Are All Objectives Necessary?

On Dimensionality Reduction in Evolutionary Multiobjective Optimization

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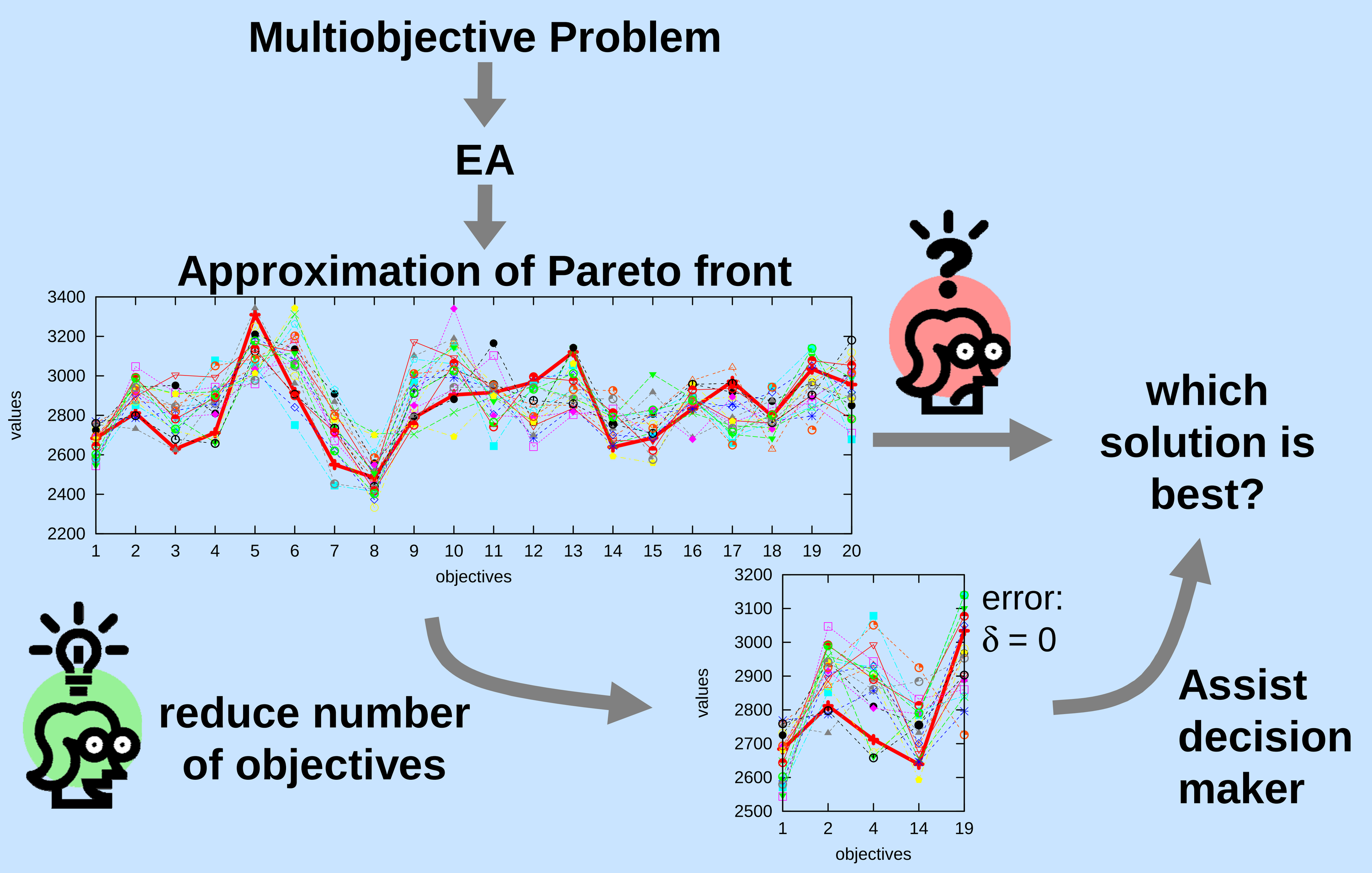
1 Motivation

Problem: Decision making with many objectives is challenging

Questions:

- Can objectives be omitted while the dominance structure is preserved/only slightly changed?
- How to compute a minimum objective set?



