GECCO 2016
Tutorial on
Evolutionary Multiobjective Optimization

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updated slides will be available at
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INVENTORS FOR THE DIGITAL WORLD

ISF
technische universität
dortmund

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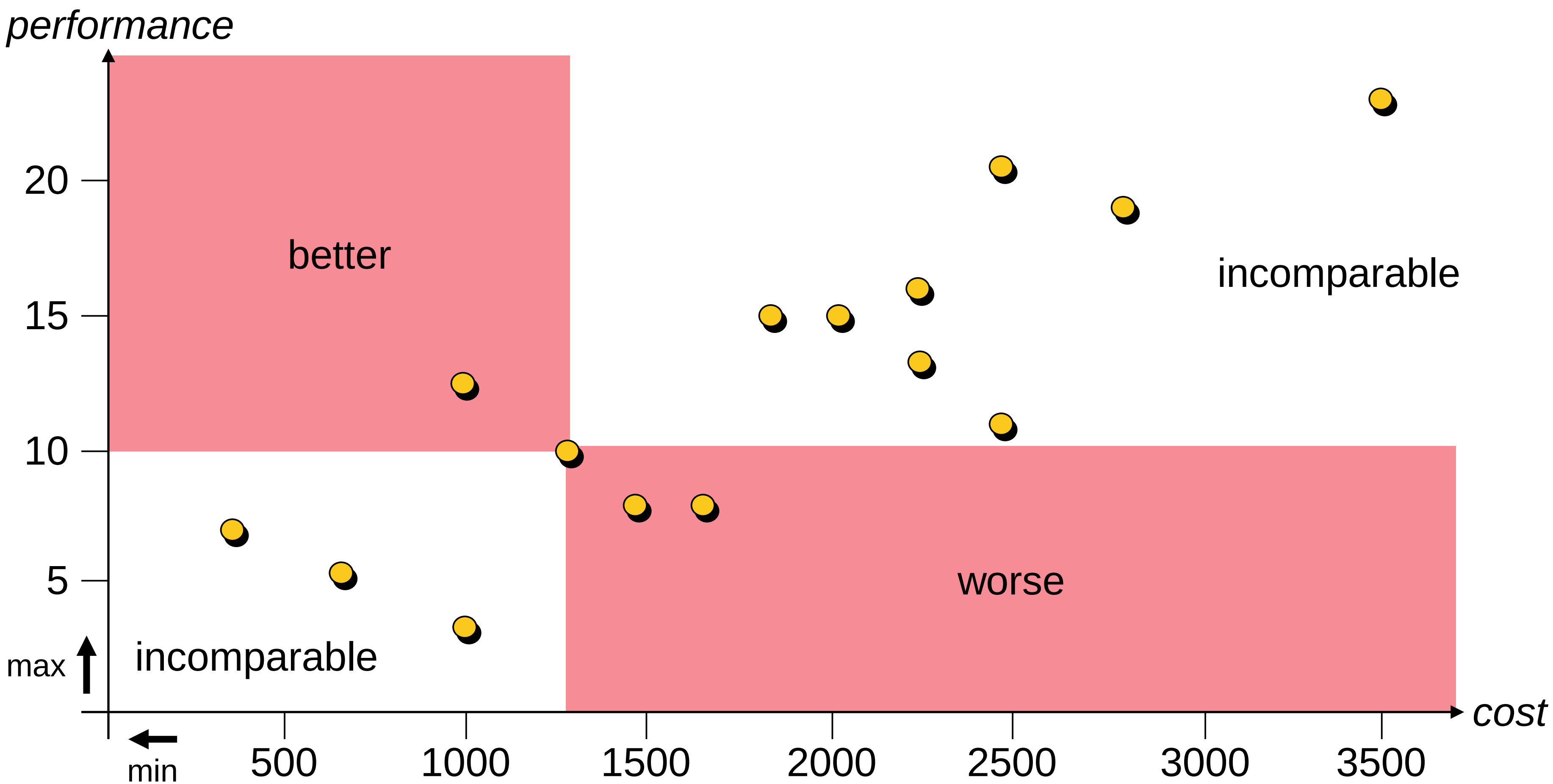
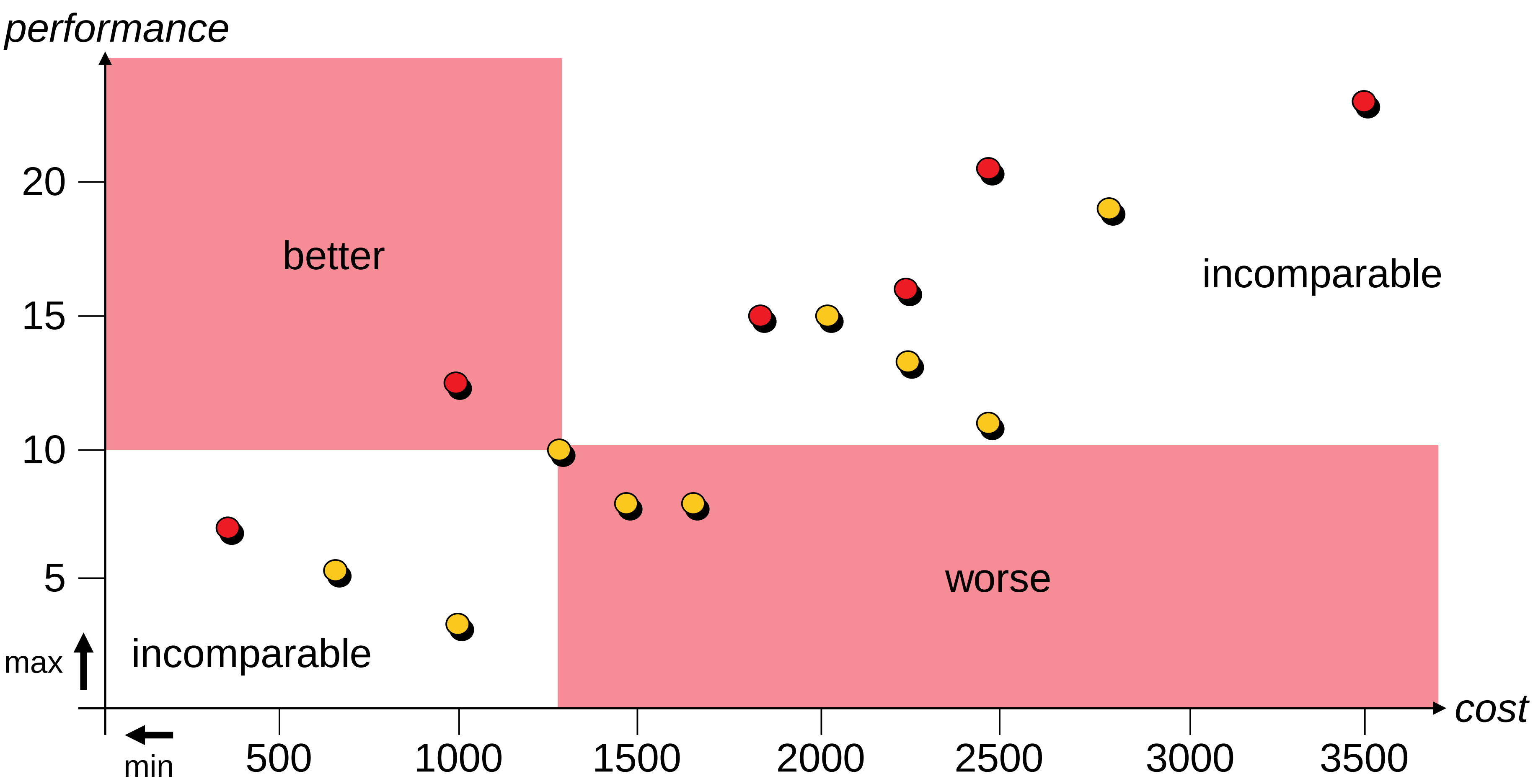
Multiobjective Optimization

Multiple objectives that have to be optimized simultaneously

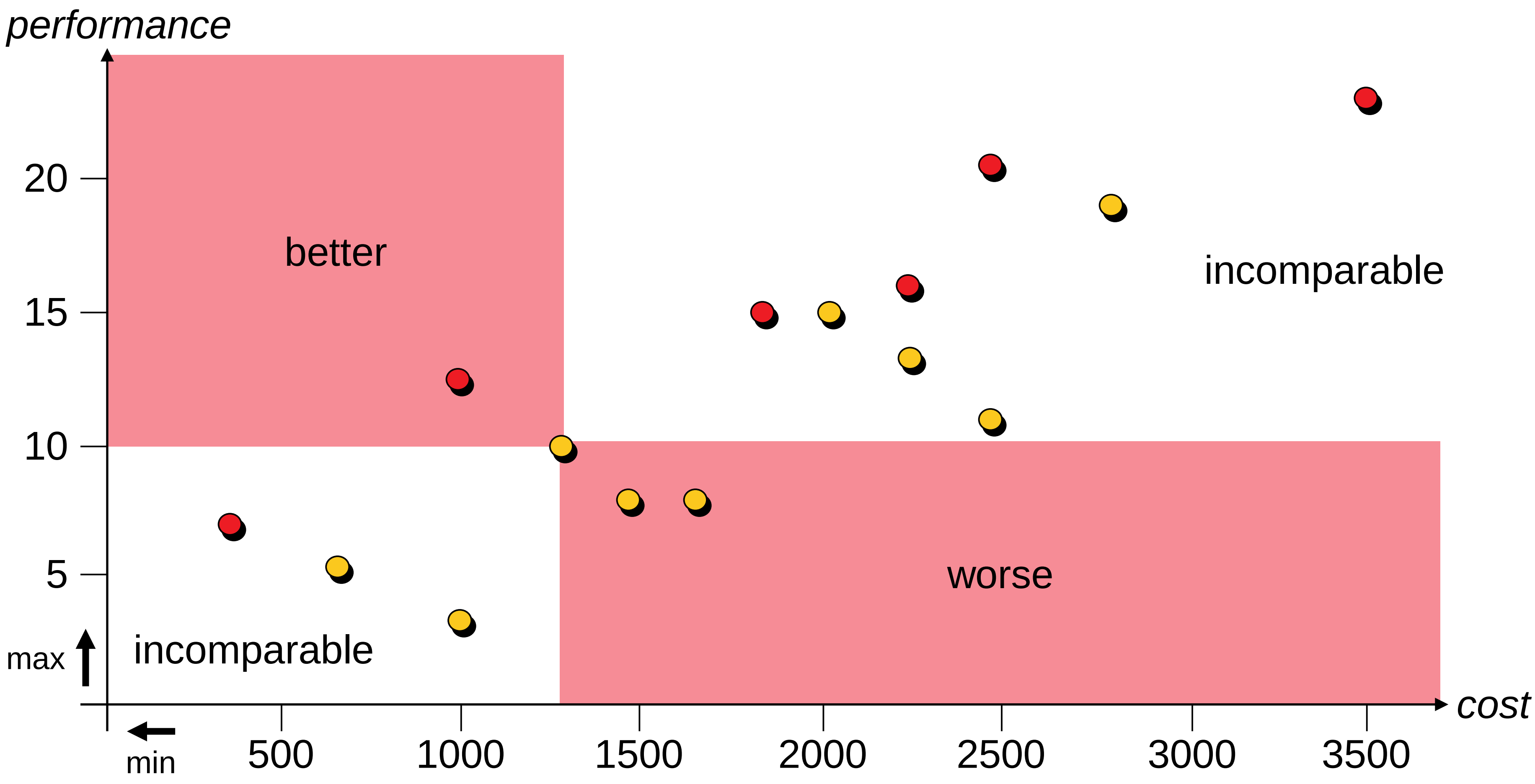


Observations: ❶ there is no single optimal solution, but
❷ some solutions (red) are better than others (yellow)



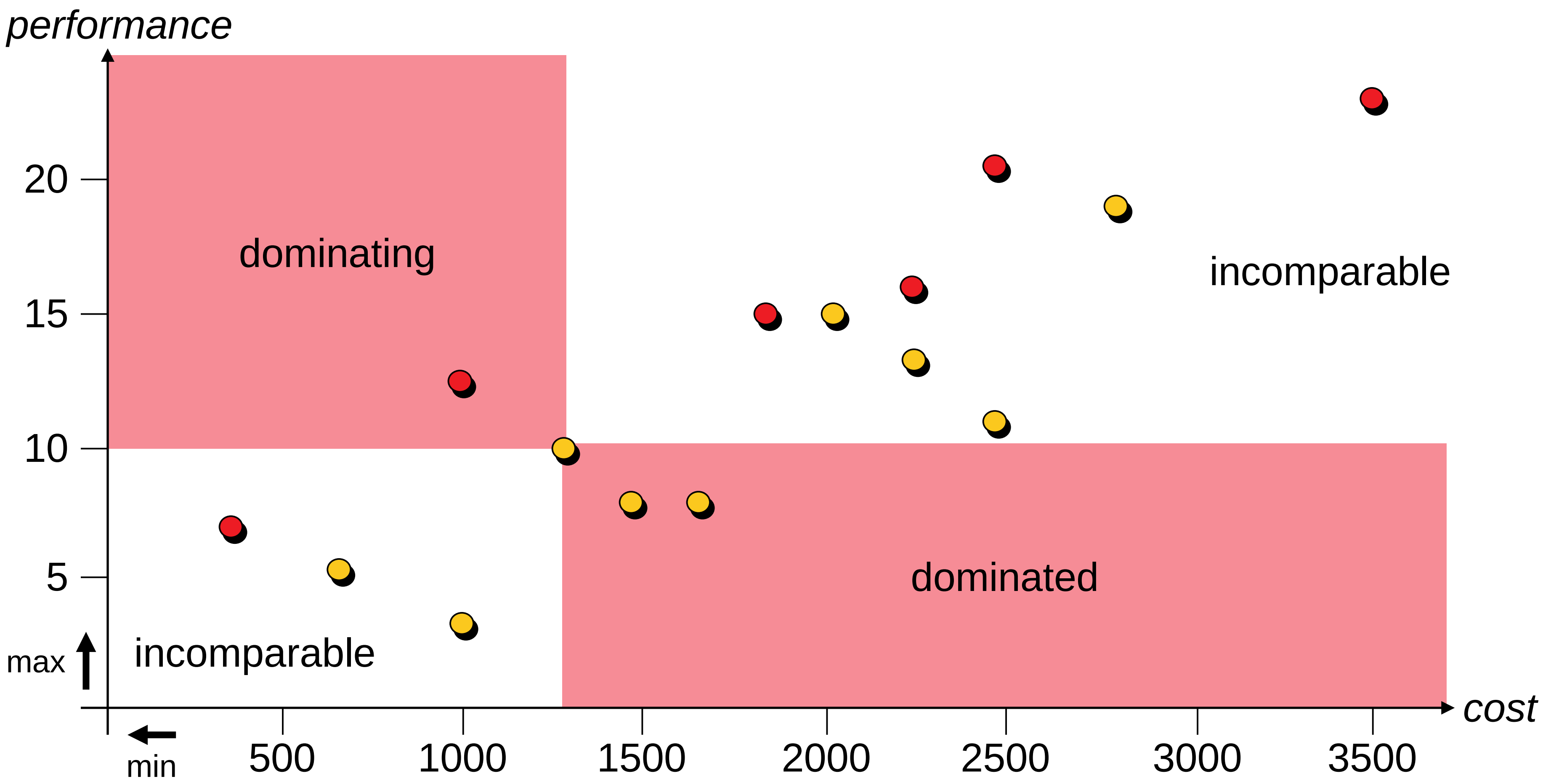
$$u \text{ weakly Pareto dominates } v \; (u \leqslant_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

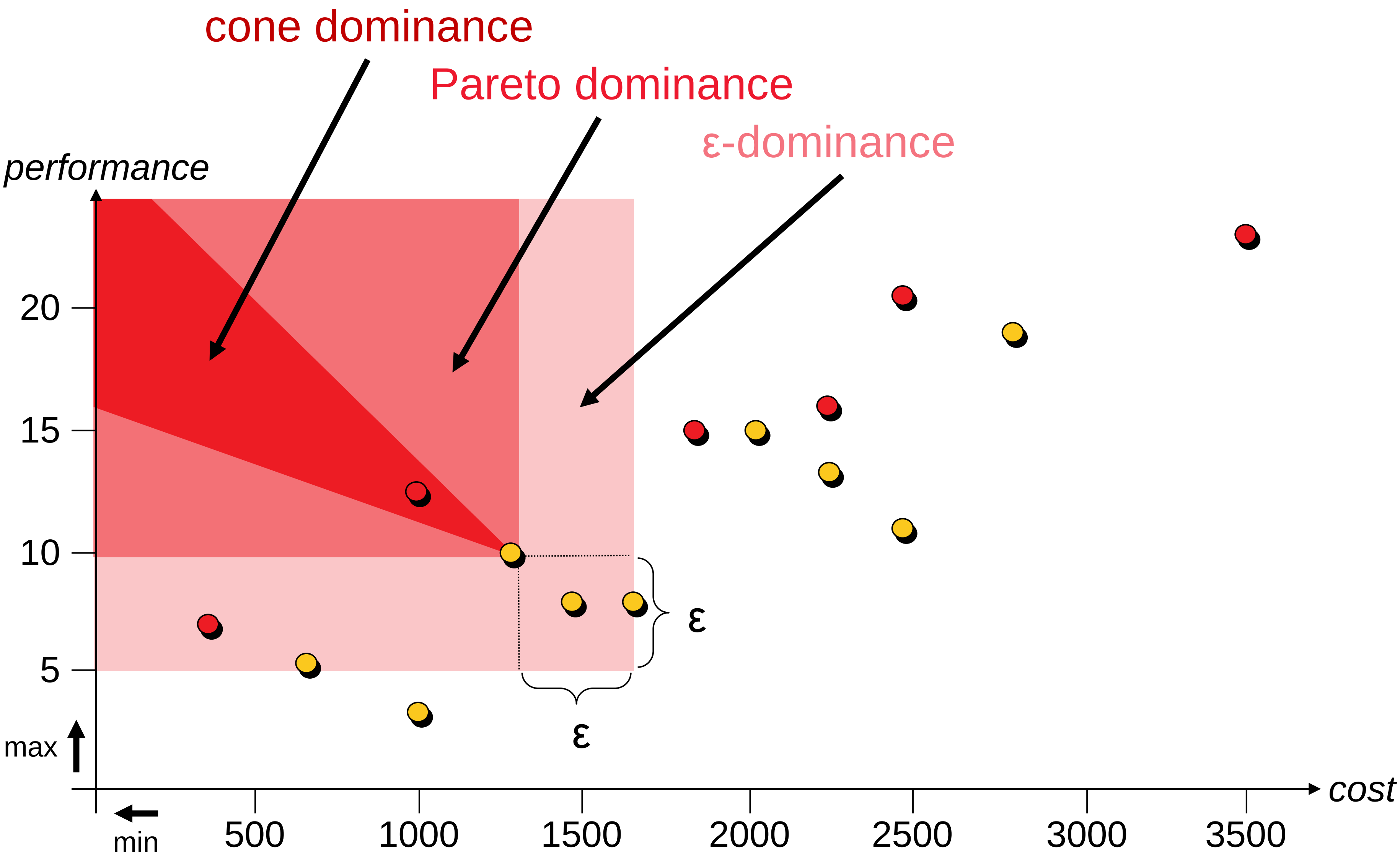
$$u \text{ Pareto dominates } v \; (u <_{par} v): \quad u \leqslant_{par} v \; \wedge \; v \nleqslant_{par} u$$



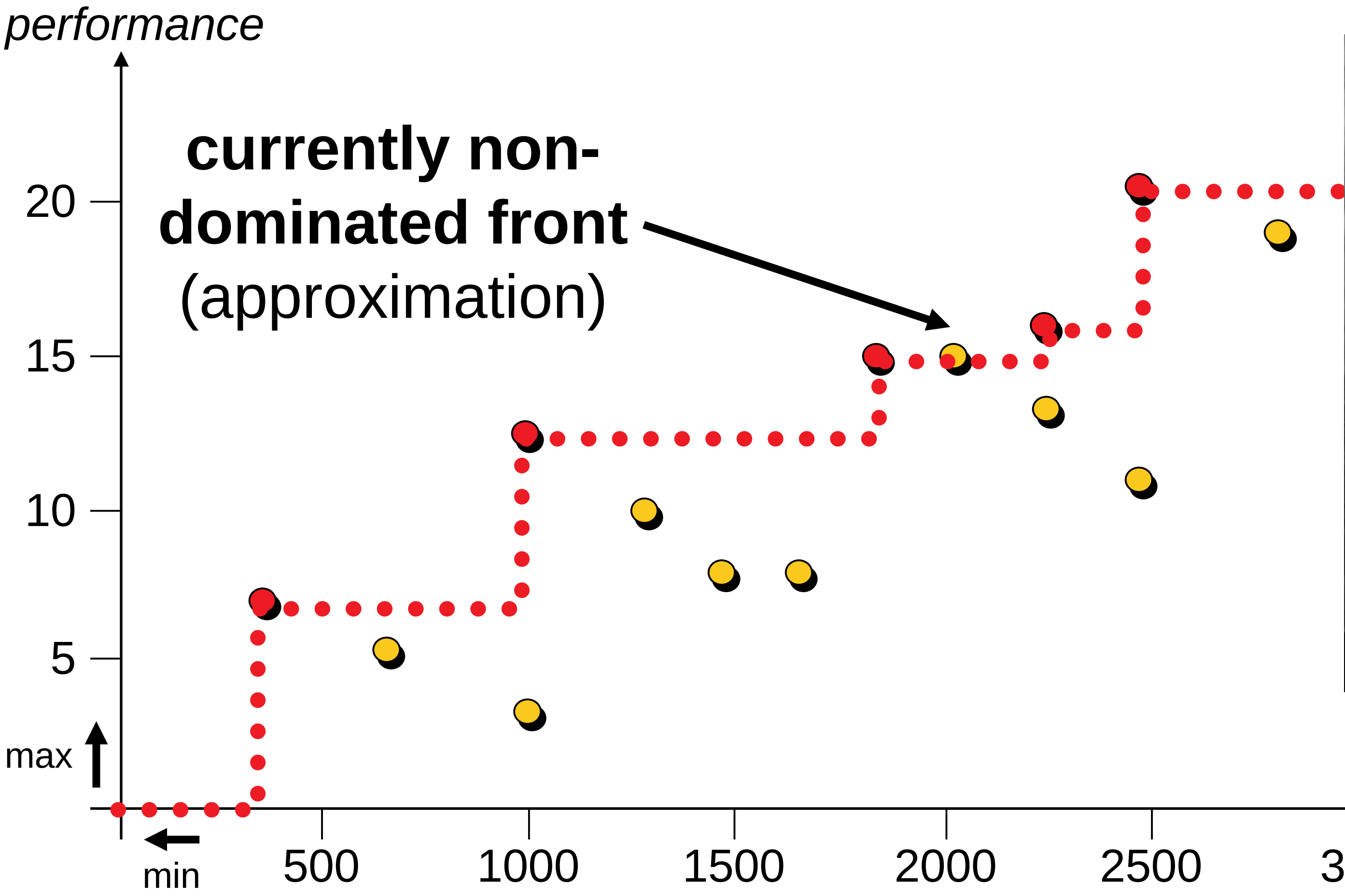
$$u \text{ weakly Pareto dominates } v \ (u \leqslant_{par} v): \quad \forall 1 \leq i \leq k : f_i(u) \leq f_i(v)$$

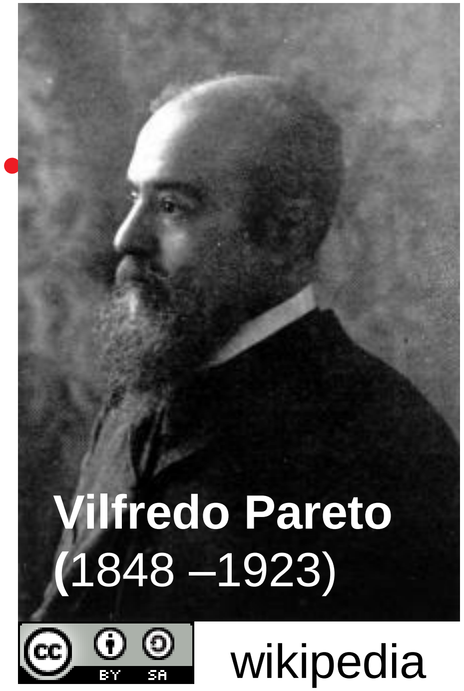
$$u \text{ Pareto dominates } v \ (u <_{par} v): \quad u \leqslant_{par} v \ \wedge \ v \not\leqslant_{par} u$$



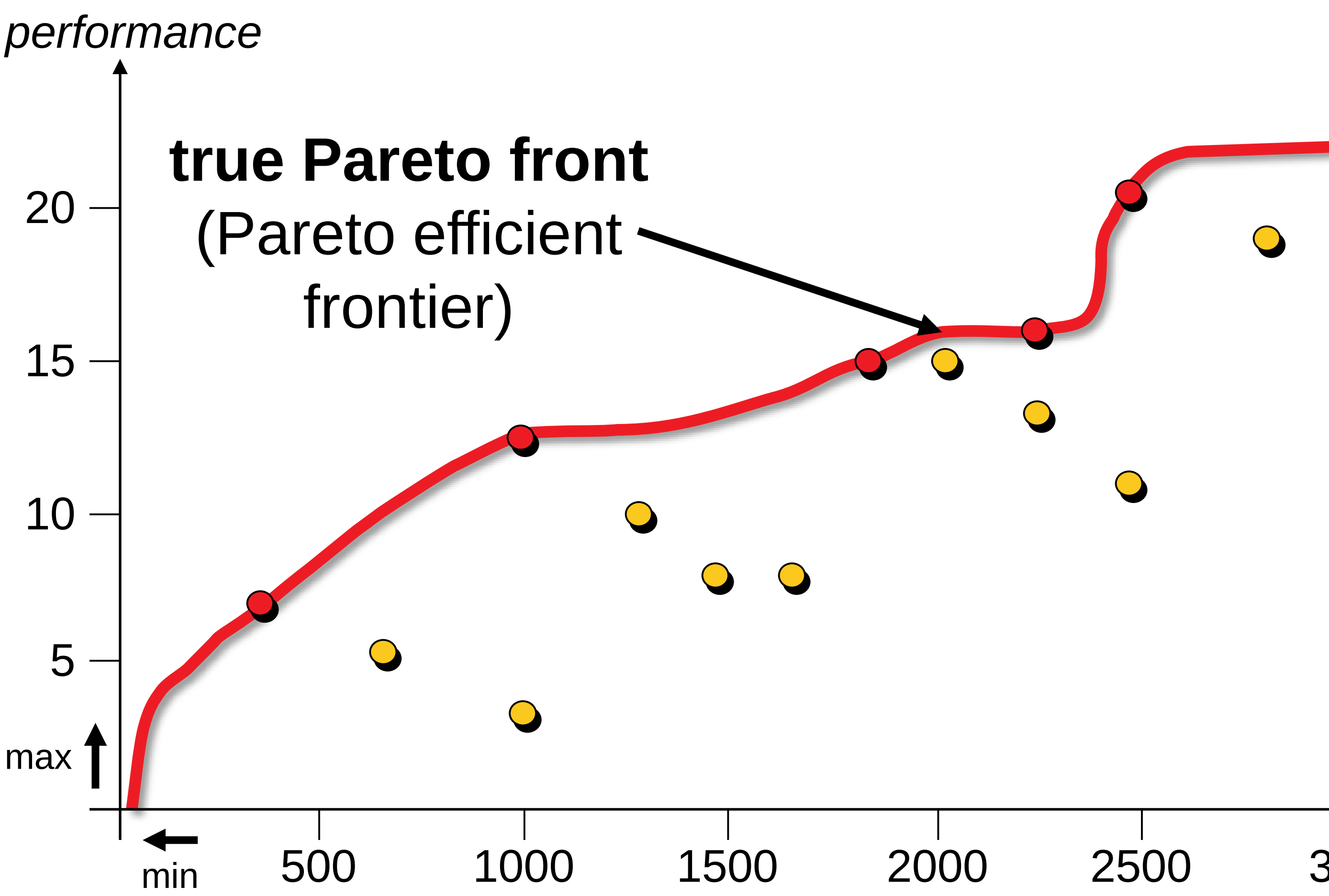

cone dominance
Pareto dominance
ε-dominance
performance
20
15
10
5
max
min
500
1000
1500
2000
2500
3000
3500
cost
ε
ε

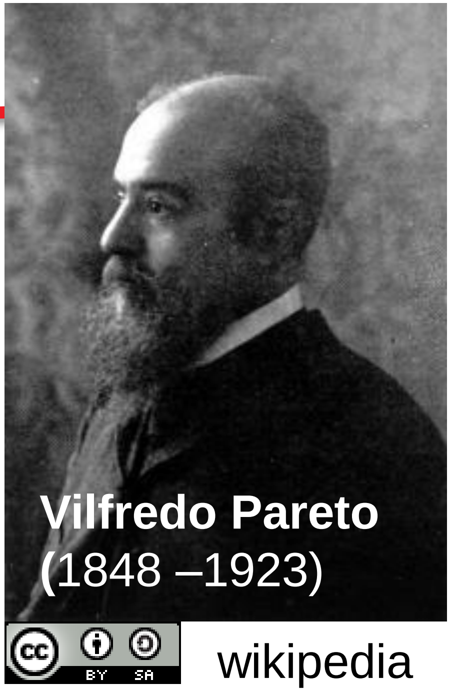
Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space





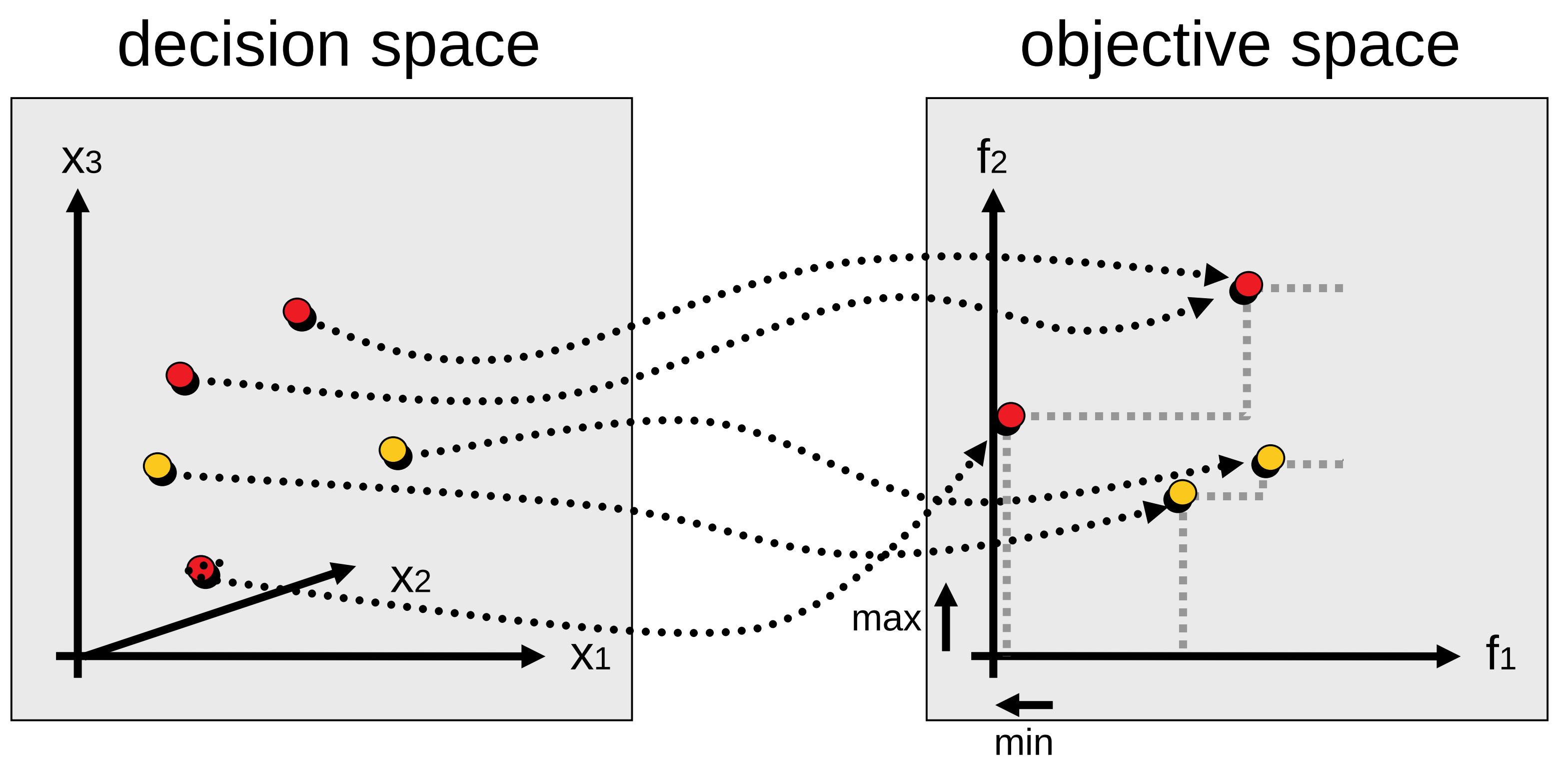
Pareto set: set of all non-dominated solutions (decision space)
Pareto front: its image in the objective space





