapes

Low-dimensional search spaces

- Dominance ratio
- Local dominance
- Gradient path length

Any-dimensional search spaces

- Line cuts

Some notes on problem properties

Test Problems and Their Visualizations

Test Problems (2)

Artificial problems (other)

- Constrained problems
- Mixed-integer problems

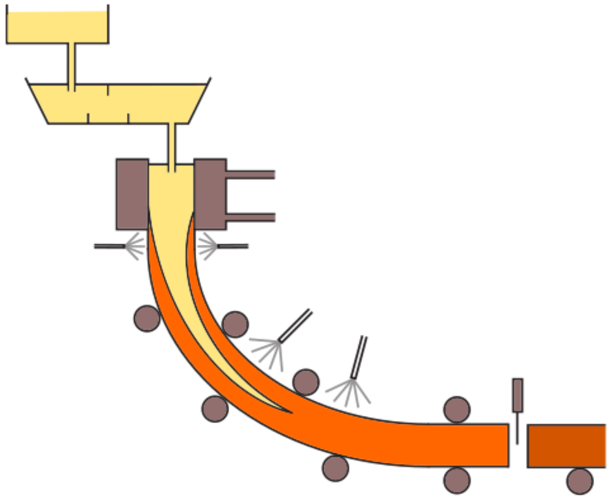
Real-world problems

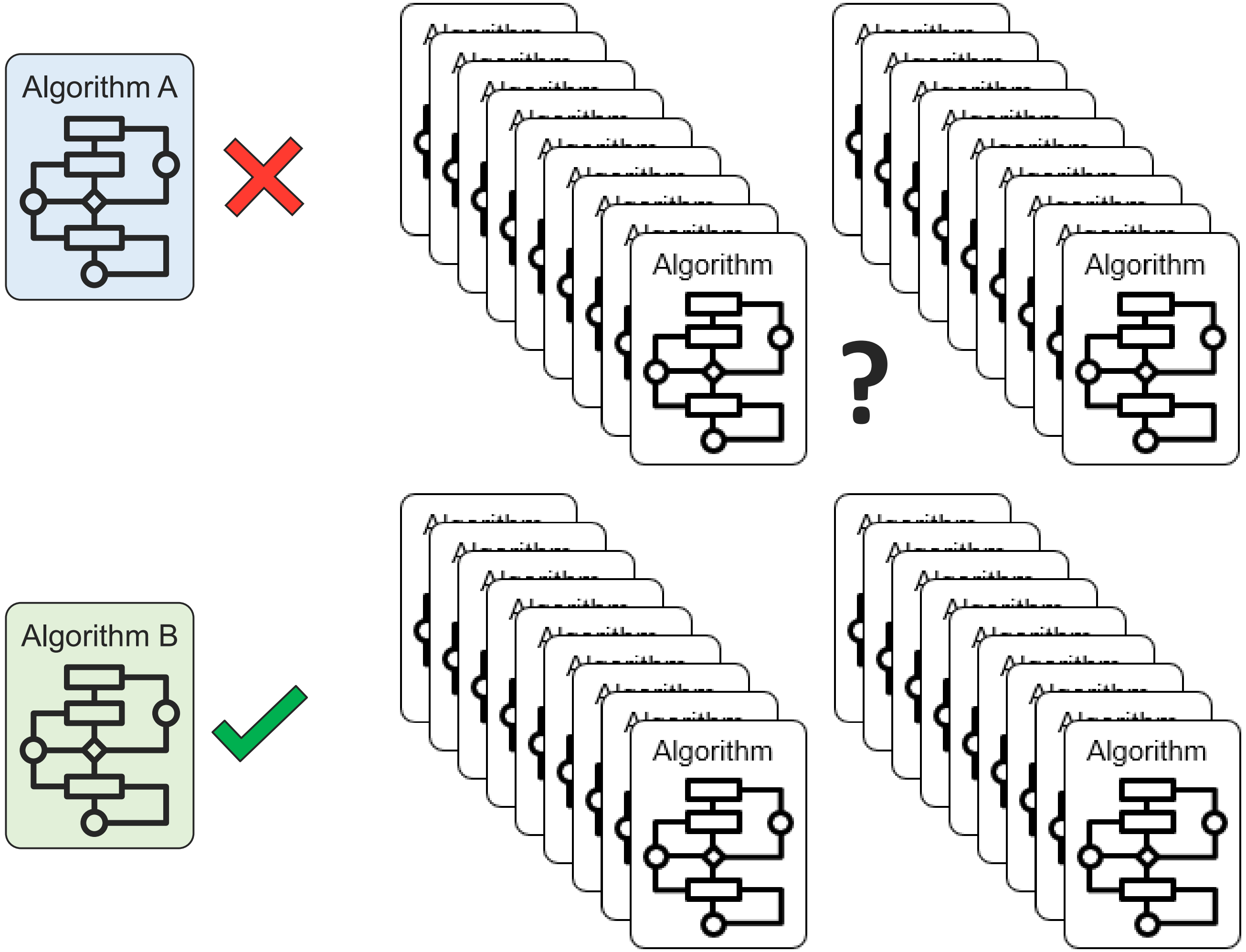
- **v0.1:** Individual problems
- **v0.2:** Suites of unscalable problems
- **v0.5:** Suites of scalable problems (in the number of variables)

Conclusions

Introduction

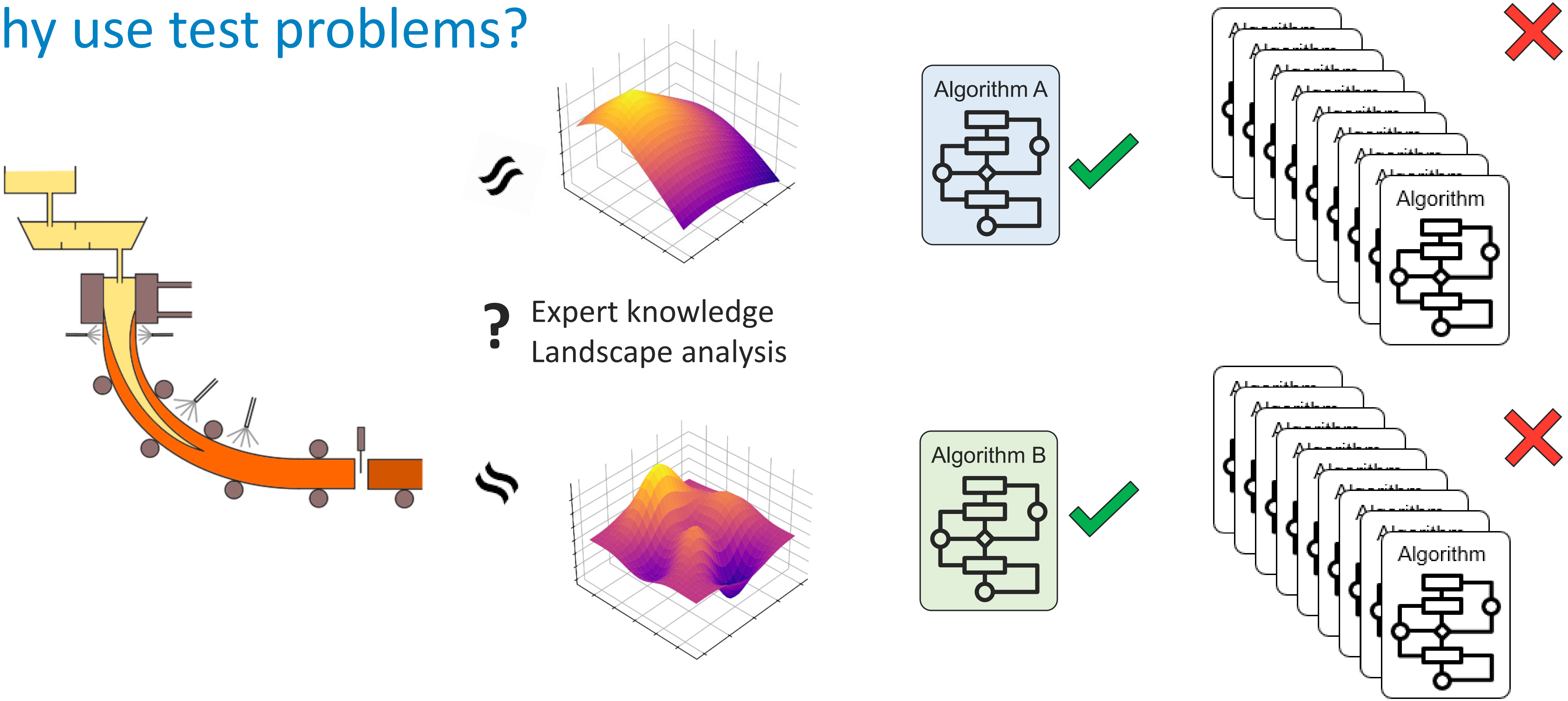
Why use test problems?





Introduction

Why use test problems?



Introduction

Desirable characteristics of a benchmark problem set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]

Introduction

Recommendations for multiobjective test suites

Adapted from [Huband et al. 2006]

1. A few “easy” (unimodal) test problems
2. The majority of problems should be hard (multimodal, nonseparable and both multimodal and nonseparable)
3. Diverse Pareto front geometries (including degenerate fronts, disconnected fronts) and disconnected Pareto sets

Introduction

Additional recommendations for multiobjective test problems

Adapted from [Huband et al. 2006]

1. No extremal variables
2. No medial variables
3. Dissimilar variable domains
4. Dissimilar objective ranges

Introduction

Problem Design Approaches

[Deb et al. 2005]

1. Multiple single-objective functions approach
2. Bottom-up approach
   1. Choose a Pareto front
   2. Build the objective space
   3. Construct the search space

Artificial Problems (Continuous and Unconstrained)

v0.1

Artificial Problems (Continuous and Unconstrained)

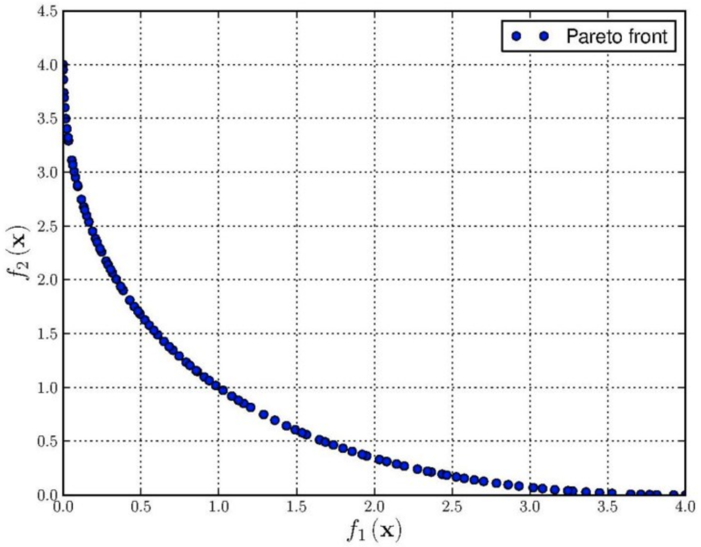
Individual Problems

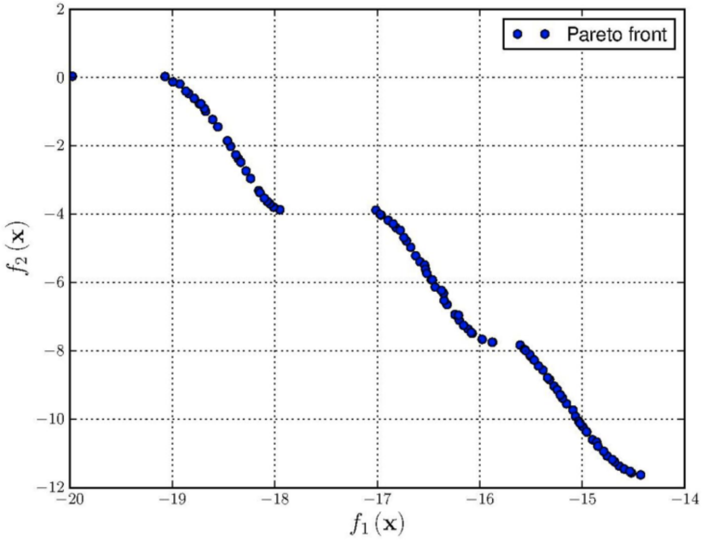
$$\text{Minimize} = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \end{cases}$$

[Schaffer 1985]

$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = \sum_{i=1}^{2} \left[-10 \exp\left(-0.2 \sqrt{x_i^2 + x_{i+1}^2} \right) \right] \\ f_2(\boldsymbol{x}) = \sum_{i=1}^{3} \left[|x_i|^{0.8} + 5 \sin\left(x_i^3 \right) \right] \end{cases}$$

[Kursawe 1991]





Individual Problems

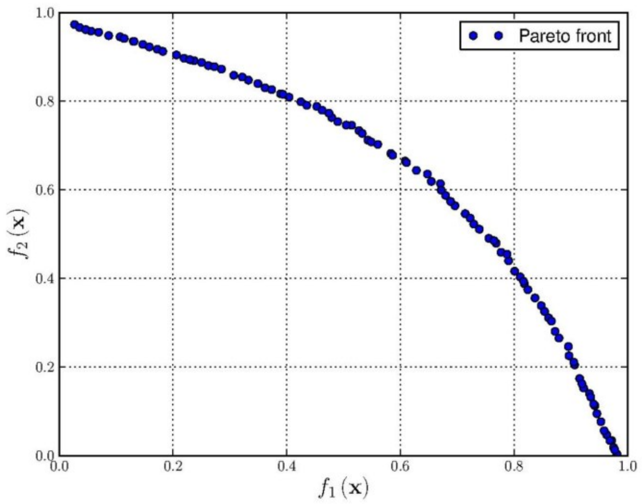
$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$$

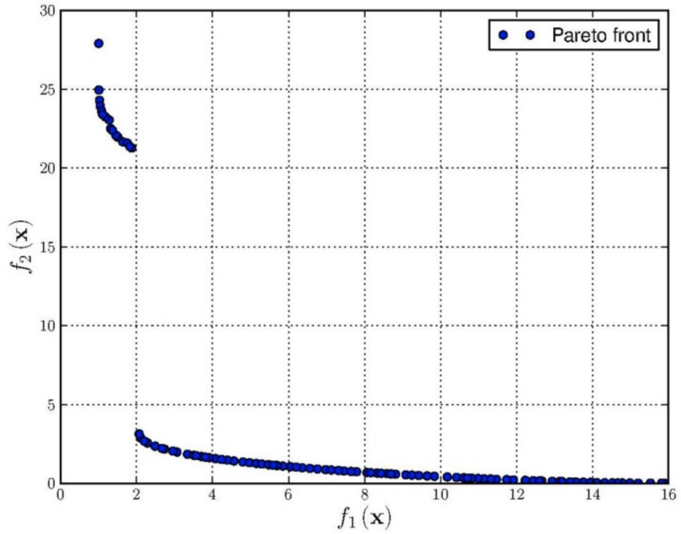
[Fonseca and Fleming 1995]

$$\text{Minimize} = \begin{cases} f_1(x,y) = \left[1 + (A_1 - B_1(x,y))^2 + (A_2 - B_2(x,y))^2\right] \\ f_2(x,y) = (x+3)^2 + (y+1)^2 \end{cases}$$

$$\text{where} = \begin{cases} A_1 = 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1(x,y) = 0.5\sin(x) - 2\cos(x) + \sin(y) - 1.5\cos(y) \\ B_2(x,y) = 1.5\sin(x) - \cos(x) + 2\sin(y) - 0.5\cos(y) \end{cases}$$

[Poloni et al. 1996]