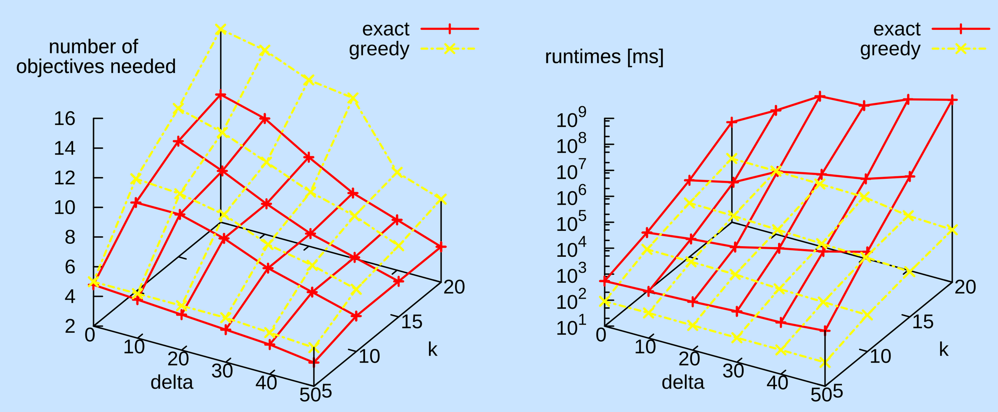
Drawbacks of known dimensionality reduction approaches:

- Not suitable for black-box optimization [Agrell 1997]
- No guarantee to preserve dominance structure [Deb and Saxena 2005]

3 Results

Exact algorithm vs. heuristic

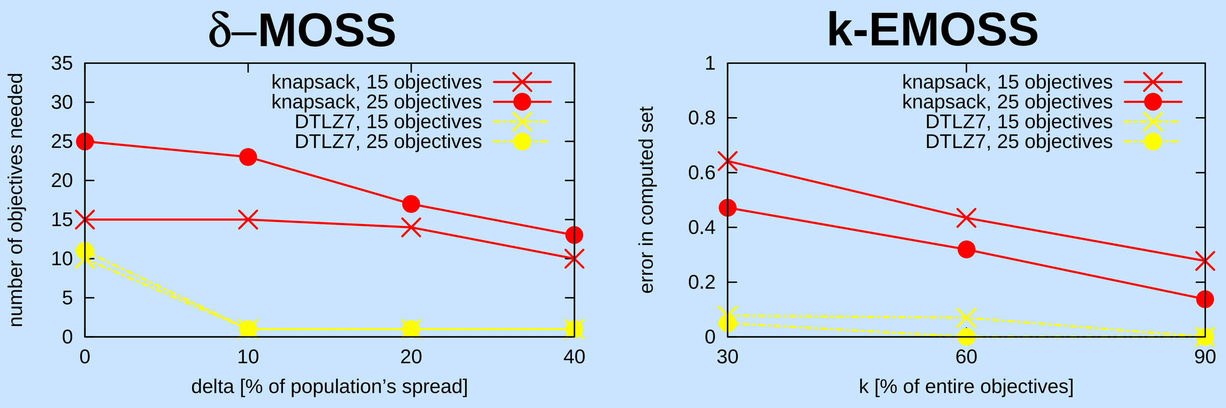
entire search space of 0-1-knapsack problem with 7 items



⇒ heuristic slightly worse results, but clearly faster

Different problems act differently

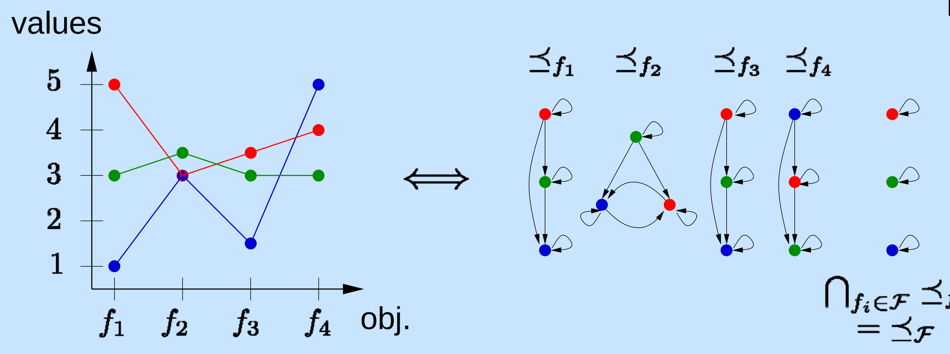
Pareto front approximations for 0-1-knapsack and DTLZ7



⇒ the smaller the objective set, the larger the error
⇒ general statements on redundancy impossible