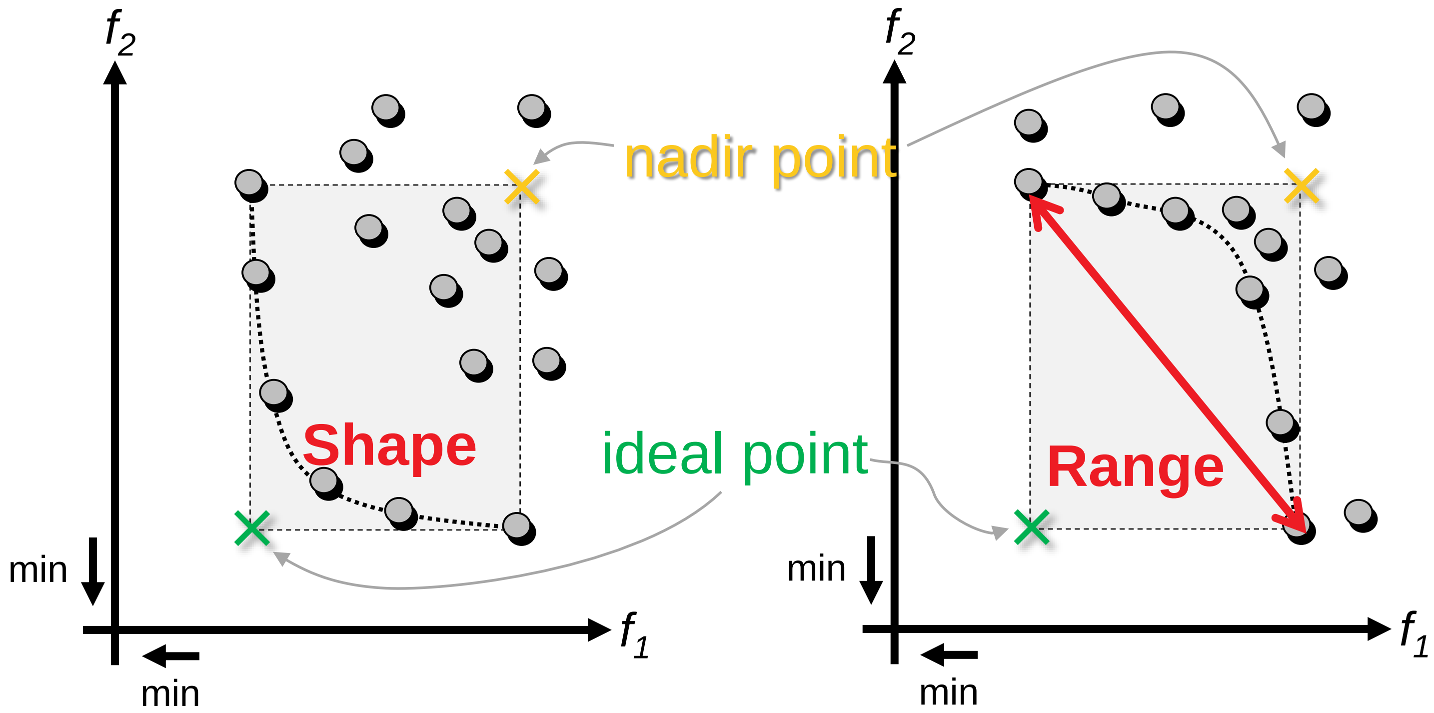
wikipedia

A Brief Introduction to Multiobjective Optimization



solution of Pareto-optimal set ● vector of Pareto-optimal front

non-optimal decision vector ● non-optimal objective vector



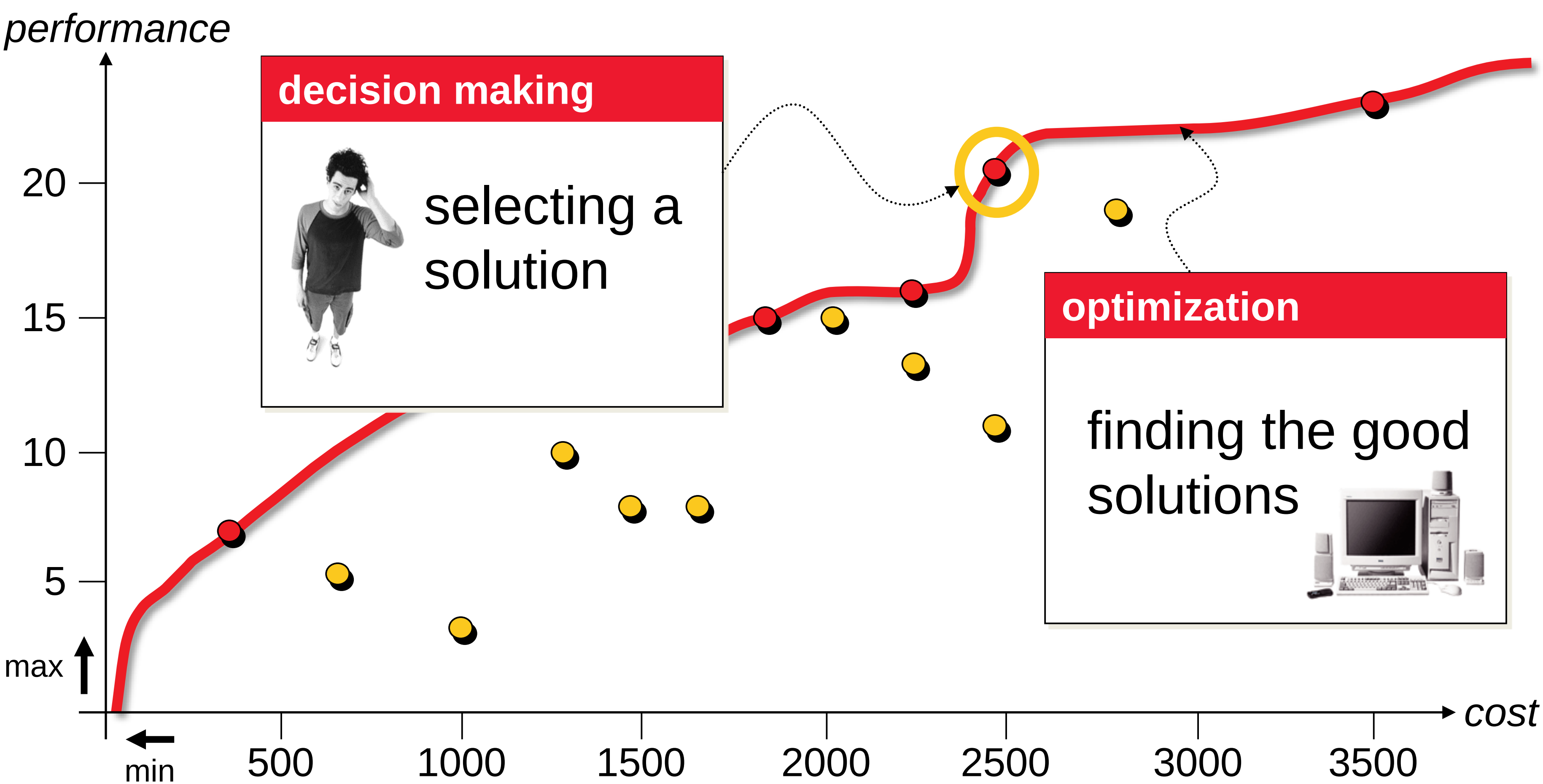
ideal point: best values
nadir point: worst values

obtained for *Pareto-optimal* points

Optimization vs. Decision Making

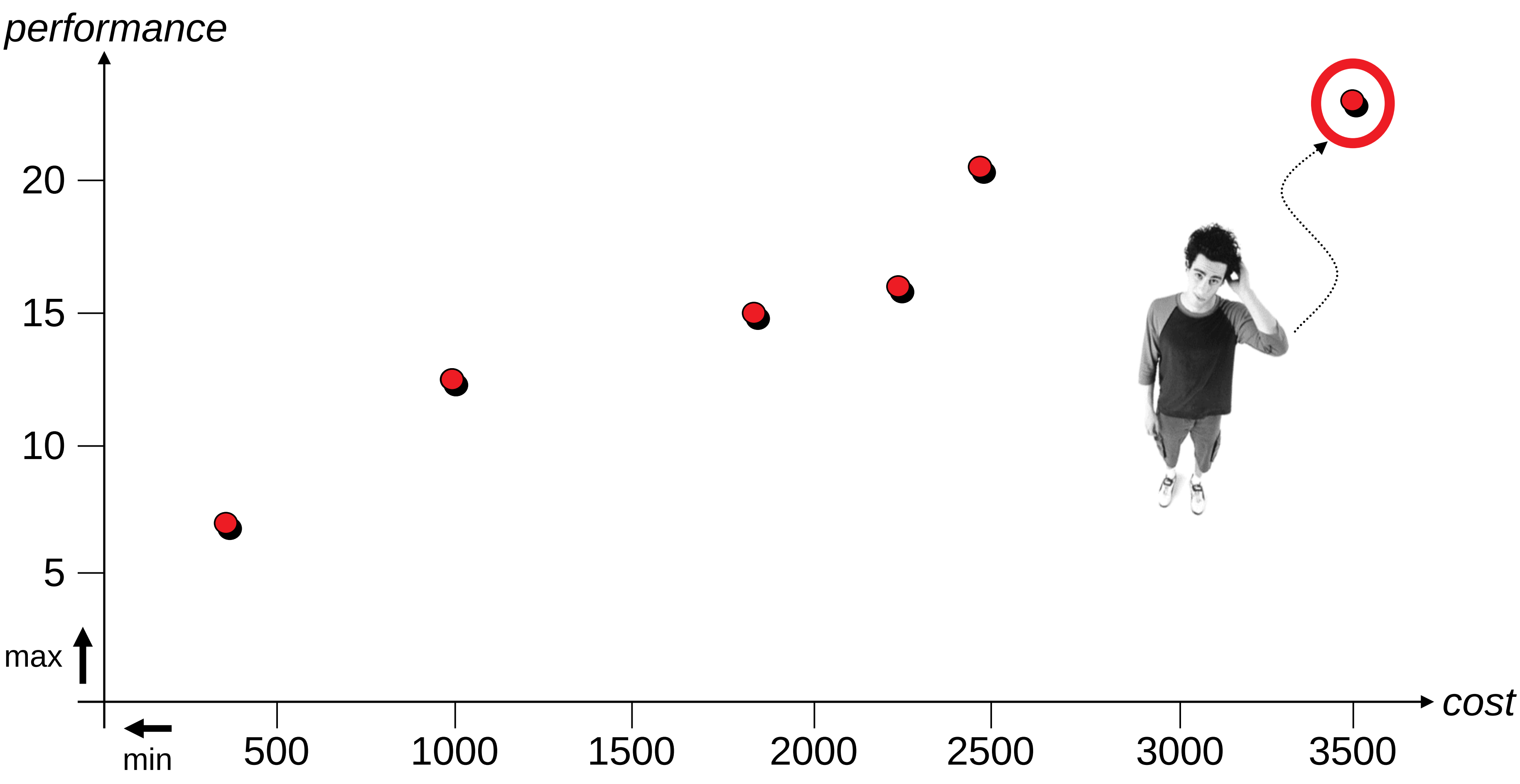
Multiobjective Optimization

combination of optimization of a set and a decision for a solution



Selecting a Solution: Examples

Possible Approaches: ❶ **ranking:** performance more important than cost

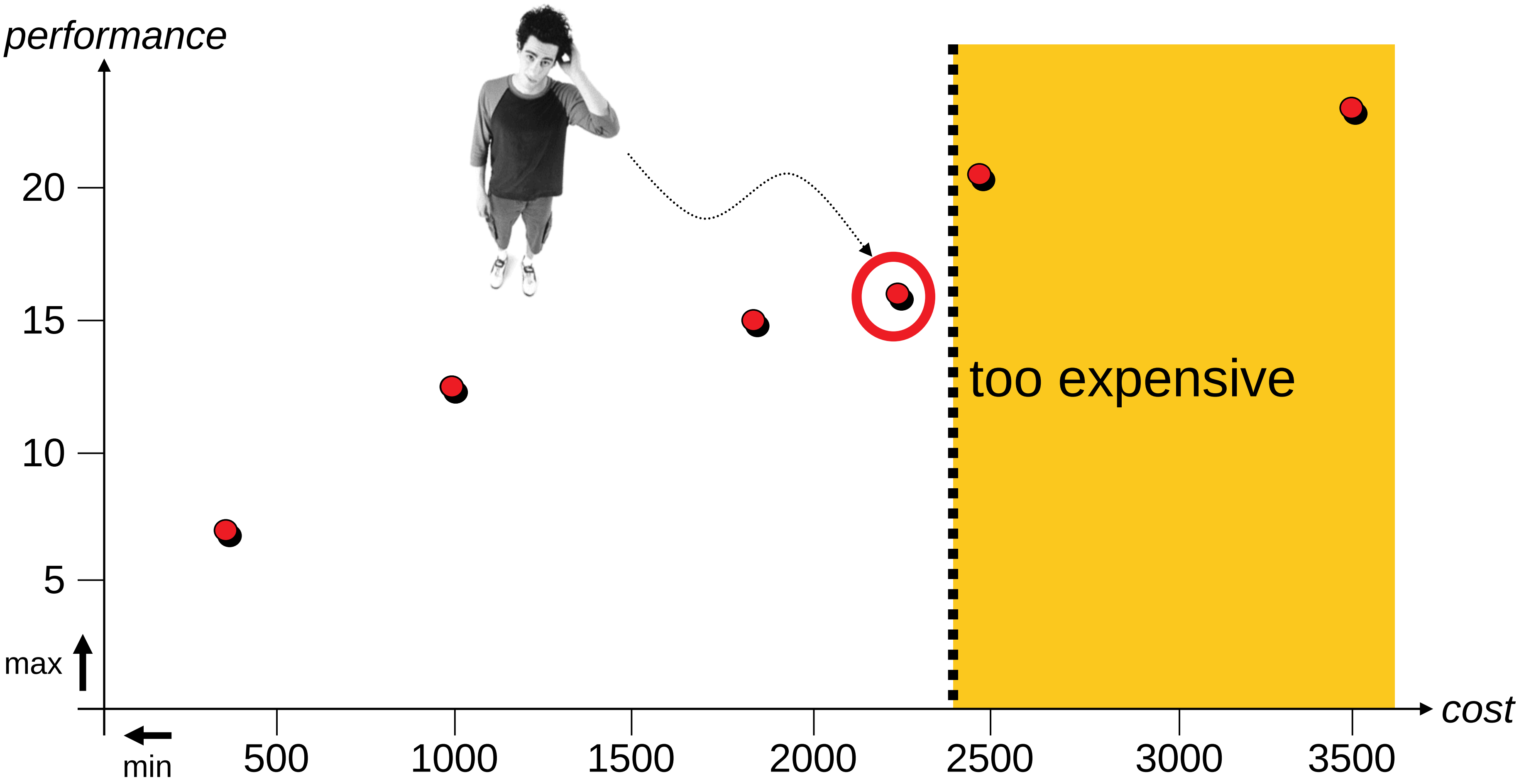


Selecting a Solution: Examples

Possible Approaches:

❶ ranking: performance more important than cost

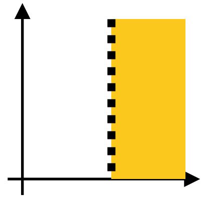
❷ **constraints:** cost must not exceed 2400



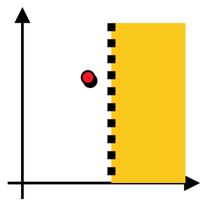
When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…



search for one
(good) solution



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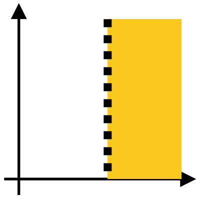

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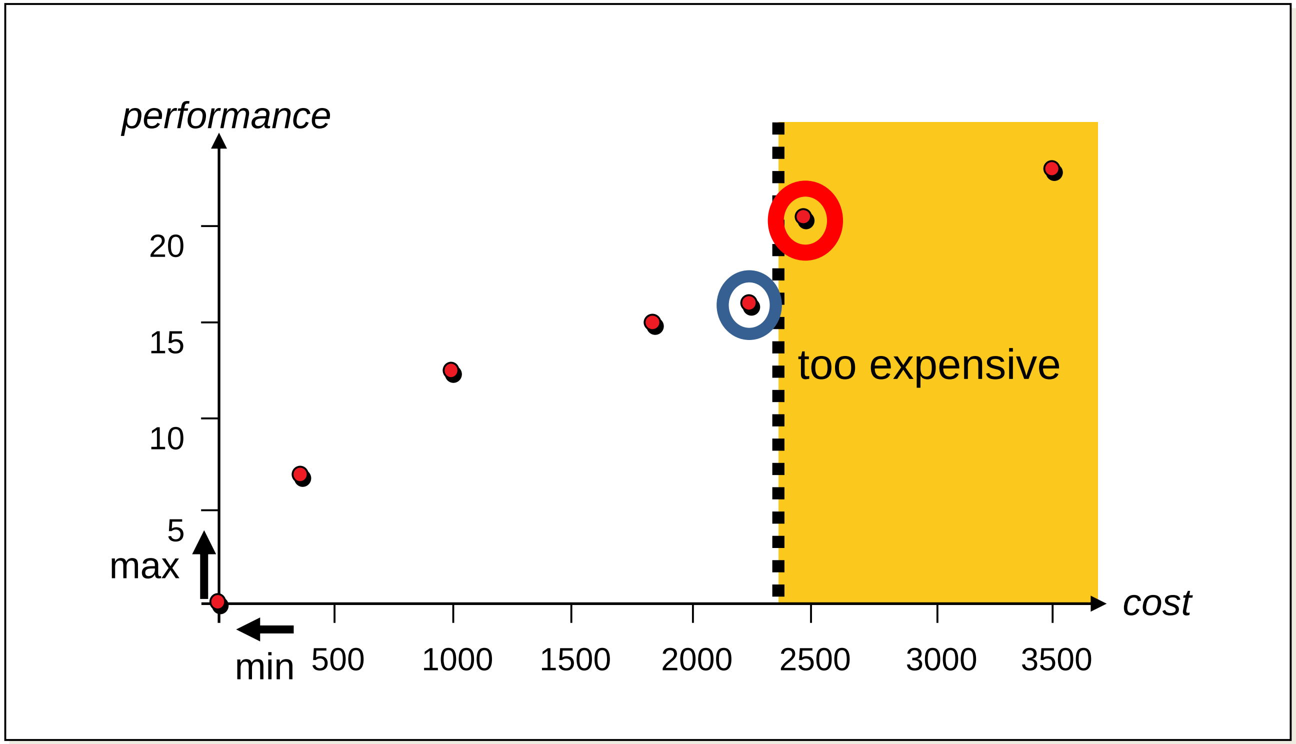

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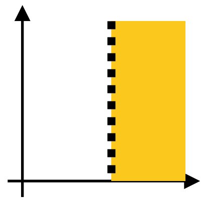
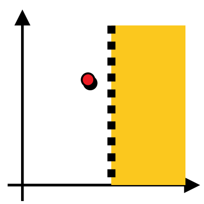
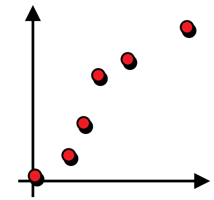


search for one
(good) solution

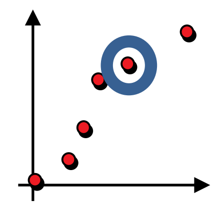


After Optimization:

search for a set of
(good) solutions



select one solution
considering
constraints, etc.



When to Make the Decision

Before Optimization:

rank objectives,
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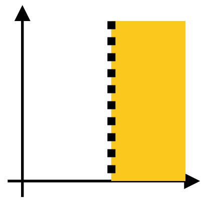

search for one
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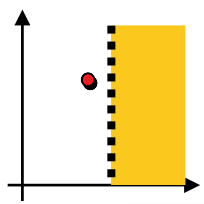

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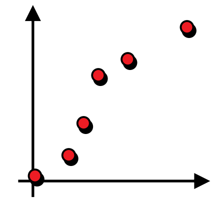

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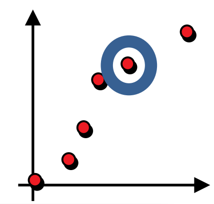


Focus: learning about a problem

- trade-off surface
- interactions among criteria
- structural information
- also: interactive optimization

Two Communities...

International Society on
Multiple Criteria Decision Making

- established field (beginning in 1950s/1960s)
- bi-annual conferences since 1975
- background in economics, math, management and social sciences
- focus on optimization and decision making

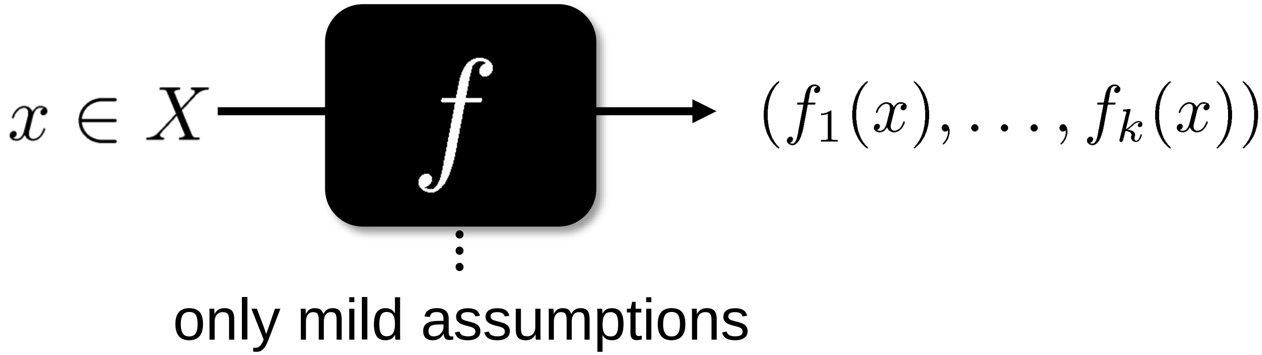
- quite young field (first papers in mid 1980s)
- bi-annual conference since 2001
- background in computer science, applied math and engineering
- focus on optimization algorithms

...Slowly Merge Into One

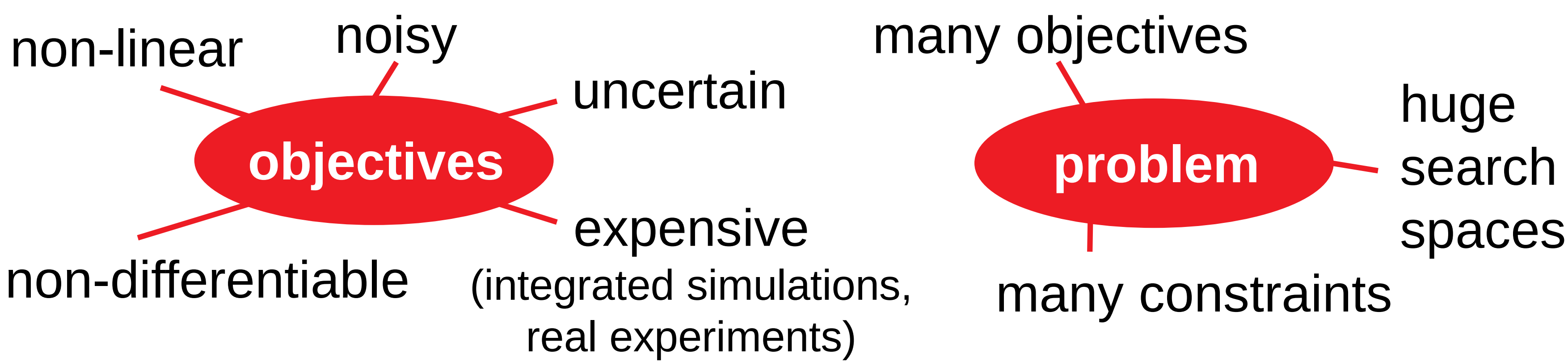
International Society on
Multiple Criteria Decision Making

- MCDM track at EMO conference since 2009
- special sessions on EMO at the MCDM conference since 2008
- joint Dagstuhl seminars since 2004

Blackbox optimization



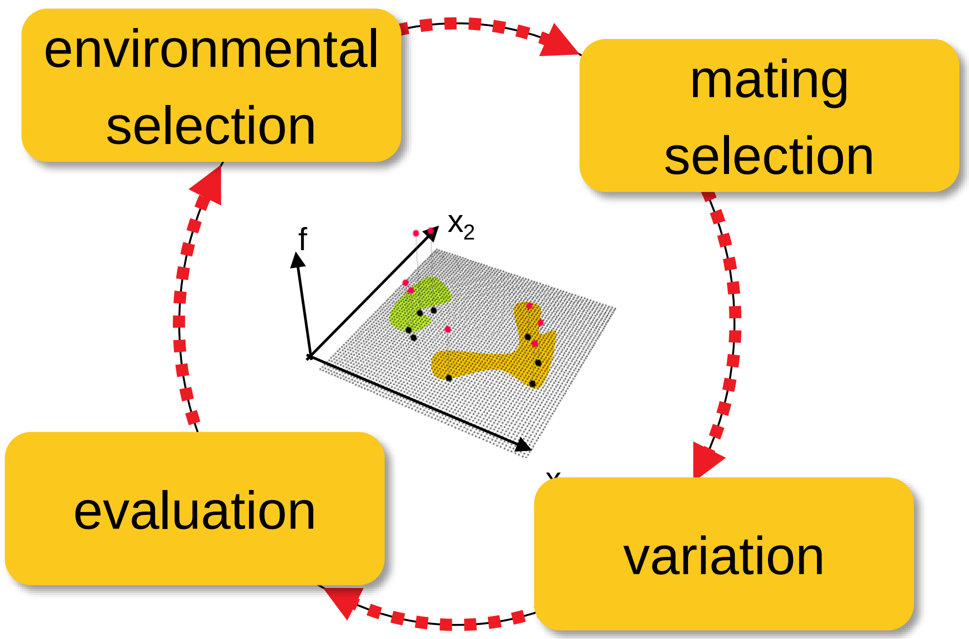
→ EMO therefore well-suited for real-world engineering problems

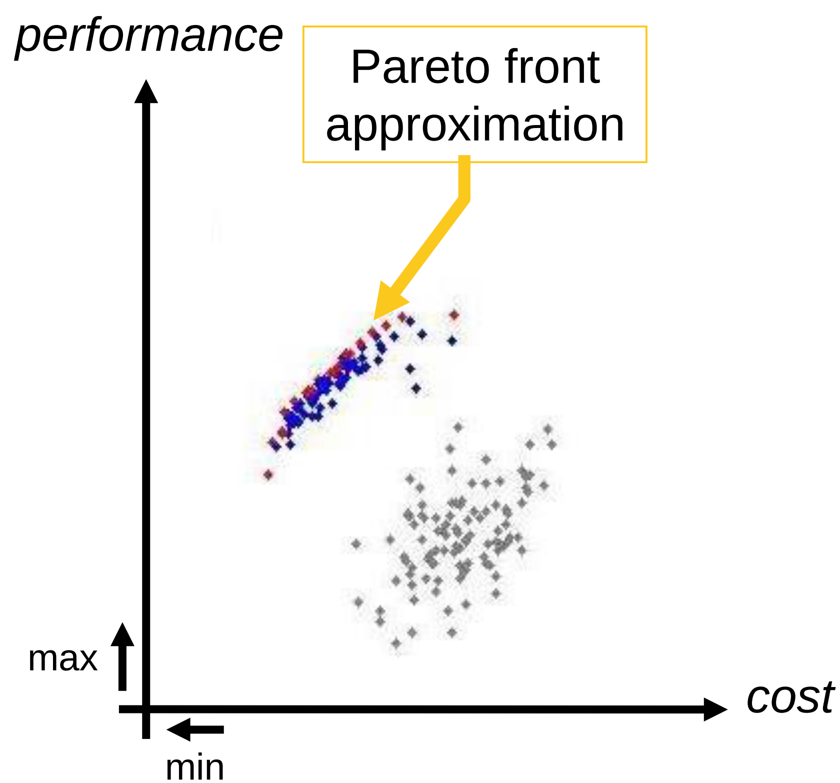


The Other Main Difference

Evolutionary Multiobjective Optimization

- set-based algorithms
- therefore possible to approximate the Pareto front in one run

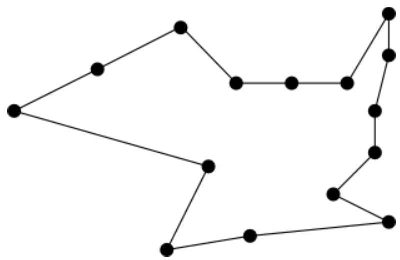


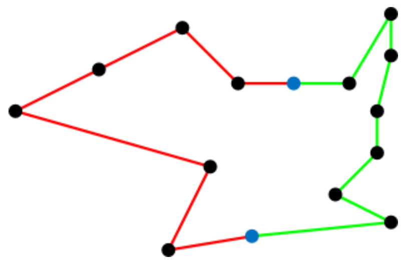


Some problems are easier to solve in a multiobjective scenario

example: TSP

[Knowles et al. 2001]





$\pi \in S_n \rightarrow f(\pi)$ $\qquad$ $\pi \in S_n \rightarrow (f_1(\pi, a, b), f_2(\pi, a, b))$

Multiobjectivization

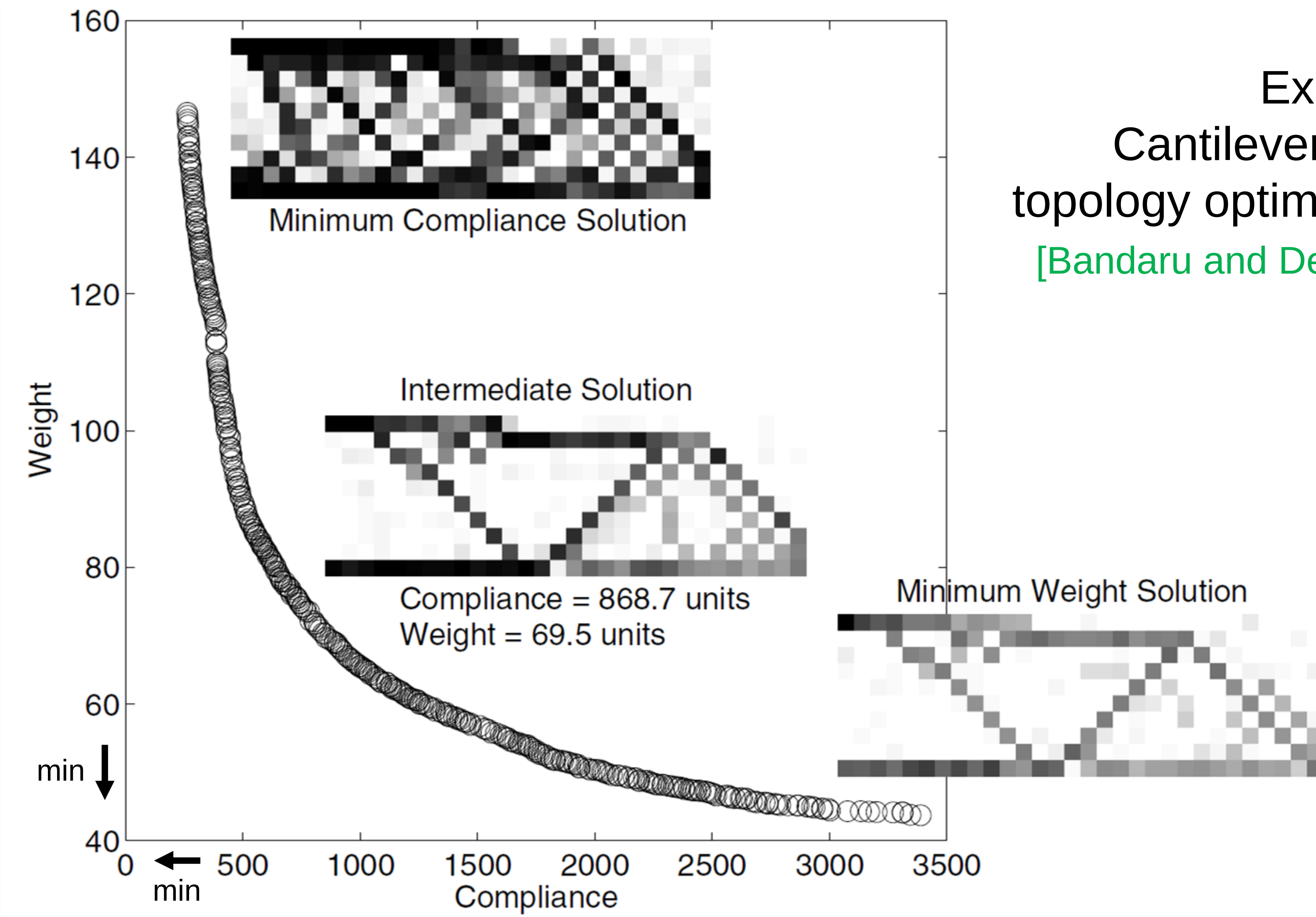
- by addition of new "helper objectives" [Jensen 2004]
  - job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], VRP [Watanabe and Sakakibara 2007], ...
- by decomposition of the single objective
  - TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], ...

also backed up by theory e.g. [Brockhoff et al. 2009, Handl et al. 2008b]

related to *constrained* and *multimodal* single-objective optimization

see also this recent overview: [Segura et al. 2013]