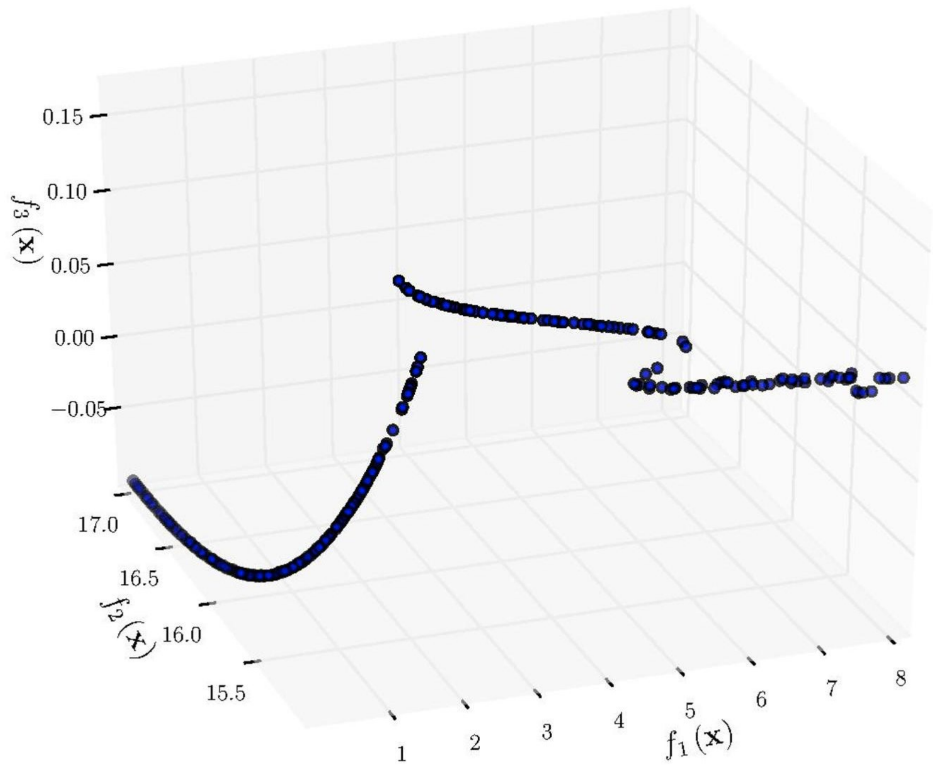




Individual Problems

$$\text{Minimize} = \begin{cases} f_1(x,y) = 0.5\left(x^2+y^2\right) + \sin\left(x^2+y^2\right) \\ f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x,y) = \frac{1}{x^2+y^2+1} - 1.1\exp\left(-\left(x^2+y^2\right)\right) \end{cases}$$

[Viennet et al. 1996]



v0.2

Artificial Problems (Continuous and Unconstrained)

MOP Suite

MOP = Multi-Objective Problem [Van Veldhuizen 1999]

Properties

- A collection of 7 test problems from the literature
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries

Issues

- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- In many problems the optima lie on the boundary or in the middle of the search space
- The Pareto set is hard to compute for some problems

v0.5

Artificial Problems (Continuous and Unconstrained)

ZDT Suite

ZDT = Zitzler, Deb, Thiele [Zitzler et al. 2000]

Construction with the bottom-up approach (following Deb's toolkit [Deb 1999])

Given $\mathbf{x} = \{x_1, \ldots, x_n\}$

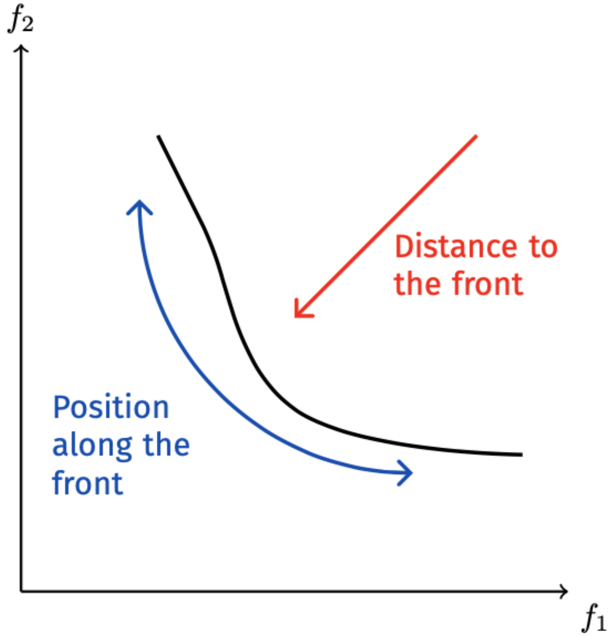
Minimise $f_1(\mathbf{y})$ — Distribution f. (f_1)

$$f_2(\mathbf{y}, \mathbf{z}) = g(\mathbf{z}) h(f_1(\mathbf{y}), g(\mathbf{z}))$$

Distance f. (g), Front shape (h)

where $\mathbf{y} = \{x_1, \ldots, x_j\}$ Position variable(s) (j = 1 for ZDT)

$\mathbf{z} = \{x_{j+1}, \ldots, x_n\}$ Distance variables



The separation of variables was done to simplify problem construction

ZDT Suite

Properties

- Scalable in the number of (distance) variables
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known

Issues

- Not scalable in the number of objectives (2 objectives)
- 4 problems have optima on the boundary of the search space
- 1 problem has optima in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)

v1.0

Artificial Problems (Continuous and Unconstrained)

DTLZ Suite

DTLZ = Deb, Thiele, Laumanns, Zitzler [Deb et al. 2005]

Improvement over ZDT

- Scalable number of objectives
- Optima do not lie on the boundary of the search space

Remaining issues

- Most problems have optima in the middle of the search space
- Problems still separable in practice (optimizing one variable at a time will yield at least one global optimum)

v1.2

Artificial Problems (Continuous and Unconstrained)

WFG Suite

WFG = Walking Fish Group [Huband et al. 2006]

Improvement over DTLZ

- Optima do not lie in the middle of the search space
- Some nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts

Remaining issues

- The Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables

v1.3

Artificial Problems (Continuous and Unconstrained)

Other Suites and Problems

Problems constructed with the bottom-up approach [Zapotecas et al. 2019]

- L-ZDT and L-DTLZ problems with linkages [Deb et al. 2006]
- IHR test suite of 5 rotated ZDT problems [Igel et al. 2007]
- ED problems based on Lamé superspheres [Emmerich and Deutz 2007]
- LZ test suite of 9 problems with complicated Pareto sets [Li and Zhang 2009]
- SZDT test suite of 7 scalable problems with complicated Pareto sets [Saxena et al. 2011]
- Convex DTLZ problem [Deb and Jain 2014]
- Inverted DTLZ problem [Jain and Deb 2014]
- MNI test suite of 2 problems with diverse shapes of the Pareto front [Masuda et al. 2016]
- LSMOP test suite of 9 problems for large-scale optimization with variable dependencies [Cheng et al. 2017b]

Other Suites and Problems

Problems constructed with the bottom-up approach

- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]
- GPD (Generalized Position-Distance) benchmark problem generator (problems can have various difficulties) [Meneghini et al. 2020]
- Suite of 10 ZCAT problems with various difficulties [Zapotecas et al. 2023]

CEC Competition Suites

Information about all CEC competitions:
https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm

13 test problems for CEC 2007 [Huang et al. 2007]

- OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
- 4 shifted ZDT, 1 rotated ZDT
- 2 shifted DTLZ, 1 rotated DTLZ
- 3 WFG

CEC Competition Suites

13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]

- 10 with complicated Pareto sets (4 from the LZ suite)
- 2 extended rotated DTLZ
- 1 WFG

15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]

- 7 modified DTLZ problems
- 2 distance minimization problems
- 3 WFG problems
- 1 SZDT problem
- 2 LSMOP problem

CEC Competition Suites

22 test problems for CEC 2019 [Liang et al. 2019]

- 2 SYM-PART
- Omni-test [Deb and Tiwari 2008]
- 19 MMF problems

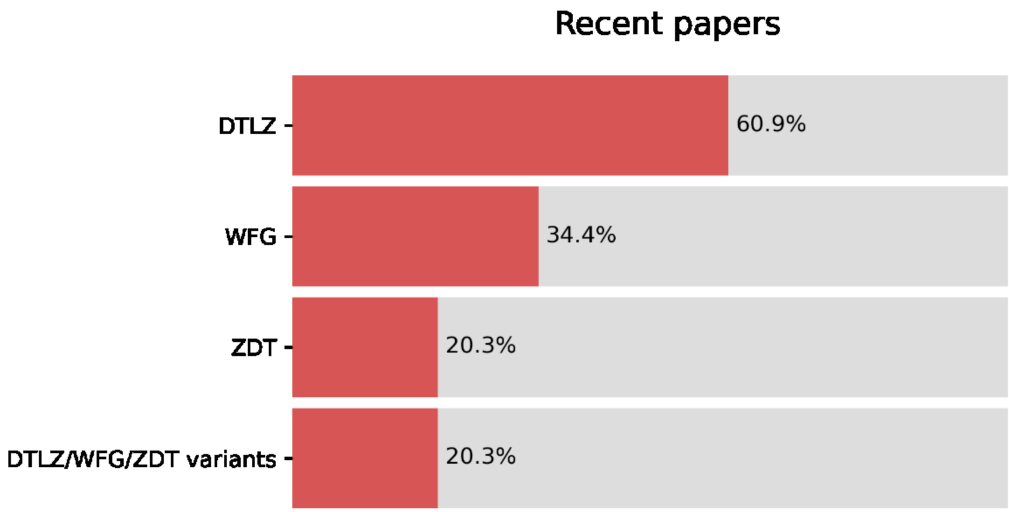
24 test problems for CEC 2020 [Liang et al. 2020]

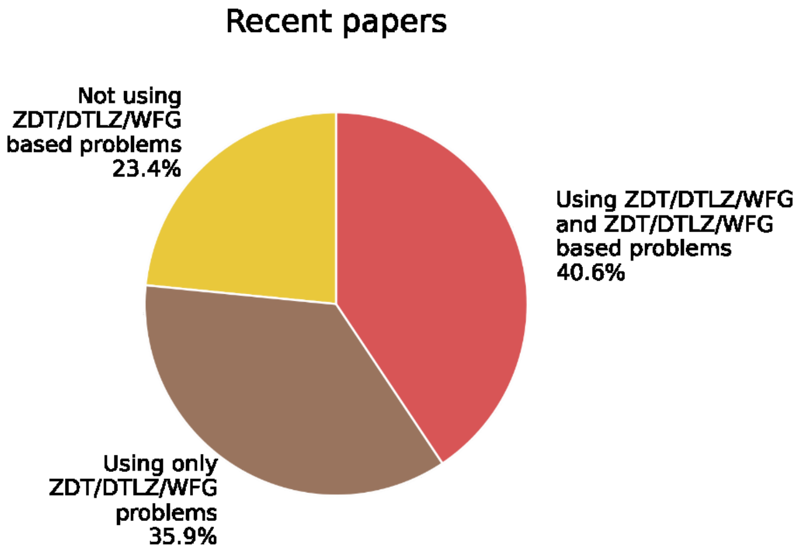
- 24 MMF problems

Survey of Recent Papers

64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)

- CEC 2020
- GECCO 2020
- PPSN 2
- EMO 2(





v1.5

Artificial Problems (Continuous and Unconstrained)

Distance-Based Problems

General idea [Ishibuchi et al. 2010]

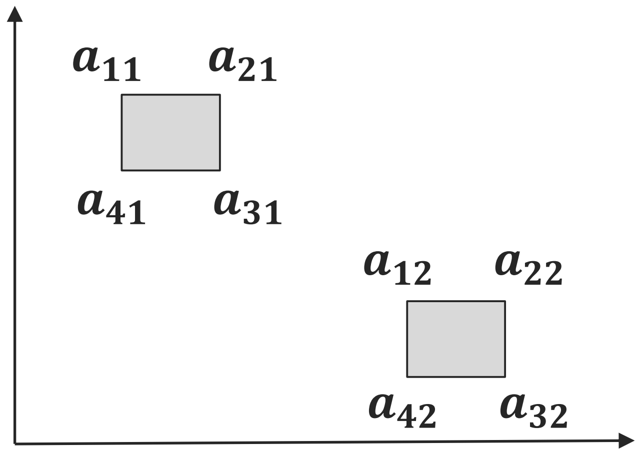
- Based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ \ldots,\ f_k(\mathbf{x}))$$

$$f_i(\mathbf{x}) = \min\{\text{dis}(\mathbf{x}, \mathbf{a}_{i1}), \text{dis}(\mathbf{x}, \mathbf{a}_{i2}), \ldots, \text{dis}(\mathbf{x}, \mathbf{a}_{im})\}$$

Properties

- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions



Issues

- Simple objective functions

Distance-Based Problems

Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]

v2.0

Artificial Problems (Continuous and Unconstrained)

bbob-biobj Suite

Motivation [Brockhoff et al. 2022]

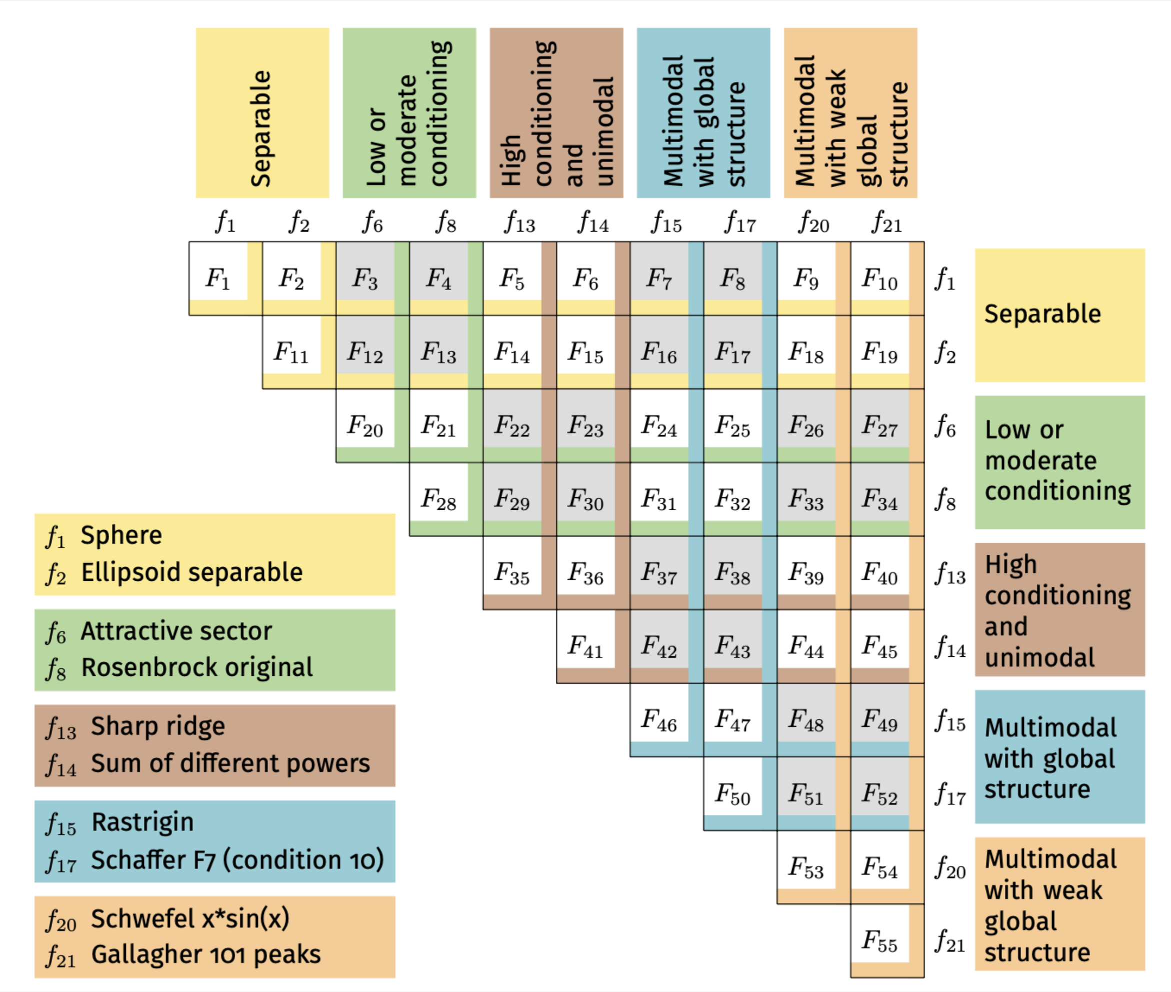
- Real-world problems are not constructed using the bottom-up approach
- Go back to basics – use single-objective functions for each objective
- Idea not new [Schaffer 1985, Igel et al. 2007, Kerschke et al. 2016]