2 Approach

Objective Conflicts



Preservation of dominance structure

$\mathcal{F}_1$ nonconfl. with $\mathcal{F}_2$ $\iff \preceq_{\mathcal{F}_1} = \preceq_{\mathcal{F}_2}$

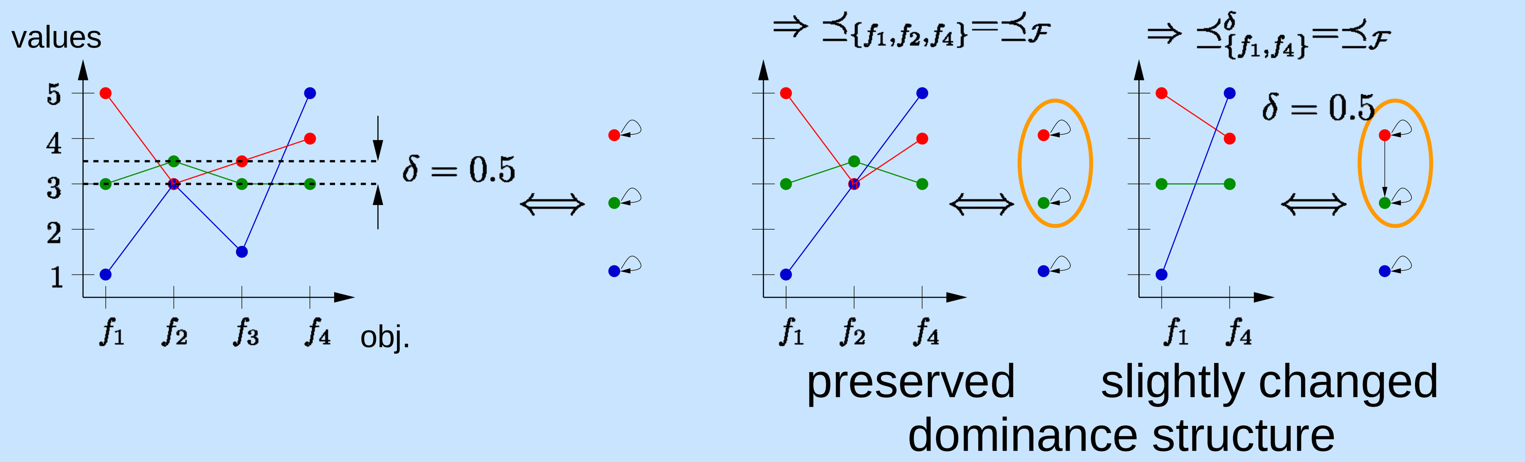
Changes in dominance structure

$\mathcal{F}_1$ δ-nonconfl. with $\mathcal{F}_2$ $\iff \preceq^{\delta}_{\mathcal{F}_1} \subseteq \preceq_{\mathcal{F}_2} \wedge \preceq^{\delta}_{\mathcal{F}_2} \subseteq \preceq_{\mathcal{F}_1}$

($\preceq^{\varepsilon}_{\mathcal{F}} \equiv \varepsilon$-dominance)

- Pairwise objective conflicts $\not\Rightarrow$ non-redundancy
- Omission of objectives $\Rightarrow$ possibly additional edges

Dimensionality Reduction



The Miminum Objective Subset Problems

Given: Solution set A with objective values $f_1(x), \dots, f_k(x)$

δ–**MOSS:** Compute a minimum objective set, yielding a slightly changed relation with error $\leq \delta$

k-**EMOSS:** Compute an objective set with k objectives, changing the relation least

Algorithms

exact

- $O(|A|^2 \cdot k \cdot 2^k)$, and $\Omega(|A|^2 \cdot 2^{k/3})$ resp., for δ-MOSS and k-EMOSS

greedy

- $O(\min\{|A|^2 \cdot k^3, |A|^4 \cdot k^2\})$ for δ-MOSS
- $O(|A|^2 \cdot k^3)$ for k-EMOSS

4 Conclusions

Key Contributions:

- Generalization of conflict between objective sets
- Framework for objective reduction to assist the decision maker

Benefits of the Approach:

- Definition of conflict between objective sets can detect redundancy
- Objective reduction is adjustable by defining error threshold or largest allowed objective set size
- Approach guarantees maximal error in dominance structure change

Take Home Message: Given a set of solutions, objective reduction is possible by preserving or only slightly changing the dominance structure. The omission of redundant information can assist the decision maker.