Often innovative design principles among solutions are found



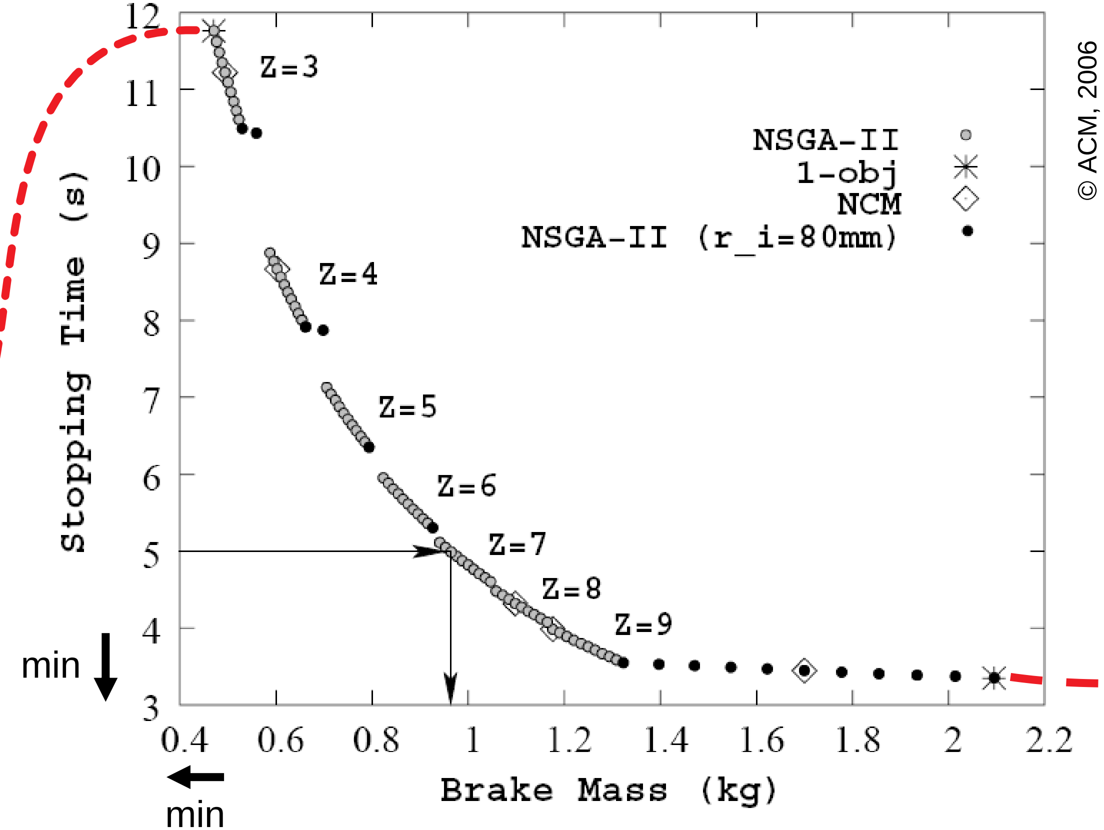
Example:
Cantilever beam
topology optimization
[Bandaru and Deb 2015]

Often innovative design principles among solutions are found

Example:

Clutch brake design

[Deb and Srinivasan 2006]



Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Innovization

Often innovative design principles among solutions are found

Innovization [Deb and Srinivasan 2006]

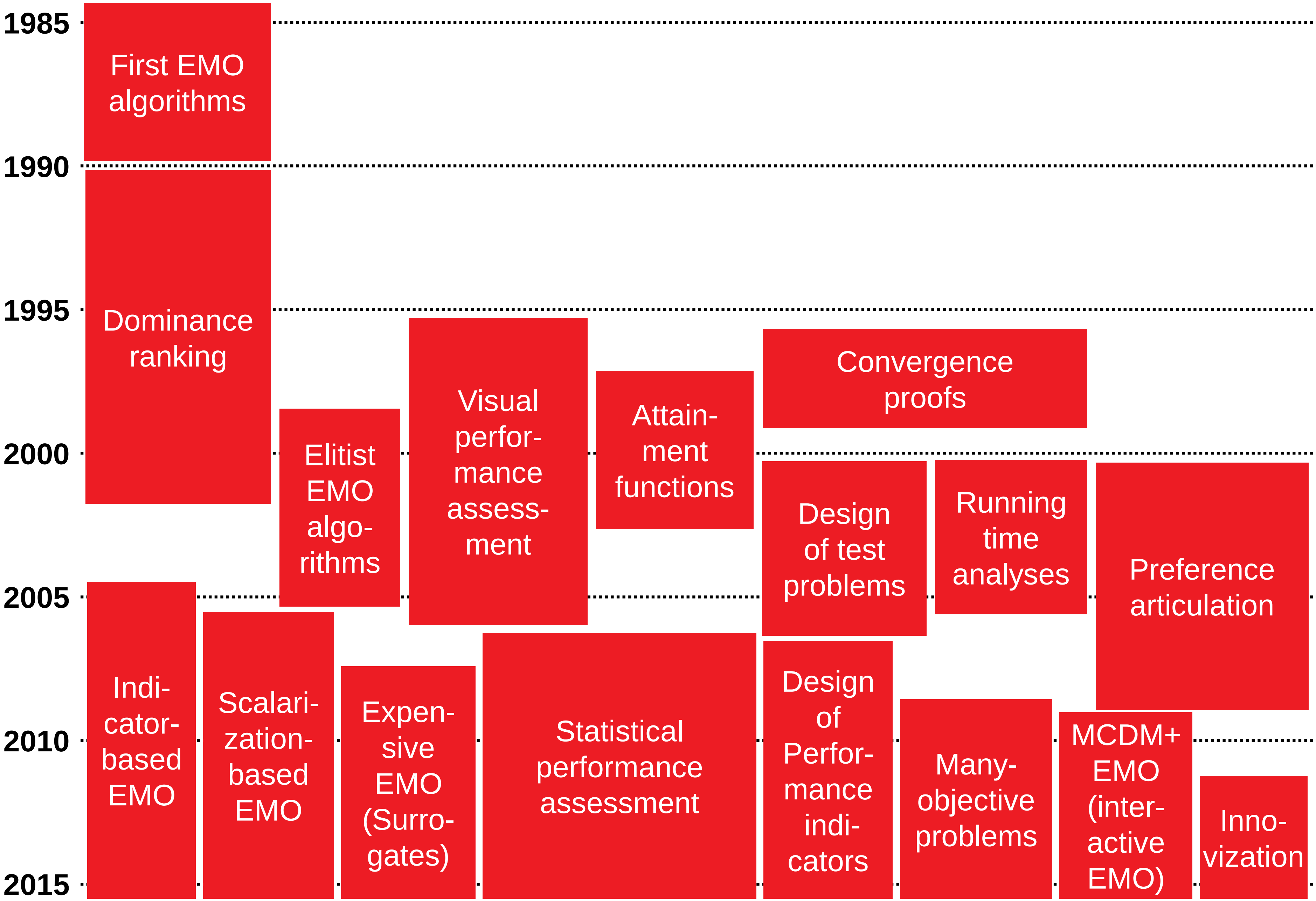
= using machine learning techniques to find new and innovative design principles among solution sets

= learning from/about a multi-objective optimization problem

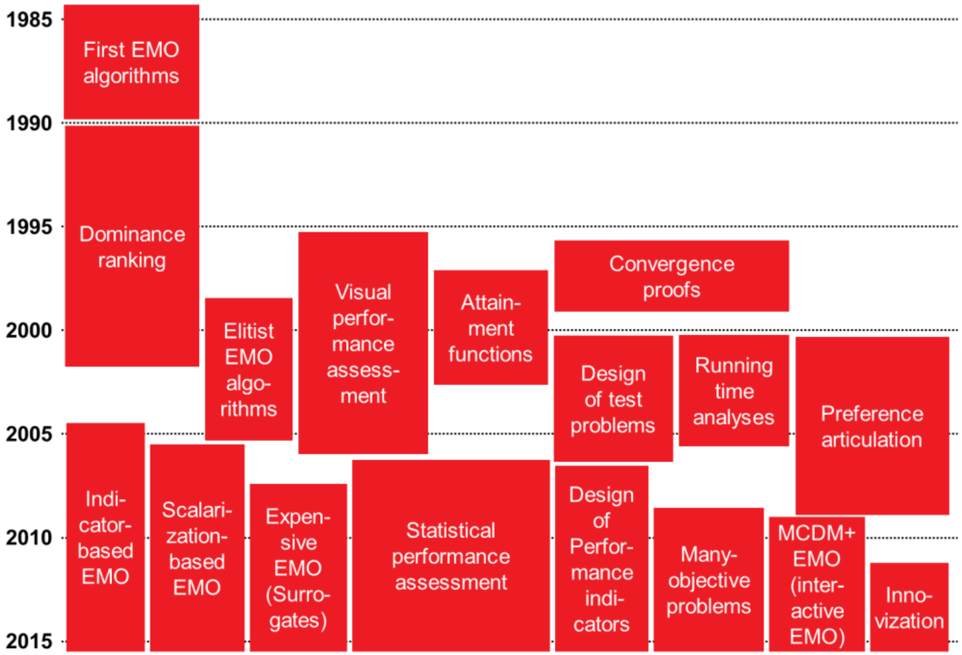
Other examples:

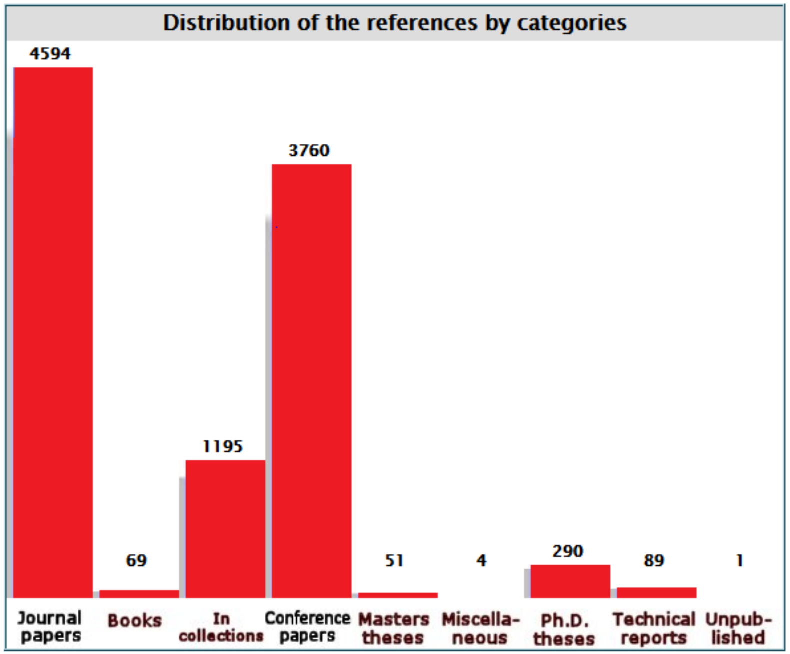
- SOM for supersonic wing design [Obayashi and Sasaki 2003]
- Biclustering for processor design and knapsack [Ulrich et al. 2007]
- Successful case studies in engineering
  (noise barrier design, polymer extrusion, friction stir welding)
  [Deb et al. 2014]

The History of EMO At A Glance

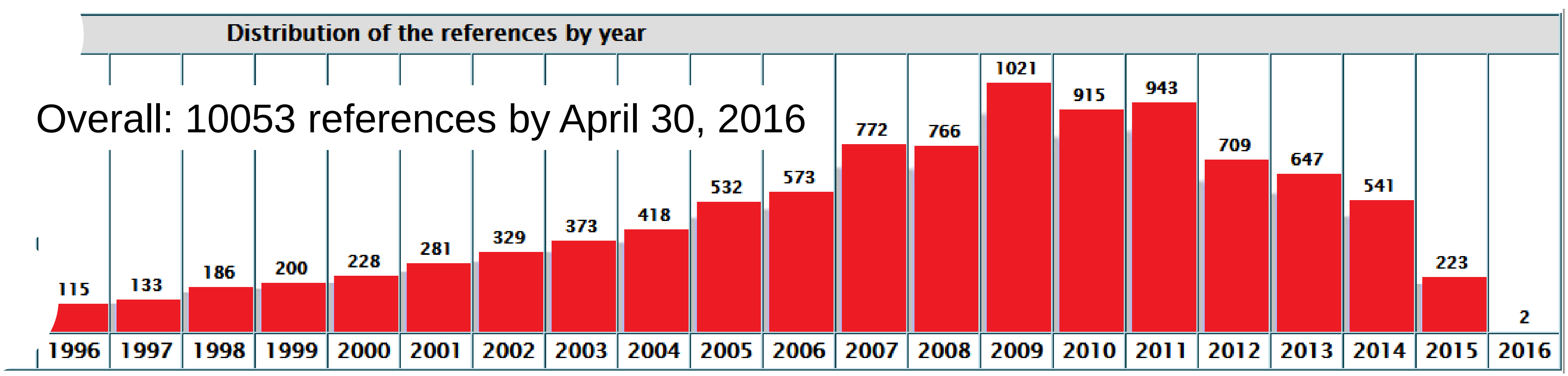


The History of EMO At A Glance



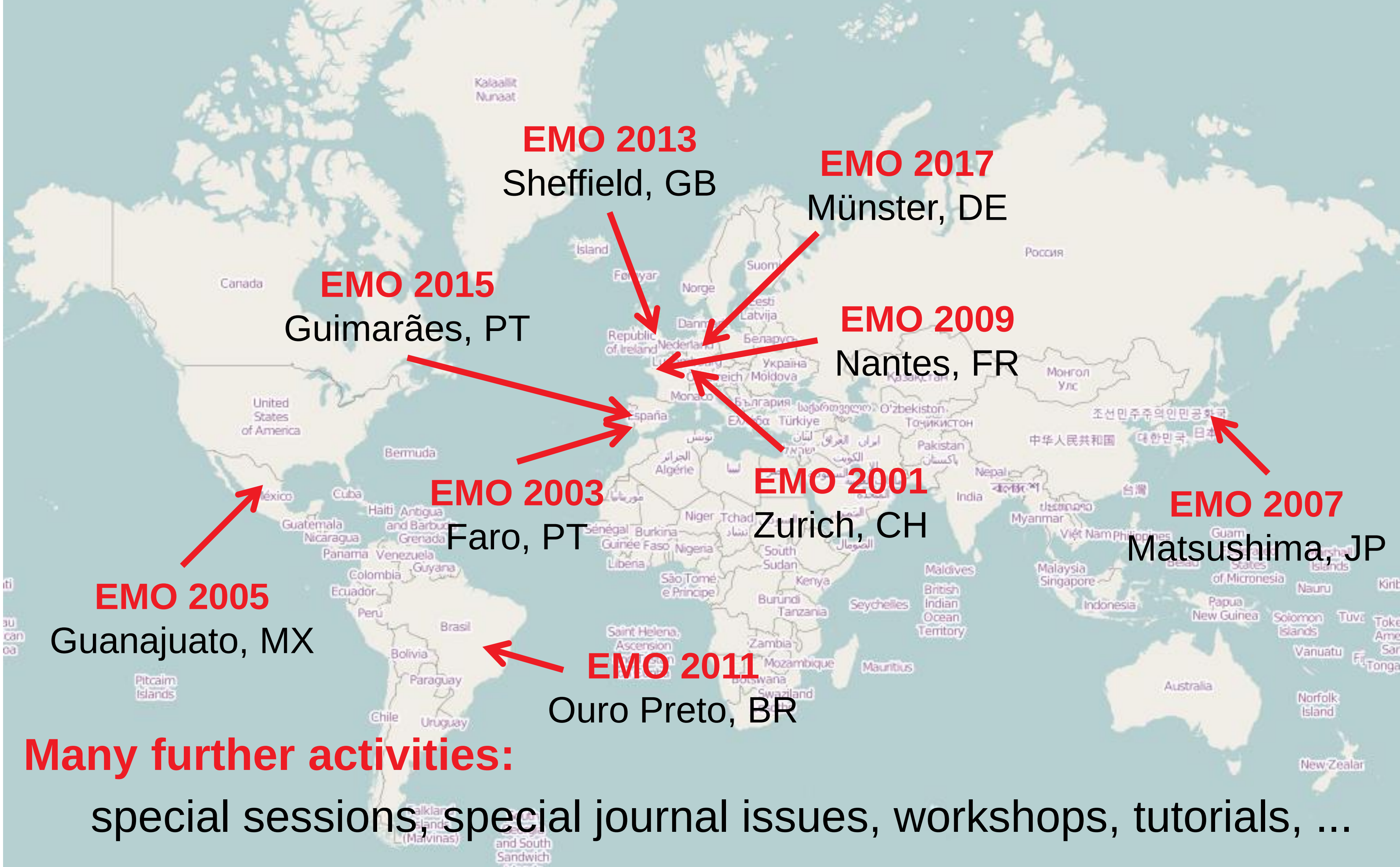


http://delta.cs.cinvestav.mx/~ccoello/EMOO



Overall: 10053 references by April 30, 2016 *from Google maps*

The EMO conference series:



Many further activities:

special sessions, special journal issues, workshops, tutorials, ...

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

Selected Advanced Concepts

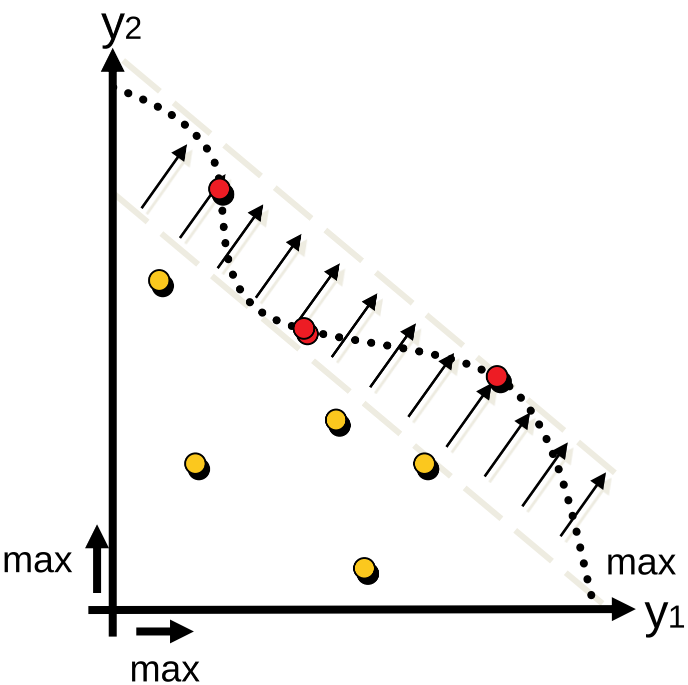
- preference articulation
- surrogate-based EMO

A Few Examples From Practice

Fitness Assignment: Principal Approaches

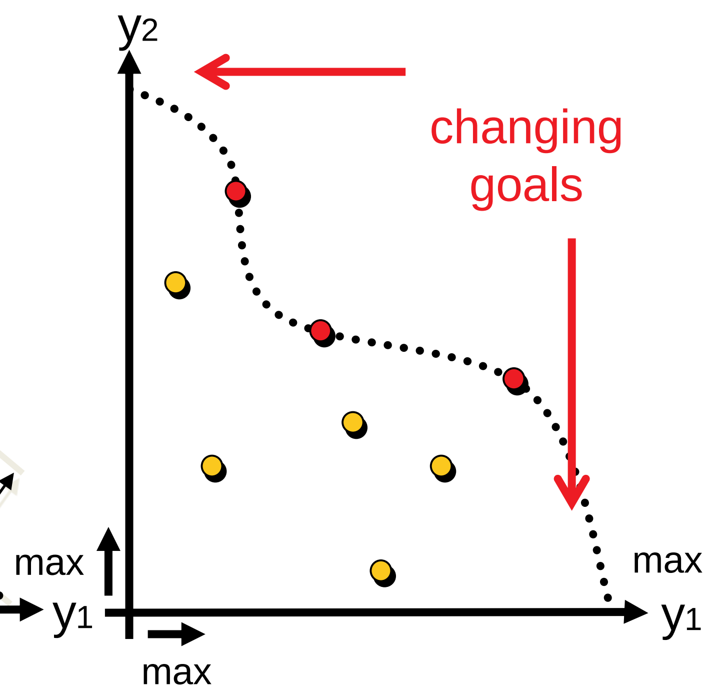
aggregation-based

problem decomposition (multiple single-objective optimization problems)



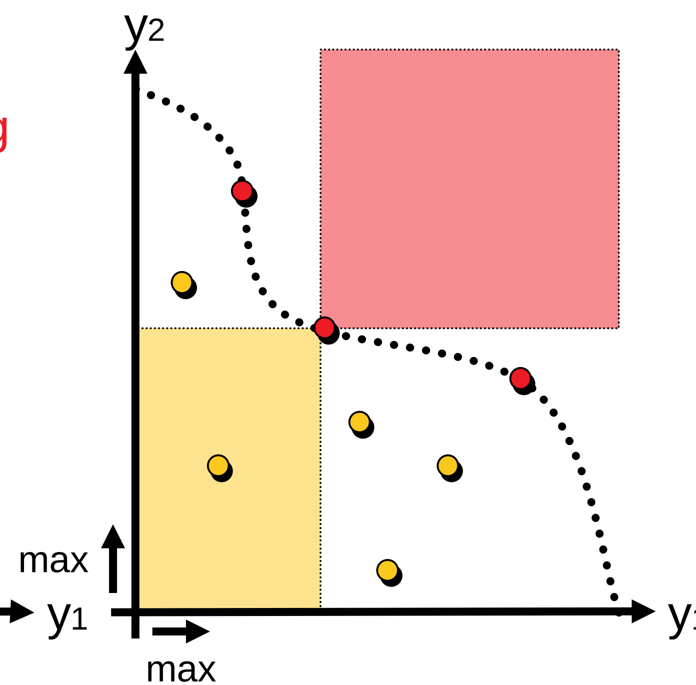
criterion-based

VEGA



dominance-based

SPEA2, NSGA-II
“modern” EMOA



solution-oriented
scaling-dependent

set-oriented
scaling-independent

Solution-Oriented Problem Transformations

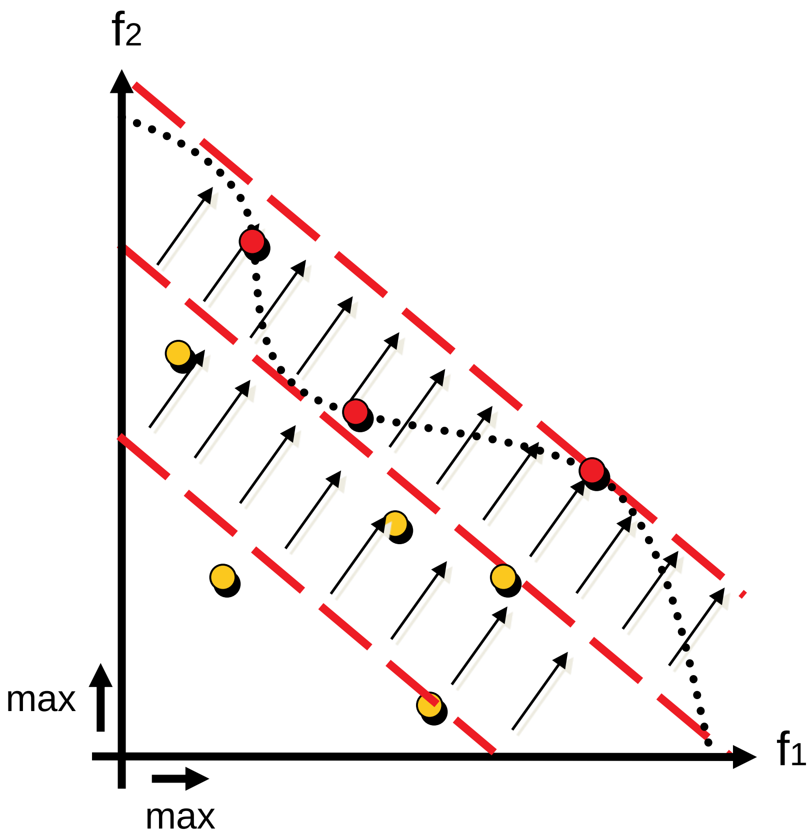
$$\text{multiple objectives } (f_1(x), f_2(x), \ldots, f_k(x)) \longrightarrow \boxed{\text{transformation}} \longrightarrow \text{single objective } s(x)$$

parameters $\downarrow$ transformation

A scalarizing function s is a function $s: Z \rightarrow \mathbb{R}$ that maps each objective vector $u = (u_1, \ldots, u_n) \in Z$ to a real value $s(u) \in \mathbb{R}$

parameters

multiple objectives $(f_1(x), f_2(x), \ldots, f_k(x))$ → **transformation** → single objective $s(x)$



Example 1: weighted sum approach

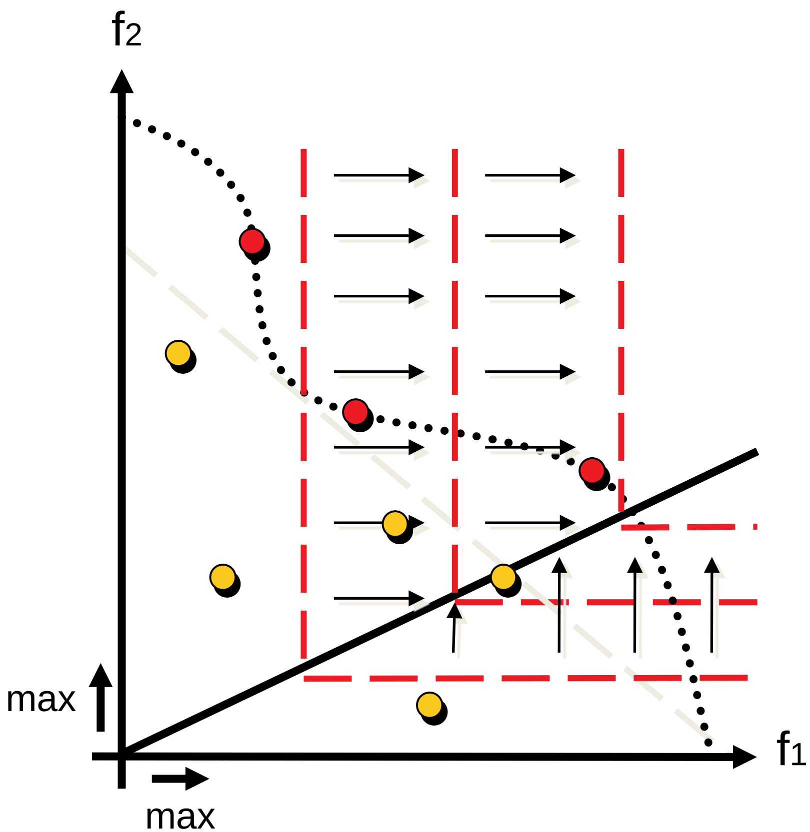
$(w_1, w_2, \ldots, w_k)$

→ $y = w_1 y_1 + \ldots + w_k y_k$ →

Disadvantage: not all Pareto-optimal solutions can be found if the front is not convex

Solution-Oriented Problem Transformations

multiple objectives $(f_1(x), f_2(x), \ldots, f_k(x))$ → transformation (parameters) → single objective $s(x)$



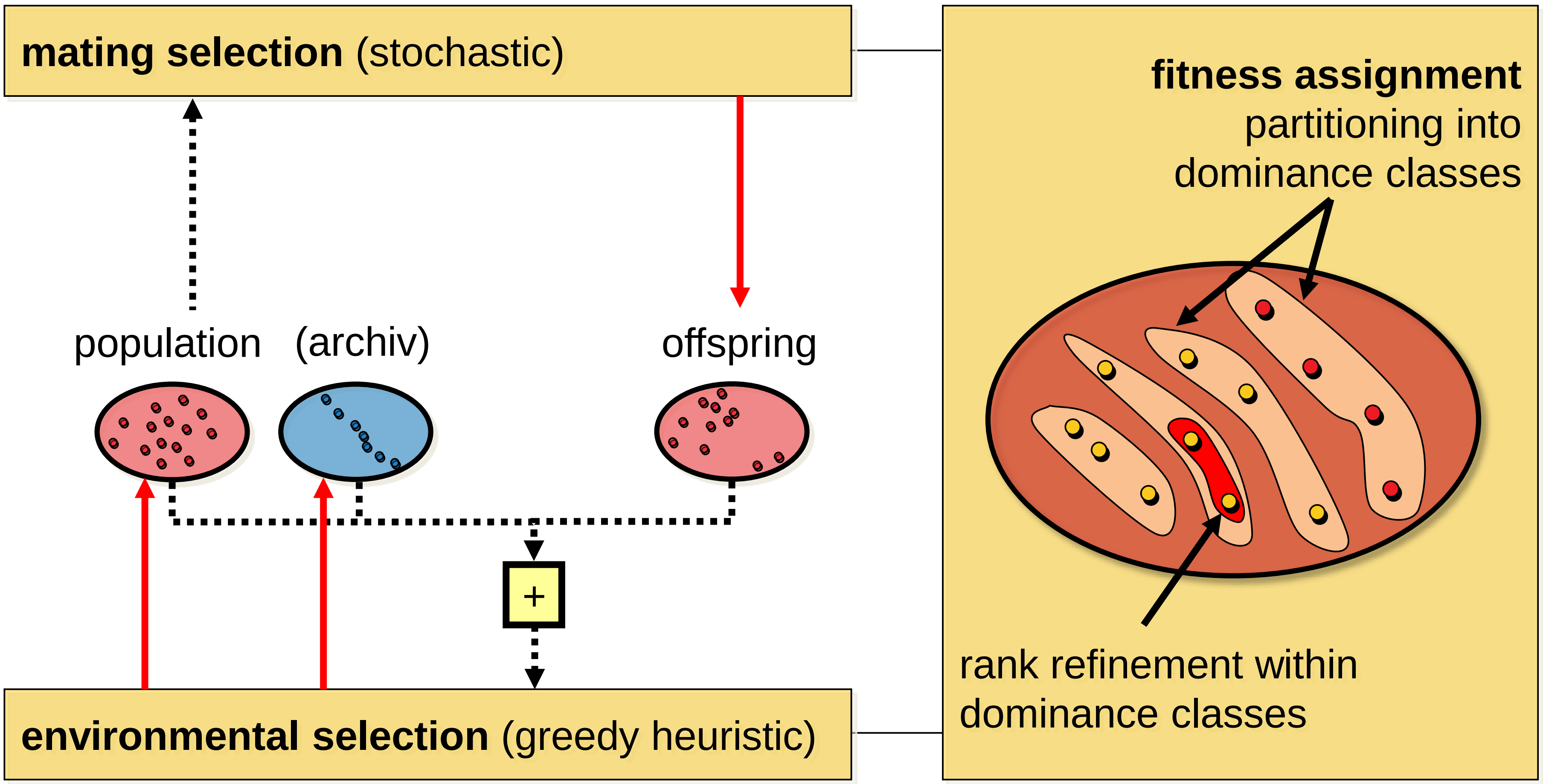
Example 2: weighted Tchebycheff

$(\lambda_1, \lambda_2, \ldots, \lambda_k)$

$$y = \max_i | \lambda_i(u_i - z_i)|$$

Several other scalarizing functions are known, see e.g. [Miettinen 1999]

General Scheme of Most Set-Oriented EMO



Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II, most of the recently proposed algorithms