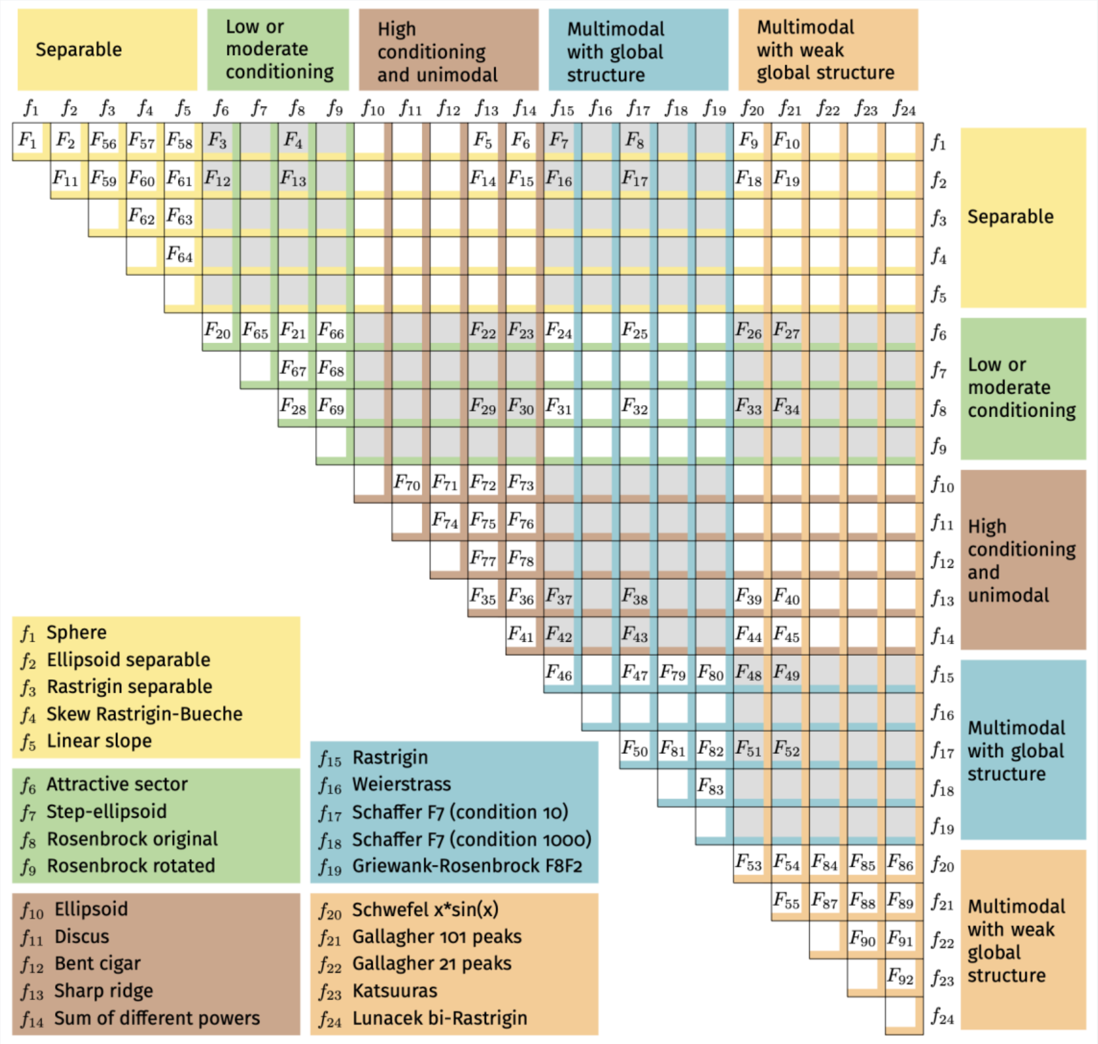
bbob-biobj Suite

Construction

- Use the functions from the bbob suite [Finck et al. 2009]
  - Well-understood
  - Scalable in the number of variables and parametrized (instances)
  - 24 functions categorized in 5 groups based on their properties
    - Separable
    - Low or moderate conditioning
    - High conditioning and unimodal
    - Multimodal with global structure
    - Multimodal with weak global structure

bbob-biobj(-ext) Suites





bbob-biobj(-ext) Suites

Properties

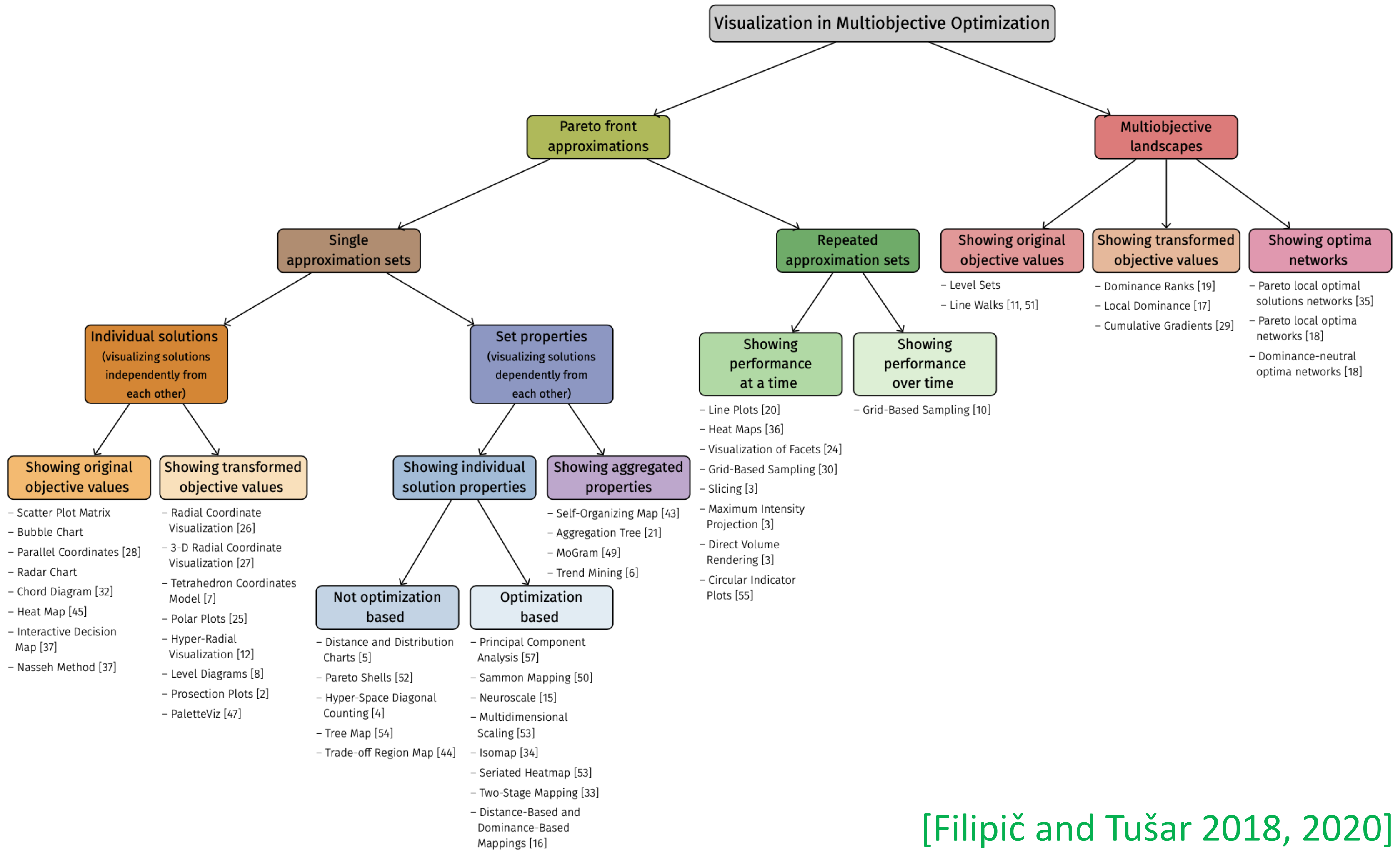
- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Included in the COCO benchmarking platform [Hansen et al. 2021]
- Problem instances can be quite diverse

Issues

- Only 2 objectives
- Unknown Pareto set and front, but known single-objective optima and available approximations of the Pareto fronts (and sets for lower-dimensional problems)

Visualization of Multiobjective Landscapes

Visualization in Multiobjective Optimization



[Filipič and Tušar 2018, 2020]

Visualization of Multiobjective Problem Landscapes

Low-dimensional search spaces

- Dominance ratio [Fonseca 1995]
- Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])
- Local dominance [Fieldsend et al. 2019]
- PLOT [Schäpermeier et al. 2020]

Any-dimensional search spaces

- Line cuts [Brockhoff et al. 2022, Volz et al. 2019]
- Optima network [Liefooghe et al. 2018, Fieldsend and Alyahya 2019]

Visualization of Multiobjective Problem Landscapes

Various visualizations of bbob-biobj-ext problems

https://numbbo.github.io/bbob-biobj/

Visualizations of bbob-biobj and other multi-objective suites using PLOT

https://schaepermeier.shinyapps.io/moPLOT/

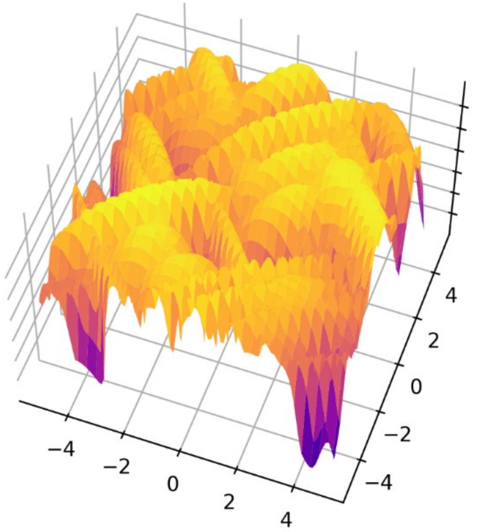
Visualization of Multiobjective Problem Landscapes

Problems for demonstration

- Double sphere problem bbob-biobj $F_1 = (f_1, f_1)$, instance 1
- Sphere-Gallagher problem bbob-biobj $F_{10} = (f_1, f_{21})$, instance 1
- Double Gallagher problem bbob-biobj $F_{55} = (f_{21}, f_{21})$, instance 1

Gallagher = Gallagher's Gaussian

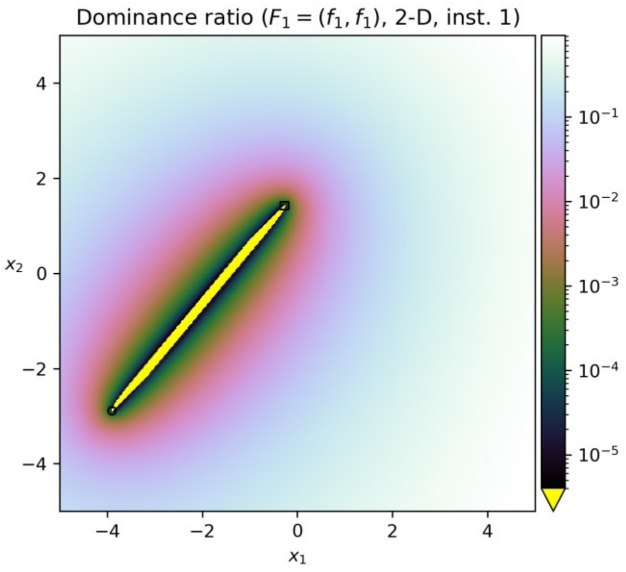
101-me Peaks Function

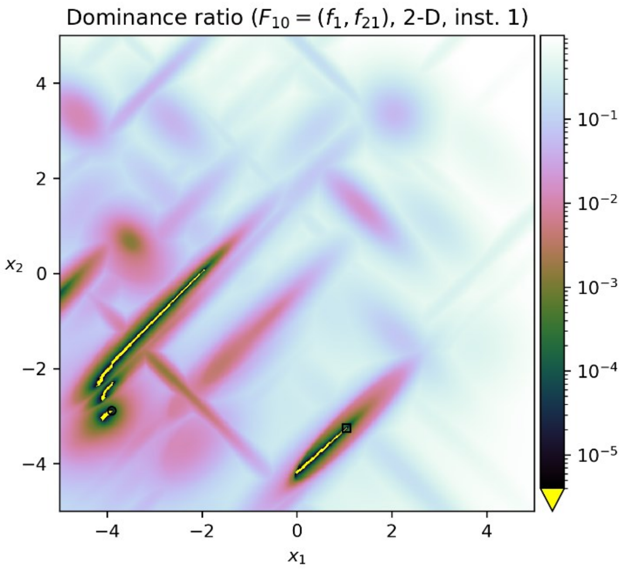


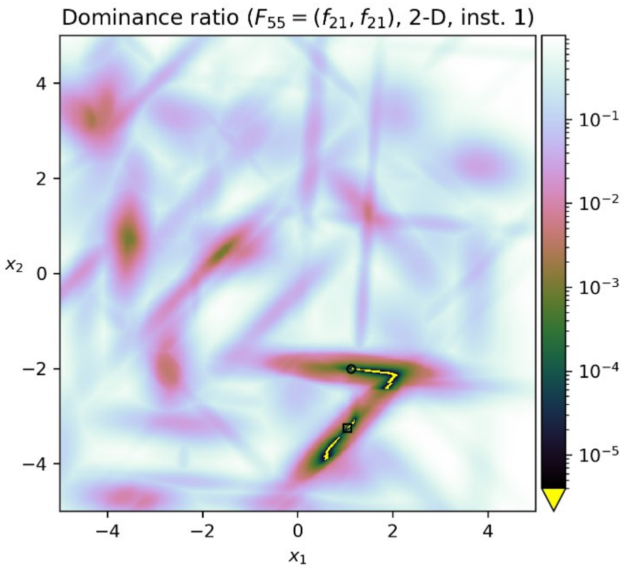
Dominance Ratio

- Discretized search space (501 x 501 grid)
- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale

[Fonseca 1995]







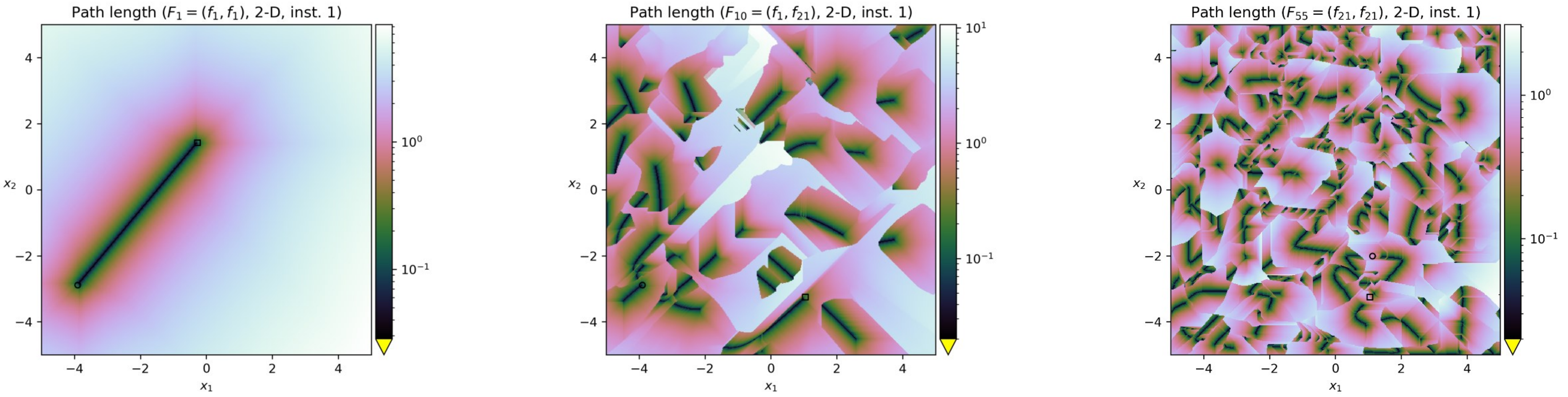
Gradient Path Length

- Compute the bi-objective gradient for all grid points

$$v = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum

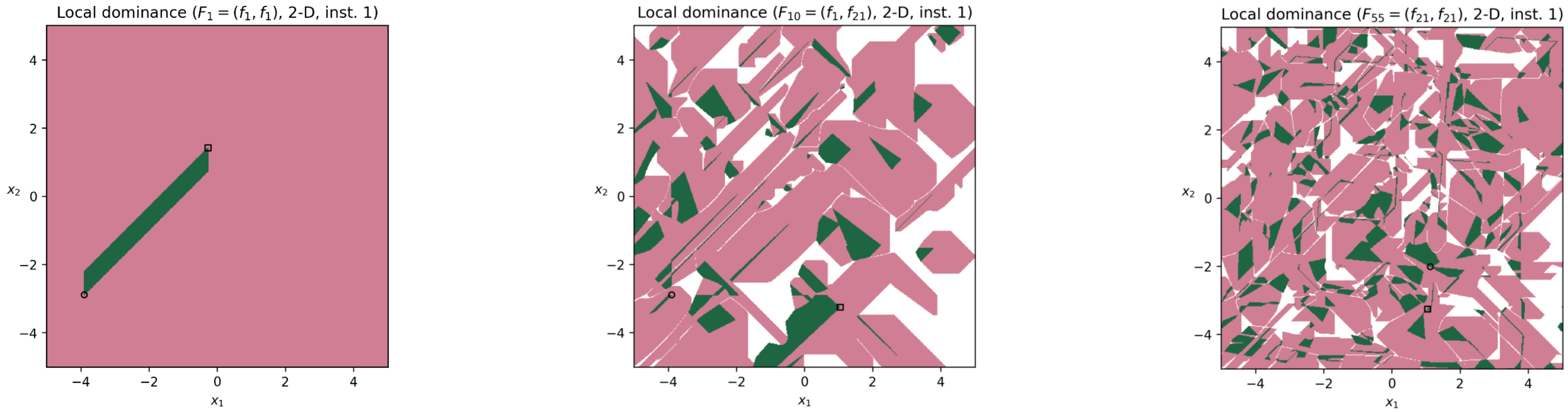
Adapted from [Kerschke and Grimme 2017]