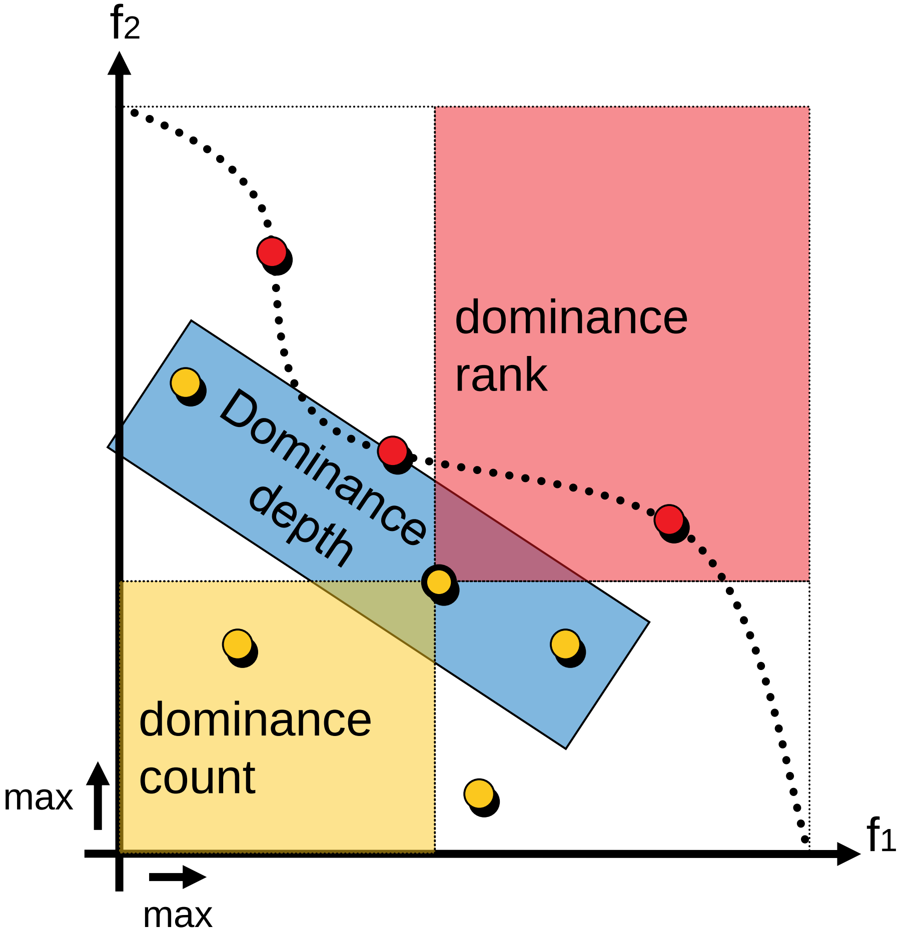
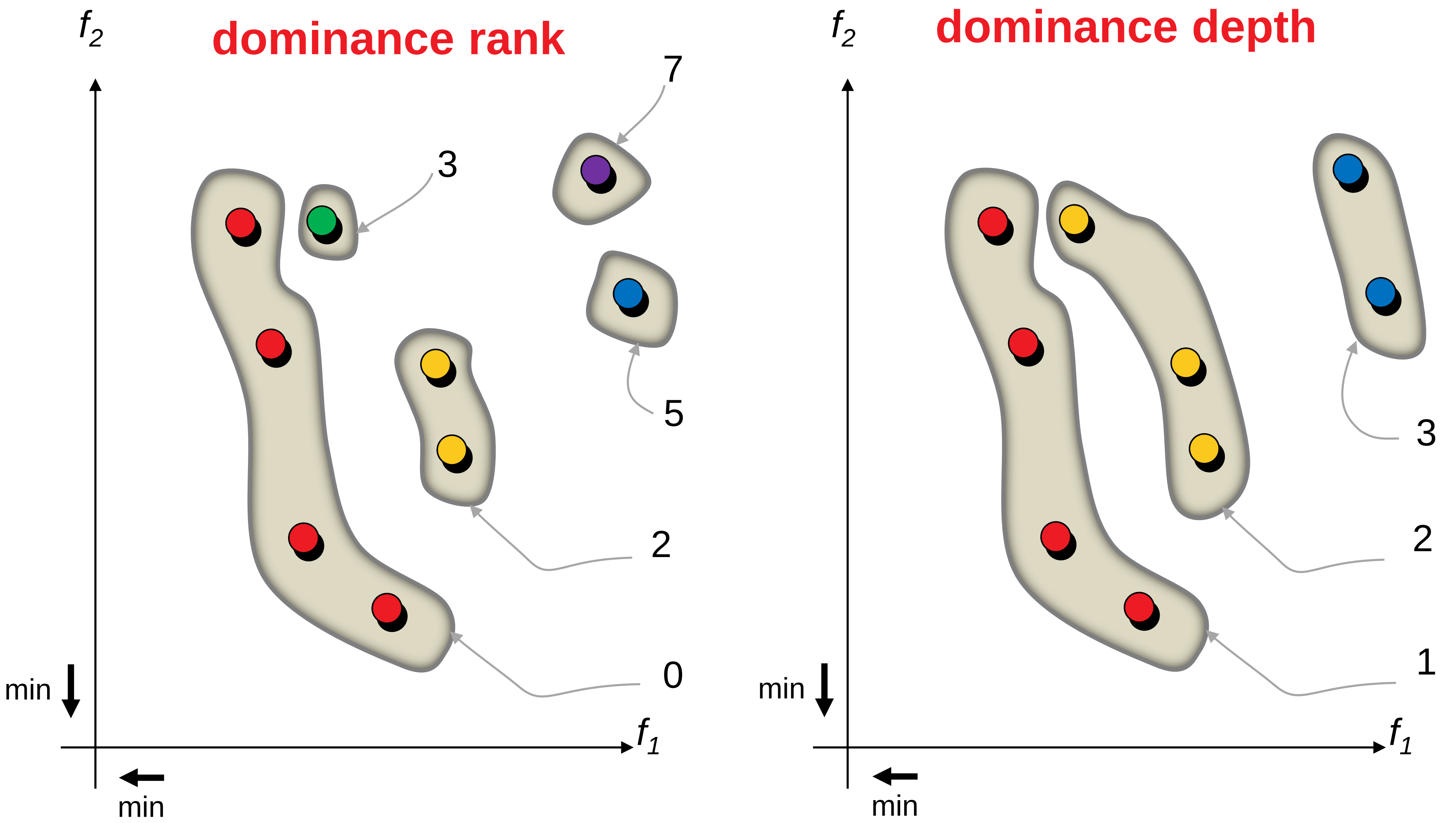


Illustration of Dominance-Based Partitioning

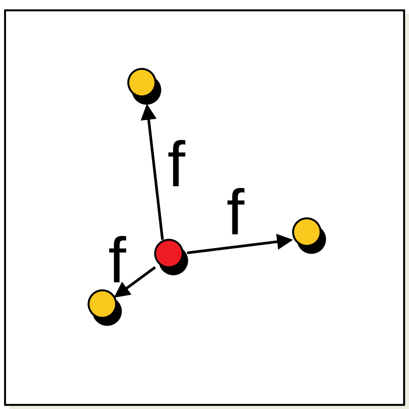


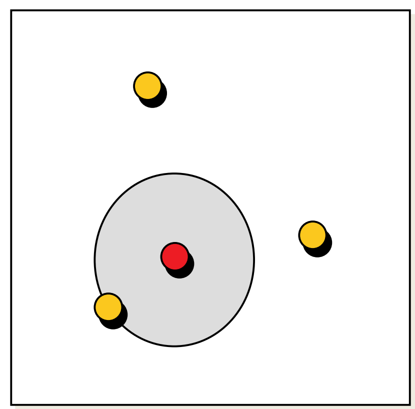
Refinement of Dominance Rankings

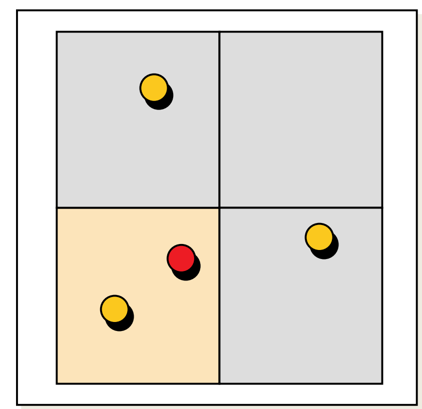
Goal: rank incomparable solutions within a dominance class

❶ Diversity information

Kernel method	k-th nearest neighbor	Histogram method
diversity = function of the distances	diversity = function of distance to k-th nearest neighbor	diversity = number of elements within box(es)





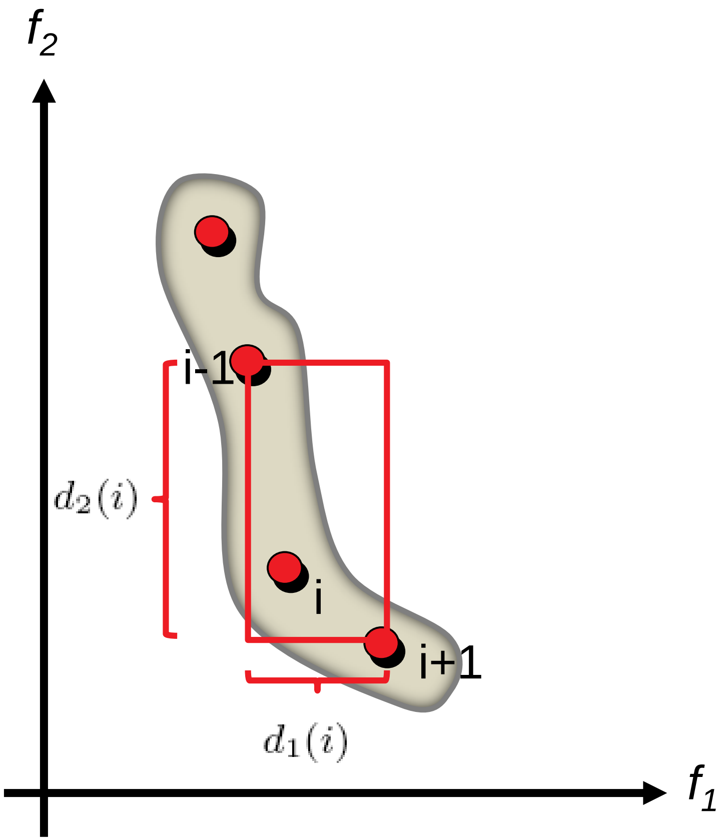


❷ (Contribution to a) quality indicator

Example: NSGA-II Diversity Preservation

Crowding Distance (CD)

- sort solutions with regard to each objective
- assign CD maximum value to extremal objective vectors
- compute CD based on the distance to the neighbors in each objective



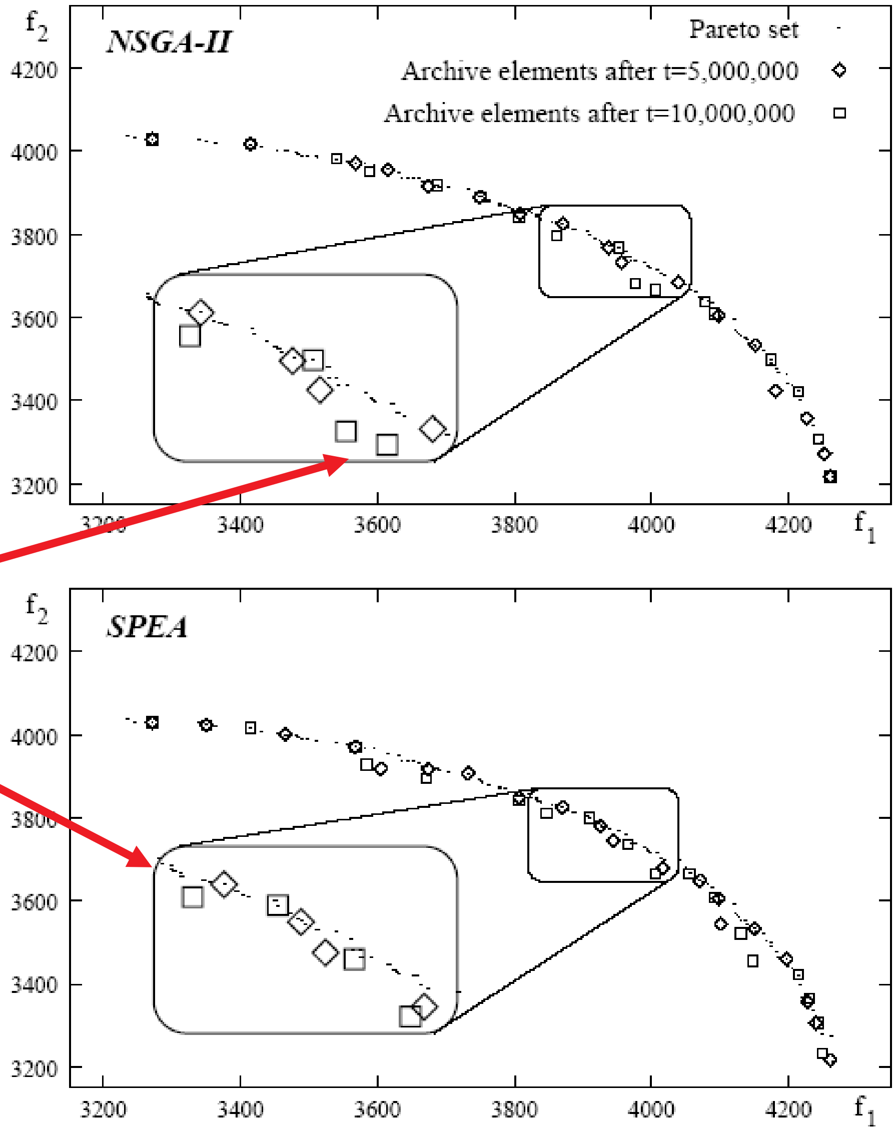
$$\mathrm{CD}(i) = \frac{d_1(i)}{f_{1,\max} - f_{1,\min}} + \cdots + \frac{d_m(i)}{f_{m,\max} - f_{m,\min}}$$

SPEA2 and NSGA-II: Deteriorative Cycles

Selection in SPEA2 and NSGA-II can result in

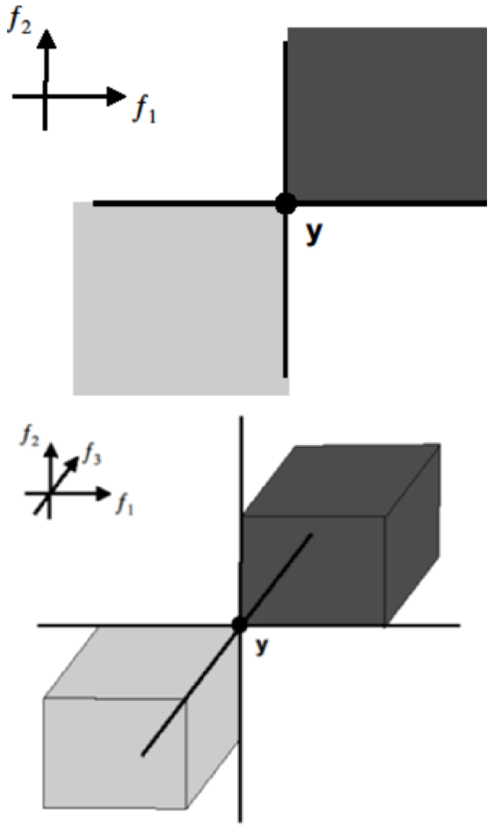
deteriorative cycles

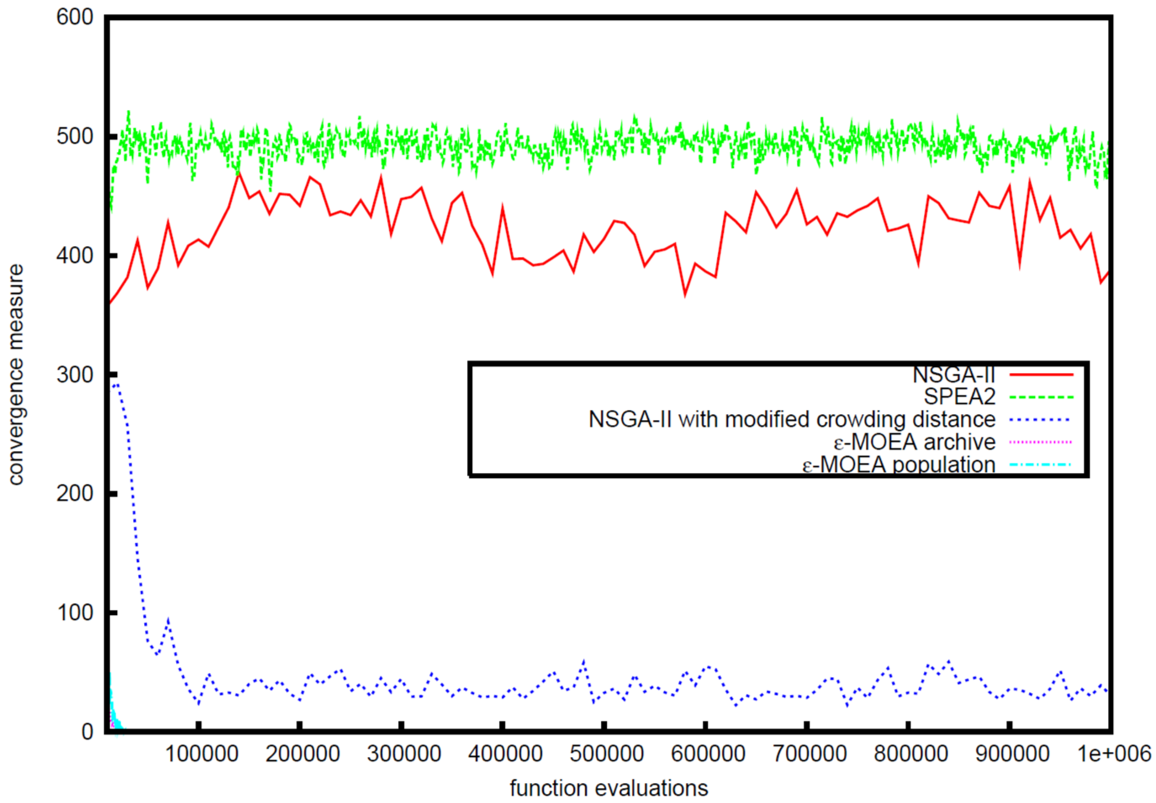
non-dominated solutions already found can be lost



Remark: Many-Objective Optimization

- high number of objectives
  → percentage of non-dominated solutions within a random sample quickly approaches 100 %
  → optimization is mainly guided by diversity criterion
  → apply secondary criterion compliant with dominance relation





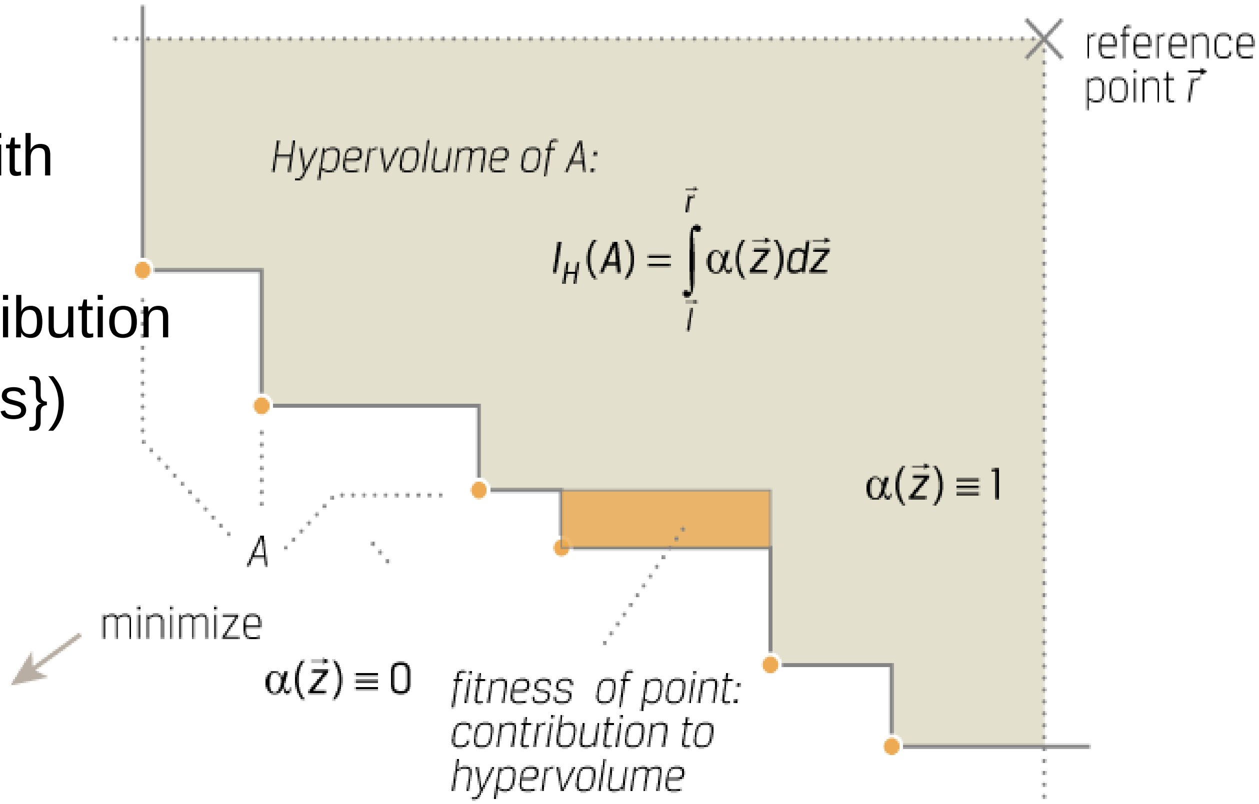
Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)

use hypervolume indicator to guide the search: refines dominance

Main idea

Delete solutions with the smallest hypervolume contribution $d(s) = I_H(P) - I_H(P / \{s\})$ iteratively



But: can also result in cycles if reference point is not constant [Judt et al. 2011] and is expensive to compute exactly [Bringmann and Friedrich 2009]

Indicator-Based Selection

- Concept can be generalized to any quality indicator

A (unary) quality indicator I is a function $I : \Psi = 2^X \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

Multiobjective Problem $\xrightarrow{\text{Indicator}}$ Single-objective Problem

- for example: R2-indicator [Brockhoff et al. 2012], [Trautmann et al. 2013], [Díaz-Manríquez et al. 2013]

- Generalizable also to contribution to larger sets

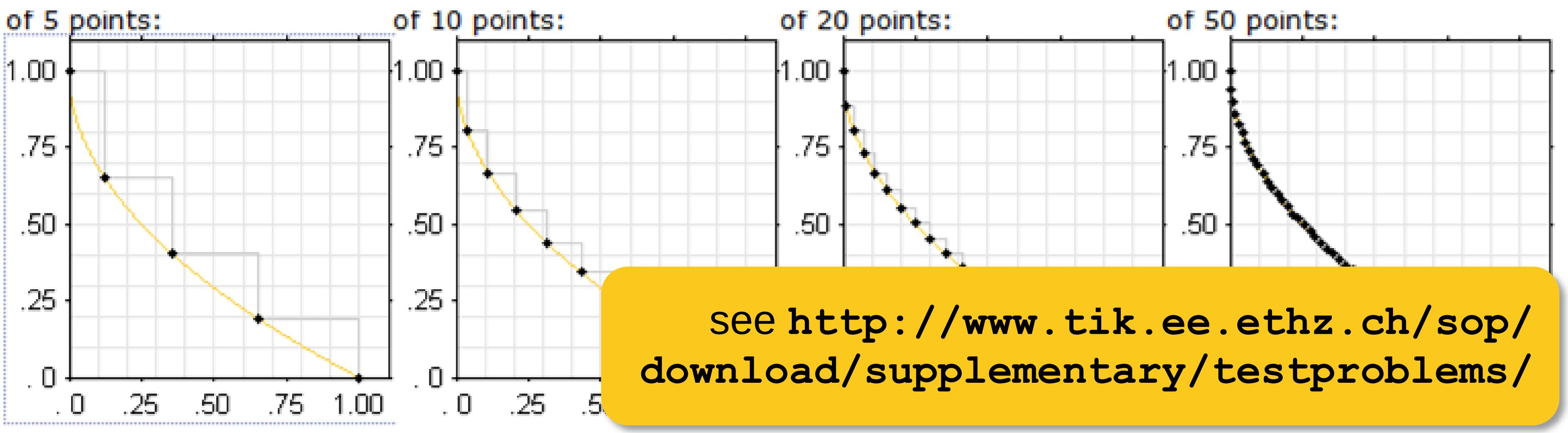
  HypE [Bader and Zitzler 2011]: Hypervolume sampling + contribution if more than 1 (random) solution deleted

When the goal is to maximize a unary indicator…

- we have a single-objective problem on sets
- but what is the optimum?
- important: population size μ plays a role!

Optimal μ-Distribution:

A set of μ solutions that maximizes a certain unary indicator I among all sets of μ solutions is called optimal μ-distribution for I. [Auger et al. 2009a]



see **http://www.tik.ee.ethz.ch/sop/download/supplementary/testproblems/**

Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal μ-distributions for hypervolume

Optimal μ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with f' the slope of the front

[Friedrich et al. 2011]:

optimal μ-distributions for the hypervolume correspond to ε-approximations of the front

OPT	$1 + \frac{\log(\min\{A/a, B/b\})}{n}$
HYP	$1 + \frac{\sqrt{A/a} + \sqrt{B/b}}{n-4}$
logHYP	$1 + \frac{\sqrt{\log(A/a)\log(B/b)}}{n-2}$

! (probably) does not hold for > 2 objectives

Indicator-Based EMO

Open Questions:

- How do the optimal μ-distributions look like for >2 objectives?
- how to compute certain indicators quickly in practice?
  - several recent improvements for the hypervolume indicator
    [Yildiz and Suri 2012], [Bringmann 2012], [Bringmann 2013]
- how to do indicator-based subset selection quickly?
  - also here several recent improvements
    [Kuhn et al. 2014], [Bringmann et al. 2014], [Guerreiro et al. 2015]
- what is the best strategy for the subset selection?

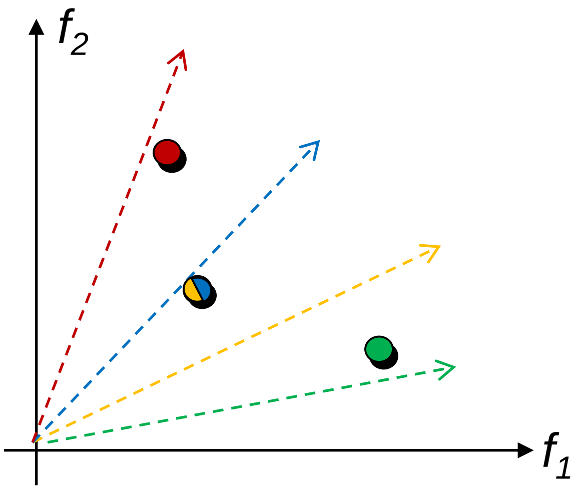
further open questions on indicator-based EMO available at
http://simco.gforge.inria.fr/doku.php?id=openproblems

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- optimize N scalarizing functions in parallel
- use best solutions of neighbor subproblems for mating
- keep the best solution for each scalarizing function
- update neighbors
- use external archive for non-dominated solutions
- several variants and enhancements



Remark: Variation in EMO

- at first sight not different from single-objective optimization
- most research on selection mechanisms (until now)
- but: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a *set*?

Related work:

- set-based gradient of the HV [Emmerich et al. 2007]
- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- RM-MEDA [Zhang et al. 2008]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]
- offline and online configuration based on libraries of variation operators [Bezerra et al. 2015] [Hadka and Reed 2013]

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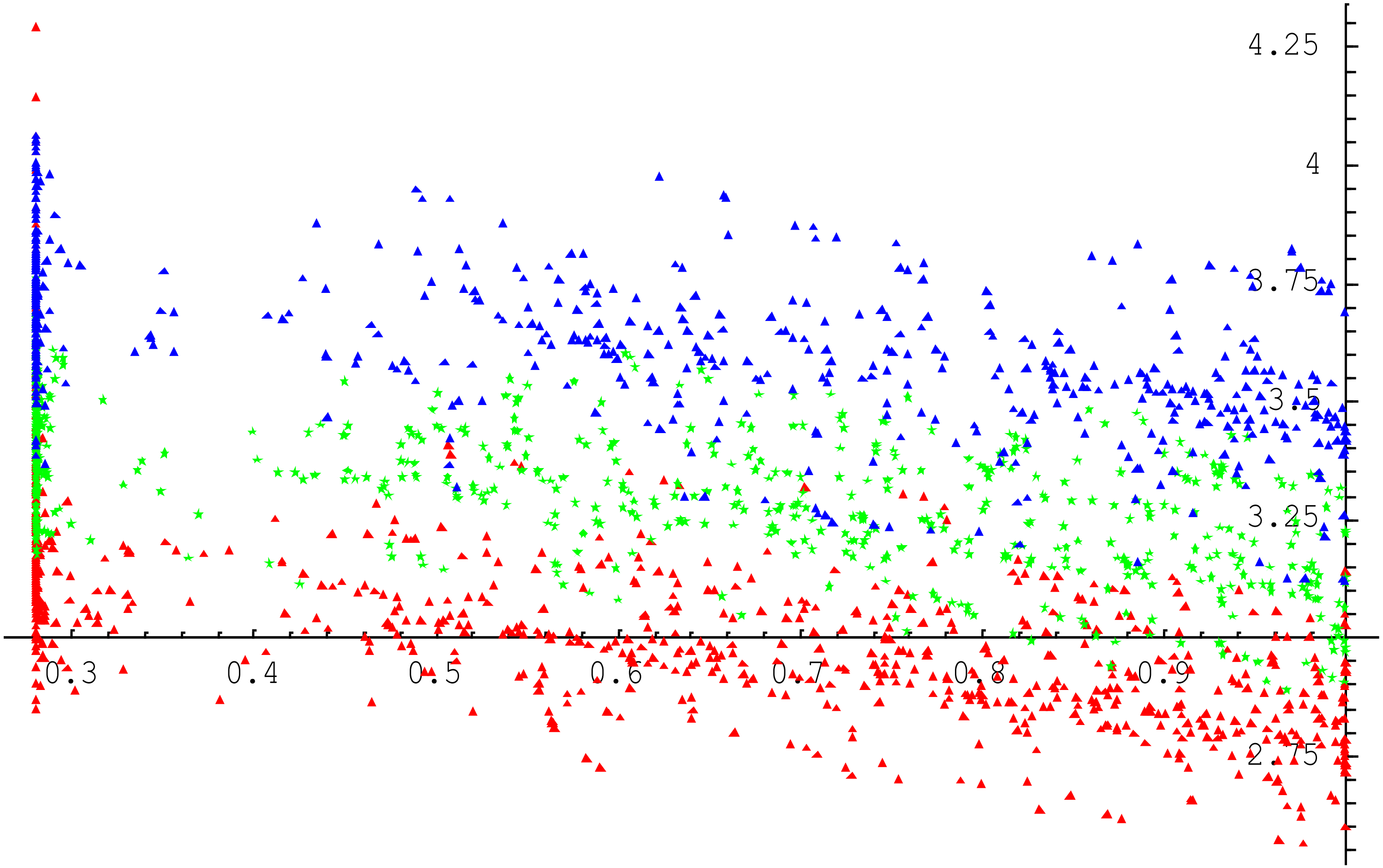
A Few Examples From Practice

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

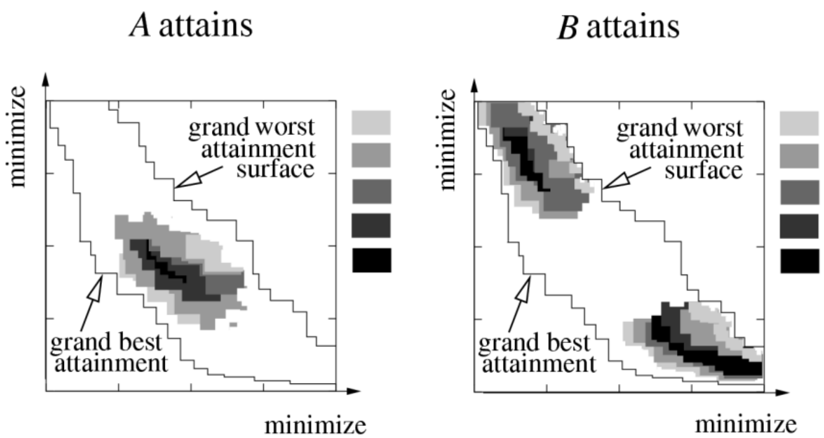
Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

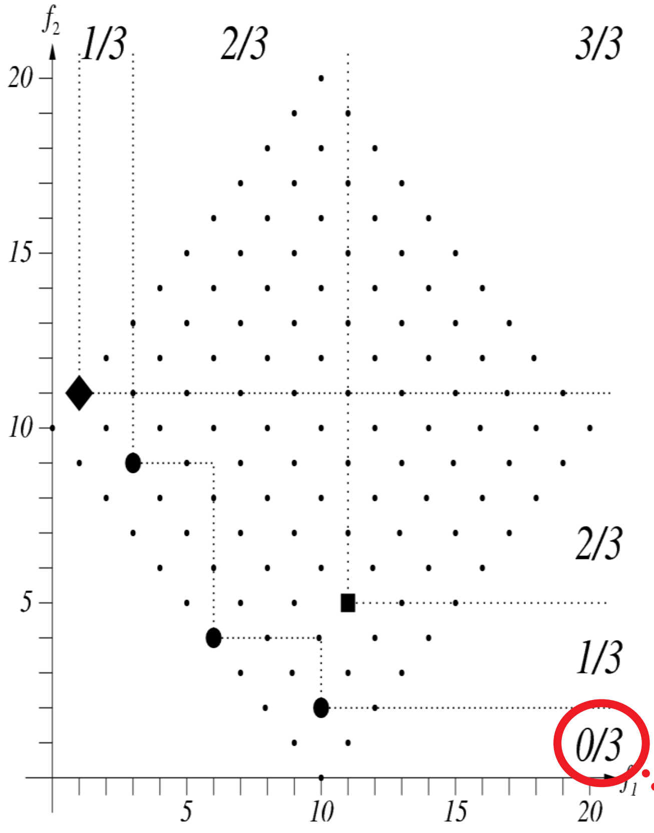
- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

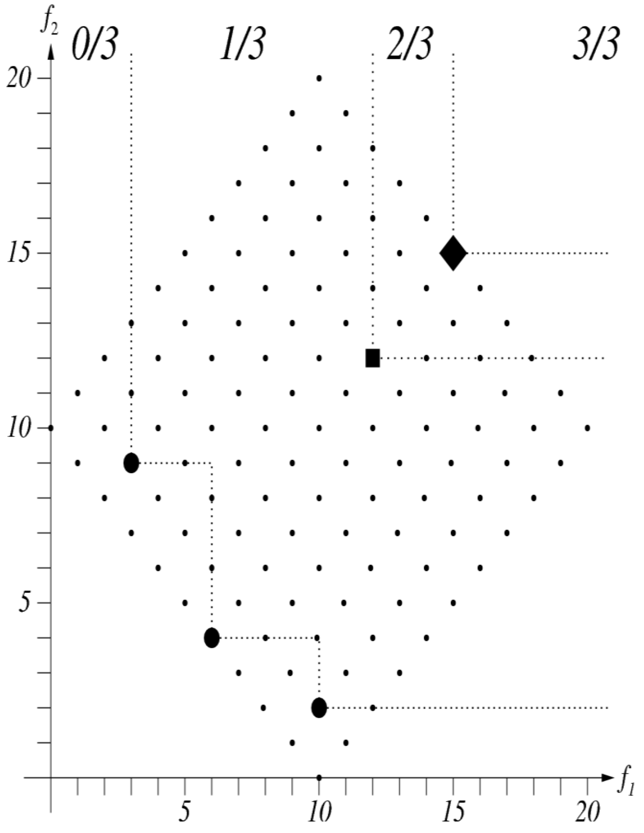


Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

three runs of two multiobjective optimizers

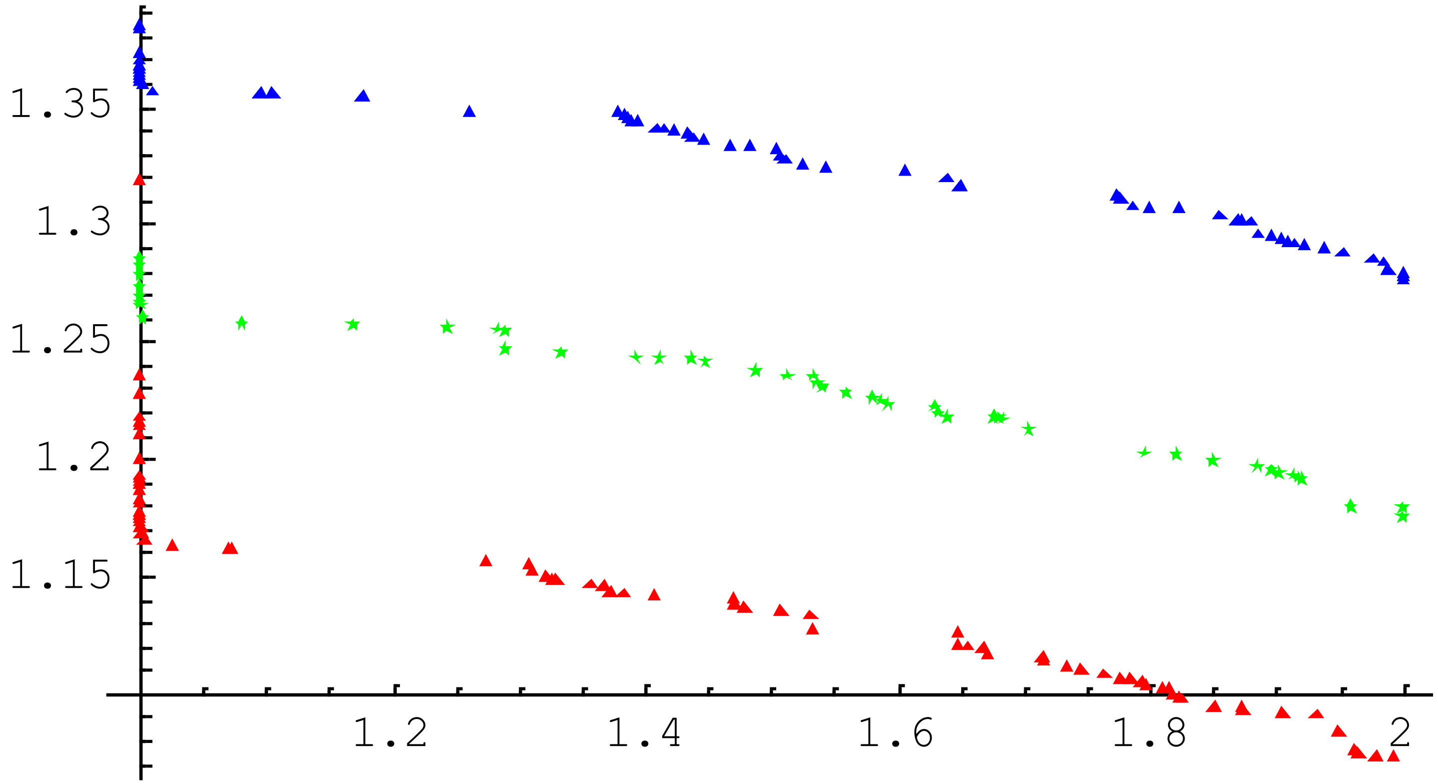




frequency of attaining regions

Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)



latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
see [Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preceq B :\Leftrightarrow \forall_{y \in B} \exists_{x \in A} x \leq_{par} y$$

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

$\Rightarrow$ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

$\Rightarrow$ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

! sought are total refinements… [Zitzler et al. 2010]

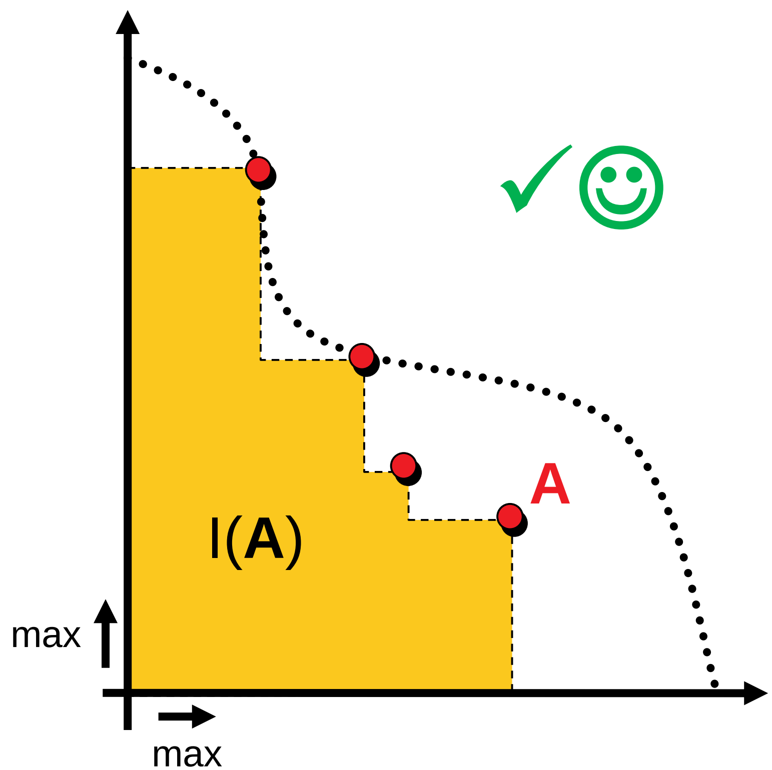
Example: Refinements Using Indicators

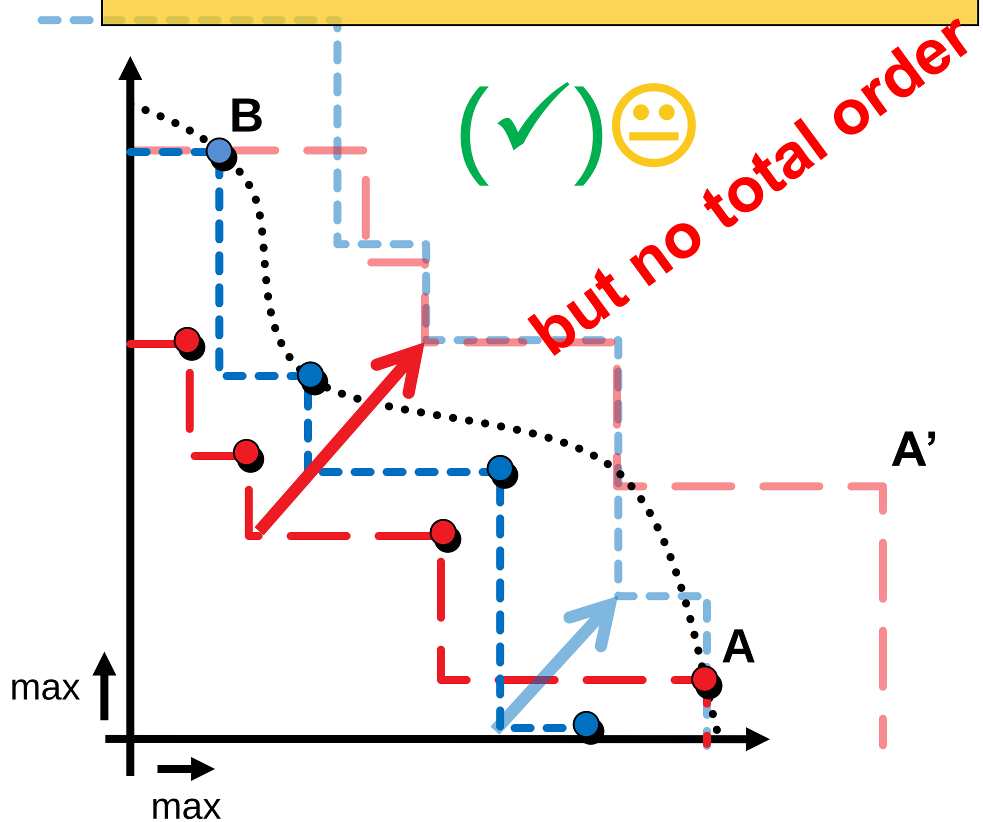
$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \geq I(B)$$

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,B) \leq I(B,A)$$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B





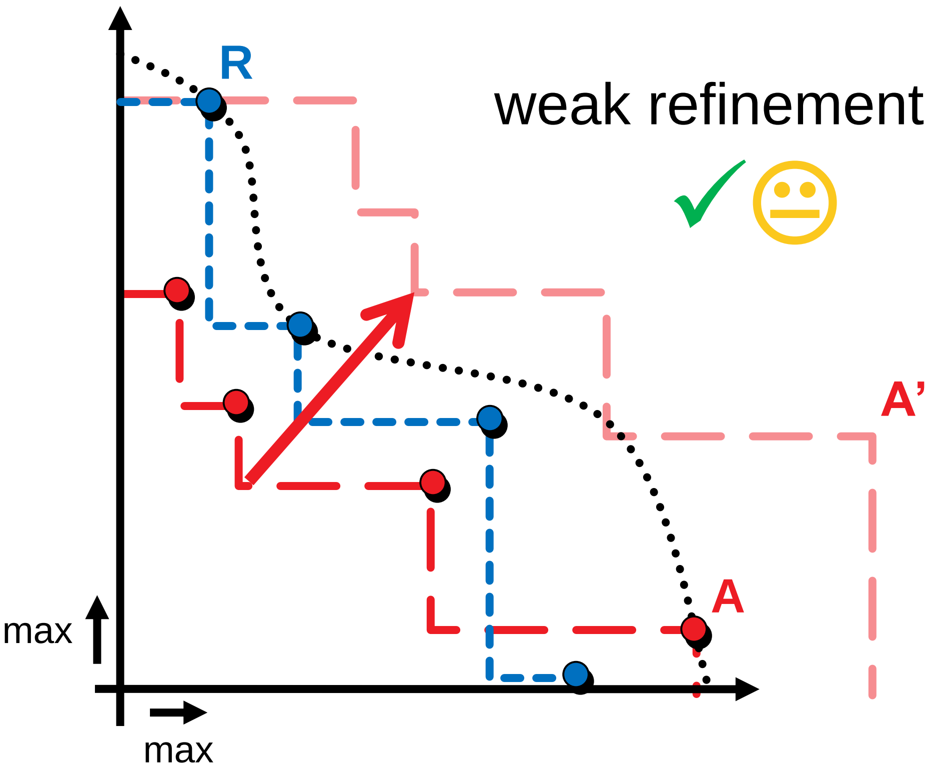
unary hypervolume indicator

binary epsilon indicator

Example: Weak Refinement / No Refinement

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A,R) \le I(B,R)$$

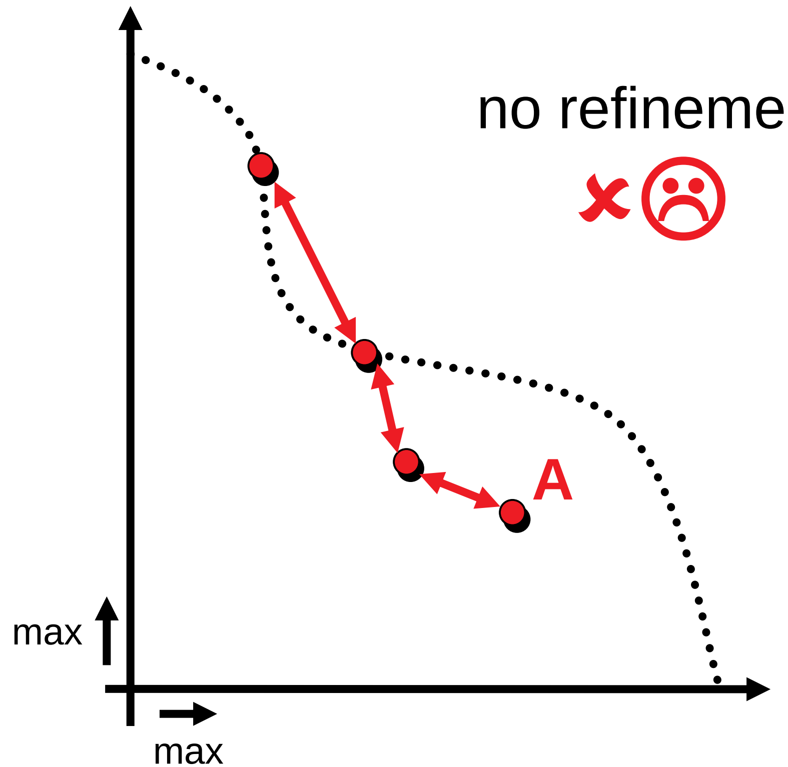
I(A,R) = how much needs A to be moved to weakly dominate R



unary epsilon indicator

$$A \overset{\text{ref}}{\preccurlyeq} B :\Leftrightarrow I(A) \le I(B)$$

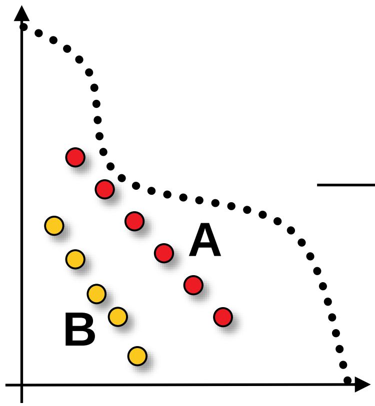
I(A) = variance of pairwise distances



unary diversity indicator

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



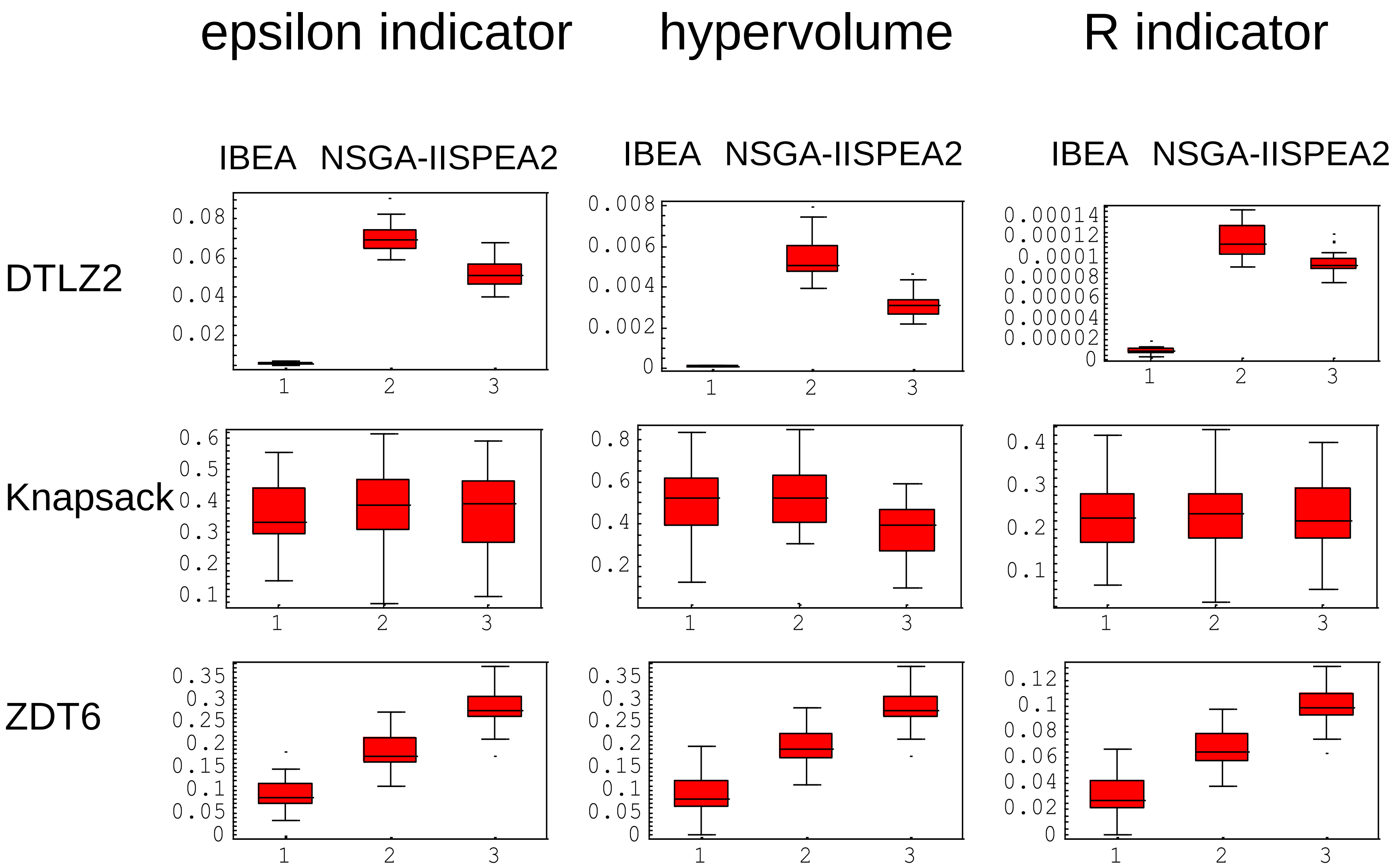
	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow[\textbf{reduction}]{\text{quality measure}} \mathbb{R}^n \xrightarrow[\textbf{interpretation}]{\text{Boolean function}} \text{statement}$$

Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

Set Quality Indicators

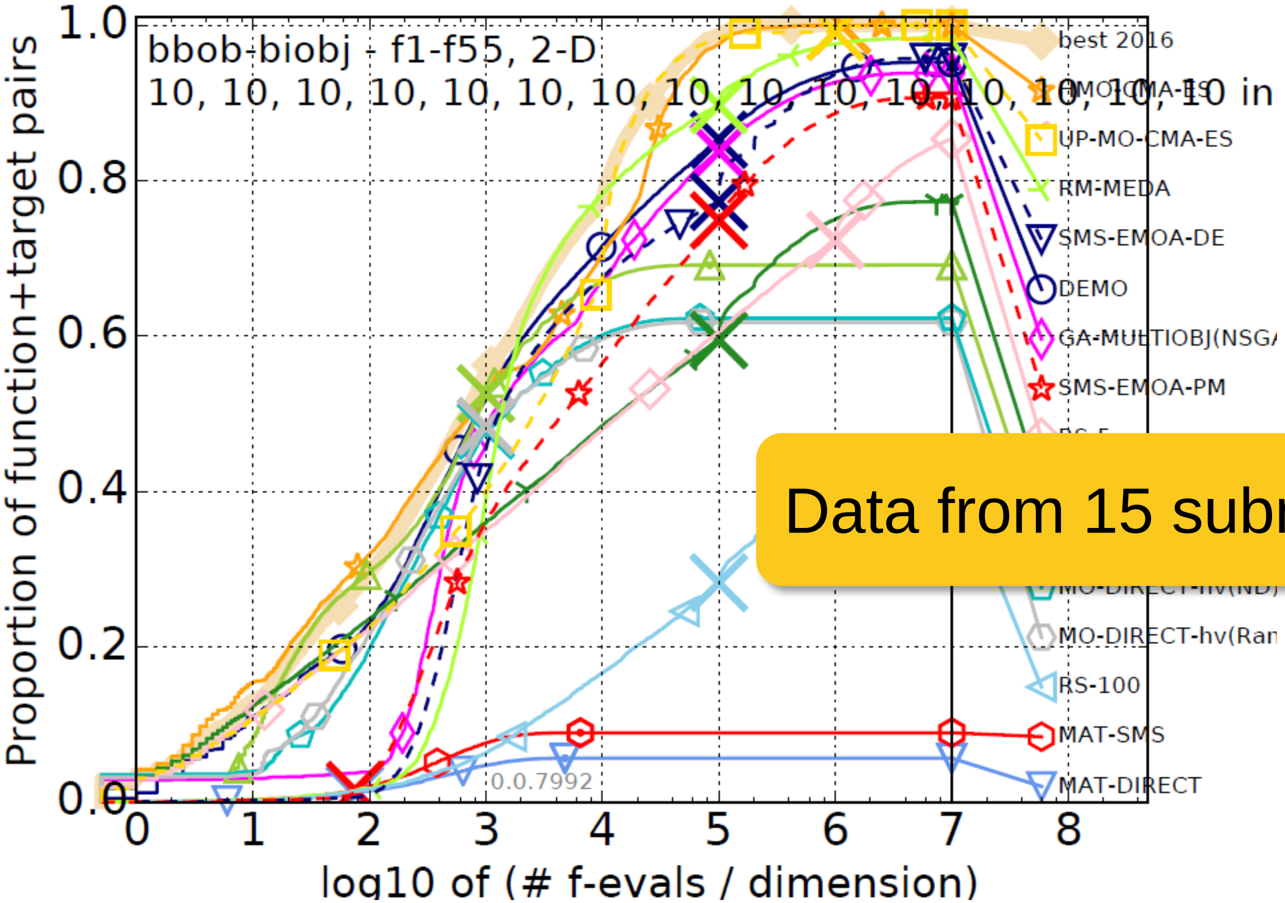
Open Questions:

- are there other unary indicators that are (weak) refinements?
- how to compute indicators efficiently (enough for practice)?
  - especially for >3 objective functions
- how to achieve good indicator values?

Automated Benchmarking

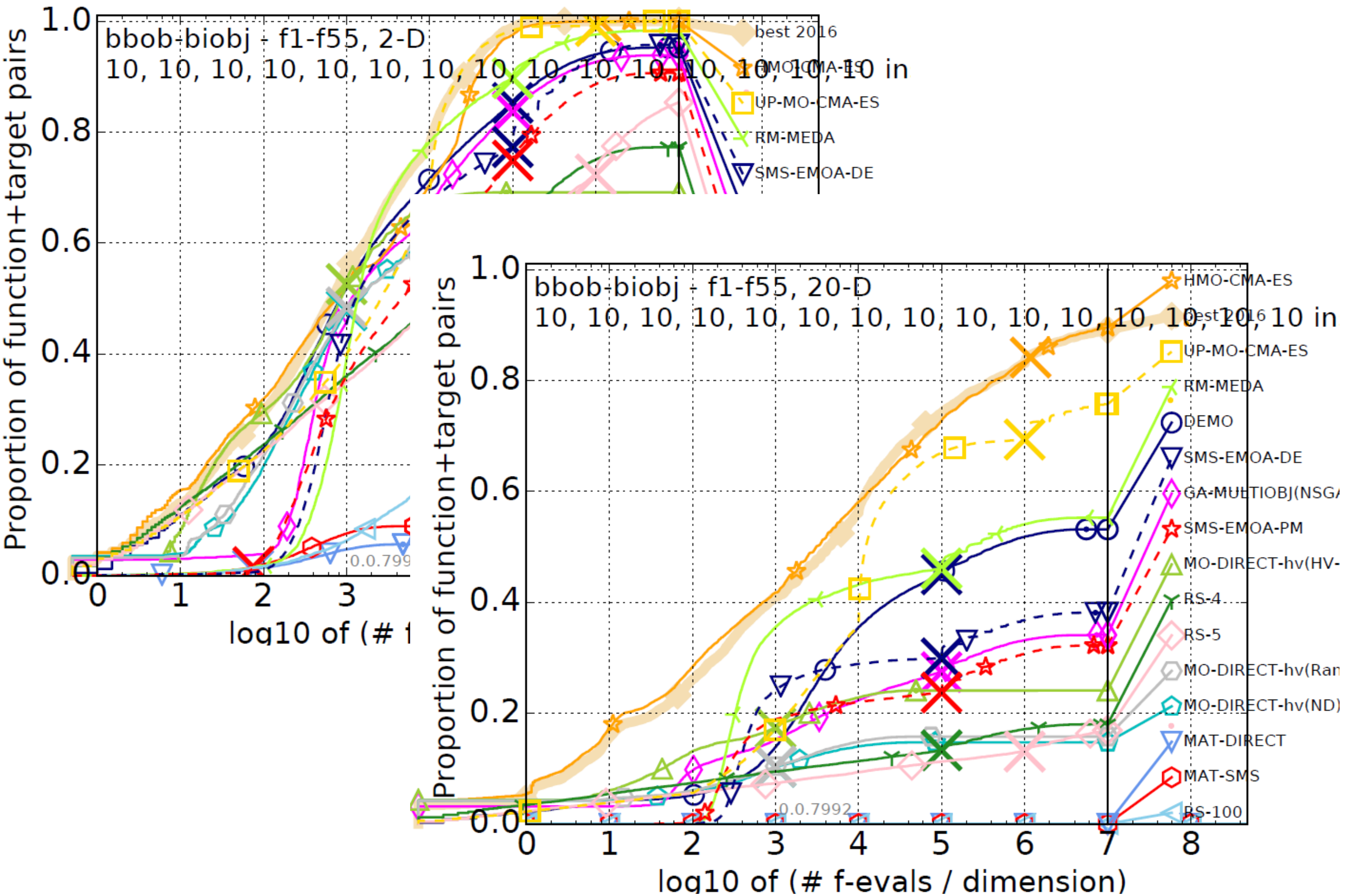
- State-of-the-art in single-objective optimization: Blackbox Optimization Benchmarking (BBOB) with COCO platform
  `https://github.com/numbbo/coco`
- This year: first release of a bi-objective test suite and corresponding BBOB-2016 workshop @ GECCO
- Focus on target-based runlengths
  - gives (nearly) anytime, interpretable results
  - defines problem=(test function instance, single-objective goal e.g. min. indicator difference to reference set, target precision)
  - reports average runtimes (aRT) to reach target precision
- COCO provides data profiles, scaling plots, scatter plots, tables, statistical tests, etc. automatically

Exemplary BBOB-2016 Results



Data from 15 submitted algorithms

Exemplary BBOB-2016 Results



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A Few Examples From Practice

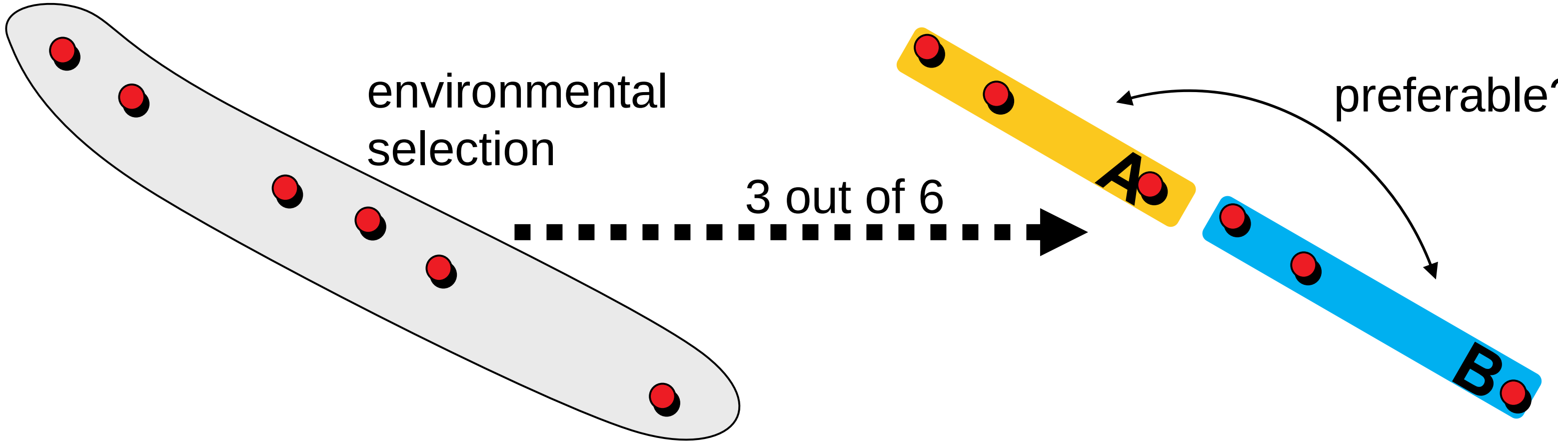
Articulating User Preferences During Search

What we thought: EMO is preference-less

Search before decision making: Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

[Zitzler 1999]

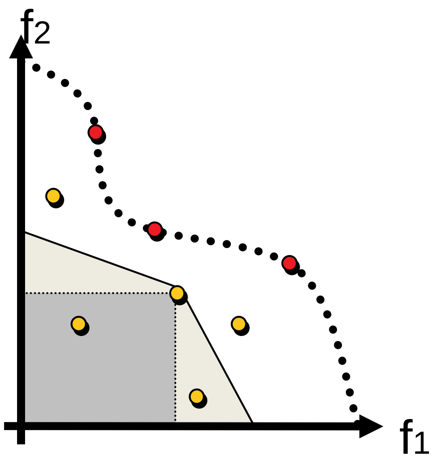
What we learnt: EMO just uses weaker preference information



Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is large

[Branke and Deb 2004] [Branke 2008] [Bechikh et al. 2015]



❶ Refine/modify dominance relation, e.g.:

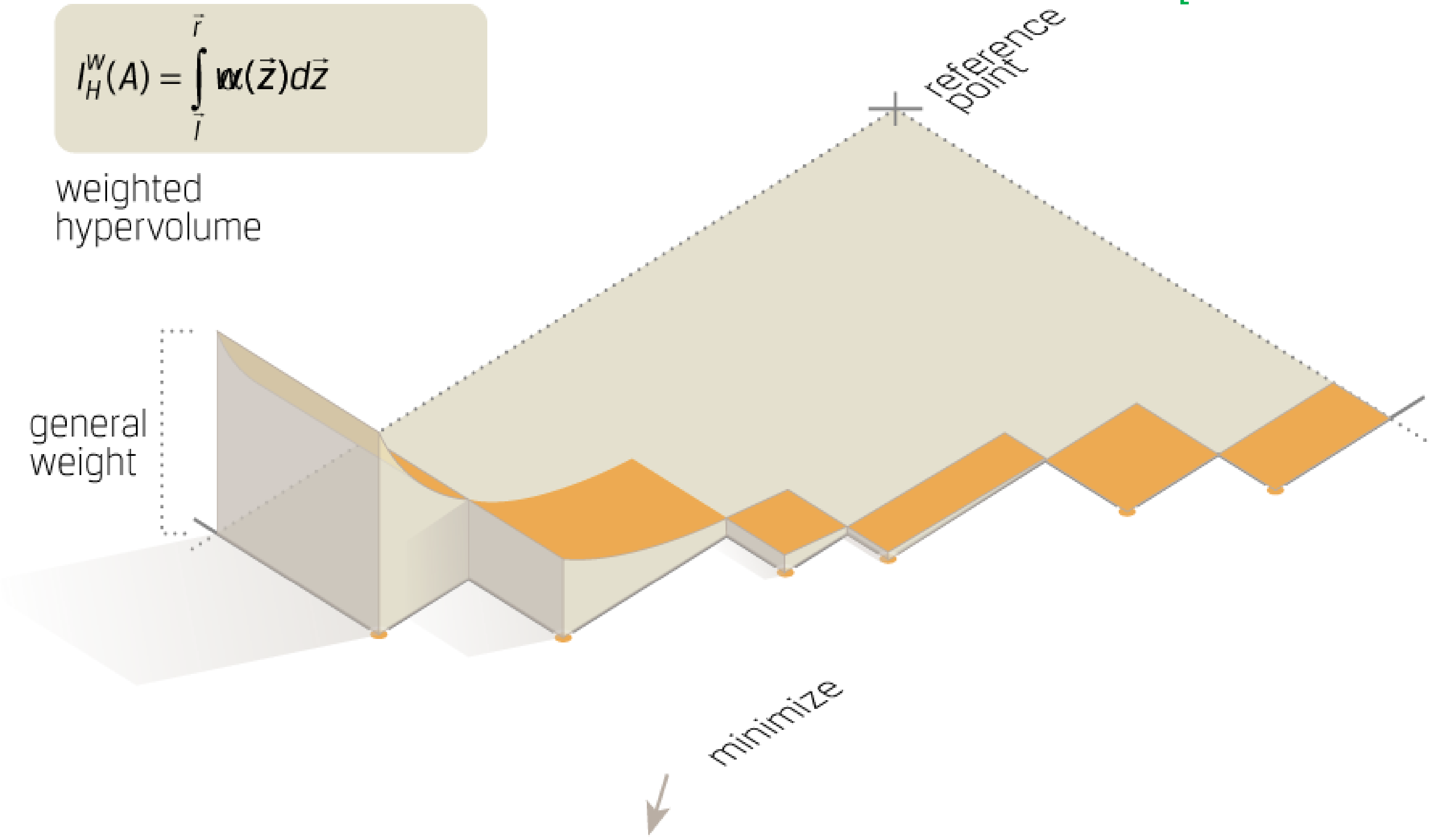
- using goals, priorities, constraints
  [Fonseca and Fleming 1998a,b]
- using different types of dominance cones
  [Branke and Deb 2004]