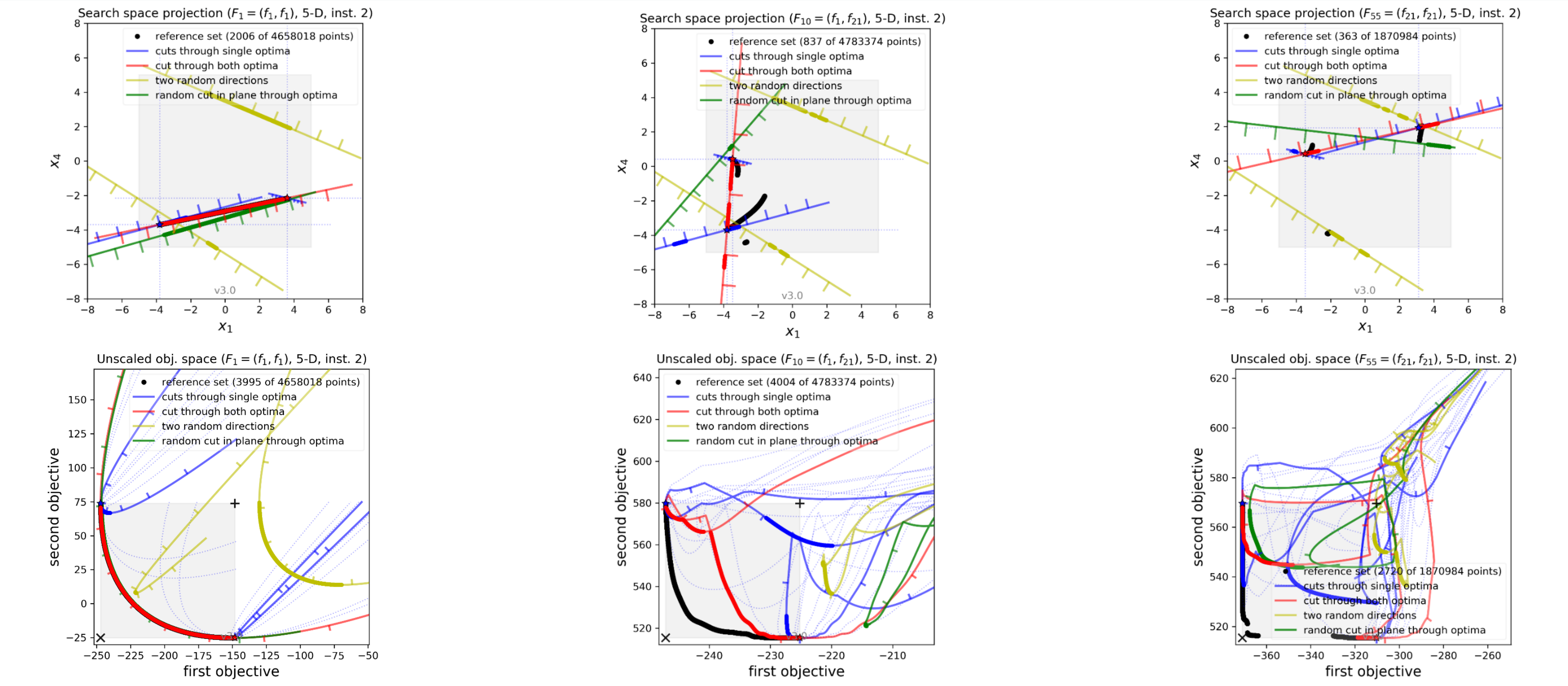
Local Dominance

[Fieldsend et al. 2019]

- Green: Dominance-neutral local optima regions
  - Points that are mutually nondominated with all their 8 neighbors
- Pink: Basins of attraction
  - Points dominated by at least one neighbor, their dominating paths lead to the same green region
- White: Boundary regions
  - Points whose dominating paths lead to different green regions



Line Cuts

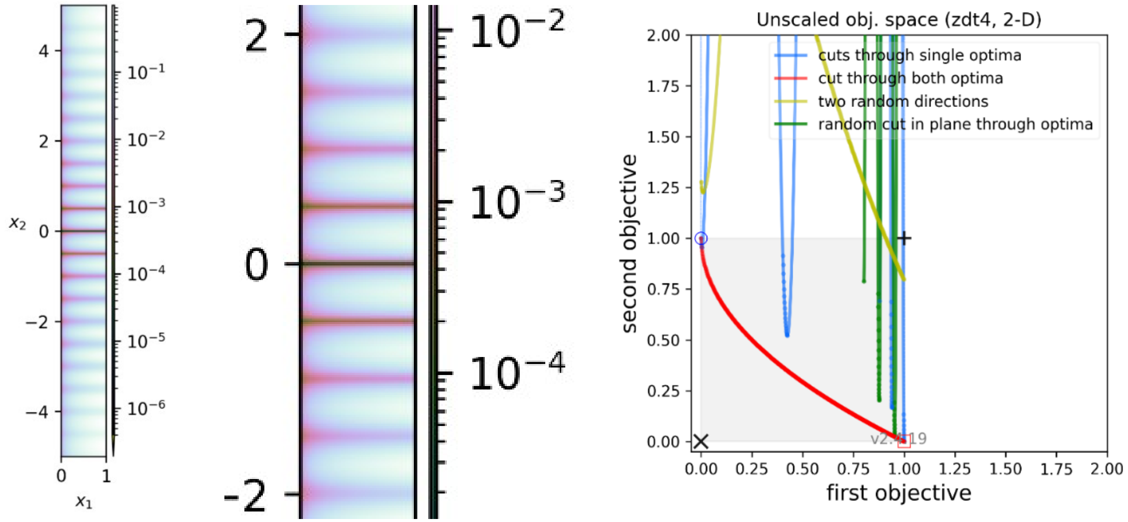


Search space projection ($F_1 = (f_1, f_1)$, 5-D, inst. 2)
reference set (2006 of 4658018 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
v3.0
x_1
x_4
Search space projection ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)
reference set (837 of 4783374 points)
Search space projection ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)
reference set (363 of 1870984 points)
Unscaled obj. space ($F_1 = (f_1, f_1)$, 5-D, inst. 2)
reference set (3995 of 4658018 points)
second objective
first objective
Unscaled obj. space ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)
reference set (4004 of 4783374 points)
Unscaled obj. space ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)
reference set (2720 of 1870984 points)

Comparison of Problem Landscapes

Two problems where both objectives are separable, first is unimodal and second is multimodal