❷ Use quality indicators, e.g.:

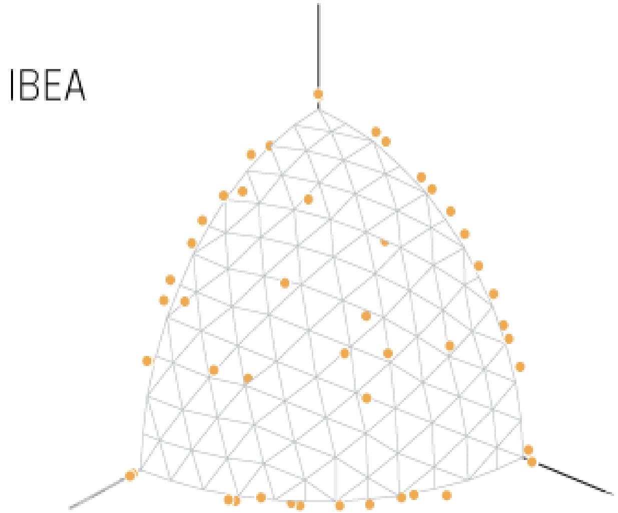
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on the hypervolume indicator
  [Brockhoff et al. 2013] [Wagner and Trautmann 2010]
- based on the R2 indicator [Trautmann et al. 2013]

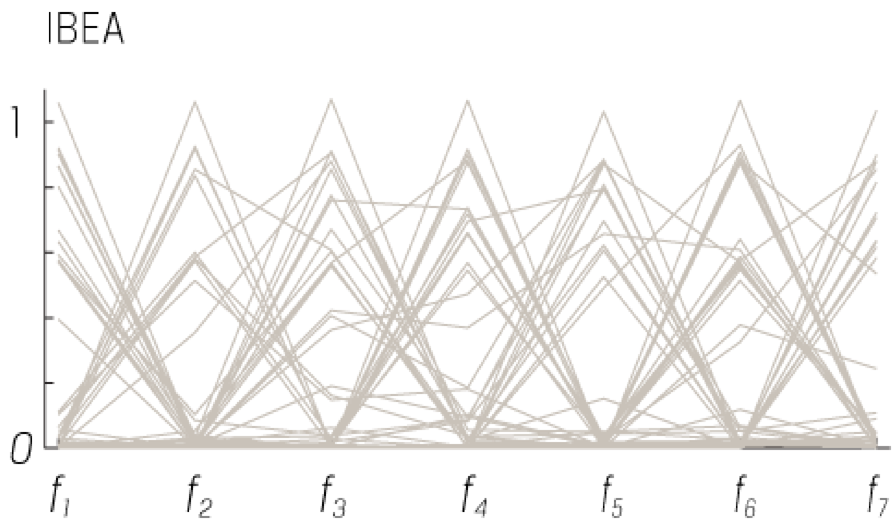
Example: Weighted Hypervolume Indicator

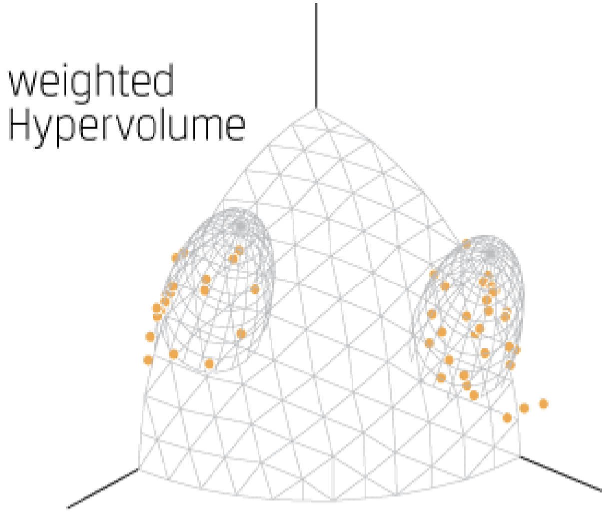
[Brockhoff et al. 2013]

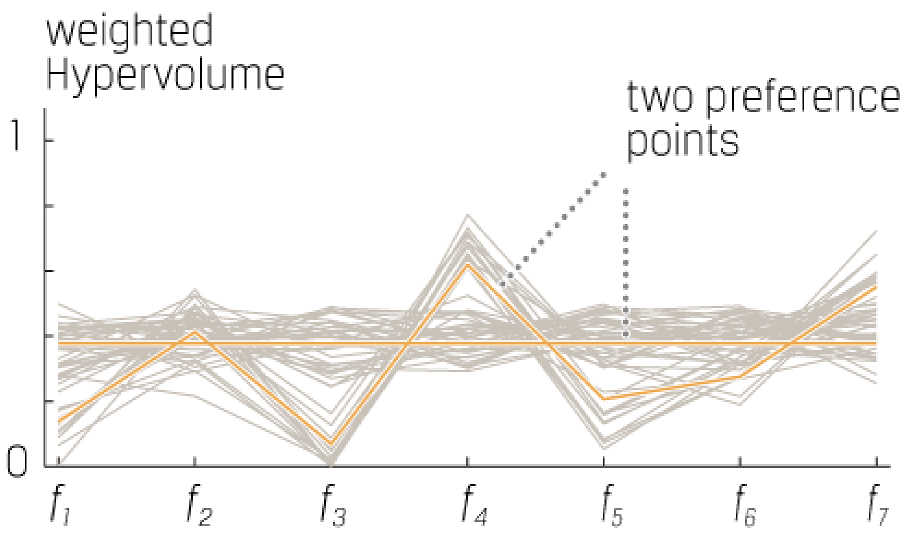


Weighted Hypervolume in Practice







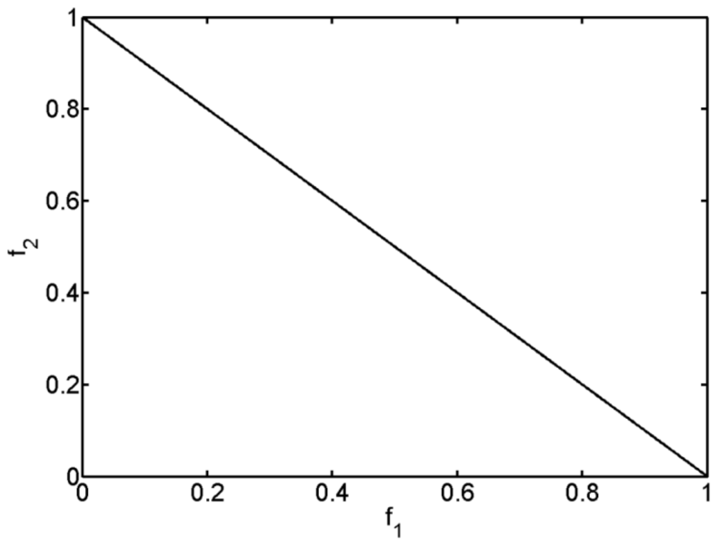


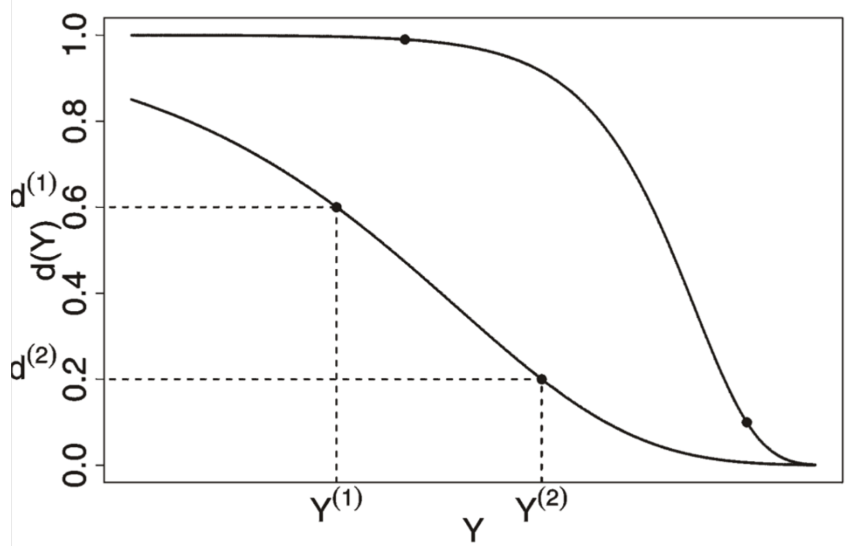
[Auger et al. 2009b]

Example: Desirability Function (DF)-SMS-EMOA

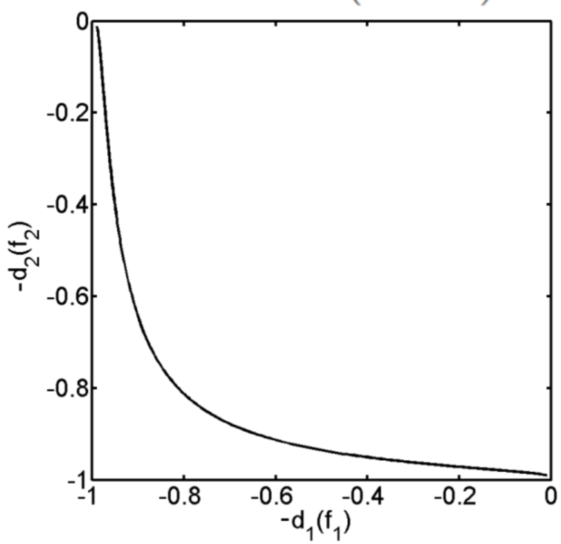
[Wagner and Trautmann 2010]

Shape of the untransformed Pareto front

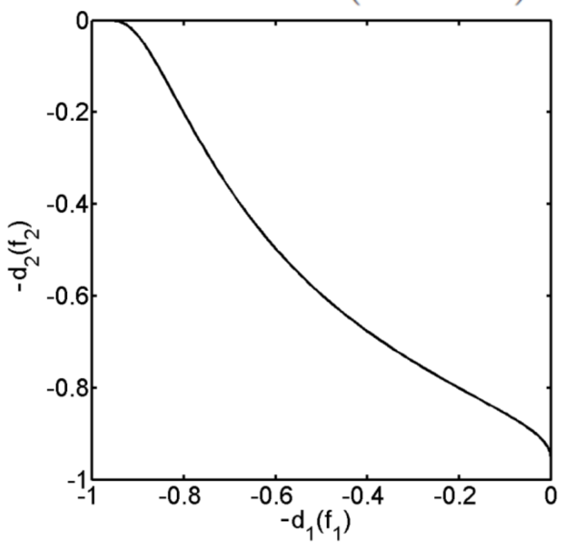




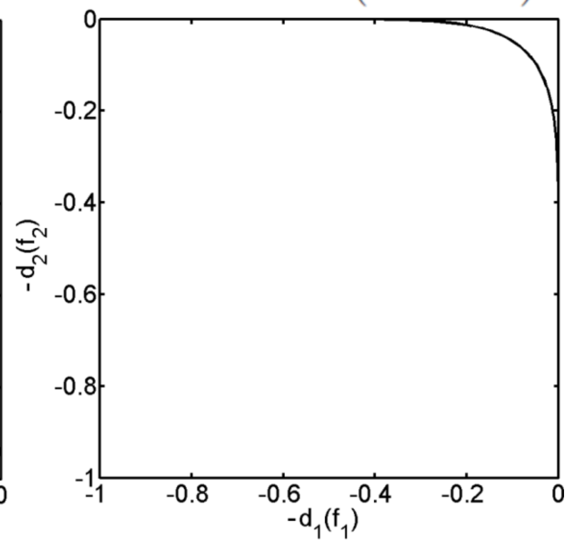
Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 1 & 0.01 \end{pmatrix}$



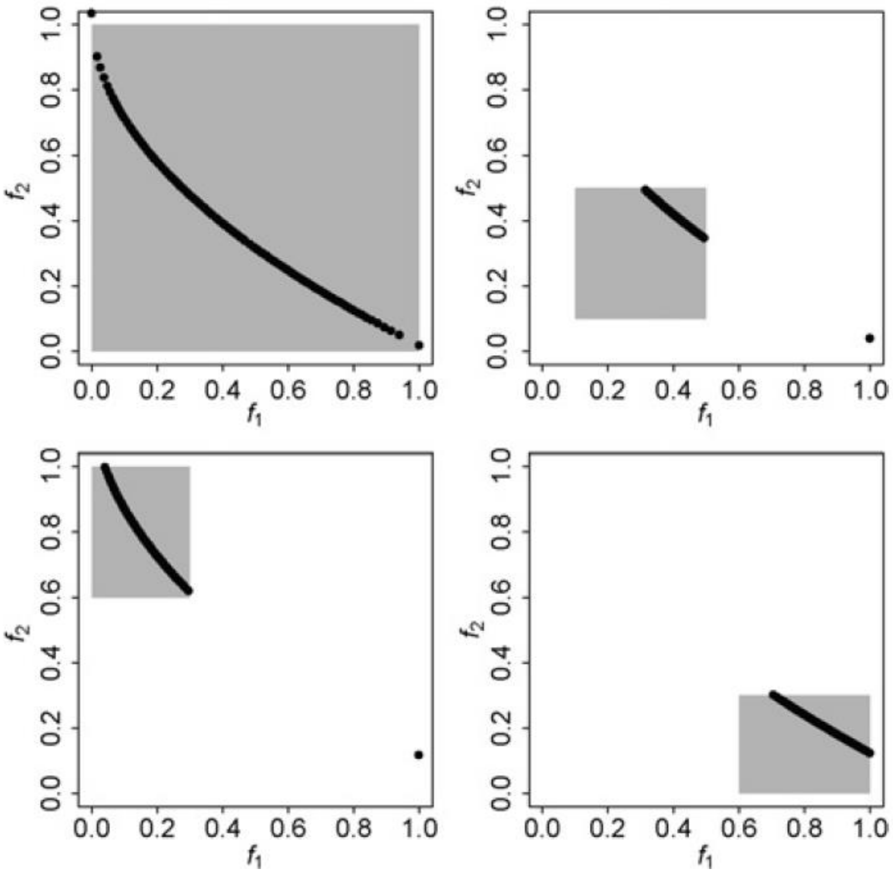
Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.75 & 0.01 \end{pmatrix}$

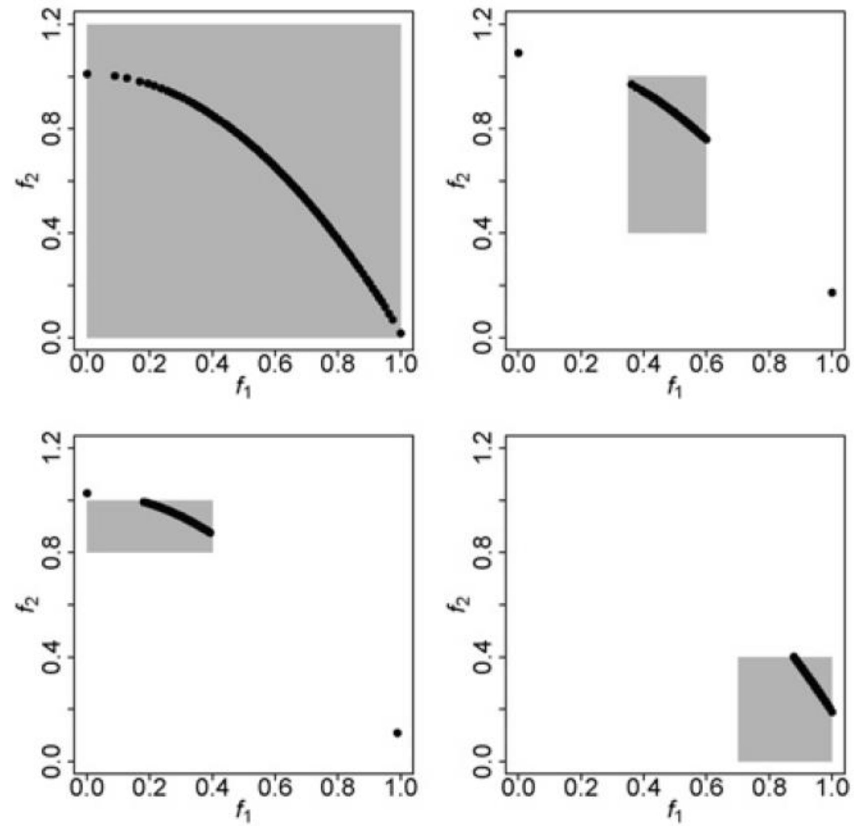


Shape of the transformed front for identical DFs with $\begin{pmatrix} 0 & 0.99 \\ 0.55 & 0.01 \end{pmatrix}$



DF-SMS-EMOA in Practice



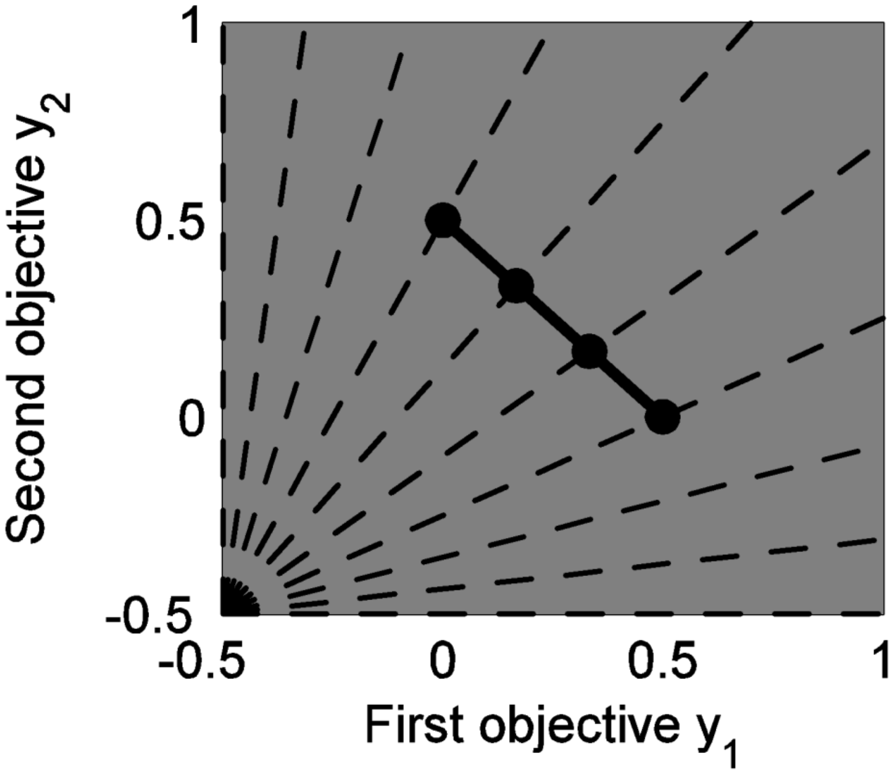


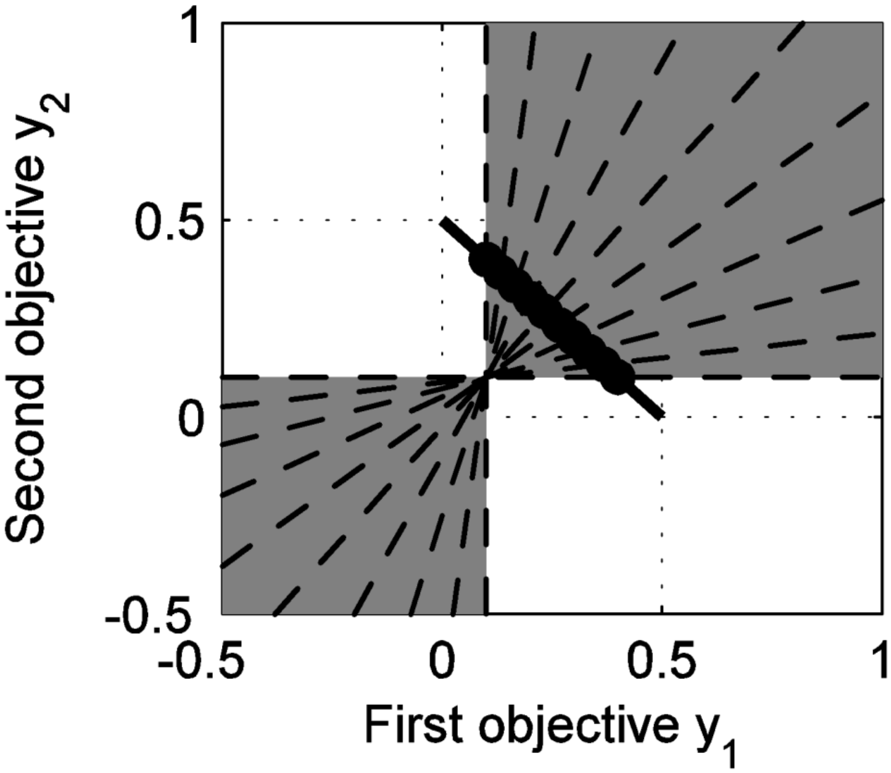
Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Position of ideal point



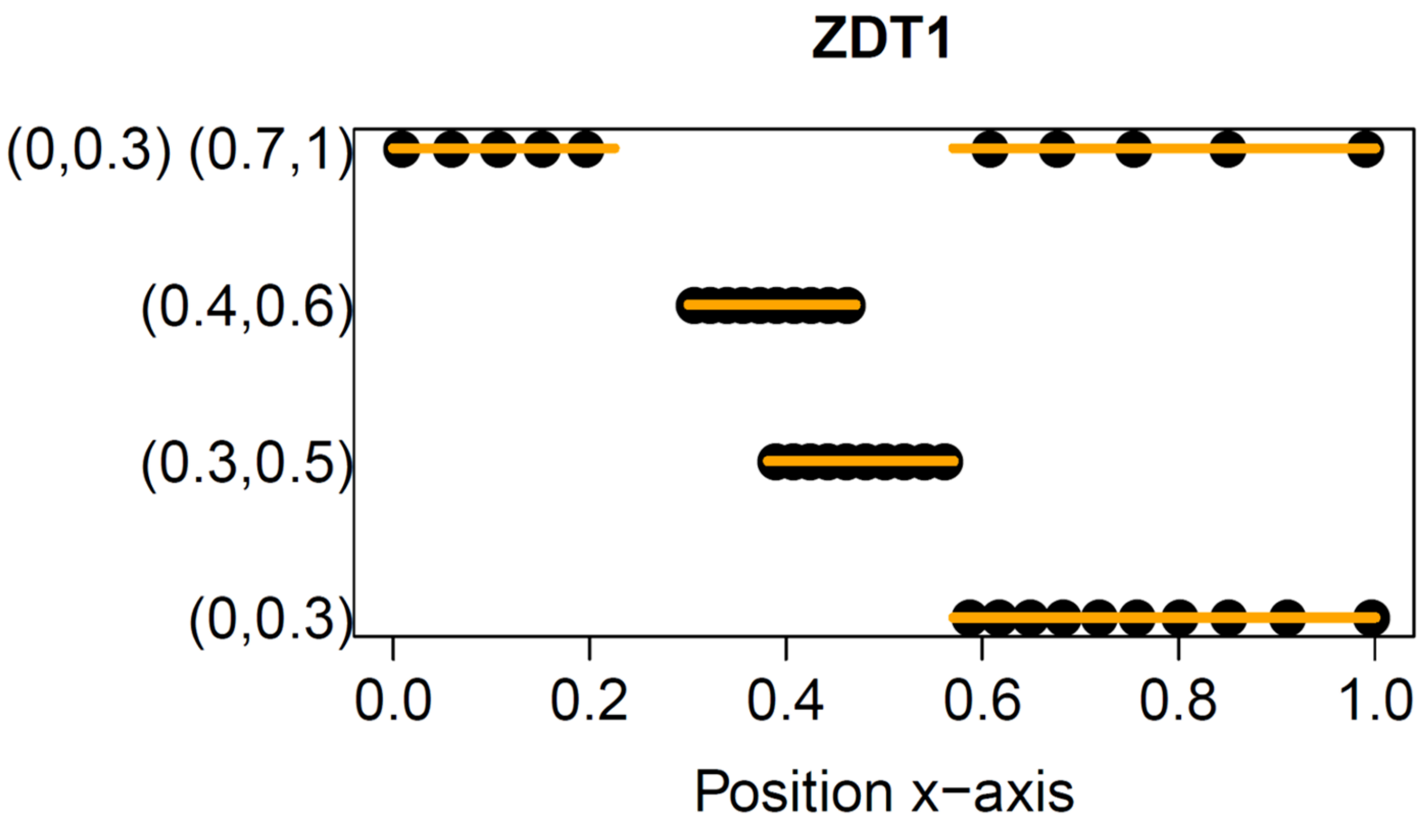


Example: R2-EMOA

Concept

Integration of preferences by varying the scalarizing functions

Restriction of the weight space



Successive Preference Articulation = Interactive EMO

- recent interest of both EMO and MCDM community
- important in practice

Examples

- first interactive EMO: [Tanino et al. 1993]
- good overview: [Jaszkiewicz and Branke 2008]
- more recent work: [Brockhoff et al. 2014] [Branke et al. 2014]

Issues/Open Questions

- realistic scenarios/ value functions
- evaluation of interactive algorithms [López-Ibáñez and Knowles 2015]

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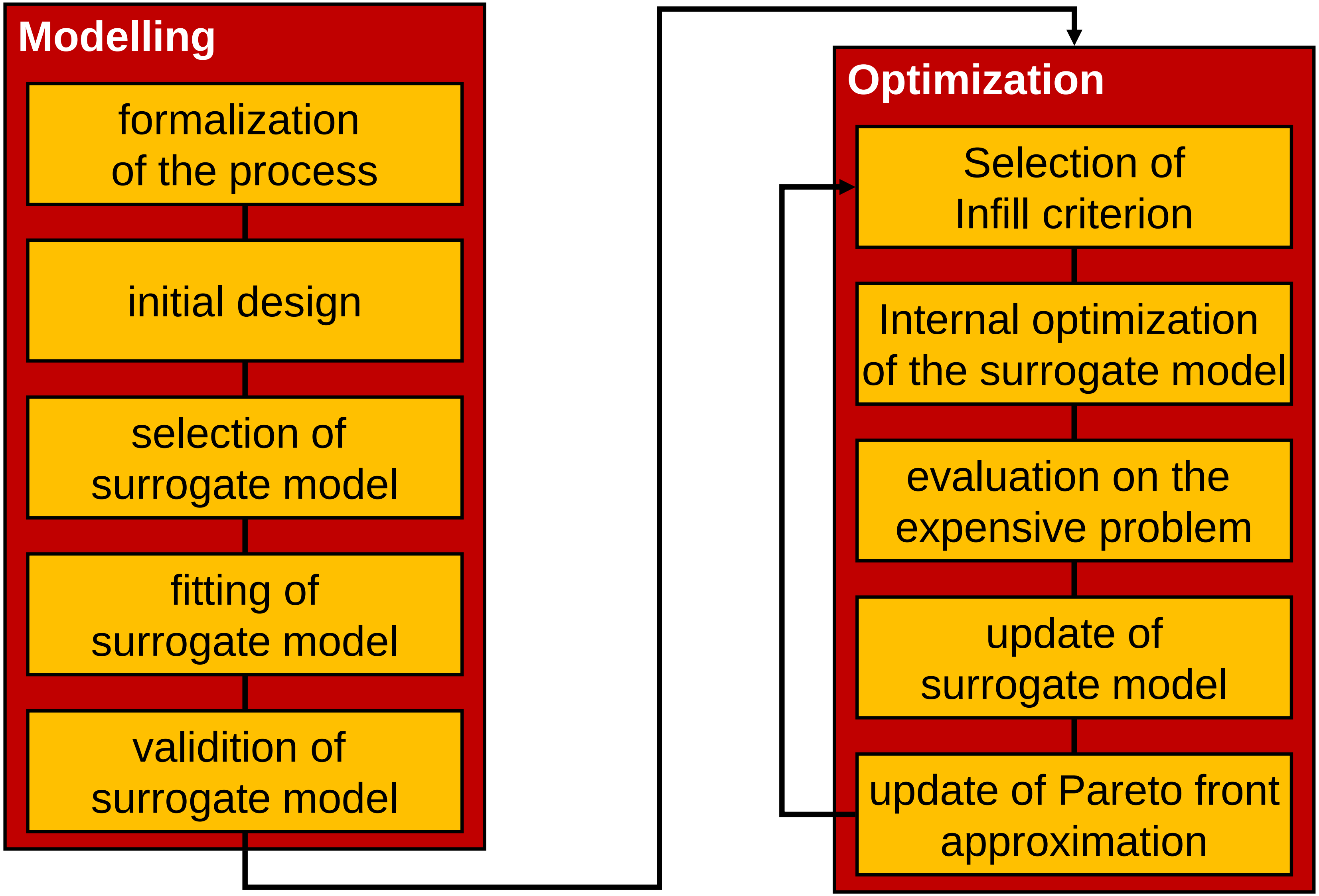
Surrogate-Based EMO

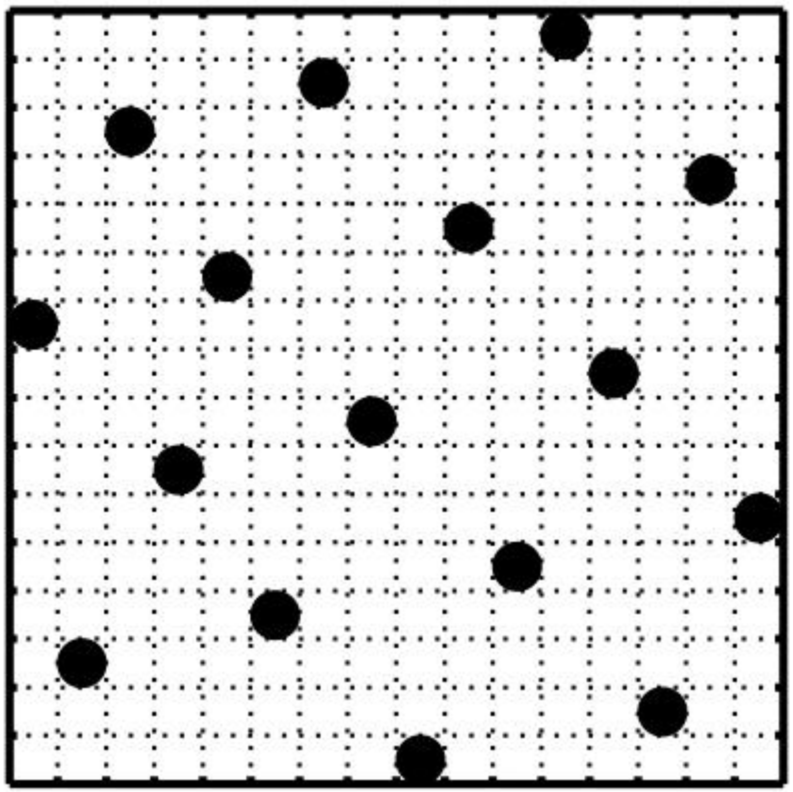
EMO + modeling and sequential experimental design



Initial design

Latin Hypercube Sampling (LHS)

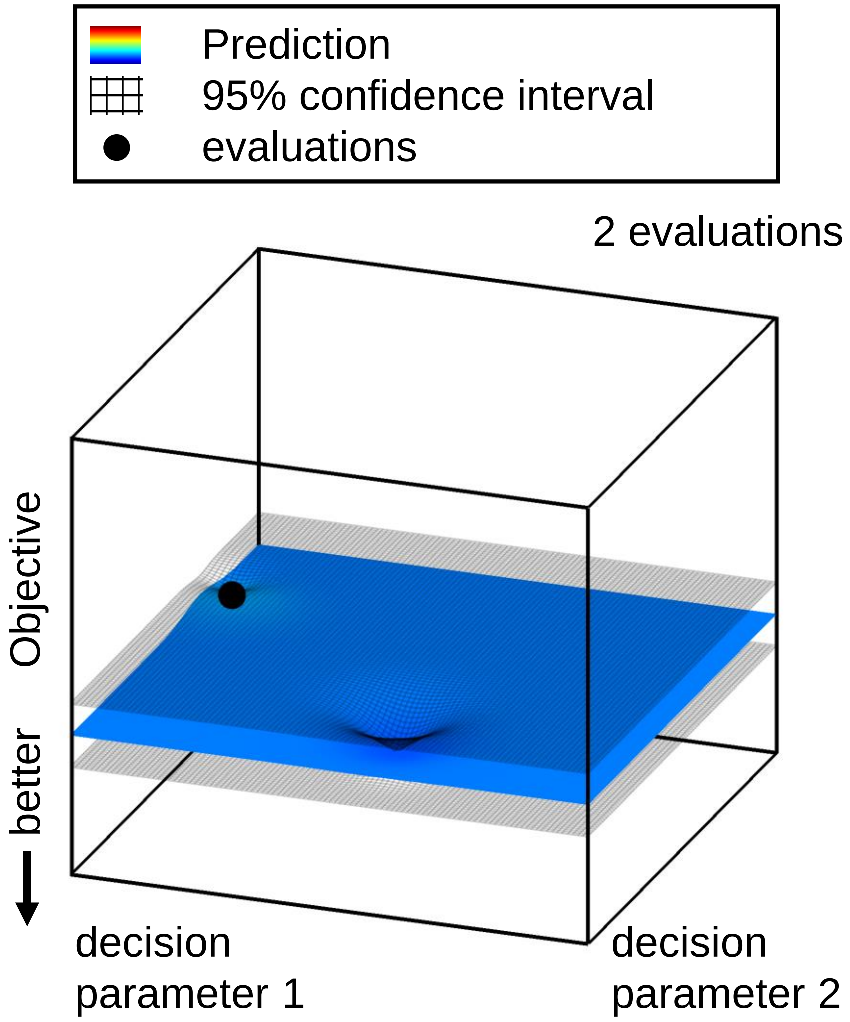
- Space-filling coverage of the decision space
- Maximum resolution for each parameter



Design and Analysis of Computer Experiments (DACE)

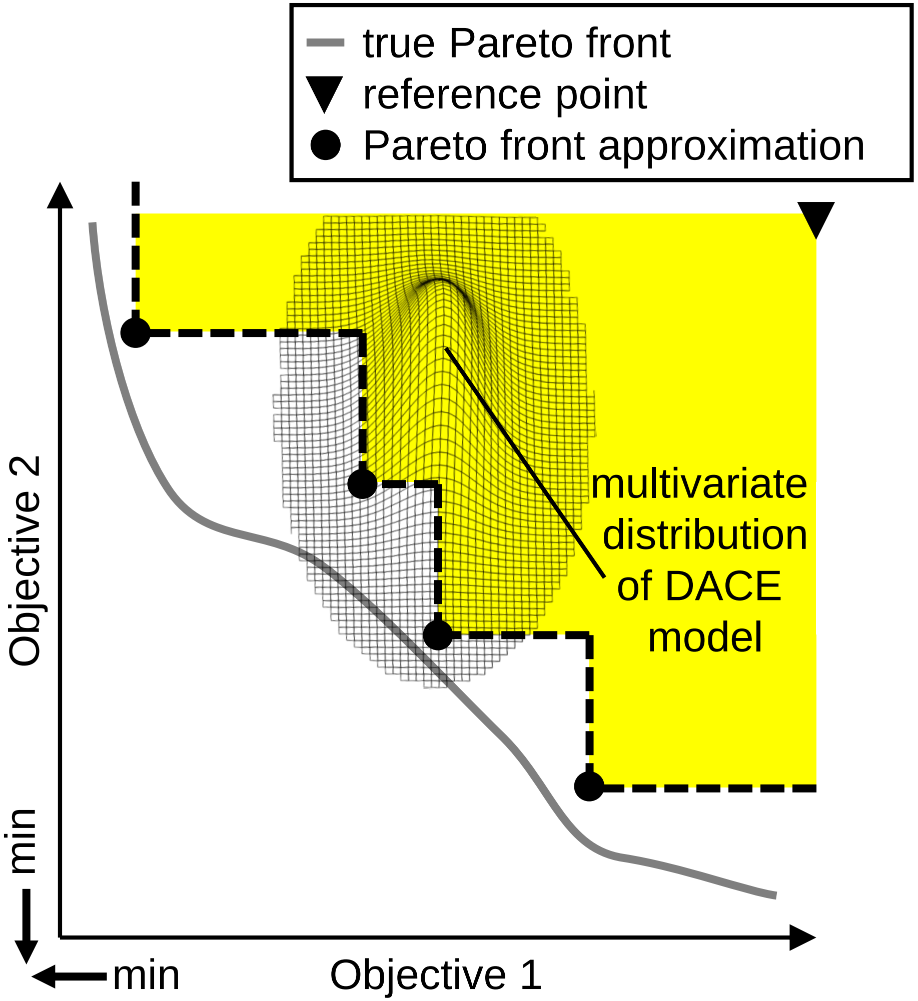
[Sacks et al. 1989]

- based on kriging models
- local modeling of the available evaluations
- possibility of adaptive refinement



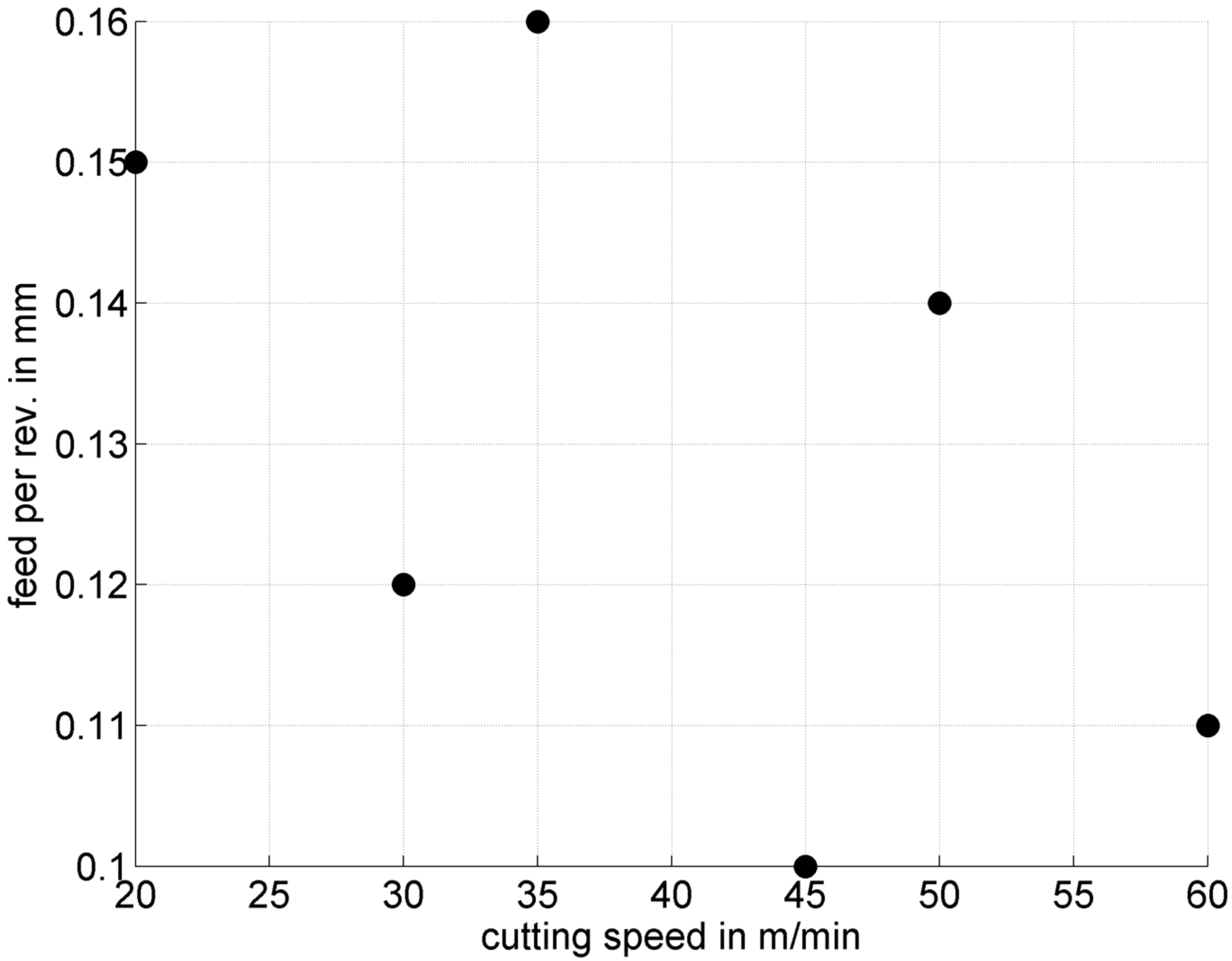
Aim

- refinement of the Pareto front approximation
- maximization of the dominated hypervolume
- improvements through experiments in the currently nondominated area
- see [Wagner et al. 2010] for a survey and first theoretical results



Initial design

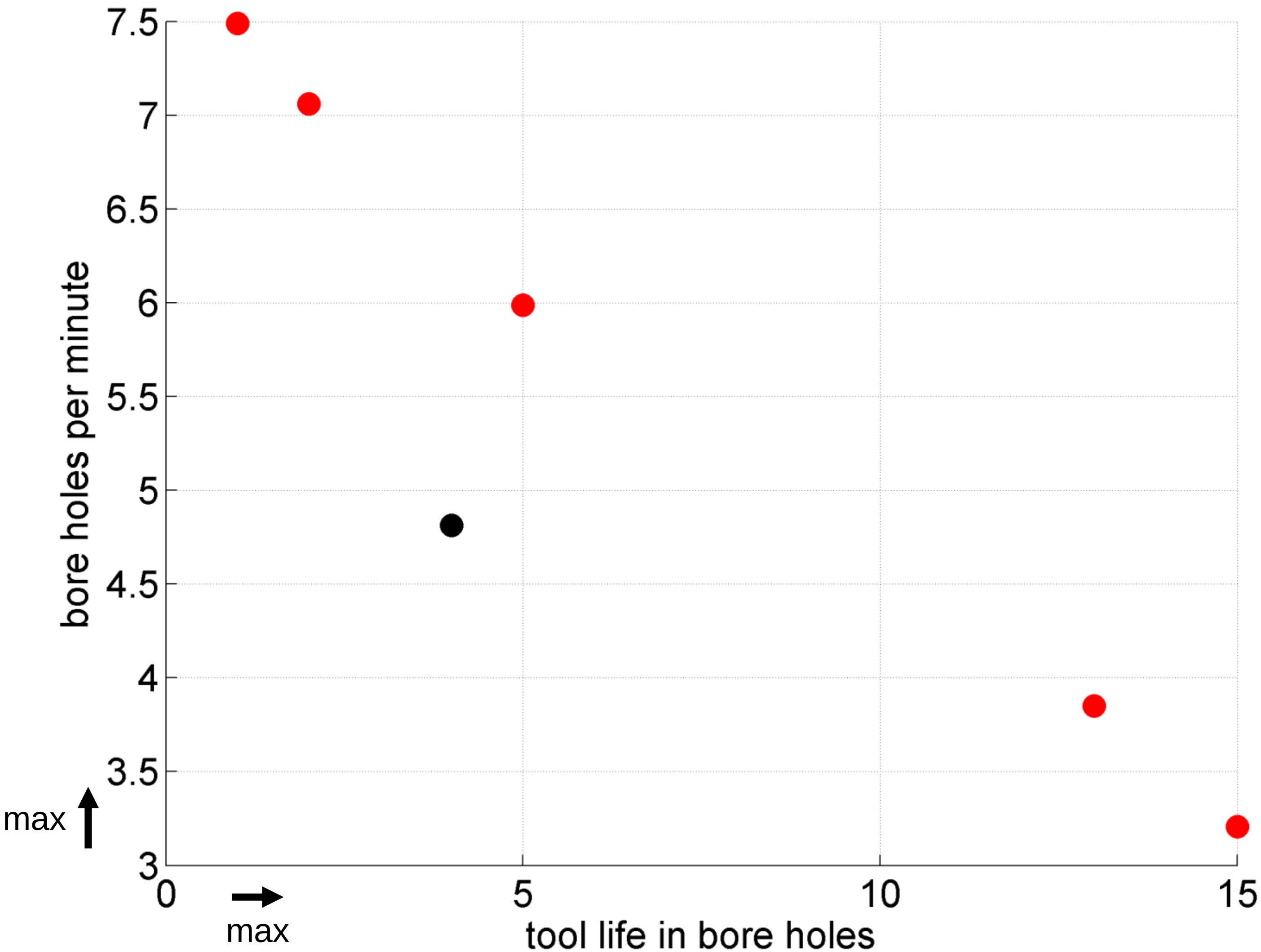
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Initial Pareto front approximation

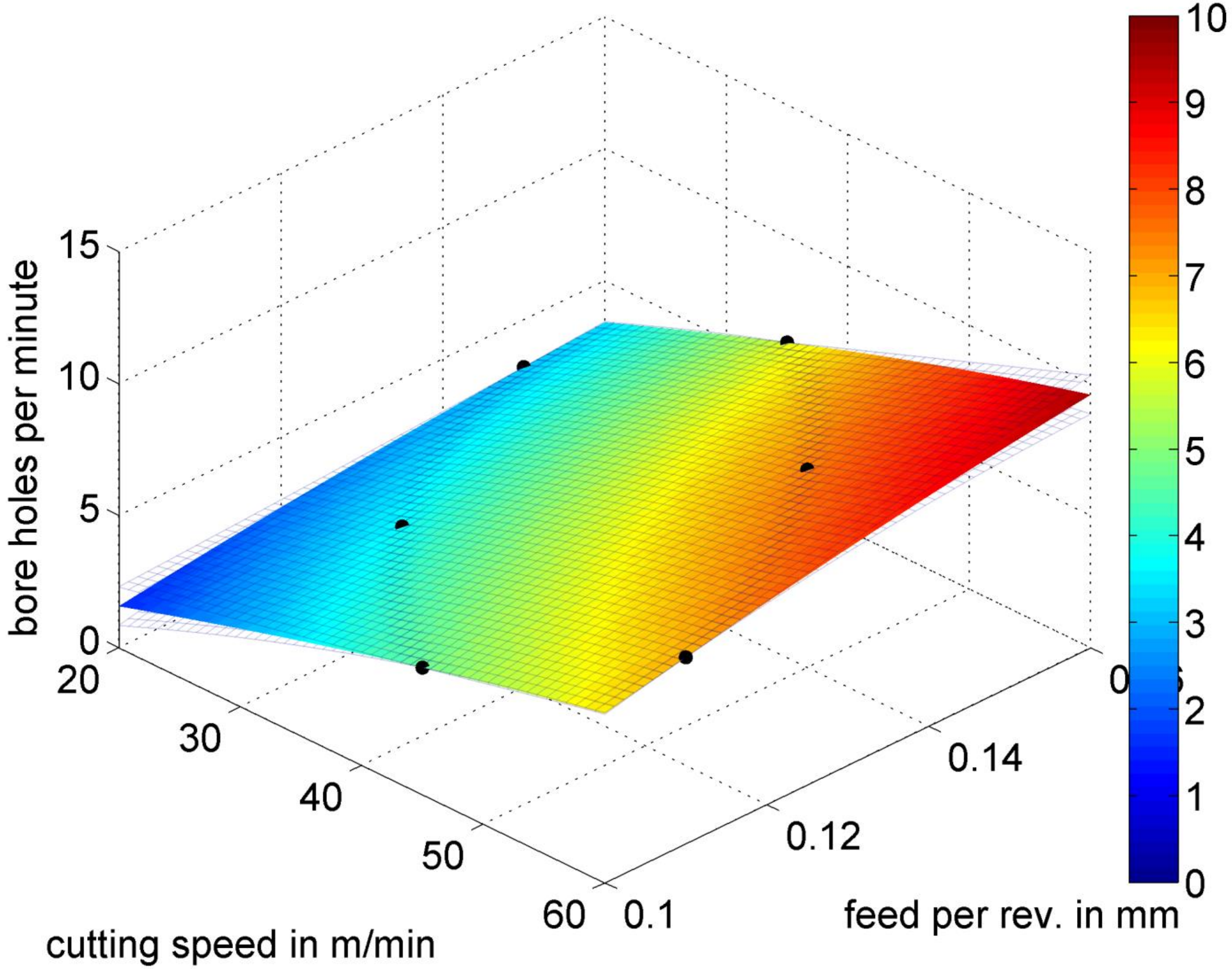
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Initial model for objective 1 (productivity)

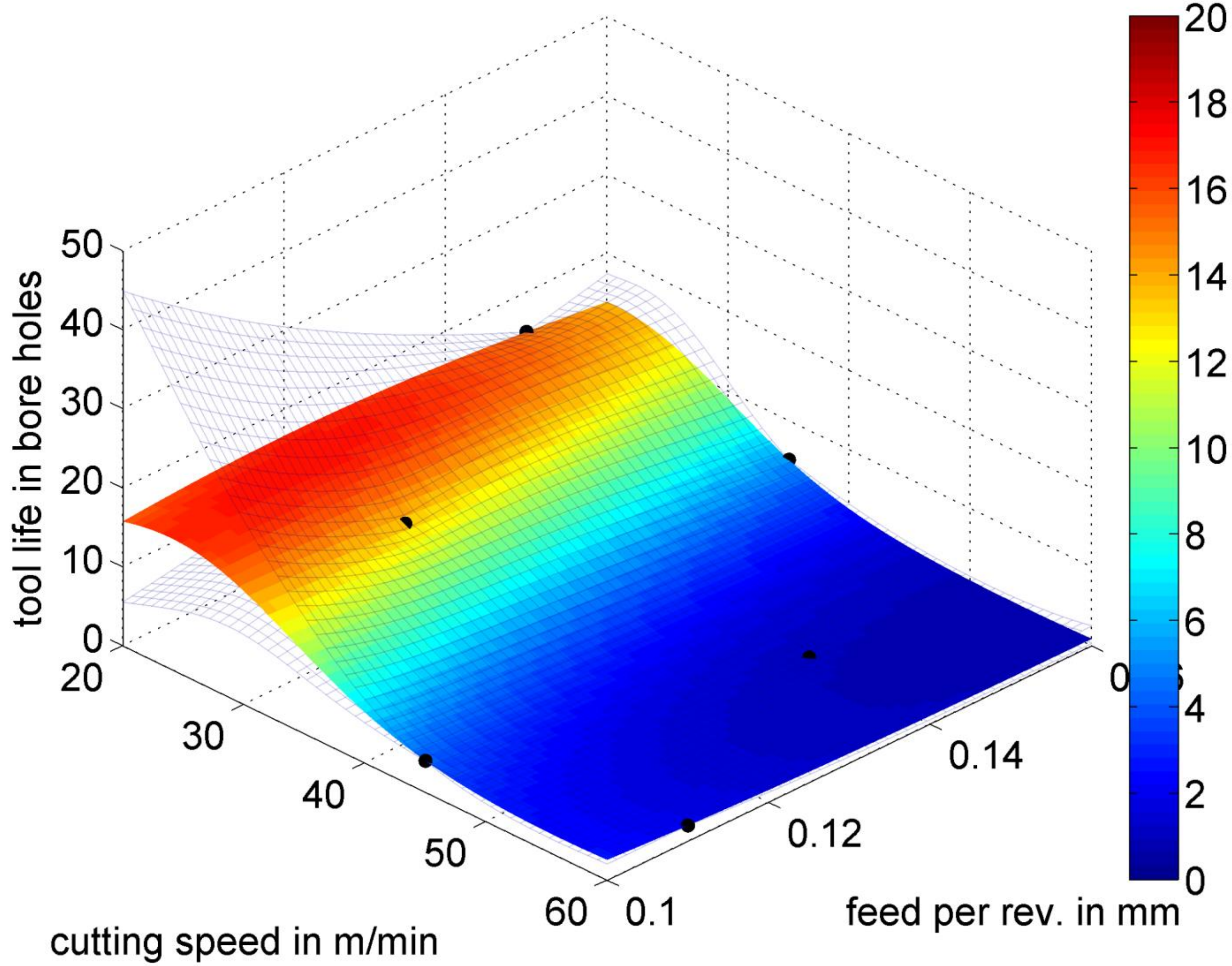
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

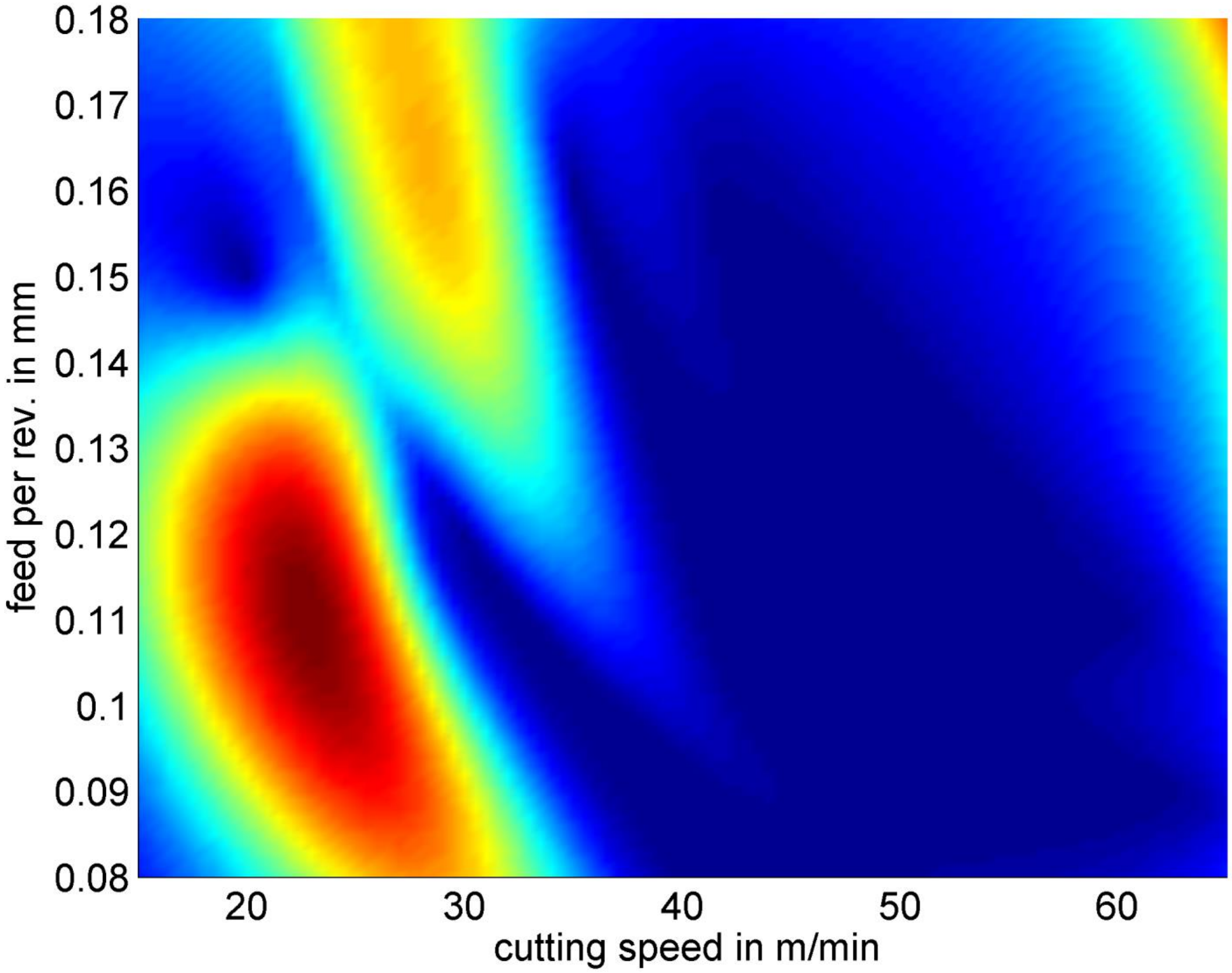
Initial model for objective 2 (tool life)

[Zhang et al. 2012]



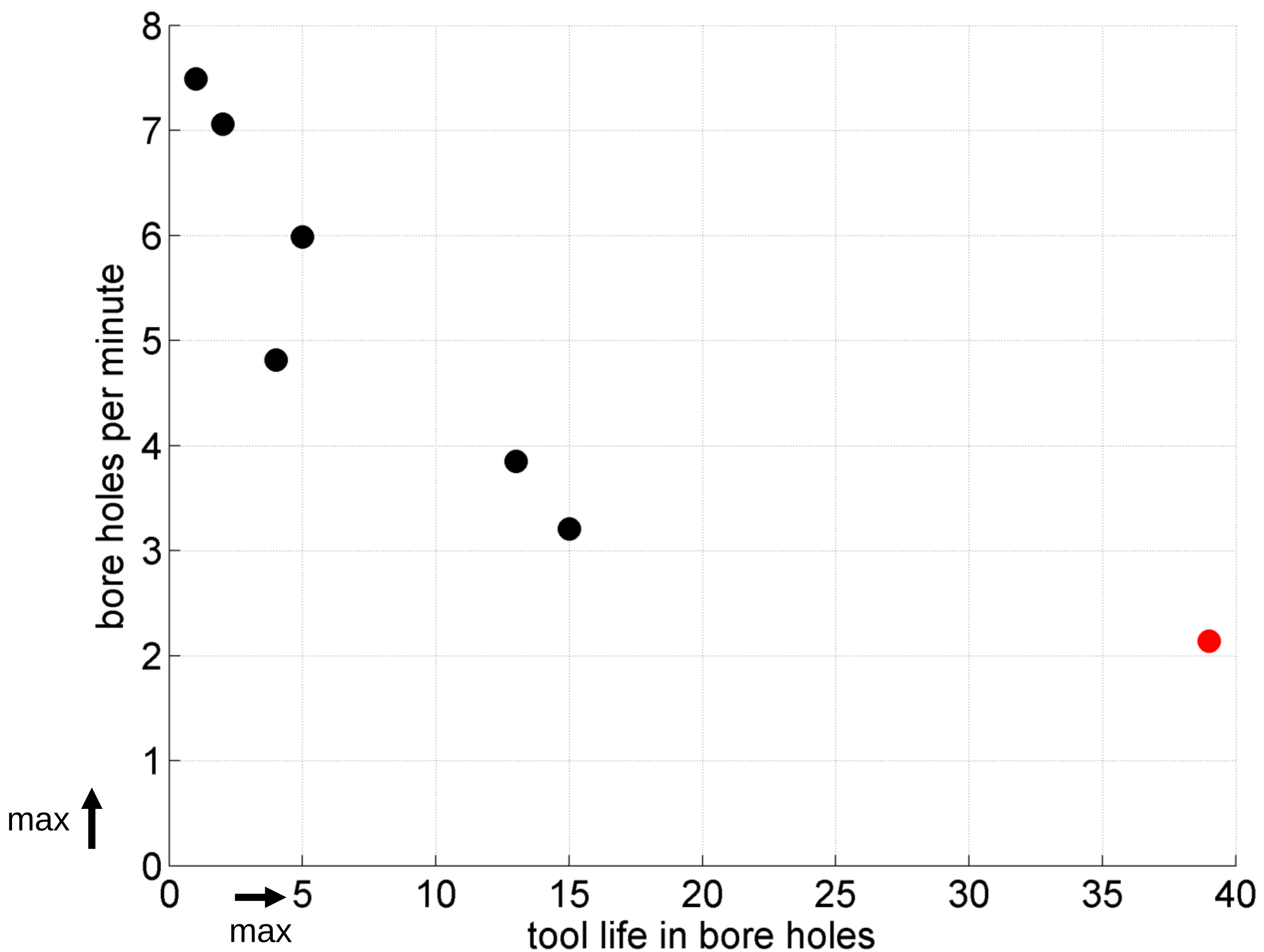
Practical application (drilling of Inconel 708)

Optimization of Infill criterion



Updated Pareto front approximation

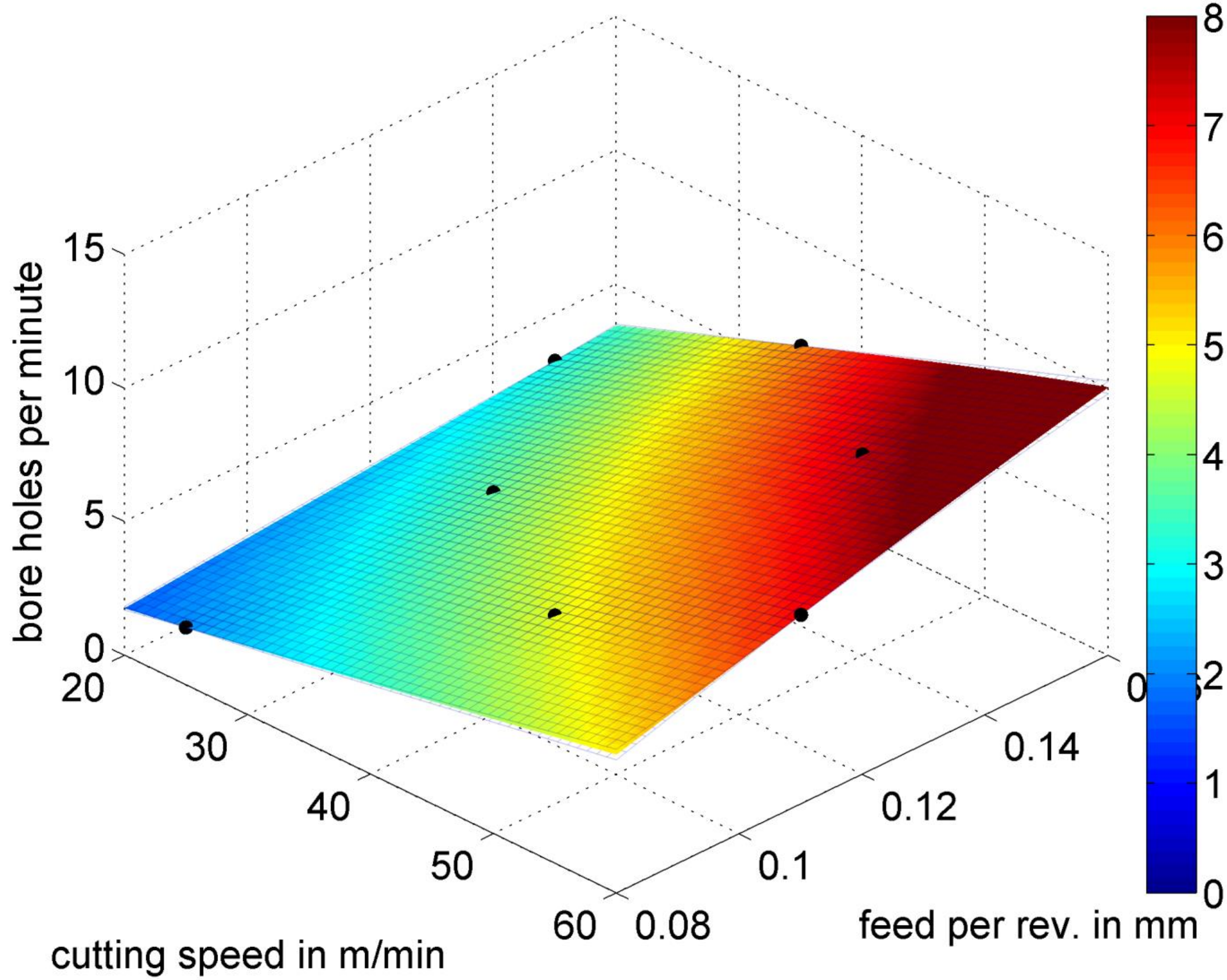
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Updated model for objective 1 (productivity)

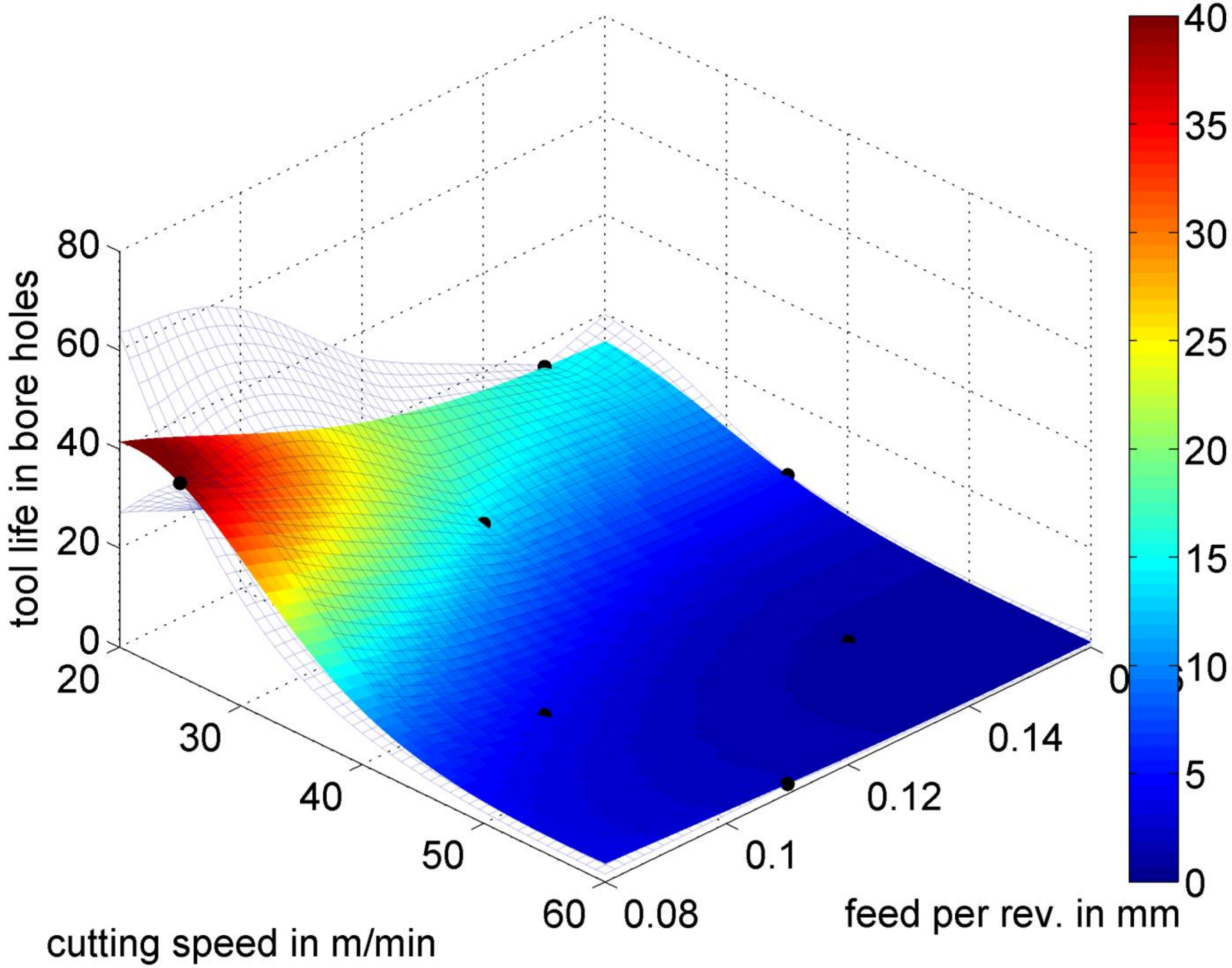
[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