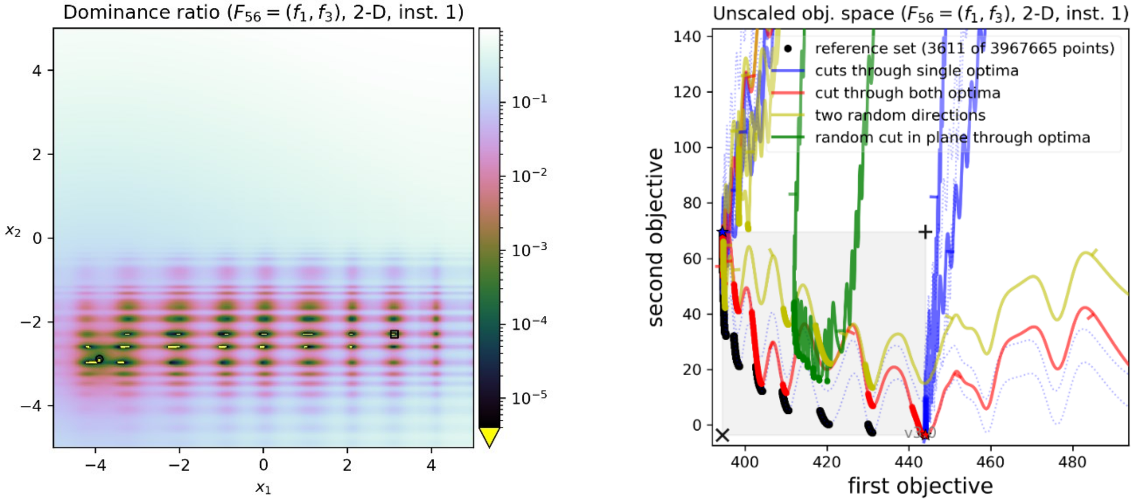
ZDT4



bbob-biobj-ext F_{56}

f_1 Sphere function

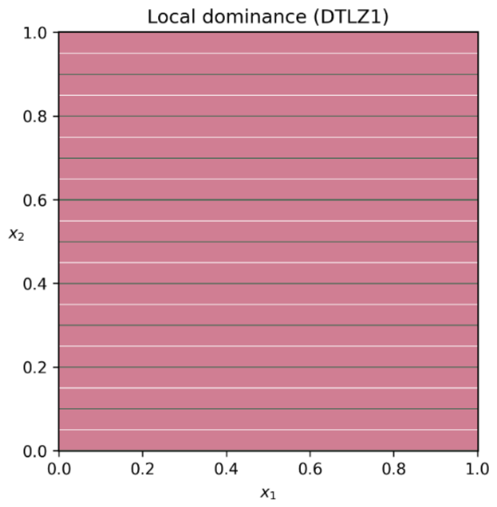
f_3 Rastrigin function

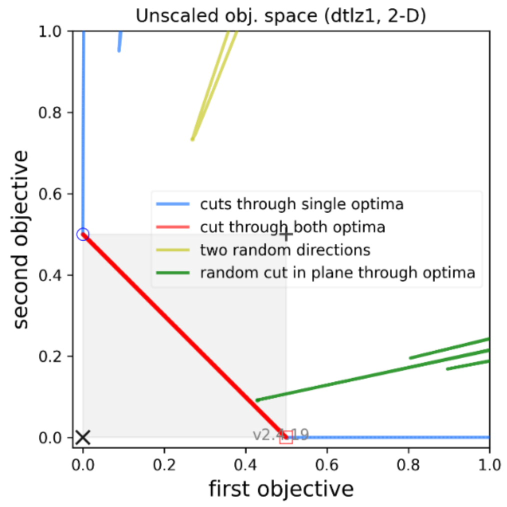


Comparison of Problem Landscapes

Two problems where both objectives are separable and multimodal

DTLZ1

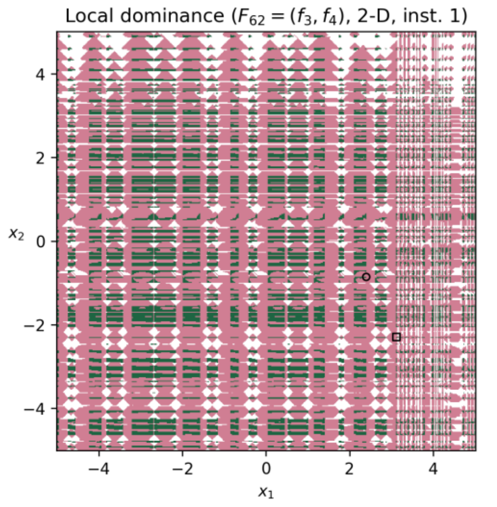


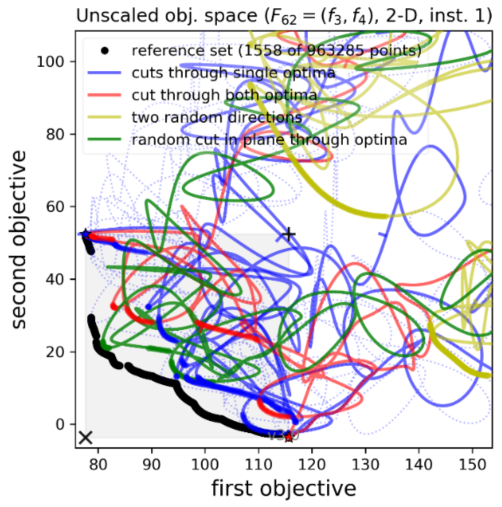


bbob-biobj-ext F_{62}

f_3 Rastrigin function

f_4 Skew Rastrigin-Bueche

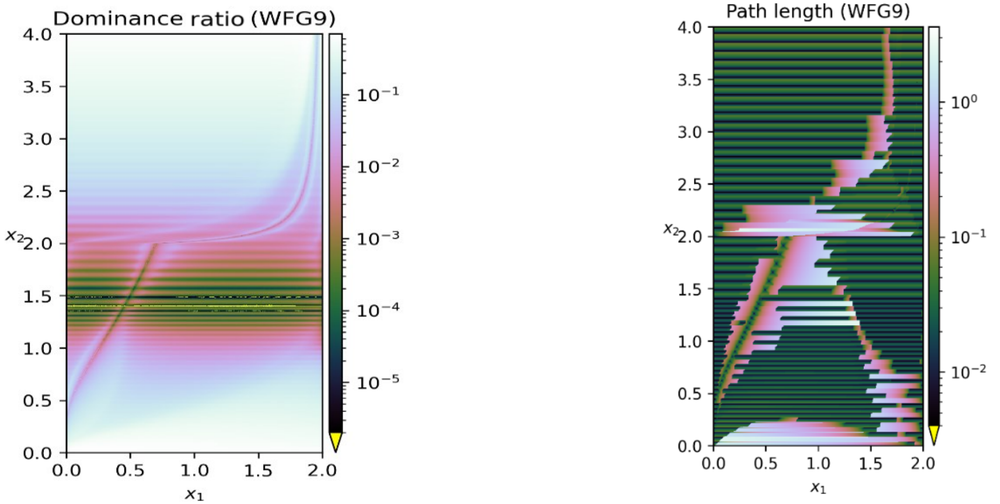




Comparison of Problem Landscapes

Two problems where both objectives are nonseparable and multimodal

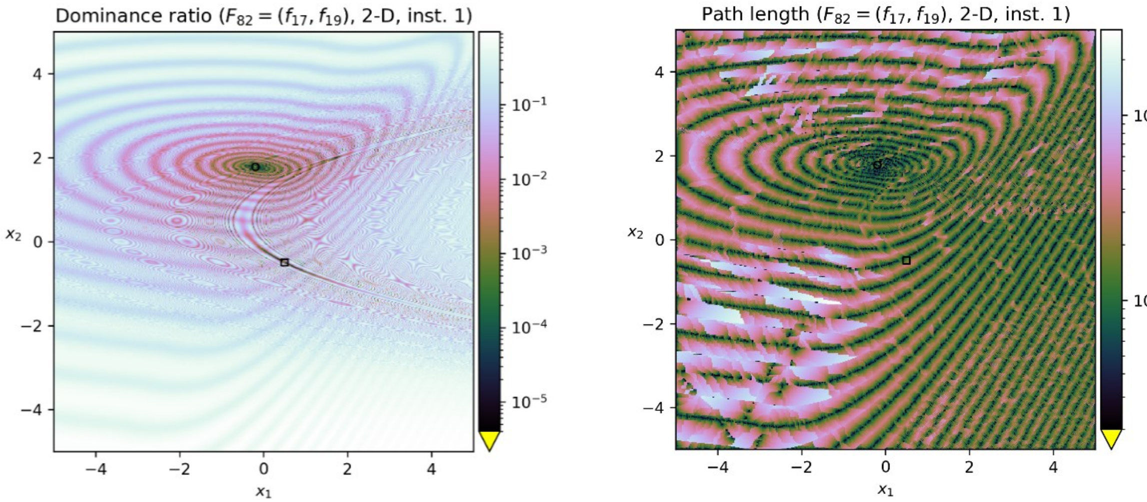
WFG9



bbob-biobj-ext F_{82}

f_{17} Schaffer F7

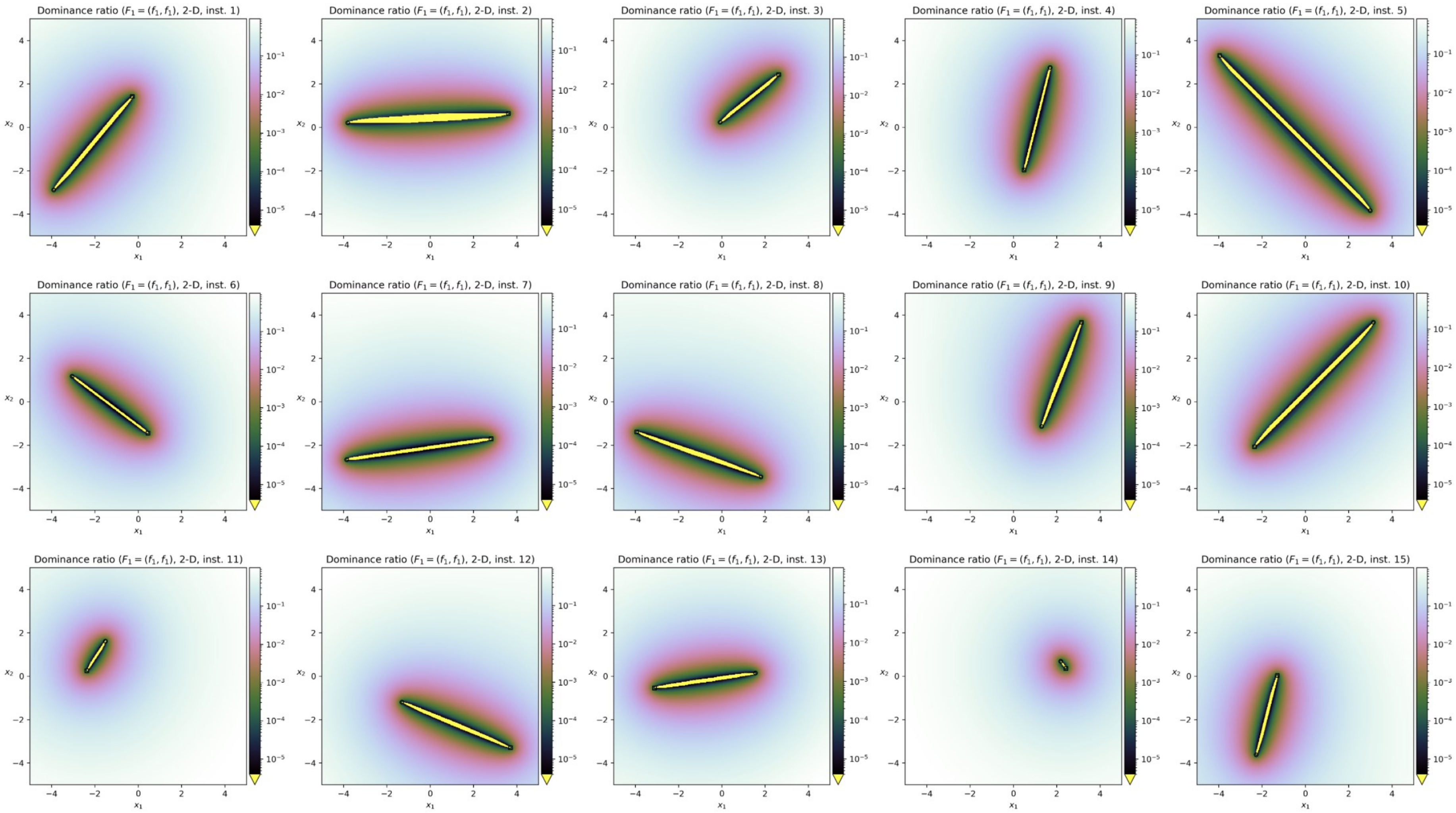
f_{19} Griewank-Rosenbrock



Some Notes on Problem Properties

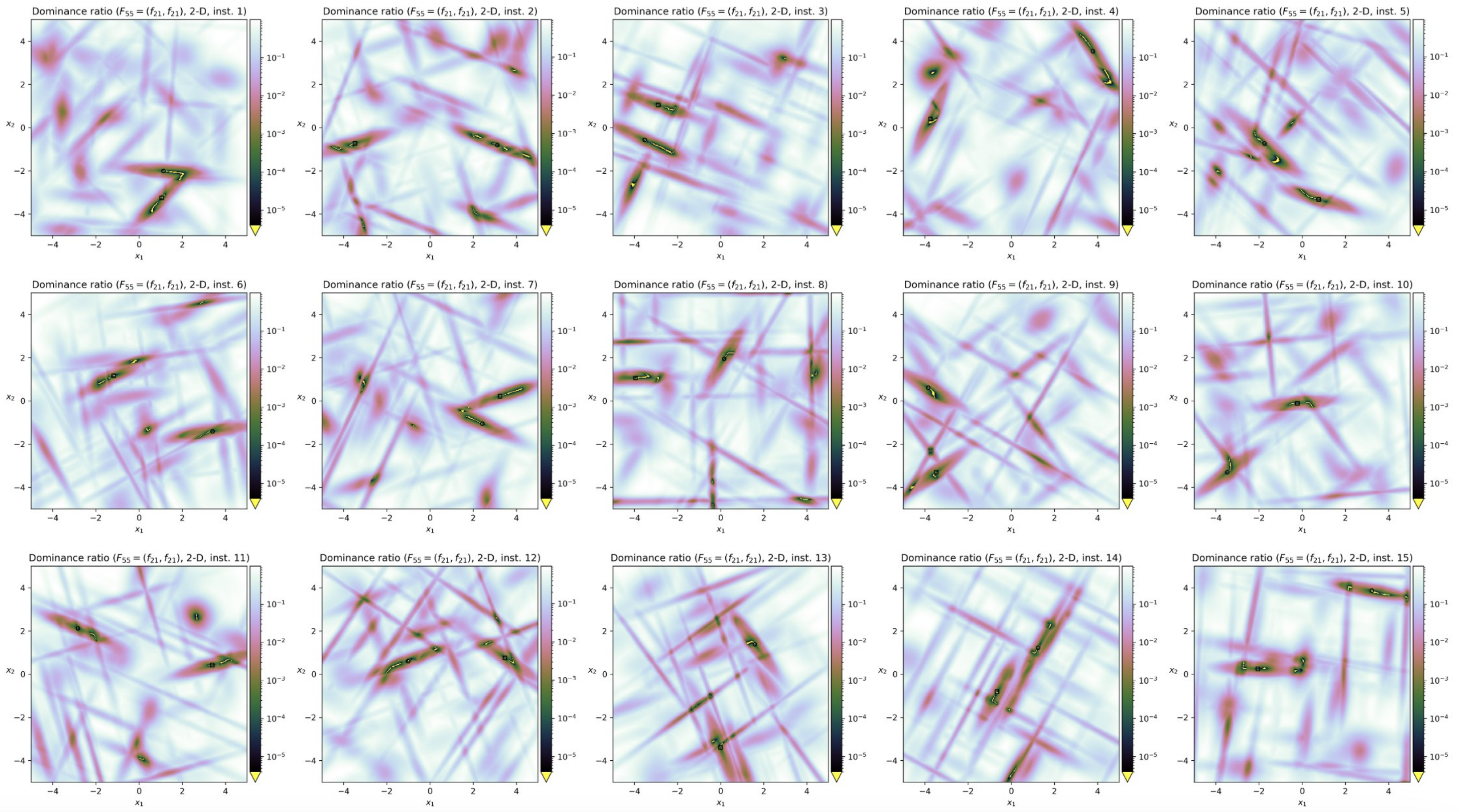
Problem Instances

15 instances of the double sphere problem bbob-biobj F_1



Problem Instances

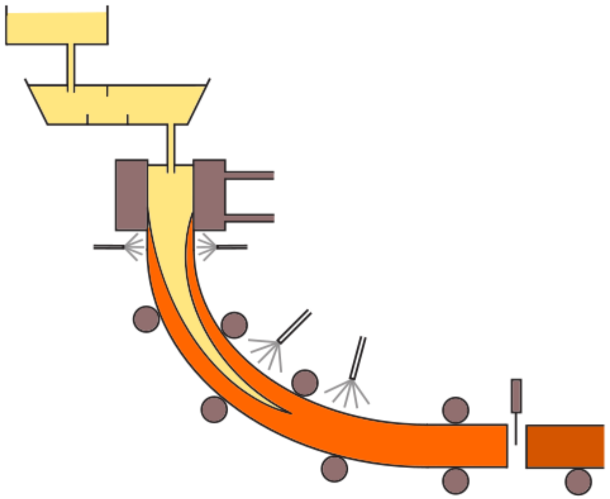
15 instances of the double Gallagher problem bbob-biobj F_{55}



Problem Instances

Real-world problems have instances

- Same problem with varying variable bounds
- Very similar problem (casting steel 31CrV3 vs. 51CrV4)

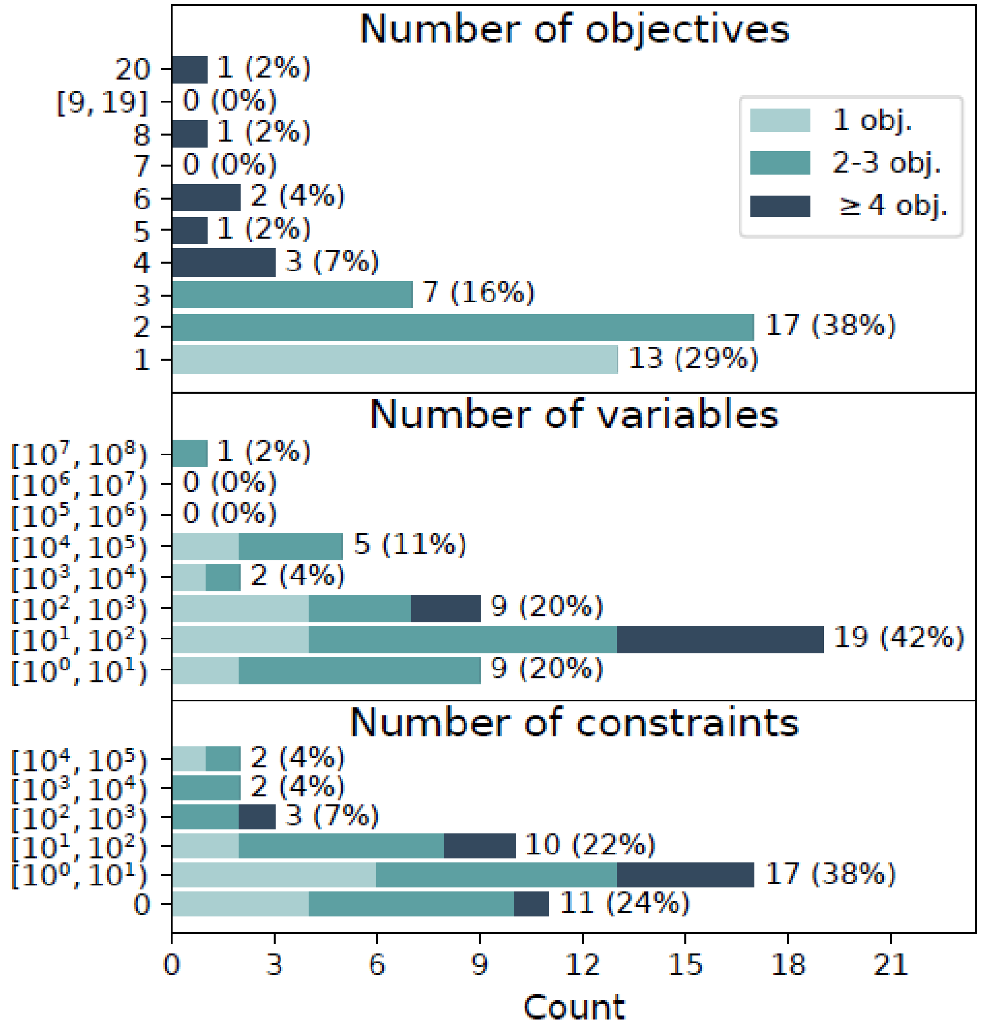


Scalability in the Number of Objectives

Questionnaire on the properties of real-world problems [van der Blom et al. 2020]
https://sites.google.com/view/macoda-rwp/home

- Only 45 problems (!)
- Of these, only 9% have more than 4 objectives

Are we over-emphasizing many-objective problems?



Scalability in the Number of Objectives

Large number of possible combinations

- ZDT/DTLZ/WFG and other suites have a fixed number of problems
- Example for 10 functions
  - 2 objectives –> 55 combinations
  - 3 objectives –> 220 combinations
  - 5 objectives –> 715 combinations
- Problem selection [Andova et al. 2023]
  - Goal: choose only a limited number of most diverse problems
  - Diversity measured in problem feature space
  - Proof of concept on bbob-biobj problems

Other Artificial Problems

Other Artificial Problems

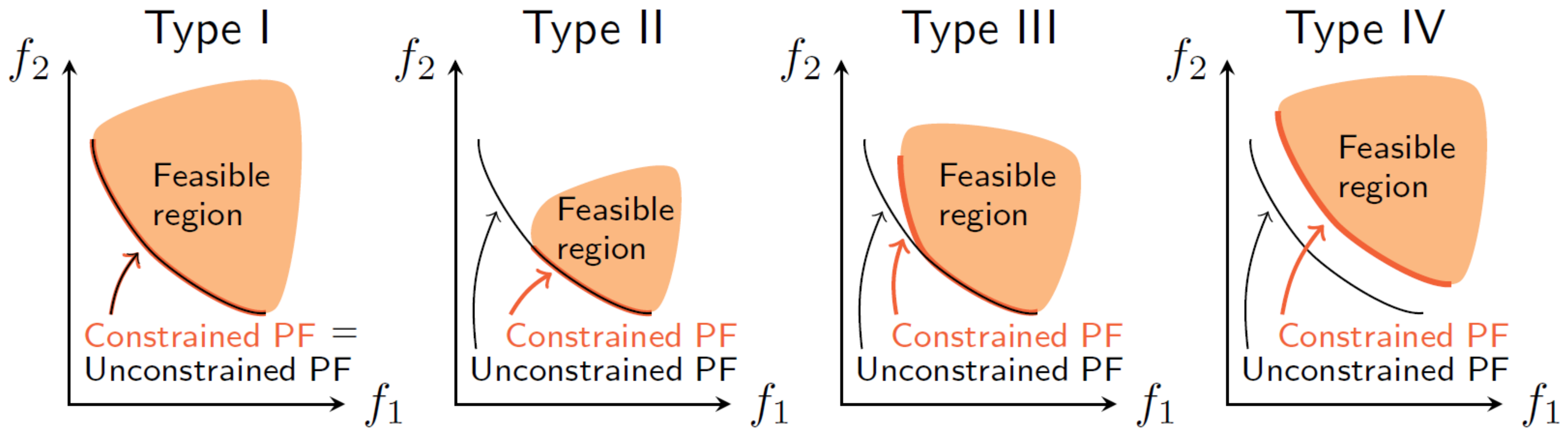
Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- DTLZ (problems DTLZ8-9) [Deb et al. 2005]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019]
- MW [Ma and Wang 2019]
- DOC [Liu and Wang 2019]
- DAS-CMOP and DAS-CMaOP [Fan et al. 2020]
- Eq-DLTZ and Eq-IDTLZ [Cuate et al. 2020]
- CLSMOP [He et al. 2021]

Other Artificial Problems

Problem types

Depending on how the constraints affect the Pareto set/front

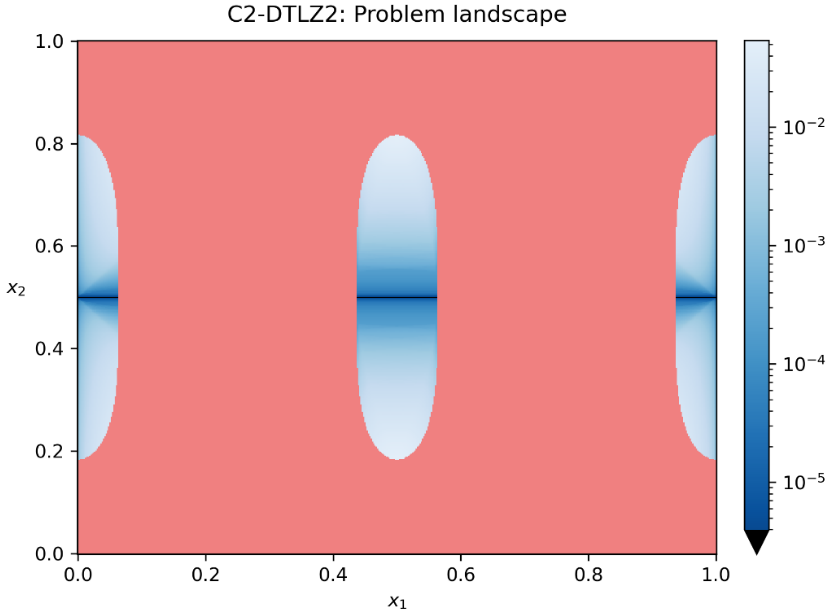


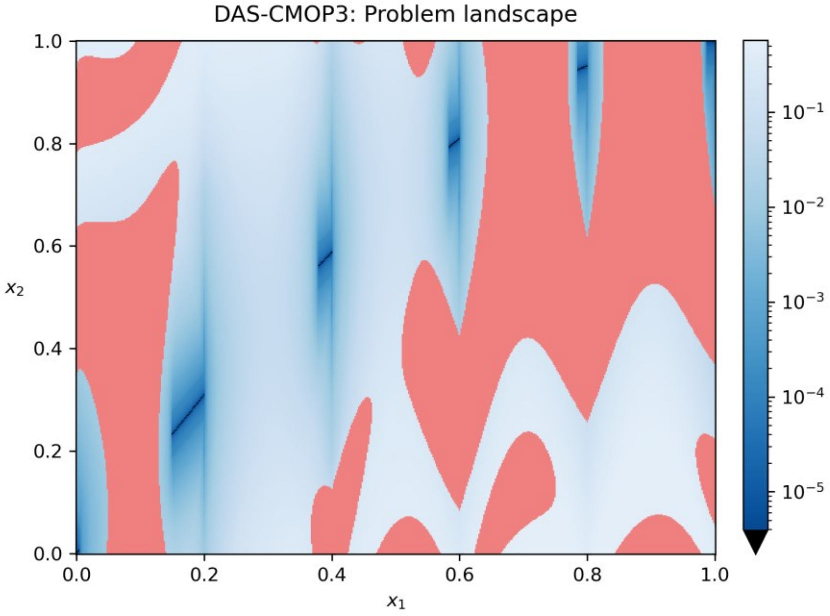
Adapted from [Ma and Wang 2019]

Problems of Type I not useful for benchmarking constraint handling techniques

Other Artificial Problems

Analysis of multiobjective problems with constraints [Vodopija et al. 2022]





https://vodopijaaljosa.github.io/cmop-web/

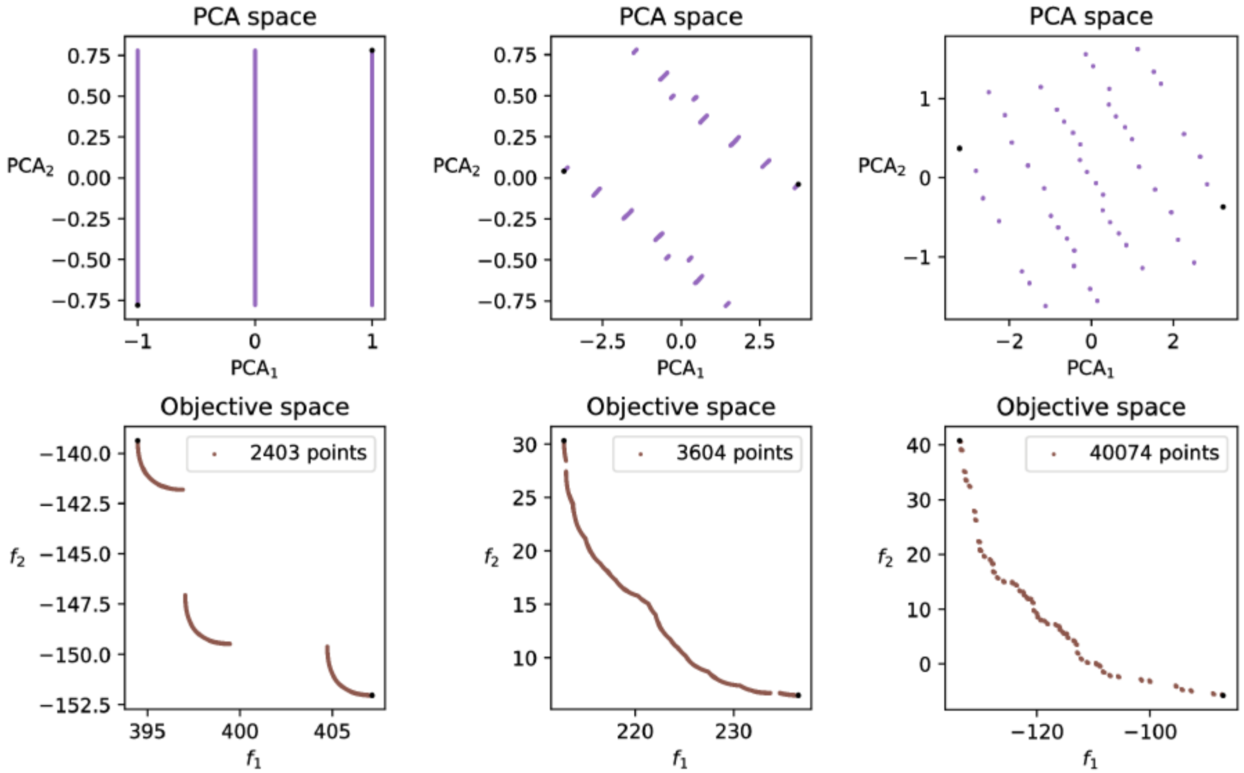
Tutorial on multiobjective optimization in the presence of constraints:
https://dis.ijs.si/filipic/wcci2022tutorial/

Other Artificial Problems

Suites of multiobjective mixed-integer problems

- 3 bi-objective problems summing one discrete and one continuous function [Sadowski et al. 2021]
- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbob-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function



Real-World Problems

v0.1

Real-World Problems

Real-World Problems

Individual problems (white box)

- Water resource planning problem with 3 variables, 5 objectives and 7 constraints [Musselman and Talavage 1980]
- Two bar truss design problem with 2 variables, 2 objectives and 2 constraints [Rao 1987]
- Vibrating platform design problem with 3 variables, 2 objectives and 5 constraints [Ray et al. 2001]
- Welded beam design problem with 4 variables, 2 objectives and 5 constraints [Ray and Liew 2002]
- Multi-speed gearbox design problem with 10 variables, 2 objectives and 38 constraints [Deb and Jain 2003]
- Mineral processing production planning problem with 6 variables, 5 objectives and 9 constraints [Yu et al. 2011]
- Car side impact problem with 11 variables, 3 objectives and 10 constraints [Jain and Deb 2014]
- …

Real-World Problems

Individual problems (black box)

- Radar waveform design with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]
- Lunar lander landing site selection problem with 2 variables, 3 objectives and 2 constraints [JSEC and JAXA 2018]
- Wind turbine design with 32 variables, 5 objectives and 22 constraints [JSEC 2019]
- Trappist tour planning problem with 34 mixed-integer variables, 2 objectives and 1 constraint [ESA 2022]
- Quantum communications constellations problem with 20 mixed-integer variables, 2 objectives and 2 constraints [ESA 2023]

v0.2

Real-World Problems

Suites of Real-World Problems

Suites of unscalable problems

- DDMOP suite of 7 test problems with a different number of variables (5–17) and objectives (2–10) [He et al. 2020]
- Two suites of previously published problems [Tanabe and Ishibuchi 2020]
  - RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
    - 11 continuous, 1 integer, 4 mixed-integer
  - CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
    - 6 continuous, 1 integer, 1 mixed-integer
- RCM suite of 50 problems with a different number of variables (2–34), objectives (2–5) and constraints (1–29) [Kumar et al. 2021]

v0.5

Real-World Problems

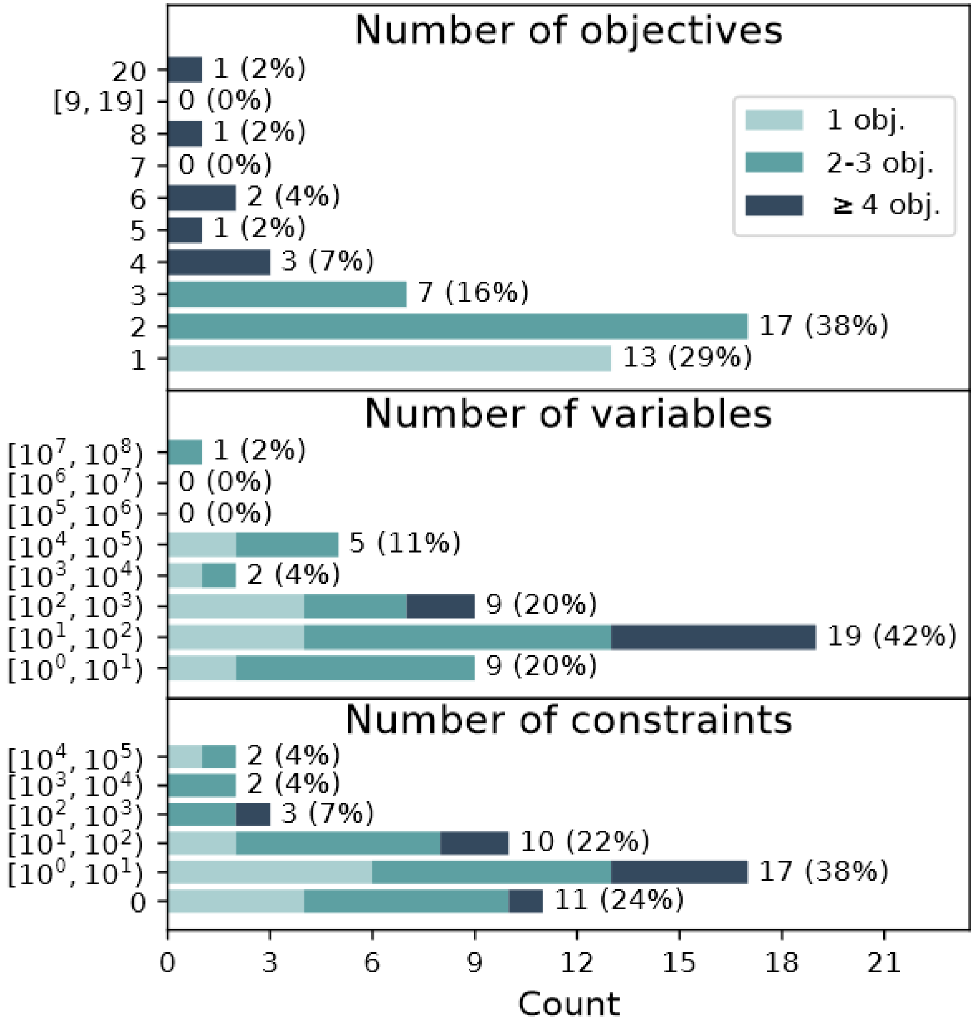
Suites of Real-World Problems

Suites of scalable problems

- Multi-observable quantum control problems with scalable variables, noise and 2 or 3 objectives [Shir et al. 2012]
- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]
- MODAct suite of 20 problems with 2+6k variables, various number of objectives (2–5) and constraints (7–10) [Picard and Schiffmann 2021]
- Scalable multi-agent pathfinding problems
  - Discrete problem formulation with 5 objectives [Weise and Mostaghim 2022]
  - Continuous problem formulation with 2 objectives [Mai et al. 2023]

Conclusions

- We should think about the usefulness of our research
- The questionnaire on the properties of real-world problems has shown their diversity
  [van der Blom et al. 2020]
- Most research is done on continuous unconstrained problems
- A lot (too much?) research on many-objective problems
- Although the test problems are scalable, most studies use a fixed number of variables



Conclusions

- Problem suites constructed with the bottom-up approach have unrealistic properties
- Algorithms are overfitting to these problems (especially the overused DTLZ and WFG) [Ishibuchi et al. 2017, 2023]
- Using separate functions for the objectives looks like a step in the right direction

Overview

❶ Performance Assessment

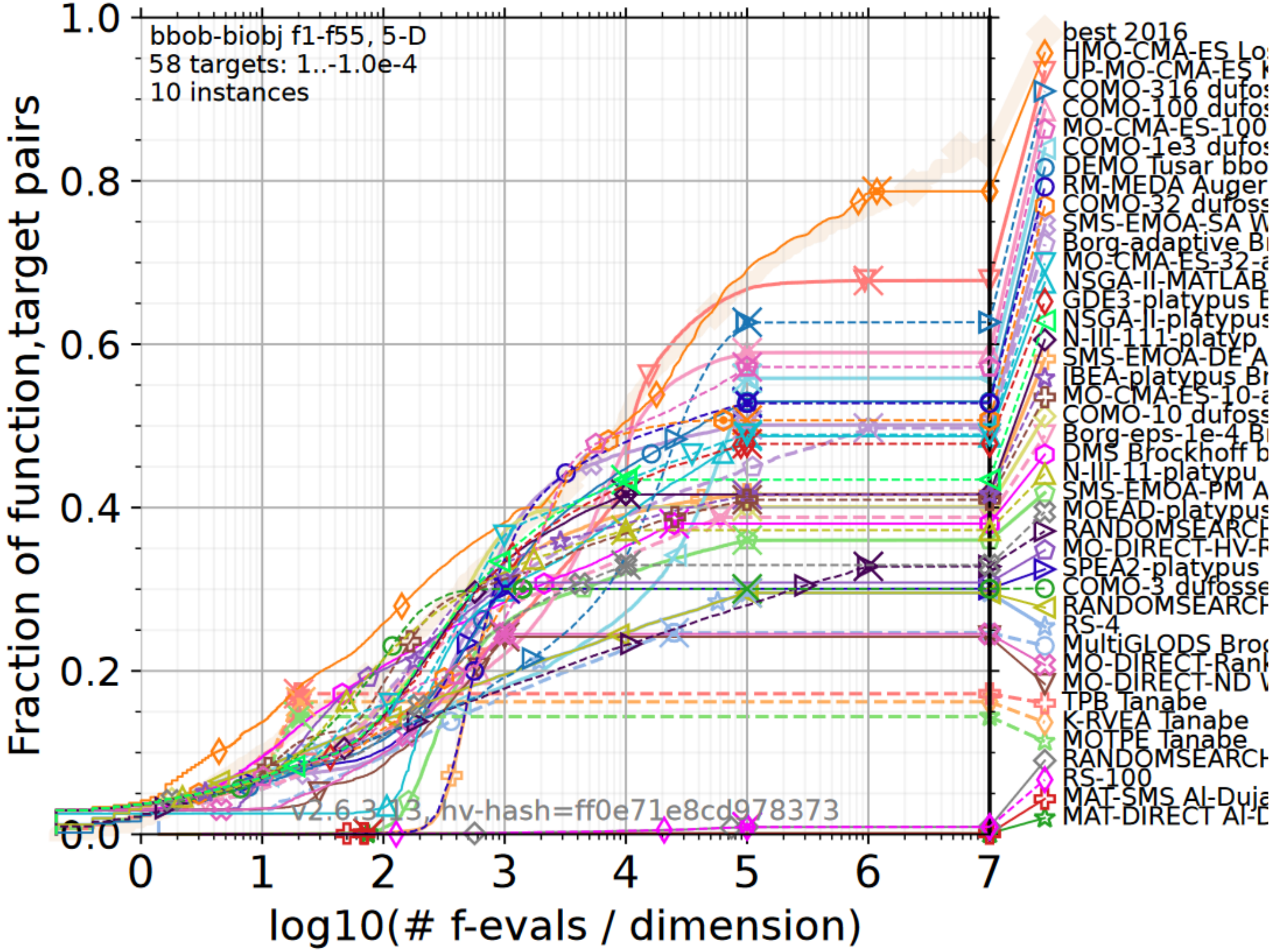
❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

```
python -m cocopp bbob-biobj*
```