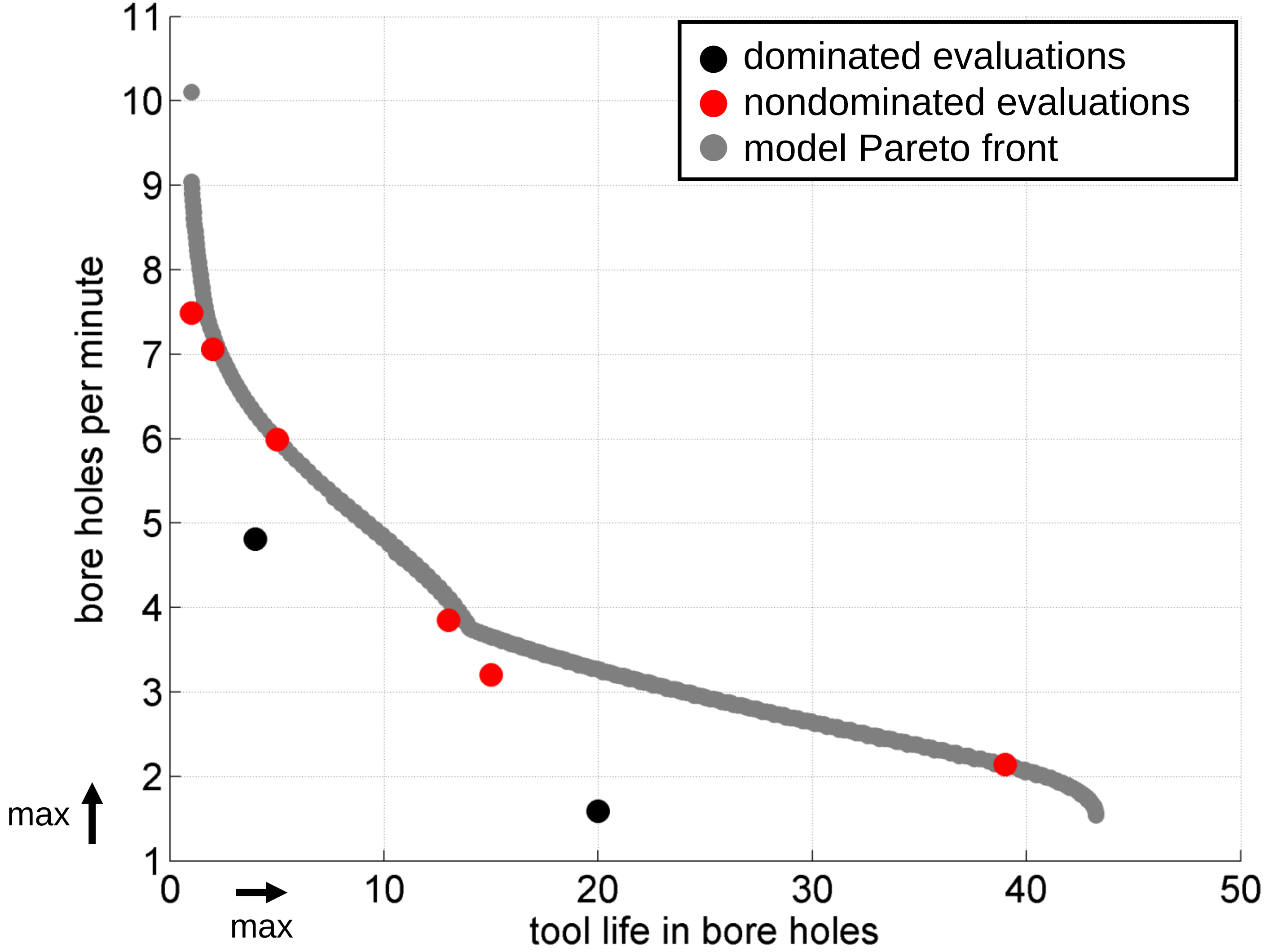
Updated model for objective 2 (tool life)

[Zhang et al. 2012]



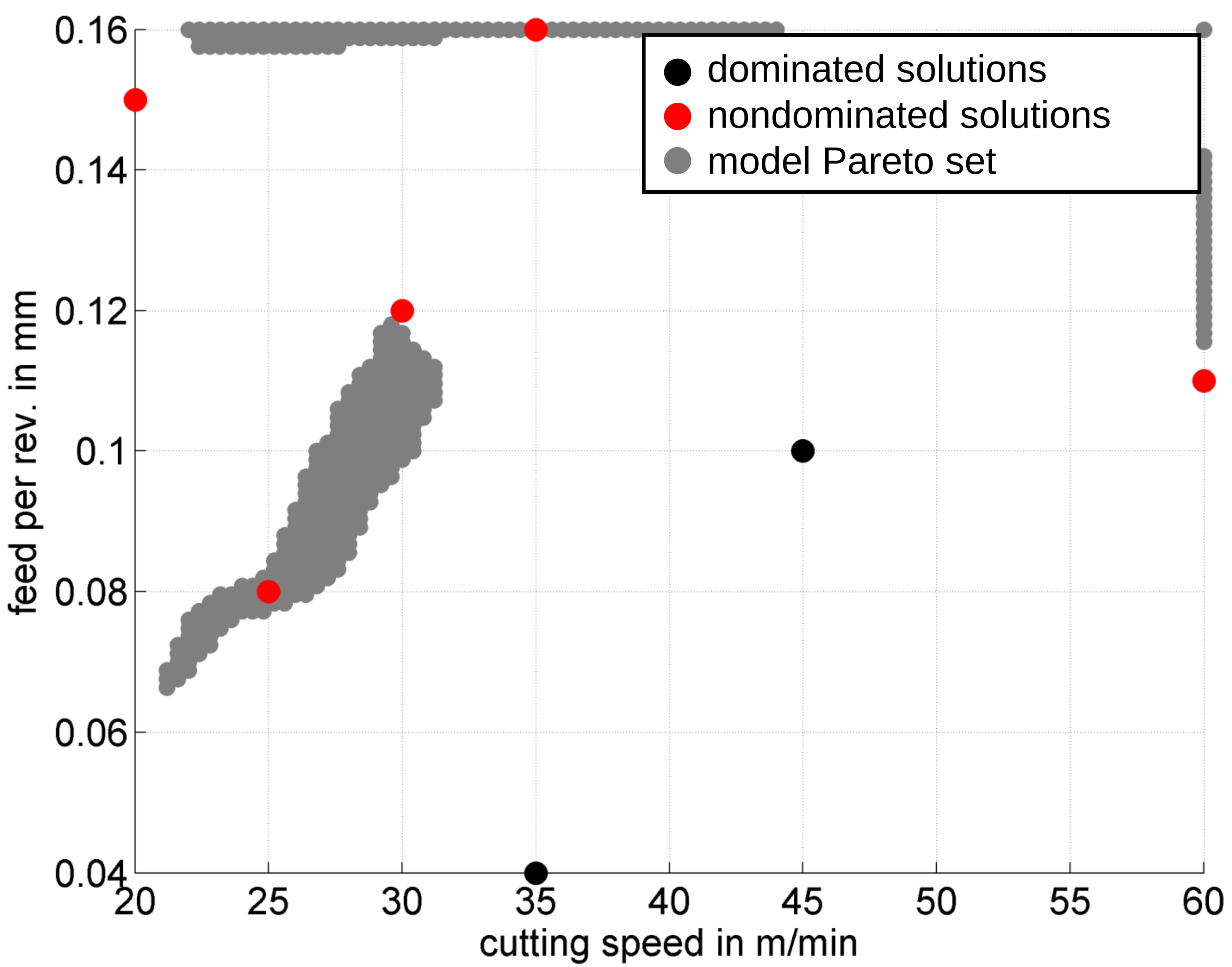
Final Pareto front approximation

[Zhang et al. 2012]



Practical application (drilling of Inconel 708)

Final Pareto set approximation (Innovization) [Zhang et al. 2012]



Overview

The Big Picture

Basic Principles of Multiobjective Optimization
- algorithm design principles and concepts
- performance assessment

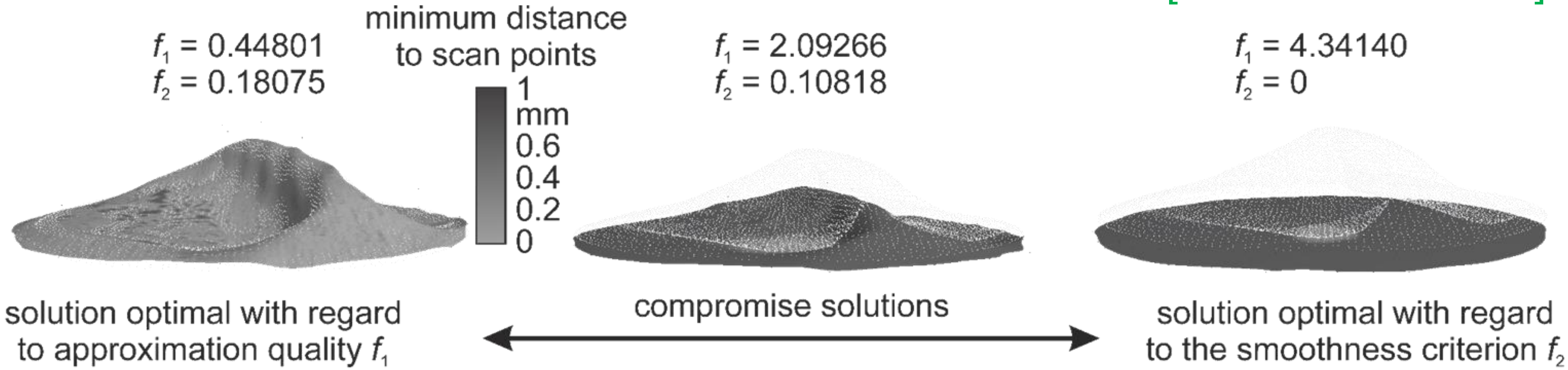
Selected Advanced Concepts
- preference articulation
- surrogate-based EMO

A Few Examples From Practice

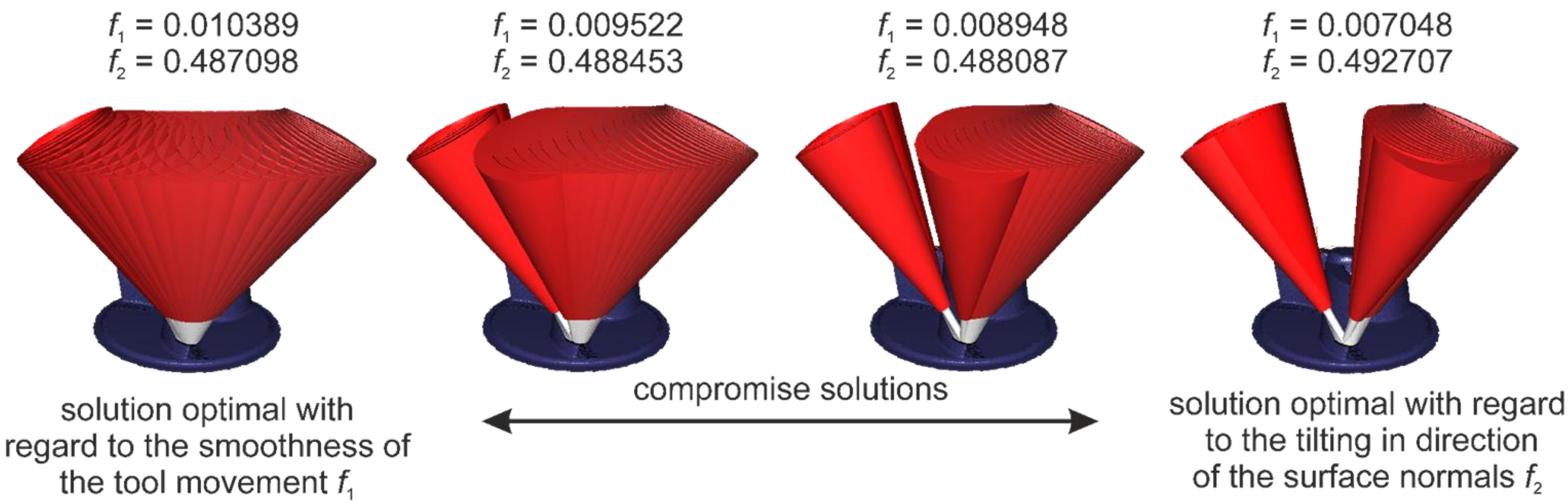
Applications of EMO

Surface reconstruction

[Weinert et al. 2009]

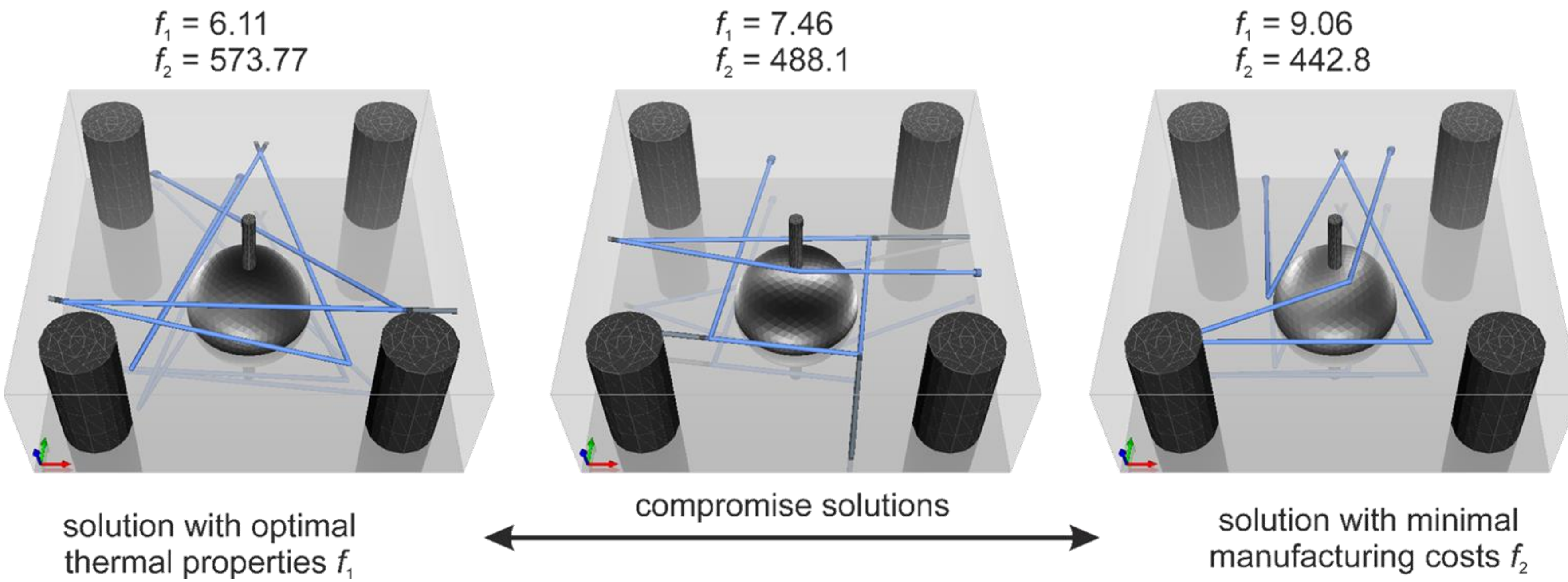


Five-Axis Milling

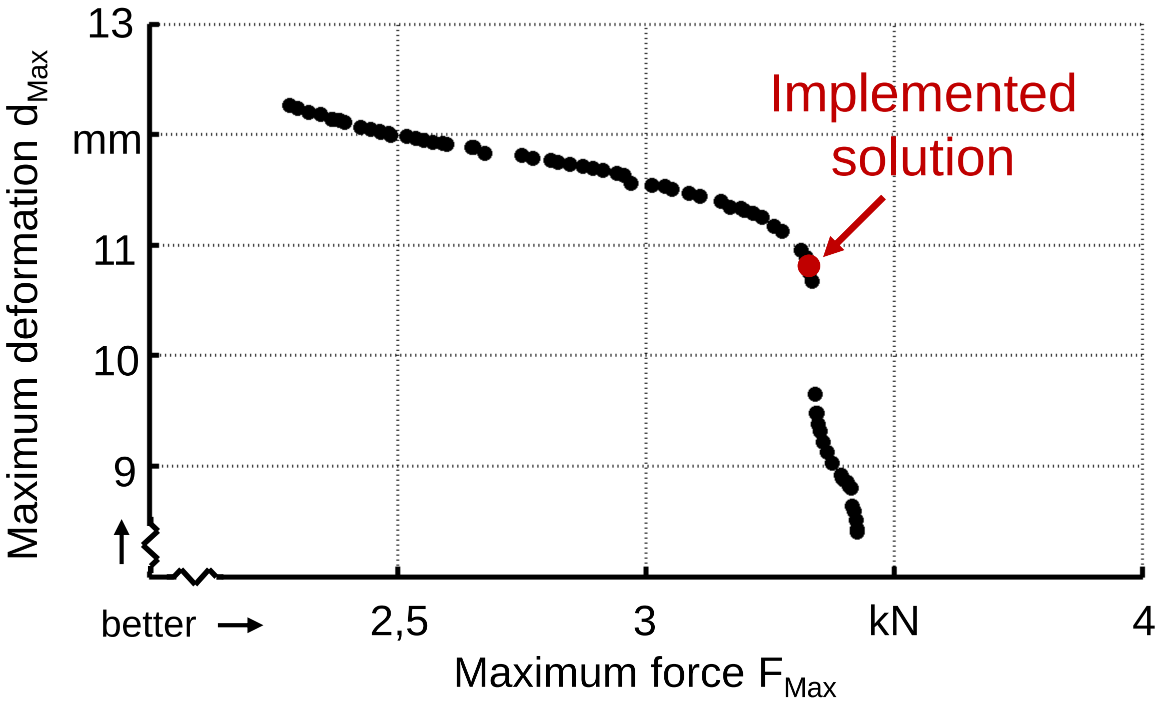


Mold Temperature Cooling Systems

[Weinert et al. 2009]

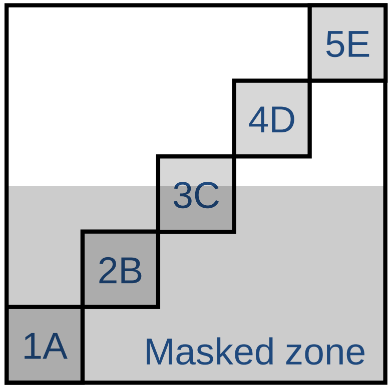


Hot Compaction of Thermoplastic Composites



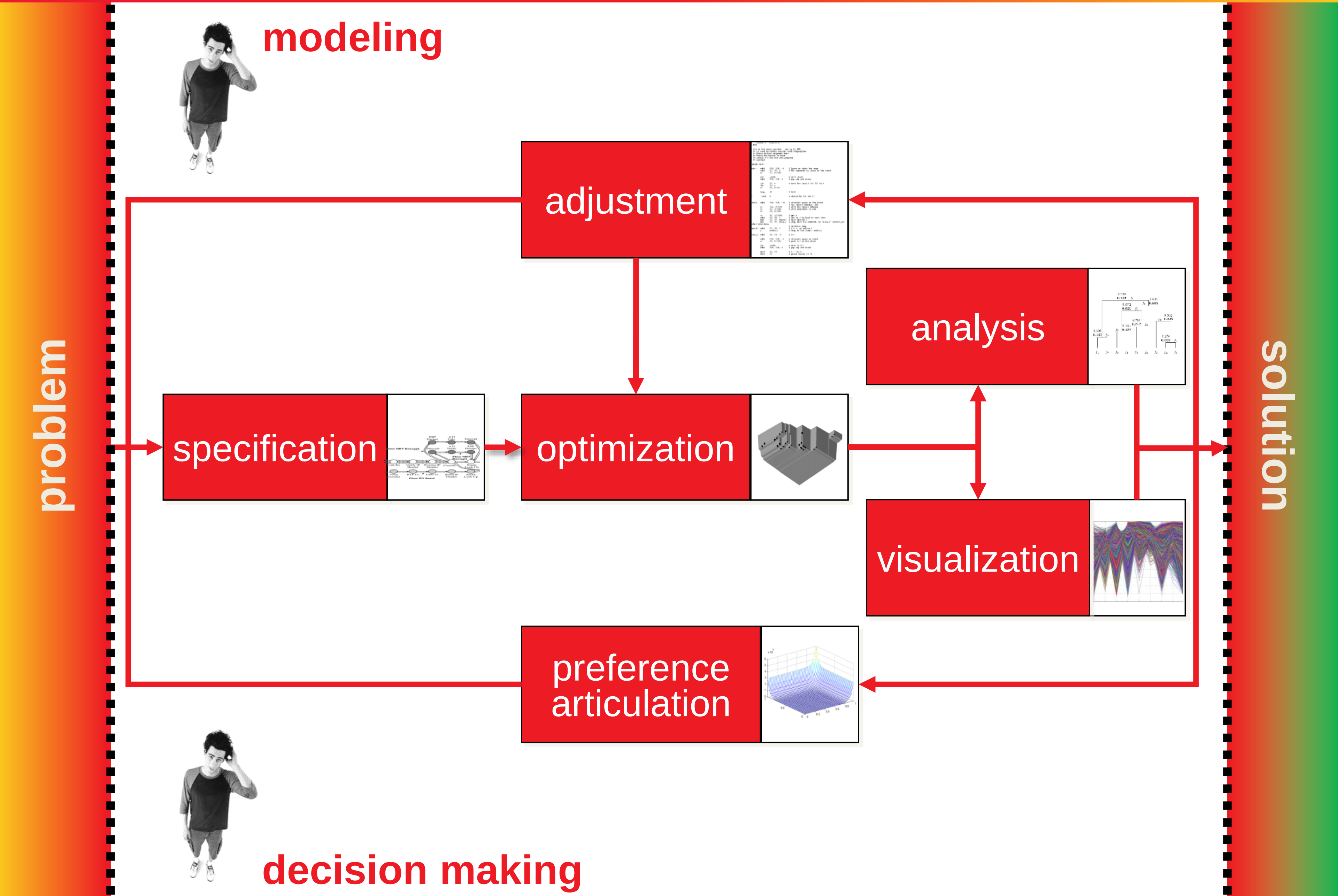
[Wagner et al. 2008]

T_{PH}^* = 146 °C,
t_{PH}^* = 174 s,
t_M^* = 170 s,
p_P^* = 1.1 MPa,
T_p^* = 169 °C,
t_P^* = 180 s



Position	T_M [°C]	F_{Max} [kN]	d_{Max} [mm]
1A	54	2.4	**10.7**
2B	84	2.8	9.8
3C	121	**3.3**	8.8
4D	147	**3.3**	8.3
5E	100	3.2	9.3

Conclusions: EMO as Interactive Decision Support



The EMO Community

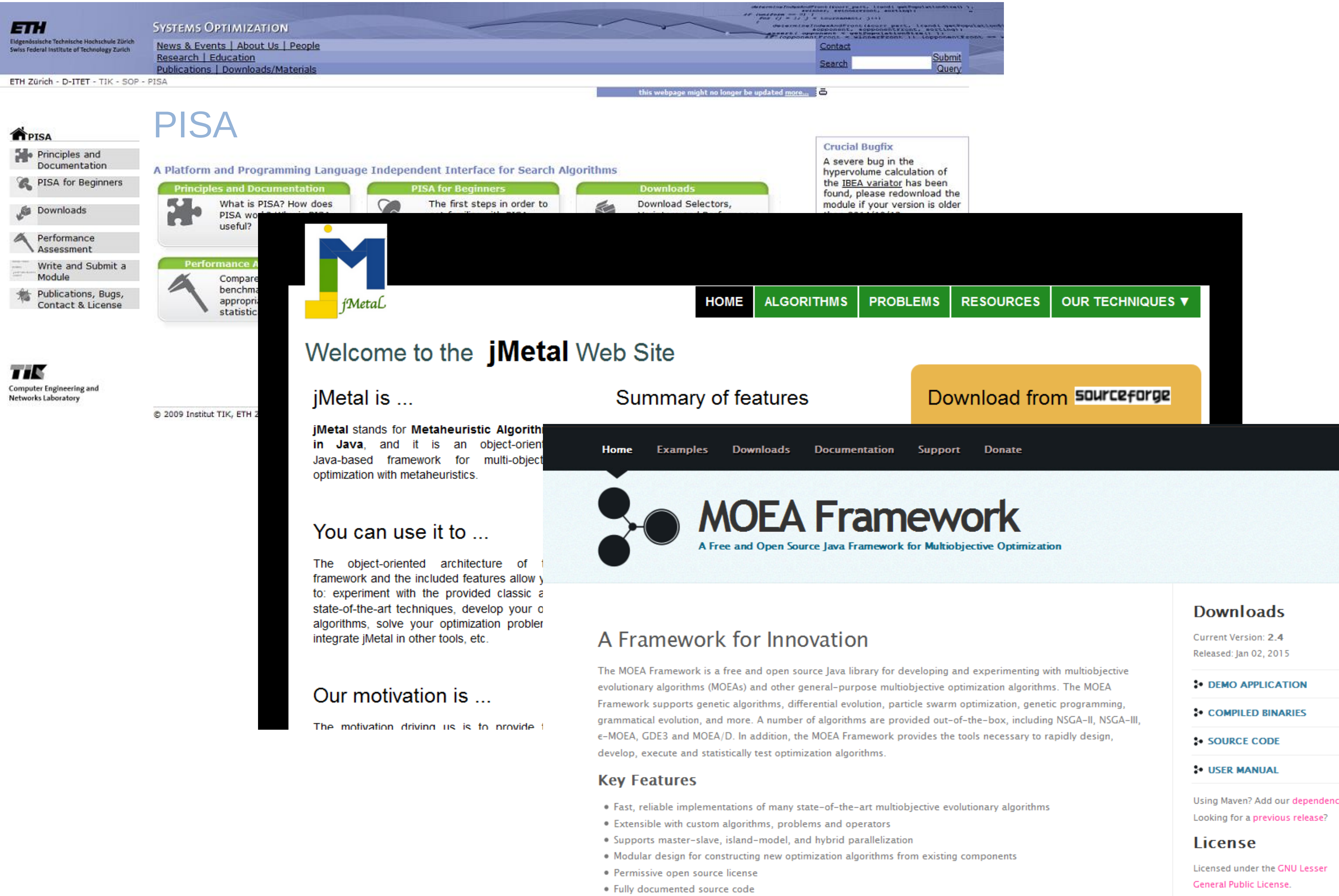
Links:

- EMO mailing list: *https://lists.dei.uc.pt/mailman/listinfo/emo-list*
- MCDM mailing list: *http://lists.jyu.fi/mailman/listinfo/mcdm-discussion*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.dep.uminho.pt/EMO2015/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [(still) many open questions!]
- and more…

Software



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to: experiment with the provided classic a
state-of-the-art techniques, develop your o
algorithms, solve your optimization proble
integrate jMetal in other tools, etc.
Our motivation is ...
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MOEA Framework
A Free and Open Source Java Framework for Multiobjective Optimization
A Framework for Innovation
The MOEA Framework is a free and open source Java library for developing and experimenting with multiobjective evolutionary algorithms (MOEAs) and other general-purpose multiobjective optimization algorithms. The MOEA Framework supports genetic algorithms, differential evolution, particle swarm optimization, genetic programming, grammatical evolution, and more. A number of algorithms are provided out-of-the-box, including NSGA-II, NSGA-III, ε-MOEA, GDE3 and MOEA/D. In addition, the MOEA Framework provides the tools necessary to rapidly design, develop, execute and statistically test optimization algorithms.
Key Features
Fast, reliable implementations of many state-of-the-art multiobjective evolutionary algorithms
Extensible with custom algorithms, problems and operators
Supports master-slave, island-model, and hybrid parallelization
Modular design for constructing new optimization algorithms from existing components
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Fully documented source code
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Current Version: 2.4
Released: Jan 02, 2015
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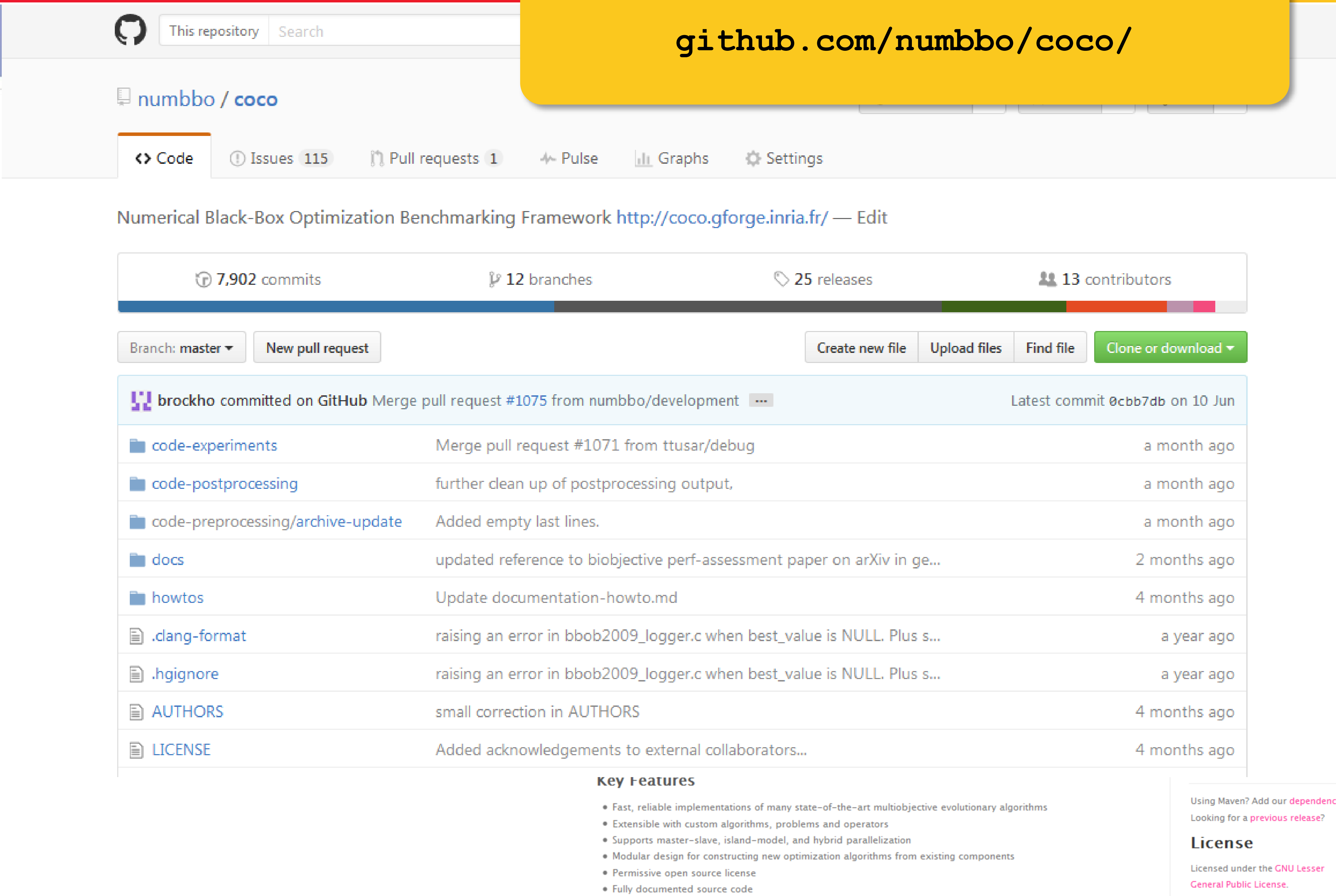
Software

github.com/numbbo/coco/



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Perspectives

Challenging Open (Research) Directions

- from algorithms to toolkits
  - libraries of modules for each task (selection, variation, etc.)
  - problem-specific algorithm configuration/ parameter tuning
- benchmarking
  - comparison with classical approaches
  - design/selection of practically relevant problems
  - Algorithm/toolkit recommendations for practice
- integration of EMO and MCDM into one field
- interactive preference articulation and learning
- interactive problem design
- integration of problem-specific knowledge

Questions?

Additional Slides

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at INRIA Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011 he has been a junior researcher (now CR1) at INRIA Lille - Nord Europe in Villeneuve d'Ascq, France. His most recent research interests are focused on evolutionary multiobjective optimization (EMO) and other (single-objective) blackbox optimization techniques, in particular with respect to benchmarking, theoretical aspects, and expensive optimization.

Instructor Biography: Tobias Wagner

Tobias Wagner

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After obtaining his diploma in computer science (Dipl.-Inform.) from the University of Dortmund, Germany in 2006, Tobias Wagner received his PhD in mechanical engineering (Dr.-Ing.) from the Technische Universität Dortmund, Germany in 2013. Between June 2006 and Sepember 2013 he held a scientific assistant position at the Institute of Machining Technology (ISF). Since October 2013 he works as a nonpermanent academic councilor at the ISF. His research is focused on surrogate-assisted single- and multi-objective optimization and sequential design techniques. With regard to EMO, he is particularly interested in the use of performance indicators and preference information within sequential design techniques.

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