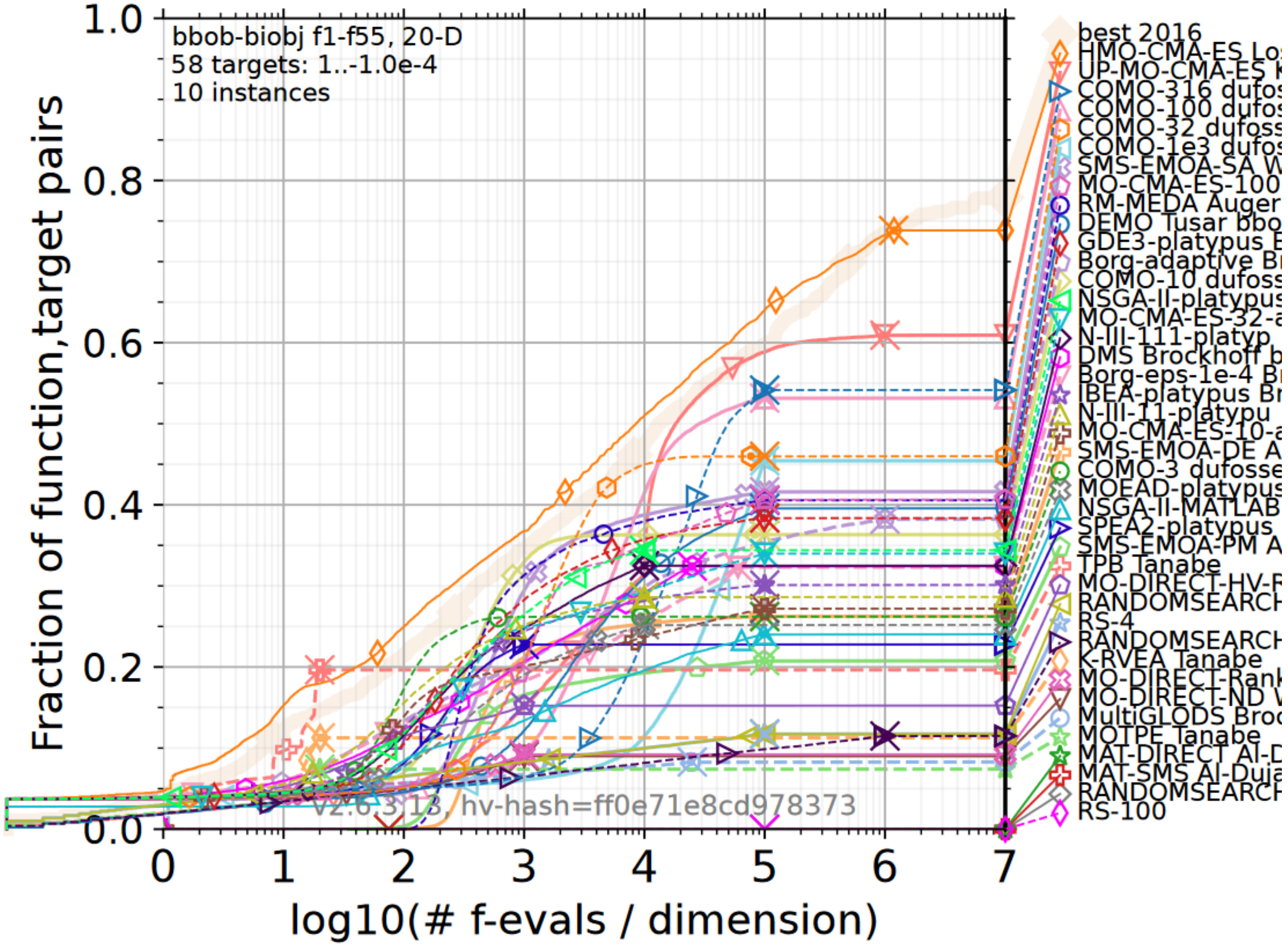
Aggregated Results Over All 55 Functions

5-D



Aggregated Results Over All 55 Functions

20-D



Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

- Many-objective problems
  - Problems/suites
  - Indicators
  - Efficient implementations

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

- Many-objective problems
  - Problems/suites
  - Indicators
  - Efficient implementations
- Constraints, mixed-integer, …

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

- Many-objective problems
  - Problems/suites
  - Indicators
  - Efficient implementations
- Constraints, mixed-integer, …
- Real-world benchmarking?
  - Simulation crashes
  - Parallelism
  - Dynamic changes
  - Interactive decision making, …

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

- Many-objective problems
  - Problems/suites
  - Indicators
  - Efficient implementations
- Constraints, mixed-integer, …
- Real-world benchmarking?
  - Simulation crashes
  - Parallelism
  - Dynamic changes
  - Interactive decision making, …
- Benchmarking results from more classical approaches

Three "New Year" Resolutions

Three "New Year" Resolutions

❶ Show convergence graphs/ECDF

Anything else than tables for fixed budget

Three “New Year” Resolutions

❶ Show convergence graphs/ECDF

Anything else than tables for fixed budget

❷ Use “most realistic” problems

Three “New Year” Resolutions

❶ Show convergence graphs/ECDF

Anything else than tables for fixed budget

❷ Use “most realistic” problems

❸ Showing scaling with (search & objective space) dimension

Three "New Year" Resolutions

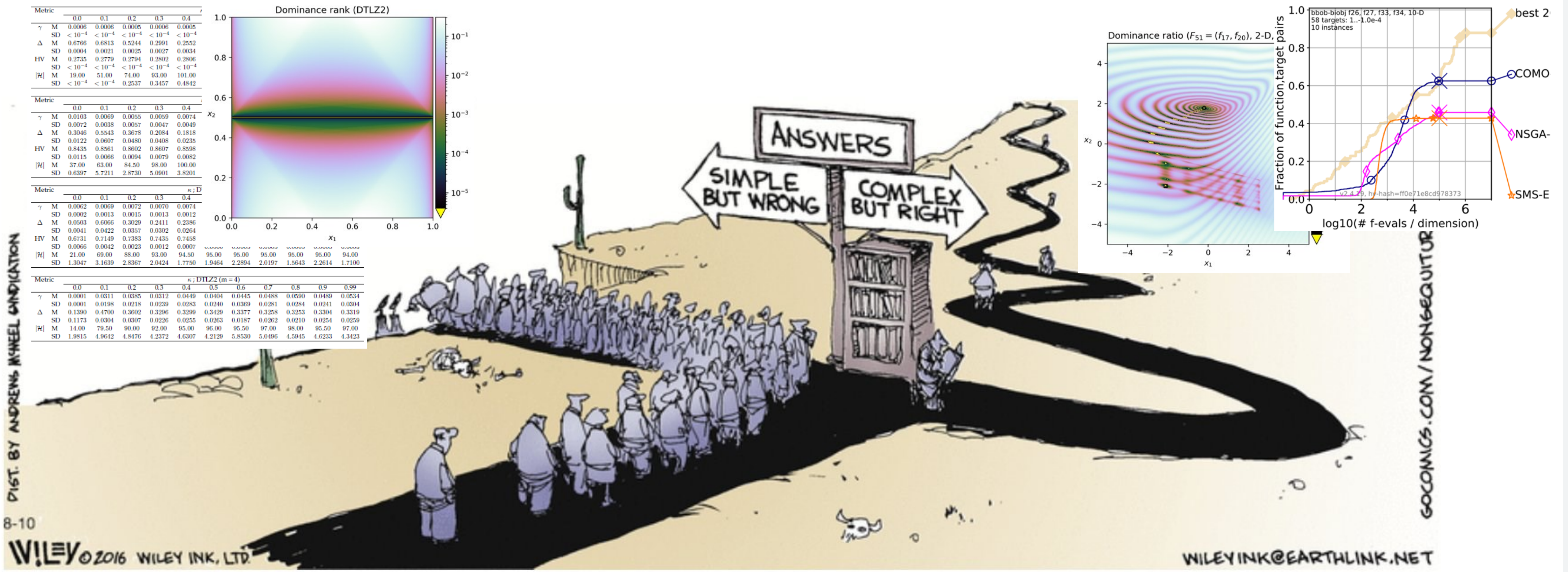
❶ Show convergence graphs/ECDF

Anything else than tables for fixed budget

❷ Use "most realistic" problems

❸ Showing scaling with (search & objective space) dimension

Thank you!



ANSWERS
SIMPLE BUT WRONG
COMPLEX BUT RIGHT
Dominance rank (DTLZ2)
Dominance ratio ($F_{51} = (f_{17}, f_{20})$, 2-D,
bbob-biobj f26, f27, f33, f34, 10-D
58 targets: 1..-1.0e-4
10 instances
Fraction of function,target pairs
log10(# f-evals / dimension)
best 2
COMO
NSGA-
SMS-E
DIST. BY ANDREWS MCMEEL SYNDICATION
GOCOMICS.COM/NONSEQUITUR
8-10
WILEY ©2016 WILEY INK, LTD.
WILEYINK@EARTHLINK.NET

Supplementary Material

Bibliography

[Andova et al. 2023] A. Andova, T. Benecke, H. Ludwig, T. Tušar. Towards Constructing a Suite of Multi-objective Optimization Problems with Diverse Landscapes. EvoApplications@EvoStar: 442–457, 2023

[Audet et al 2021] C. Audet, J. Bigeon, D. Cartier, S. Le Digabel, and L. Salomon. Performance indicators in multiobjective optimization. European Journal of Operational Research, 292(2), 2021

[Bartz-Beielstein et al. 2020] T. Bartz-Beielstein, C. Doerr, J. Bossek, S. Chandrasekaran, T. Eftimov, A. Fischbach, P. Kerschke, M. López-Ibáñez, K. M. Malan, J. H. Moore, B. Naujoks, P. Orzechowski, V. Volz, M. Wagner, T. Weise. Benchmarking in Optimization: Best Practice and Open Issues, ArXiv e-prints, arXiv:2007.03488v2, 2020

[Brockhoff et al. 2022] D. Brockhoff, A. Auger, N. Hansen and T. Tušar. Using Well-understood single-objective functions in multiobjective black-box optimization test suites, Evolutionary Computation 30(2):165–193, 2022

[Cheng et al. 2017a] R. Cheng, M. Li, Y. Tian, X. Zhang, S. Yang, Y. Jin, and X. Yao. A benchmark test suite for evolutionary many-objective optimization. Complex & Intelligent Systems 3:67–81, 2017

[Cheng et al. 2017b] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff. Test problems for large-scale Multiobjective and many-objective optimization, IEEE Transactions on Cybernetics, 47(12):4108-4121, 2017

[Cuate et al. 2020] O. Cuate, L. Uribe, A. Lara, O. Schütze. A benchmark for equality constrained multi-objective optimization. Swarm and Evolutionary Computation 52, 2020.

[Daniels et al. 2018] S. J. Daniels, A. A. M. Rahat, R. M. Everson, G. R. Tabor, and J. E. Fieldsend. A suite of computationally expensive shape optimisation problems using computational fluid dynamics. In Parallel Problem Solving from Nature (PPSN XV), volume 11102 of LNCS, pp. 296–307. Springer, 2018

Bibliography

[Deb 1999] K. Deb. Multi-objective genetic algorithms: Problem difficulties and construction of test problems, Evolutionary Computation 7(3):205–230, 1999

[Deb and Jain 2003] K. Deb, and S. Jain. Multi-speed gearbox design using multi-objective evolutionary algorithms. Journal of Mechanical Design, 125(3), 609–619, 2003

[Deb and Jain 2014] K. Deb and H. Jain. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints, IEEE Transactions on Evolutionary Computation 18(4):577–601, 2014

[Deb and Tiwari 2008] K. Deb and S. Tiwari. Omni-optimizer: A Procedure for single and multi-objective optimization. In Evolutionary Multi-Criterion Optimization (EMO 2005), volume 3410 of LNCS, pp. 47–61. Springer, 2005

[Deb et al. 2001] K. Deb, A. Pratap, and T. Meyarivan. Constrained test problems for multi-objective evolutionary optimization. In Evolutionary Multi-Criterion Optimization (EMO 2001), volume 1993 of LNCS, pp. 284–298. Springer, 2001

[Deb et al. 2005] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable test problems for evolutionary multiobjective optimization. In Evolutionary Multiobjective Optimization: Theoretical Advances and Applications, pp. 105–145. Springer, 2005

[Deb et al. 2006] K. Deb, A. Sinha and S. Kukkonen. Multi-objective test problems, linkages, and evolutionary methodologies. In Genetic and Evolutionary Computation Conference (GECCO 2006), pp. 1141–1148. ACM, 2006

[Emmerich and Deutz 2007] M. T. Emmerich and A. H. Deutz. Test problems based on Lamé superspheres. In Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pp. 922–936. Springer, 2007

Bibliography

[ESA 2022] ESA. Space competition challenge (SpOC): Trappist tour https://optimize.esa.int/challenge/spoc-trappist-tour/About, 2022

[ESA 2023] ESA. Space competition challenge 2 (SpOC 2): Quantum Communications Constellations https://optimize.esa.int/challenge/spoc-2-quantum-communications-constellations/About, 2023

[Fan et al. 2019] Z. Fan, W. Li, X. Cai, H. Huang, Y. Fang, Y. You, J. Mo, C. Wei, and E. Goodman. An improved epsilon constraint-handling method in MOEA/D for CMOPs with large infeasible regions, Soft Computing 23(23):12491–12510, 2019

[Fan et al. 2020] Z. Fan, W. Li, X. Cai, H. Li, C. Wei, Q. Zhang, K. Deb, and E. Goodman. Difficulty adjustable and scalable constrained multiobjective testproblem toolkit, Evolutionary Computation 28(3):339–378, 2020

[Fieldsend 2016] J. E. Fieldsend. Enabling dominance resistance in visualisable distance-based many-objective problems. In Genetic and Evolutionary Computation Conference Companion (GECCO 2016 Companion), pp. 1429–1436. ACM, 2016

[Fieldsend and Alyahya 2019] J. E. Fieldsend and K. AlYahya. Visualising the landscape of multi-objective problems using local optima networks. In Genetic and Evolutionary Computation Conference Companion (GECCO 2019 Companion), pp. 1421–1429. ACM, 2019

[Fieldsend et al. 2019] J. E. Fieldsend, T. Chugh, R. Allmendinger, and K. Miettinen A feature rich distance-based many-objective visualisable test problem generator. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2019), pp. 541–549. ACM, 2019

[Filipič and Tušar 2018] B. Filipič and T. Tušar. A Taxonomy of Methods for Visualizing Pareto Front Approximations. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2018), pp. 649–656. ACM, 2018

[Filipič and Tušar 2020] B. Filipič and T. Tušar. Visualization in multiobjective optimization. In Genetic and Evolutionary Computation Conference Companion (GECCO 2018 Companion), pp. 775–800. ACM, 2020

Bibliography

[Finck et al. 2009] S. Finck, N. Hansen, R. Ros and A. Auger.Real-parameter black-box optimization benchmarking 2010: Presentation of the noiseless functions. Tech. Rep. 2009/20, Research Center PP, 2009 (version from February 2019)

[Fonseca 1995] C. M. Fonseca. Multiobjective Genetic Algorithms with Application to Control Engineering Problems. Ph.D. thesis, University of Sheffield, 1995

[Fonseca and Fleming 1995] C. M. Fonseca and P. J. Fleming. An overview of evolutionary algorithms in multiobjective optimization. Evolutionary Computation 3:1–16, 1995

[Fonseca and Fleming 1996] C. M. Fonseca and P. J. Fleming. On the performance assessment and comparison of stochastic multiobjective optimizers. In Parallel Problem Solving from Nature (PPSN 1996), pp. 584-593. Springer, 1996

[Fonseca et al. 2011] C. M. Fonseca, A. P. Guerreiro, M. López-Ibáñez, and L. Paquete. On the computation of the empirical attainment function. In Evolutionary Multi-Criterion Optimization (EMO 2011), volume 6576 of LNCS, pp. 106–120. Springer, 2011

[He et al. 2020] C. He, Y. Tian, H. Wang, Y. Jin. A repository of real-world datasets for data-driven evolutionary multiobjective optimization. Complex I& Intelligent Systems, 6:189–197, 2020

[He et al. 2021] C. He, R. Cheng, Y. Tian, X. Zhang, K. C. Tan and Y. Jin. Paired Offspring Generation for Constrained Large-Scale Multiobjective Optimization. IEEE Transactions on Evolutionary Computation 25(3):448–462, 2021

[Hansen et al. 2021] N. Hansen, A. Auger, R. Ros, O. Mersmann, T. Tušar and D. Brockhoff. COCO: A platform for comparing continuous optimizers in a black-box setting. Optimization Methods and Software, 36(1), pp.114–144. 2021

[Hansen and Jaszkiewicz 1998] M. P. Hansen and A. Jaszkiewicz. Evaluating The Quality of Approximations of the Non-Dominated Set. Technical report, Institute of Mathematical Modeling, Technical University of Denmark, 1998. IMM Technical Report IMM-REP-1998-7

Bibliography

[Huang et al. 2007] V. L. Huang, A. K. Qin, K. Deb, E. Zitzler, P. N. Suganthan, J. J .Liang, M. Preuss, and S. Huband. Problem definitions for performance assessment on multi-objective optimization algorithms, Technical Report, 2007

[Huband et al. 2006] S. Huband, P. Hingston, L. Barone, and L. While. A review of multiobjective test problems and a scalable test problem toolkit, IEEE Transactions on Evolutionary Computation 10(5):477–506, 2006

[Hughes 2007] E. J. Hughes, Radar waveform optimisation as a many-objective application benchmark. In Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pp. 700–714. Springer, 2007

[Igel et al. 2007] C. Igel, N. Hansen, and S. Roth. Covariance matrix adaptation for multi-objective optimization. Evolutionary Computation, 15(1):1–28, 2007

[Ishibuchi et al. 2010] H. Ishibuchi, Y. Hitotsuyanagi, N. Tsukamoto, and Y. Nojima. Many-objective test problems to visually examine the behavior of multiobjective evolution in a decision space. In Parallel Problem Solving from Nature (PPSN XI), volume 6239 of LNCS, part II, pp. 91-100, Springer, 2010

[Ishibuchi et al. 2015] H. Ishibuchi, H. Masuda, Y. Tanigaki, Y. Nojima. Modified distance calculation in generational distance and inverted generational distance. In Evolutionary Multi-Criterion Optimization (EMO 2015), pp. 110–125. Springer, 2015

[Ishibuchi et al. 2017] H. Ishibuchi, Y. Setoguchi, H. Masuda, and Y. Nojima. Performance of decomposition-based many-objective algorithms strongly depends on Pareto front shapes. IEEE Transactions on Evolutionary Computation 21(2):169–190, 2017

[Ishibuchi et al. 2023] H. Ishibuchi, Y. Nan, and L. M. Pang. Performance evaluation of multi-objective evolutionary algorithms using artificial and real-world problems. In Evolutionary Multi-Criterion Optimization (EMO 2023), volume 13970 of LNCS, pp. 333–347. Springer, 2023.

Bibliography

[Jain and Deb 2014] H. Jain and K. Deb. An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, part II: Handling constraints and extending to an adaptive approach, IEEE Transactions on Evolutionary Computation 18(4):602–622, 2014

[JSEC and JAXA 2018] JSEC and JAXA. Second evolutionary computation competition: Lunar lander landing site selection problem, http://www.jpnsec.org/files/competition2018/EC-Symposium-2018-Competition-English.html, 2018.

[JSEC 2019] JSEC. Third evolutionary computation competition: Wind turbine design optimization problem, http://www.jpnsec.org/files/competition2019/EC-Symposium-2019-Competition-English.html, 2019.

[Kerschke and Grimme 2017] P. Kerschke and C. Grimme. An expedition to multi-modal multi-objective optimization landscapes. In Evolutionary Multi-Criterion Optimization (EMO 2017), volume 10173 of LNCS, pp. 329–343. Springer, 2017

[Kerschke et al. 2016] P. Kerschke, H. Wang, M. Preuss, C. Grimme, A. H. Deutz, H. Trautmann, and M. Emmerich. Towards analyzing multimodality of continuous multiobjective landscapes. In International Conference on Parallel Problem Solving from Nature (PPSN XIV), volume 9921 of LNCS, pp. 962–972. Springer, 2016

[Kohira et al. 2018] T. Kohira, H. Kemmotsu, A. Oyama, T. Tatsukawa. Proposal of benchmark problem based on real-world car structure design optimization. In Genetic and Evolutionary Computation Conference Companion (GECCO 2018 Companion), pp. 183–184. ACM, 2018

[Köppen et al. 2005] M. Köppen, R. Vicente-Garcia, and B. Nickolay. Fuzzy-Pareto-dominance and its application in evolutionary multi-objective optimization. In Evolutionary Multi-Criterion Optimization (EMO 2005), volume 3410 of LNCS, pp. 399–412. Springer, 2005

Bibliography

[Kumar et al. 2021] A. Kumar, G. Wu, M. Z. Ali, Q. Luo, R. Mallipeddi, P. N. Suganthan, S. Das. A benchmark suite of real-world constrained multi-objective optimization problems and some baseline results. Swarm and Evolutionary Computation 67, 2021.

[Kursawe 1991] F. Kursawe. A variant of evolution strategies for vector optimization. In International Conference on Parallel Problem Solving from Nature (PPSN I), volume 496 of LNCS, pp. 193–197, 1991

[Li and Yao 2019] M. Li and X. Yao. Quality evaluation of solution sets in multiobjective optimisation: A survey. ACM Computing Surveys (CSUR), 52(2):1-38. 2019

[Li and Zhang 2009] H. Li and Q. Zhang. Multiobjective optimization problems with complicated Pareto sets, OEA/D and NSGA-II, IEEE Transactions on Evolutionary Computation 13(2):284–302, 2009

[Li et al. 2014] M. Li, S. Yang, and X. Liu. A test problem for visual investigation of high-dimensional multi-objective search. In Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2014), pp. 2140–2147. IEEE, 2014

[Li et al. 2016] J. P. Li, Y. Wang, S. Yang, and Z. Cai. A comparative study of constraint-handling techniques in evolutionary constrained multiobjective optimization. In IEEE Congress on Evolutionary Computation (CEC 2016), pp. 4175–4182. IEEE, 2016

[Li et al. 2018] M. Li, C. Grosan, S. Yang, X. Liu, and X. Yao. 2018. Multiline distance minimization: A visualized many-objective test problem suite. IEEE Transactions on Evolutionary Computation 22(1):61–78, 2018

[Liang et al. 2019] J. J. Liang, B. Y. Qu, D. W. Gong, and C. T. Yue. Problem definitions and evaluation criteria for the CEC 2019 special session on multimodal multiobjective optimization, Technical Report, 2019

[Liang et al. 2020] J. J. Liang, P. N. Suganthan, B. Y. Qu, D. W. Gong, and C. T. Yue. Problem definitions and evaluation criteria for the CEC 2020 special session on multimodal multiobjective optimization, Technical Report 201912, Zhengzhou University, China, 2020

Bibliography

[Liefooghe et al. 2018] A. Liefooghe, B. Derbel, S. Vérel, M. López-Ibáñez, H. E. Aguirre, and K. Tanaka. On Pareto local optimal solutions networks. In International Conference on Parallel Problem Solving from Nature (PPSN XV), volume 11102 of LNCS, pp. 232-244. Springer, 2018

[Liu and Wang 2019] Z. -Z. Liu and Y. Wang. Handling Constrained Multiobjective Optimization Problems With Constraints in Both the Decision and Objective Spaces. IEEE Transactions on Evolutionary Computation 23(5):870–884, 2019.

[Liu et al. 2018] Y. Liu, H. Ishibuchi, Y. Nojima, N. Masuyama, and K. Shang. A Double niched evolutionary algorithm and its behavior on polygon-based problems. In International Conference on Parallel Problem Solving from Nature (PPSN XV), volume 11101 of LNCS, pp. 262–273. Springer, 2018

[López-Ibáñez et al. 2010] M. López-Ibáñez, L. Paquete, and T. Stützle. Exploratory analysis of stochastic local search algorithms in biobjective optimization. In Experimental Methods for the Analysis of Optimization Algorithms, pp. 209–222. Springer, 2010

[Ma and Wang 2019] Z. Ma and Y. Wang. Evolutionary constrained multiobjective optimization: Test suite construction and performance comparisons, IEEE Transactions on Evolutionary Computation 23(6):972–986, 2019

[Mai et al. 2023] S. Mai, T. Benecke, and S. Mostaghim. MACO: A real-world inspired benchmark for multi-objective evolutionary algorithms. In Evolutionary Multi-Criterion Optimization (EMO 2023), volume 13970 of LNCS, pp. 305–318. Springer, 2023.

[Masuda et al. 2014] H. Masuda, Y. Nojima, and H. Ishibuchi. Visual examination of the behavior of EMO algorithms for many-objective optimization with many decision variables. In IEEE Congress on Evolutionary Computation (CEC 2014), pp. 2633–2640. IEEE, 2014

[Masuda et al. 2016] H. Masuda, Y. Nojima, and H. Ishibuchi. Common properties of scalable multiobjective problems and a new framework of test problems. In Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2016), pp. 3011-3018. IEEE, 2016

[McClymont and Keedwell 2011] K. McClymont and E. Keedwell. Benchmark multi-objective optimisation test problems with mixed encodings. In Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2011), pp. 2131–2138. IEEE, 2011

Bibliography

[Meneghini et al. 2020] I. R. Meneghini, M. A. Alves, A. Gaspar-Cunha, and F. G. Guimaraes. Scalable and customizable benchmark problems for many-objective optimization. Applied Soft Computing, 90:106139, 2020

[Moré and Wild 2009] J.J. Moré and S. M. Wild. Benchmarking derivative-free optimization algorithms. SIAM Journal on Optimization, 20(1), 172-191. 2009

[Musselman and Talavage 1980] K. Musselman and J. Talavage. A trade-off cut approach to multiple objective optimization. Operations Research 28(6):1424–1435, 1980

[Okabe et al. 2004] T. Okabe, Y. Jin, M. Olhofer, and B. Sendhoff. On test functions for evolutionary multi-objective optimization. In Parallel Problem Solving From Nature (PPSN VIII), volume 3242 of LNCS, pp. 792–802, 2004

[Picard and Schiffmann 2021] C. Picard and J. Schiffmann. Realistic constrained multi-objective optimization benchmark problems from design. IEEE Transactions on Evolutionary Computation 25(2):234–246, 2021

[Poloni et al. 1996] C. Poloni, G. Mosetti, and S. Contessi. Multi objective optimization by GAs: Application to system and component design. In Proceedings of Computational Methods in Applied Sciences ’96, pp. 258–264, 1996

[Ray et al. 2001] T. Ray, K. Tai and K. C. Seow. Multiobjective design optimization by an evolutionary algorithm. Engineering Optimization 33:399–424, 2001

[Ray and Liew 2002] T. Ray and K. M. Liew. A swarm metaphor for multiobjective design optimization. Engineering Optimization 34(2):141–153, 2002

[Rao 1987] S. S. Rao. Game theory approach for multiobjective structural optimization. Computers and Structures 26(1):119–127, 1987

[Reed et al. 2013] P. M. Reed, D. Hadka, J. D. Herman, J. R. Kasprzyk, and J. B. Kollat. Evolutionary multiobjective optimization in water resources: The past, present, and future. Advances in Water Resources, 51:438-456, 2013

Bibliography

[Rudolph et al. 2007] G. Rudolph, B. Naujoks, and M. Preuss. Capabilities of EMOA to detect and preserve equivalent Pareto subsets, In Evolutionary Multi-Criterion Optimization (EMO 2007), volume 4403 of LNCS, pp. 36–50. Springer, 2007

[Sadowski et al. 2021] K. L. Sadowski, D. Thierens, and P. A. N. Bosman. Optimization of multi-objective mixed-integer problems with a model-based evolutionary algorithm in a black-box setting. In Genetic and Evolutionary Computation Conference Companion (GECCO 2021 Companion), pp. 227–228. ACM, 2021

[Saxena et al. 2011] D. K. Saxena, Q. Zhang, J. Ao, A. Duro, and A. Tiwari. Framework for many-objective test problems with both simple and complicated Pareto set shapes. In Evolutionary Multi-Criterion Optimization (EMO 2011), volume 6576 of LNCS, pp. 197–211. Springer, 2011

[Schäpermeier et al. 2020] L. Schäpermeier, C. Grimme, P. Kerschke. One PLOT to show them all: Visualization of efficient sets in multi-objective landscapes. In Parallel Problem Solving From Nature (PPSN XVI), volume 12270 of LNCS, pp. 154–167. Springer, 2020

[Schaffer 1985] J. D. Schaffer. Multiple objective optimization with vector evaluated genetic algorithms. In John J. Grefenstette, editor, Conference on Genetic Algorithms and Their Applications, pp. 93–100, 1985

[Shir et al. 2012] O. M. Shir, J. Roslund, Z. Leghtas, and H. Rabitz. Quantum control experiments as a testbed for evolutionary multi-objective algorithms. Genetic Programming and Evolvable Machines 13:445–491, 2012

[Tanabe and Ishibuchi 2020] R. Tanabe and H. Ishibuchi. An easy-to-use real-world multi-objective optimization problem suite. Applied Soft Computing 89:106078, 2020

Bibliography

[Tušar et al. 2019] T. Tušar, D. Brockhoff, and N. Hansen. Benchmark problems for single- and bi-objective optimization. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2019), pp. 718-726. ACM, 2019

[van der Blom et al. 2020] K. van der Blom, T. M. Deist, V. Volz, M. Marchi, Y. Nojima, B. Naujoks, A. Oyama, and T. Tušar. Identifying properties of real-world optimisation problems through a questionnaire. CoRR abs/2004.06395, 2020

[Van Veldhuizen 1999] D. A. Van Veldhuizen. Multiobjective evolutionary algorithms: Classifications, analyses, and new innovations, Ph.D. dissertation, Air Force Institute of Technology, Ohio, USA, 1999

[Viennet et al. 1996] R. Viennet, C. Fonteix, and I. Marc. Multicriteria optimization using a genetic algorithm for determining a Pareto set, International Journal of Systems Science, 27(2):255-260, 1996

[Vodopija et al. 2022] A. Vodopija, T. Tušar, and B. Filipič. Characterization of constrained continuous multiobjective optimization problems: A feature space perspective. Information Sciences 607: 244–262, 2022

[Volz et al. 2019] V. Volz, B. Naujoks, P. Kerschke, and T. Tušar. Single- and multi-objective game-benchmark for evolutionary algorithms. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2019), pp. 647-655. ACM, 2019

[Yu et al. 2011] G. Yu, T. Chai, and X. Luo. Multiobjective production planning optimization using hybrid evolutionary algorithms for mineral processing. IEEE Transactions on Evolutionary Computation 15(4):487–514, 2011.

[Yue et al. 2019] C. Yue, B. Qu, K. Yu, J. Liang, and X. Li. A novel scalable test problem suite for multimodal multiobjective optimization, Swarm and Evolutionary Computation, 48:62–71, 2019

[Weise and Mostaghim 2022] J. Weise and S. Mostaghim. A Scalable Many-Objective Pathfinding Benchmark Suite. IEEE Transactions on Evolutionary Computation 26(1): 188–194, 2022

Bibliography

[Zapotecas et al. 2019] S. Zapotecas-Martínez, C. A. Coello Coello, H. E Aguirre, and K. Tanaka. A review of features and limitations of existing scalable multi-objective test suites. IEEE Transactions on Evolutionary Computation 23(1): 130–142, 2019

[Zapotecas et al. 2023] S. Zapotecas-Martínez, C. A. Coello Coello, H. E Aguirre, and K. Tanaka. Challenging test problems for multi- and many-objective optimization. Swarm and Evolutionary Computation 81: 101350, 2023

[Zhang et al. 2009] Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari. Multiobjective optimization test instances for the CEC 2009 special session and competition. Technical Report CES-487, University of Essex, UK, 2009

[Zitzler et al. 2000] E. Zitzler, K. Deb, and L. Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. Evolutionary Computation, 8(2):173–195, 2000

[Zitzler et al. 2003] E. Zitzler, L. Thiele, M. Laumanns, C.M. Fonseca, and V. Grunert Da Fonseca. Performance assessment of multiobjective optimizers: An analysis and review. IEEE Transactions on evolutionary computation, 7(2):117-132. 2003

[Zitzler et al. 2007] E. Zitzler, D. Brockhoff, L. Thiele. The Hypervolume Indicator Revisited: On the Design of Pareto-compliant Indicators Via Weighted Integration. In Evolutionary Multi-Criterion Optimization (EMO 2007). LNCS, vol. 4403, pp. 862–876. Springer, Heidelberg, 2007

[Zitzler and Thiele 1989] E. Zitzler and L. Thiele. Multiobjective Optimization Using Evolutionary Algorithms - A Comparative case study. In PPSN-V, pages 292–301, Amsterdam, Sept. 1998

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

RandOpt team

Inria Saclay - Ile-de-France

CMAP, CNRS, Ecole Polytechnique, IP Paris

Route de Saclay

91128 Palaiseau

France

After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. After postdoctoral research positions at Inria Saclay Ile-de-France in Orsay and at Ecole Polytechnique in Palaiseau, both in France, Dimo has been a permanent researcher at Inria: from 2011 till 2016 with the Inria Lille - Nord Europe research center and since October 2016 with the Saclay - Ile-de-France research center, co-located with CMAP, Ecole Polytechnique, IP Paris. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

Instructor Biography: Tea Tušar

Tea Tušar

Computational Intelligence Group

Department of Intelligent Systems

Jožef Stefan Institute

Jamova cesta 29

1000 Ljubljana

Slovenia

Tea Tušar is a senior research associate at the Department of Intelligent Systems of the Jožef Stefan Institute, and an assistant professor at the Jožef Stefan International Postgraduate School, both in Ljubljana, Slovenia. After receiving the PhD degree in Information and Communication Technologies from the Jožef Stefan International Postgraduate School for her work on visualizing solution sets in multiobjective optimization, she has completed a one-year postdoctoral fellowship at Inria Lille in France where she worked on benchmarking multiobjective optimizers. Her research interests include evolutionary algorithms for singleobjective and multiobjective optimization with emphasis on visualizing and benchmarking their results and applying them to real-world problems.